一.

① 拉格朗日插值: $g_1(x) = 1\cdot\frac{(x-1)(x-2)}{(0-1)(0-2)} + 9\cdot\frac{(x-0)(x-2)}{(1-0)(1-2)} + 23\cdot\frac{(x-0)(x-1)}{(2-0)(2-1)}$

② 牛顿差商表

x	y	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$
0	1		
1	9	$\frac{9-1}{1-0}=8$	
2	23	$\frac{23-9}{2-1}=14$	$\frac{14-8}{2-0}=3$

$g_2(x) = 1 + 8(x-0) + 3(x-0)(x-1)$

③ $g_1(x) = 3x^2+5x+1$ $\quad g_1(x) = g_2(x)$

$g_2(x) = 3x^2+5x+1$

二:

$A^T\cdot e = A^T(B-P) = 0$

$\because P = A\cdot\hat{x}$

$\therefore A^T\cdot(B-A\hat{x}) = 0$

$\hat{x} = (A^TA)^{-1}\cdot A^T\cdot B$

三:

① $A_1 = \begin{bmatrix}1\\2\\2\end{bmatrix}$ $\quad A_2 = \begin{bmatrix}-4\\3\\2\end{bmatrix}$

$y_1 = A_1 = \begin{bmatrix}1\\2\\2\end{bmatrix}$ $\quad q_1 = \frac{y_1}{\|y_1\|_2} = \frac{\begin{bmatrix}1\\2\\2\end{bmatrix}}{\sqrt{1+4+4}} = \frac{\begin{bmatrix}1\\2\\2\end{bmatrix}}{3}$

$y_2 = A_2 - q_1\boxed{q_1^T\cdot A_2} \rightarrow r_{12}$

$= \begin{bmatrix}-4\\3\\2\end{bmatrix} - \frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}\cdot\boxed{\frac{1}{3}[1,2,2]\cdot\begin{bmatrix}-4\\3\\2\end{bmatrix}}$

$= \begin{bmatrix}-4\\3\\2\end{bmatrix} - \frac{2}{3}\begin{bmatrix}1\\2\\2\end{bmatrix} = \begin{bmatrix}-\frac{14}{3}\\\frac{5}{3}\\\frac{2}{3}\end{bmatrix}$ $\quad q_2 = \frac{\begin{bmatrix}-\frac{14}{3}\\\frac{5}{3}\\\frac{2}{3}\end{bmatrix}}{\sqrt{(-\frac{14}{3})^2+(\frac{5}{3})^2+(\frac{2}{3})^2}} = \frac{\begin{bmatrix}-\frac{14}{3}\\\frac{5}{3}\\\frac{2}{3}\end{bmatrix}}{5} = \frac{y_2}{\|y_2\|_2}$

$\therefore \begin{bmatrix}1&-4\\2&3\\2&2\end{bmatrix} = \begin{bmatrix}\frac{1}{3}&-\frac{14}{15}\\\frac{2}{3}&\frac{1}{3}\\\frac{2}{3}&\frac{2}{15}\end{bmatrix}\cdot\begin{bmatrix}3&2\\0&5\end{bmatrix}$

$= Q\cdot R$

② 由①得 $q_1 = \frac{\begin{bmatrix}1\\2\\2\end{bmatrix}}{3}$ $\quad q_2 = \frac{\begin{bmatrix}-\frac{14}{3}\\\frac{5}{3}\\\frac{2}{3}\end{bmatrix}}{5}$

加上第三个向量 $A_3 = \begin{bmatrix}1\\0\\0\end{bmatrix}$ 得:

$y_3 = A_3 - q_1q_2^TA_3 - q_2q_2^TA_3$

得: $y_3 = \begin{bmatrix}1\\0\\0\end{bmatrix} - \frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}\times\frac{1}{3}[1\ 2\ 2]\times\begin{bmatrix}1\\0\\0\end{bmatrix} - \frac{1}{5}\begin{bmatrix}-\frac{14}{3}\\\frac{5}{3}\\\frac{2}{3}\end{bmatrix}\times\frac{1}{5}[-\frac{14}{3}\ \ \frac{5}{3}\ \ \frac{2}{3}]\times\begin{bmatrix}1\\0\\0\end{bmatrix}$

$= \begin{bmatrix}1\\0\\0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1\\2\\2\end{bmatrix} + \frac{11}{75}\begin{bmatrix}-\frac{14}{3}\\\frac{5}{3}\\\frac{2}{3}\end{bmatrix} = \frac{2}{225}\begin{bmatrix}2\\10\\-11\end{bmatrix}$

