MATH 2800–01 - Fall 2025 - Assignment 02 - Due 09/24/2025 at 11:59PM

Instructions: Please follow the rules stated in the syllabus. Submit only one pdf file to WyoCourses.

1. Which of the following sets are equal: $A = \{n \in \mathbb{Z} : |n| < 2\}$, $B = \{n \in \mathbb{Z} : n^3 = n\}$, $C = \{n \in \mathbb{Z} : n^2 \leq n\}$, $D = \{n \in \mathbb{Z} : n^2 \leq 1\}$, $E = \{-1, 0, 1\}$.

2. Determine whether the following statements are true or false.
   (a) If $\{1\} \in \mathcal{P}(A)$, then $1 \in A$ but $\{1\} \notin A$.
   (b) If A, B and C are sets such that $A \subset \mathcal{P}(B) \subset C$ and $|A| = 2$, then $|C|$ can be 5 but $|C|$ cannot be 4.
   (c) If a set B has one more element than a set A, then $\mathcal{P}(B)$ has at least two more elements than $\mathcal{P}(A)$.
   d If four sets A,B, C and D are subsets of $\{1, 2, 3\}$ such that $|A| = |B| = |C| = |D| = 2$, then at least two of these sets are equal.

3. Three subsets A, B and C of $\{1, 2, 3, 4, 5\}$ have the same cardinality. Furthermore,
   (a) $1 \in A \cap B$ and $1 \notin C$.
   (b) $2 \in A \cap C$ and $2 \notin B$.
   (c) Either $3 \in A \cap B$ or $3 \in A \cap C$.
   (d) Either 4 belongs to *none* of the sets A, B and C or 4 belongs to *two* of the sets A, B and C.
   (e) Either 5 belongs to *only one* of the sets A, B and C or 5 belongs to *all* of the sets A, B and C.
   (f) The sums of the elements in two of the sets A, B and C differ by 1.

   Determine A, B and C. Attempt to write your argument leading to your answer in clear sentences.

4. Which of the following are partitions of $A = \{a, b, c, d, e, f, g\}$? Explain your answer.
   (a) $S_1 = \{\ \{a, c, e, g\}, \{b, f\}, \{d\}\ \}$.
   (b) $S_2 = \{\ \{a, b, c, d\}, \{e, f\}\ \}$.
   (c) $S_3 = \{A\}$.
   (d) $S4 = \{\ \{a\}, \emptyset, \{b, c, d\}, \{e, f, g\}\ \}$.
   (e) $S5 = \{\ \{a, c, d\}, \{b, g\}, \{e\}, \{b, f\}\ \}$.

5. Let $A = \{1, 2, \cdots, 12\}$. Give an example of a partition S of A satisfying the following requirements: (i) $|S| = 5$, (ii) there is a subset T of S such that $|T| = 4$ and $\left| \bigcup_{X \in T} X \right| = 10$ and (iii) all the subsets in the partition S cannot have cardinality equals 3. For the proposed example, demonstrate that all the requirements are satisfied.

6. For a real number r, let $A_r = \{r, r + 1\}$. Let $S = \{x \in \mathbb{R} : x^2 + 2x - 1 = 0\}$.
   (a) Determine $B = A_s \times A_t$ for the distinct elements $s, t \in S$, where $s < t$.
   (b) Let $C = \{ab : (a, b) \in B\}$. Determine the sum of the elements of C.

7. Let $A = \{x \in \mathbb{R} : |x - 1| \leq 2\}$, $B = \{x \in \mathbb{R} : |x| \geq 1\}$ and $C = \{x \in \mathbb{R} : |x + 2| \leq 3\}$.
   (a) Express A, B and C using interval notation.
   (b) Determine each of the following sets using interval notation: $A \cup B$, $A \cap B$, $B \cap C$, $B - C$.