Chapter - 6 13/6/22

Triangles

★ Exercise - 6.1

Q1 Fill in the blanks :-

i) All circles are similar.

ii) All squares are similar.

iii) All equilateral triangles are similar.

iv) Two polygons of the same no. of sides are similar, if (a) their corresponding angles are equal and (b) if their corresponding sides are proportional.

★ Notes :-

→ Two polygon of same number of sides are similar then :

1. Their corresponding angles are equal
2. Their corresponding sides are in equal ratio.

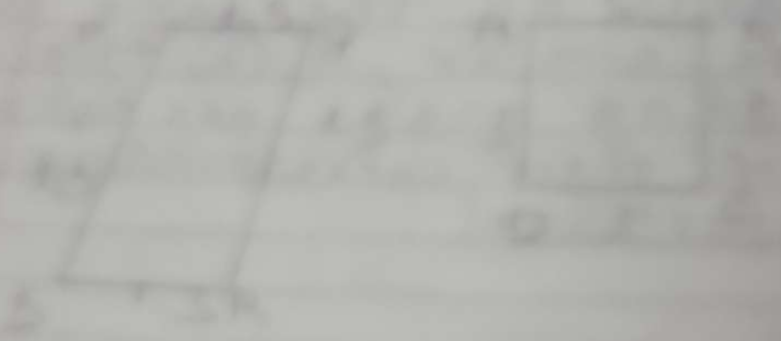
→ All congruent figures are similar but similar figures need not to be congruent.

Q2. Give 2 different examples of pair of :

i) similar figures

Ans. Circles, Squares

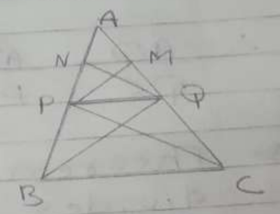
Q3. No, they are not similar as the ratio of the angles and sides are not similar.



Theorem 6.1 :-

Basic Proportionality theorem :- If a line is drawn parallel to one side of a triangle to intersect other two sides in distinct points, the other two sides are divided in the same ratio.

$\frac{9-8}{12}$



Given :- $PQ \parallel BC$

P.T. :- $\frac{AP}{PB} = \frac{AQ}{QC}$

Const. :- Join BE and CD ; and const. $QN \perp AP$ and $PM \perp AQ$.

(1) Area of $\triangle PBQ = \frac{1}{2} \times PB \times QN$

(2) Area of $\triangle APQ = \frac{1}{2} \times AP \times QN$

(3) Area of $\triangle PQC = \frac{1}{2} \times QC \times PM$

(4) Area of $\triangle APQ = \frac{1}{2} \times AQ \times PM$

Divide (2) by (1)

$$= \frac{Ar \triangle APQ}{Ar \triangle PBQ} = \frac{\cancel{\frac{1}{2}} \times AP \times \cancel{QN}}{\cancel{\frac{1}{2}} \times PB \times \cancel{QN}} = \frac{AP}{PB}$$

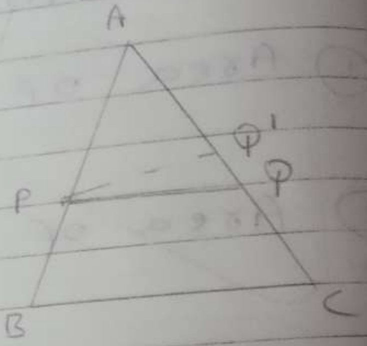
Divide (4) by (3)

$$\frac{Ar\,\Delta APQ}{Ar\,\Delta PQC} = \frac{\frac{1}{2} \times AQ \times \cancel{PM}}{\frac{1}{2} \times QC \times \cancel{PM}} = \frac{AQ}{QC}$$

Area of ΔPBQ and ΔPQC is same as they lie on same base.

$$\therefore \frac{AQ}{QC} = \frac{AP}{PB}$$

Theorem 6.2 :- If a line divides two sides of a Δ in the same ratio, then the line is $\parallel$ to the third side.



