

# GSPO

GRPO

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E} \left[ q \sim P(Q), \{o_i\}_{i=1}^G \sim \pi_{\theta_{old}}(O|q) \right]$$
$$\frac{1}{G} \sum_{i=1}^G \frac{1}{|o_i|} \sum_{t=1}^{|o_i|} \left\{ \min \left[ \frac{\pi_{\theta}(o_{i,t}|q, o_{i,<t})}{\pi_{old}(o_{i,t}|q, o_{i,<t})} \hat{A}_{i,t}, \text{clip} \left( \frac{\pi_{\theta}(o_{i,t}|q, o_{i,<t})}{\pi_{old}(o_{i,t}|q, o_{i,<t})}, 1 - \epsilon, 1 + \epsilon \right) \hat{A}_{i,t} \right] - \beta \mathbb{D}_{KL} [\pi_{\theta} || \pi_{ref}] \right\}$$

GSPO

$$\mathcal{J}_{GSPO}(\theta) = \mathbb{E}_{x \sim \mathcal{D}, \{y_i\}_{i=1}^G \sim \pi_{\theta_{old}}(\cdot|x)} \left[ \frac{1}{G} \sum_{i=1}^G \min \left( s_i(\theta) \hat{A}_i, \text{clip}(s_i(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_i \right) \right]$$
$$s_i(\theta) = \frac{\pi_{\theta}(y_i|x)^{\frac{1}{|y_i|}}}{\pi_{old}(y_i|x)^{\frac{1}{|y_i|}}} = \exp \left( \frac{1}{|y_i|} \sum_{t=1}^{|y_i|} \log \frac{\pi_{\theta}(y_{i,t}|x, y_{i,<t})}{\pi_{old}(y_{i,t}|x, y_{i,<t})} \right)$$

## token-level to seq-level

1. 高方差
2. 优化目标(token-level) 和 奖励(seq-level) 不匹配

GSPO在seq-level粒度计算重要性权重，和奖励粒度对齐

$$s_i(\theta) = \frac{\pi_{\theta}(y_i|x)^{\frac{1}{|y_i|}}}{\pi_{old}(y_i|x)^{\frac{1}{|y_i|}}}$$

分子\分母 表示给定序列 $x$ 输出序列 $y$ 的概率, 展开:

$$\pi(y_i|x) = \prod_{t=1}^{|y_i|} \pi(y_{i,t}|x, y_{i,<t})$$

## 长度归一化

$$s_i(\theta) = (\cdot)^{\frac{1}{|y_i|}}$$

## clip粒度

GRPO: 考虑min操作, 当 $A < 0$ 时, 重要性范围波动大, 噪声累计, 可能导致MoE模型崩溃.

GSPO: seq-level, 避免token裁剪带来的噪声和偏差.

## 消除Routing Replay依赖

Routing Replay

- MoE模型在梯度更新后, 专家激活模式会发生变化.

这导致token重要性权重波动, 触发clip使其梯度消失; 而保留梯度的token带有噪音影响训练.

- Routing Replay强制**当前策略**路由和**旧策略**相同的专家.

GSPO: seq-level, 对单个token波动不敏感.