
MATH 2800–01 - Fall 2025 - Assignment 08 - Due 12/12/2025 at 11:59PM

Instructions: Please follow the rules stated in the syllabus. Submit only one pdf file to WyoCourses.
Start every problem below on a new page and use the following format.

Result. *Write the statement you want to proof.*

Proof. *Compose the proof. At its completion, end it with the box (see the image at the right end corner).* □

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1. Suppose that $\lim_{x\to a} f(x) = L$, where $L > 0$. Utilize $\epsilon - \delta$ argument to prove that $\lim_{x\to a} \sqrt{f(x)} = \sqrt{L}$.
   Hint: in your scratch work, you may need to use conjugation of an expression.

2. Let $Q : \mathbb{R} \setminus \{-1\}$ be defined as

$$Q(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x^2 - 1}, & \text{for } x \in \mathbb{R} \setminus \{-1, 1\}, \\ \\ -\dfrac{1}{2}, & \text{for } x = 1. \end{cases}$$

   Utilize $\epsilon - \delta$ argument to prove that Q is continuous at $x = 1$.
   Hint: in your scratch work, you may need to use the fact that $|x - 1| < 1$ *implies* $-1 < x - 1 < 1$. *What does it say about* $|x + 1|$*?*

3. A function $g : \mathbb{R} \to \mathbb{R}$ is bounded if there exists a positive real number B such that $|g(x)| < B$ for each $x \in \mathbb{R}$.
   - Let $g : \mathbb{R} \to \mathbb{R}$ be a bounded function and suppose that $f : \mathbb{R} \to \mathbb{R}$ and $a \in \mathbb{R}$ such that $\lim_{x\to a} f(x) = 0$. Utilize $\epsilon - \delta$ argument to prove that $\lim_{x\to a} f(x)g(x) = 0$.
     Hint: in your scratch work, you may need to use identity $f(x)g(x) = (f(x) - 0)g(x)$.
   - Use the above result to determine $\lim_{x\to 0} x^2 \sin(\dfrac{1}{x})$.

4. Let $f : [1, \infty) \to [0, \infty)$ be defined by $f(x) = \sqrt{x - 1}$. Utilize $\epsilon - \delta$ argument to prove that
   - f is continuous at $x = 10$.
   - f is differentiable at $x = 10$.