Given :- $\frac{AP}{PB} = \frac{AQ}{QC}$

P.T. :- $PQ \parallel BC$

Const :- $PQ' \parallel BC$

By BPT, $\frac{AP}{PB} = \frac{AQ'}{Q'C}$ — (1)

$$\frac{AQ}{QC} = \frac{AQ'}{Q'C}$$

$$\frac{AQ}{QC} + 1 = \frac{AQ'}{Q'C} + 1$$

$$\frac{AQ+QC}{QC} = \frac{AQ'+Q'C}{Q'C}$$

$$\frac{AC}{QC} = \frac{AC}{Q'C}$$

$$AC.Q'C = AC.QC$$

$$Q'C = QC$$

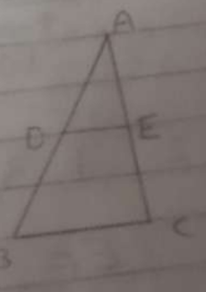
As $Q'C = QC$, then Q' and Q must coincide.

$\therefore Q' = Q$

$\therefore PQ \parallel BC$

Ex 6.2

1) Given - $DE \parallel BC$



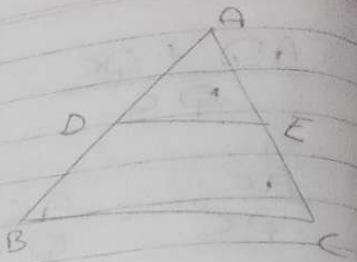
By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{\cancel{1.5}}{\cancel{3}\;2} = \frac{1}{EC}$$

$$\frac{1}{2} = \frac{1}{EC}$$

$\therefore EC = 2cm$

ii) DE ∥ BC (Given)



$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

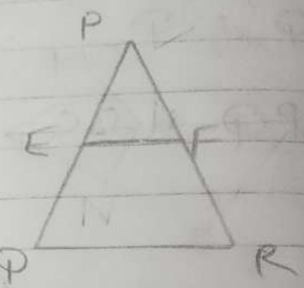
$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$AD = \frac{\cancel{18}}{\cancel{54}_{3}} \times \frac{\cancel{72}^{\cancel{36}\ 13}}{\cancel{10}_{5}}$$

$$AD = 2.6\ cm$$

$$5\overline{)13}\ = 2.6$$

Q2) If EF ∥ QR,



Then $\frac{PE}{EQ} = \frac{PE}{FR}$

$$= \frac{3.9}{3} = \frac{3.6}{2.4}$$

$$= \frac{\cancel{39}^{13}}{\cancel{30}_{10}} = \frac{\cancel{36}^{\cancel{6}\ 3}}{\cancel{24}_{\cancel{4}\ 2}}$$

$$= 1.3 \neq 1.5$$

EF is not parallel to QR.

ii) $\frac{4}{4.5} = \frac{8}{9}$

$= \frac{\cancel{40}^{8}}{\cancel{45}_{9}} = \frac{8}{9}$

$\frac{8}{9} = \frac{8}{9}$

Yes

iii) $\frac{1.28}{2.56} = \frac{0.18}{0.36}$

$= \frac{\cancel{128}^{1}}{\cancel{256}_{2}} = \frac{\cancel{18}^{1}}{\cancel{36}_{2}}$

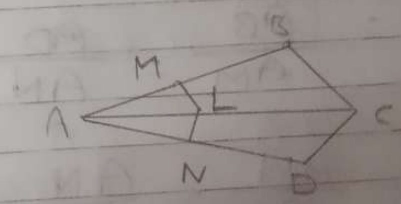
$\frac{1}{2} = \frac{1}{2}$

(B)

Yes

Q3 Given :- $LM \parallel CB$

$LM \parallel CD$



In $\triangle ABC$,

$LM \parallel CB$

$\therefore \frac{AM}{BM} = \frac{AL}{LC}$ By BPT ~ (1)

