MATH 2800–01 - Fall 2025 - Assignment 04 - Due 10/16/2025 at 11:59PM

Instructions: Please follow the rules stated in the syllabus. Submit only one pdf file to WyoCourses.
Start every problem below on a new page and use the following format.

Result. *Write the statement you want to proof.*

Proof. *Compose the proof. At its completion, end it with the box (see the image at the right end corner).*
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1. Let $x \in \mathbb{Z}$. Prove that if $7x - 8$ is even, then x is even.
2. Give a direct proof of the following: Let $x \in \mathbb{Z}$. If $11x - 5$ is odd, then x is even.
3. Prove using contrapositive: if a and b are two distinct real numbers, then either $\dfrac{a+b}{2} > a$ or $\dfrac{a+b}{2} > b$.
4. Let $x, y \in \mathbb{Z}$. Prove that if xy and $x + y$ are even, then both x and y are even.
5. Let $x \in \mathbb{Z}$. Prove that $3x + 1$ is even if and only if $5x - 2$ is odd.
6. **Definition.** For integers a and b with $a \neq 0$, we say that a divides b if there is an integer c such that $b = ac$. In this case, we write $a \mid b$.
   Let $n \in \mathbb{Z}$. Prove that $5 \mid n^2$ if and only if $5 \mid n$.
7. Prove: if $a, b \in \mathbb{R}$, then $ab \leq \sqrt{a^2}\sqrt{b^2}$.
8. Prove: if $a > 0$ and $b > 0$, then $\dfrac{a}{b} + \dfrac{b}{a} \geq 2$. *Hint: use the expression $a = (a - b) + b$ and manipulate the fractions summation.*