MATH 2800–01 - Fall 2025 - Assignment 06 - Due 11/14/2025 at 11:59PM

Instructions: Please follow the rules stated in the syllabus. Submit only one pdf file to WyoCourses.
Start every problem below on a new page and use the following format.

Result. *Write the statement you want to proof.*

Proof. *Compose the proof. At its completion, end it with the box (see the image at the right end corner).*

$\square$

1. Use mathematical induction to prove that

$$1 + 5 + 9 + \cdots + (4n - 3) = 2n^2 - n$$

   for every positive integer n.

2. • Show that $\dfrac{k}{k+1} = \dfrac{1}{1+\frac{1}{k}} \geq \dfrac{2}{3}$ for any positive integer $k \geq 2$. You do not need induction argument to show it.
   • Use an induction argument to prove that $4^n > n^3$ for every positive integer n.

3. Prove that $7 \mid (3^{4n+1} - 5^{2n-1})$ for every positive integer n.

4. • Show that $4(k^2 + k) < (2k + 1)^2$ for every positive integer k. You do not need induction argument to show it.
   • Use an induction argument to prove that $\displaystyle\sum_{m=1}^{n} \frac{1}{\sqrt{m}} \leq 2\sqrt{n} - 1$ for every positive integer n.

5. A sequence $\{a_n\}$ is defined recursively by $a_1 = 1$, $a_2 = 2$ and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Conjecture a formula for a_n and verify that your conjecture is correct by using an induction argument.

6. Use the Strong Principle of Mathematical Induction to prove that for each integer $n \geq 28$, there are nonnegative integers x and y such that $n = 5x + 8y$.