$q_3 = \frac{y_3}{\|y_3\|} = \begin{bmatrix}\frac{2}{15}\\\frac{10}{15}\\-\frac{11}{15}\end{bmatrix}$ $\quad \therefore A = \begin{bmatrix}1&-4\\2&3\\2&2\end{bmatrix} = \begin{bmatrix}\frac{1}{3}&-\frac{14}{15}&\frac{2}{15}\\\frac{2}{3}&\frac{1}{3}&\frac{2}{3}\\\frac{2}{3}&\frac{2}{15}&-\frac{11}{15}\end{bmatrix}\cdot\begin{bmatrix}3&2\\0&5\\0&0\end{bmatrix} = Q\cdot R$

四:

由题知: $\begin{cases}k_1\cdot e^{k_2\cdot 2016} = 1000\\k_1\cdot e^{k_2\cdot 2017} = 400\\k_1\cdot e^{k_2\cdot 2018} = 100\\k_1\cdot e^{k_2\cdot 2019} = 10\end{cases}$

方程组等式两边取对数:

$\begin{cases}\ln k_1 + 2016k_2 = n1000\\\ln k_1 + 2017k_2 = \ln 400\\\ln k_1 + 2018k_2 = \ln 100\\\ln k_1 + 2019k_2 = \ln 10\end{cases}$

$\Rightarrow \begin{bmatrix}1&2016\\1&2017\\1&2018\\1&2019\end{bmatrix}\begin{bmatrix}\ln k_1\\k_2\end{bmatrix} = \begin{bmatrix}\ln 1000\\\ln 400\\\ln 100\\\ln 10\end{bmatrix}$

$\Rightarrow A\cdot x = B$

$\hat{x} = (A^T\cdot A)^{-1}\cdot A^T\cdot B$

五:

一阶向前差分: $f'(x) = \frac{f(x+h)-f(x)}{h} - \frac{h}{2!}f''(c)$ $\quad c\in(x,x+h)$

中心差分: $f'(x) = \frac{f(x+h)-f(x-h)}{2h} - \frac{h^2}{6}f'''(c)$ $\quad c\in(x-h,x+h)$

向后差分: $f'(x) = \frac{f(x)-f(x+h)}{h} + \frac{h}{2!}\cdot f''(c)$ $\quad c\in(x-h,x)$

推导中心差分公式: $f'(x) = \frac{f(x+h)-f(x-h)}{2h} - \frac{h^2}{6}f'''(c)$

由泰勒公式: $f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2}h^2 + \frac{f'''(\xi_1)}{3!}\cdot h^3$ $\quad \xi_1\in(x,x+h)$ ①

$f(x-h) = f(x) - f'(x)h + \frac{f''(x)}{2}h^2 - \frac{f'''(\xi_2)}{3!}\cdot h^3$ $\quad \xi_2\in(x-h,x)$ ②

$\therefore$ ① − ② $= f(x+h) - f(x-h) = 2f'(x)h + \frac{h^3}{3!}[f'''(\xi_1)+f'''(\xi_2)]$

$f(x+h) - f(x-h) = 2f'(x)h + \frac{h^3}{3!}\cdot 2\cdot f'''(c)$ $\quad c\in(\xi_2,\xi_1)$

$f'(x) = \frac{f(x+h)-f(x-h)}{2h} - \frac{f'''(c)}{6}\cdot h^3$

七:

$\begin{cases}x_0 = 1\\x_{i+1} = x_i + h\cdot f(t_i,x_i)\end{cases}$

$t_0 = 0$ $\quad x_0 = 1$

$t_1 = 0.4$ $\quad x_1 = 1 + 0.4\times 0 = 1$

$t_2 = 0.8$ $\quad x_2 = 1 + 0.4\times(0.4\times 1 + 0.4^3) = 1.1856$

$t_3 = 1.2$ $\quad x_2 = 1.1856 + 0.4\times(0.8\times 1.1856 + 0.8^3) = 1.7698$

$t_4 = 1.6$ $\quad x_4 = 1.7698 + 0.4\times(1.2\times 1.7698 + 1.2^3) = 3.3105$

$t_5 = 2$ $\quad x_5 = 3.3105 + 0.4\times(1.6\times 3.3105 + 1.6^3) = 7.0676$

八:

$w_i + \frac{h}{6}(S_1 + 2S_2 + 2S_3 + S_4)$

$f(t_i, w_i)$

$f(t_i+\frac{h}{2}, w_i+\frac{h}{2}\cdot S_1)$

算出 $i=1$ 的 $S_1\ S_2\ S_3\ S_4$

然后算出 $x(t_1)$

算出 $i=2$ 的 $S_1\ S_2\ S_3\ S_4$

然后算出 $x(t_2)$

⋮