MATH 2800–01 - Fall 2025 - Assignment 05 - Due 11/31/2025 at 11:59PM

Instructions: Please follow the rules stated in the syllabus. Submit only one pdf file to WyoCourses.
Start every problem below on a new page and use the following format.

Result. *Write the statement you want to proof.*

Proof. *Compose the proof. At its completion, end it with the box (see the image at the right end corner).*

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1. Prove that the equation $x^5 + 2x - 5 = 0$ has a unique real number solution between $x = 1$ and $x = 2$. You are not to compute the actual solution.
2. Disprove the statement:
   For every positive integer x and every integer $n \geq 2$, the equation $x^n + (x+1)^n = (x+2)^n$ has no solution.
3. Utilize contradiction to prove that if $a \geq 2$ and b are integers, then $a \nmid b$ or $a \nmid (b+1)$.
4. Prove that $\sqrt{3}$ is irrational. *Hint: use the following fact: for an integer a, $3 \mid a^2$ if and only if $3 \mid a$.*
5. Prove that there do not exist positive integers m and n such that $m^2 - n^2 = 1$.
6. Let x be a positive real number. Prove that if $x - \dfrac{2}{x} > 1$, then $x > 2$ by
   (a) a direct proof, (b) a proof by contrapositive and (c) a proof by contradiction.
7. Prove that the equation $4x - \cos^2(x) = 10$ has a real number solution in the interval $[0, 4]$. Is the solution unique? Explain.
8. Disprove the statement: There is a real number x such that $x^6 + x^4 - 2x^2 + 1 = 0$.
9. The king's daughter had three suitors and couldn't decide which one to marry. So, the king said to the suitors, "I have three gold crowns and two silver ones. I will put either a gold or silver crown on each of your heads. The suitor who can tell me which crown he has will marry my daughter." The first suitor looked around and said he could not tell. The second did the same. The third suitor said: " I have a gold crown." He is correct, but the daughter was puzzled: This suitor was blind. How did he know? (Reference: Ask Marilyn, Parade Magazine, July 6, 2003.)