MATH 2800–01 - Fall 2025 - Assignment 01 - Due 09/16/2025 at 11:59PM

Instructions: Please follow the rules stated in the syllabus. Submit only one pdf file to WyoCourses.

1. Determine the cardinality of each of the following sets.

   (a) $A = \{1, 2, 3, \{1, 2, 3\}, 4, \{4\}\}$.
   (b) $B = \{x \in \mathbb{R} : |x| = -1\}$.
   (c) $C = \{m \in \mathbb{N} : 2 < m \leq 5\}$.
   (d) $D = \{n \in \mathbb{N} : n < 0\}$.
   (e) $E = \{k \in \mathbb{N} : 1 \leq k^2 \leq 100\}$.
   (f) $F = \{k \in \mathbb{Z} : 1 \leq k^2 \leq 100\}$.

2. The set $E = \{\cdots, -4, -2, 0, 2, 4, \cdots\}$ of even integers can be described by means of a defining condition by
$$E = \{y = 2x : x \in \mathbb{Z}\} = \{2x : x \in \mathbb{Z}\}.$$
   Describe the following sets in a similar manner.

   (a) $A = \{\cdots, -4, -1, 2, 5, 8, \cdots\}$.
   (b) $B = \{\cdots, -10, -5, 0, 5, 10, \cdots\}$.
   (c) $C = \{1, 8, 27, 64, 125, \cdots\}$.

3. Let
$$\begin{aligned} A &= \{n \in \mathbb{Z} : 2 \leq |n| < 4\}, \\ B &= \{x \in \mathbb{Q} : 2 < x \leq 4\}, \\ C &= \{x \in \mathbb{R} : x^2 - (2 + \sqrt{2})x + 2\sqrt{2} = 0\}, \text{ and} \\ D &= \{x \in \mathbb{Q} : x^2 - (2 + \sqrt{2})x + 2\sqrt{2} = 0\}. \end{aligned}$$
   (a) Describe the set A by listing its elements.
   (b) Give an example of three elements that belong to B but do not belong to A.
   (c) Describe the set C by listing its elements.
   (d) Describe the set D in another manner.
   (e) Determine the cardinality of each of the sets A, C and D.

4. For $A = \{2, 3, 5, 7, 8, 10, 13\}$, let
$$\begin{aligned} B &= \{x \in A : x = y + z, \text{ where } y, z \in A\}, \\ C &= \{r \in B : r + s \in B \text{ for some } s \in B\}. \end{aligned}$$
   Determine B and C.

5. Give examples of three sets A, B and C such that

   (a) $A \subseteq B \subset C$.
   (b) $A \in B$, $B \in C$, and $A \notin C$.
   (c) $A \in B$ and $A \subset C$.