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**MATH 2800–01 - Fall 2025 - Assignment 03 - Due 10/01/2025 at 11:59PM**

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**Instructions:** Please follow the rules stated in the syllabus. Submit only one pdf file to WyoCourses.

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1. Determine the truth value of the following statements? Give an explanation for each false statement.  
 (a)  $\emptyset \in \emptyset$ , (b)  $\emptyset \in \{\emptyset\}$ , (c)  $\{1, 3\} = \{3, 1\}$ , (d)  $\emptyset = \{\emptyset\}$ , (e)  $\emptyset \subset \{\emptyset\}$ , (f)  $1 \subseteq \{1\}$ .

2. Consider the following statements:

$$P : \sqrt{2} \text{ is rational. } Q : \frac{2}{3} \text{ is rational. } R : \sqrt{3} \text{ is rational.}$$

Write each of the following statements in words and indicate whether the statement is true or false.

$$(a) (P \wedge Q) \implies R, (b) (P \wedge Q) \implies \sim R, (c) ((\sim P) \wedge Q) \implies R, (d) (P \vee Q) \implies (\sim R).$$

3. The instructor of a computer science class announces to her class that there will be a well-known speaker on campus later that day. Four students in the class are Alice, Ben, Cindy and Don. Ben says that he'll attend the lecture if Alice does. Cindy says that she'll attend the talk if Ben does. Don says that he will go to the lecture if Cindy does. That afternoon exactly two of the four students attend the talk. Which two students went to the lecture?

4. A college student makes the following statement:

*If I receive an A in both Calculus I and Discrete Mathematics this semester, then I'll take either Calculus II or Computer Programming this summer.*

For each of the following, determine whether the statement above is true or false.

- (a) The student doesn't get an A in Calculus I but decides to take Calculus II this summer anyway.
  - (b) The student gets an A in both Calculus I and Discrete Mathematics but decides not to take any class this summer.
  - (c) The student does not get an A in Calculus I and decides not to take Calculus II but takes Computer Programming this summer.
  - (d) The student gets an A in both Calculus I and Discrete Mathematics and decides to take both Calculus II and Computer Programming this summer.
  - (e) The student gets an A in neither Calculus I nor Discrete Mathematics and takes neither Calculus II nor Computer Programming this summer.
5. For the open sentences  $P(x) : |x - 3| < 1$ . and  $Q(x) : x \in (2, 4)$ . over the domain  $\mathbb{R}$ , state the biconditional  $P(x) \iff Q(x)$  in two different ways.
6. For statements  $P$ ,  $Q$  and  $R$ , show that

$$((P \implies Q) \wedge (Q \implies R)) \implies (P \implies R)$$

is a tautology. Then state this compound statement in words. (This is another important logical argument form, called *sylogism*.)

7. Let  $A = \{1, 2, \dots, 6\}$  and  $B = \{1, 2, \dots, 7\}$ . For  $x \in A$ , let  $P(x) : 7x + 4$  is odd. For  $y \in B$ , let  $Q(y) : 5y + 9$  is odd. Let

$$S = \{(P(x), Q(y)) : x \in A, y \in B, P(x) \implies Q(y) \text{ is false}\}.$$

What is  $|S|$ ?

8. For statements  $P$ ,  $Q$  and  $R$ , consider the following compound statements  $X$ ,  $Y$  and  $Z$ :

$$X : (P \wedge Q) \implies R, \quad Y : P \wedge (\sim R) \implies (\sim Q), \quad Z : Q \wedge (\sim R) \implies (\sim P).$$

- Show that  $X \equiv Y$  and  $X \equiv Z$ .
- For a real-valued function  $f$  and a real number  $x$ , utilize the previous item to restate the following implication in two different ways:

$$\text{If } f'(x) = 3x^2 - 2x \text{ and } f(0) = 4, \text{ then } f(x) = x^3 - x^2 + 4.$$