Q4

In $\triangle ADC$,

$LN \parallel CD$,

$\therefore \frac{AN}{DN} = \frac{AL}{LC}$ — By BPT, (2)

By (1) and (2),

$$\frac{AM}{BM} = \frac{AN}{DN}$$

$$\Rightarrow \frac{BM}{AM} = \frac{DN}{AN}$$

$$\Rightarrow \frac{BM}{AM} + 1 = \frac{DN}{AN} + 1$$

$$\frac{BM + AM}{AM} = \frac{DN + AN}{AN}$$

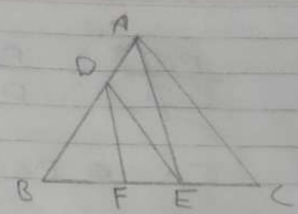
$$\frac{AB}{AM} = \frac{AD}{AN}$$

$$\frac{AM}{AB} = \frac{AN}{AD}$$ Hence Proved

1/7/22

Q4 Given :- DE ∥ AC, DF ∥ AE

P.T. :- $\frac{BF}{FE} = \frac{BE}{EC}$



In △ ABC,

DE ∥ AC,

$\therefore \frac{AD}{DB} = \frac{BE}{EC}$ — (1) (By BPT)

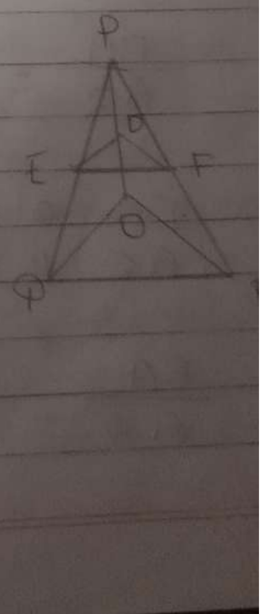
In △ ABE,
DF ∥ AE

$\therefore \frac{AD}{DB} = \frac{BF}{FE}$ — (2) (By BPT)

∴ By (1) and (2),

$\frac{BF}{FE} = \frac{BE}{EC}$

Hence proved.



Q5 Given :- DE ∥ OQ
DF ∥ OR

P.T :- EF ∥ QR

In △ POQ,
DE ∥ OQ

$\frac{PE}{EQ} = \frac{PD}{DO}$ By BPT — (1)

In $\triangle POR$,

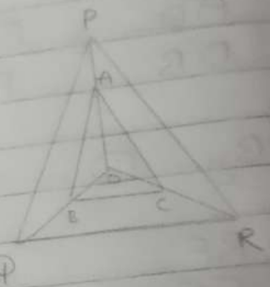
$\frac{PF}{FR} = \frac{PD}{DO}$ By BPT — (2)

By (1) and (2)

$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore$ By CBPT, $EF \parallel QR$

Q6 Given :- $AB \parallel PQ$
$AC \parallel PR$



P.T. :- $BC \parallel QR$

In $\triangle POQ$,
$AB \parallel PQ$

$\therefore \frac{PA}{AO} = \frac{QB}{BO}$ By BPT — (1)

In $\triangle POR$,
$AC \parallel PR$

$\therefore \frac{PA}{AO} = \frac{RC}{CO}$ By BPT — (2)

$\therefore$ By ① and ②,

$$\frac{QB}{BO} = \frac{RC}{CO}$$

By CBPT, $BC \parallel QR$

Hence proved,

Q7 Given - $AP = PB$
$PQ \parallel BC$

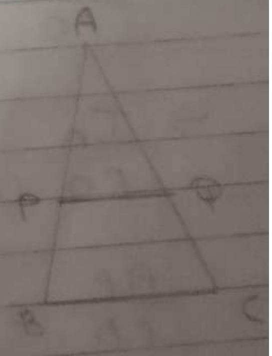
P.T. - $AQ = QC$

As $PQ \parallel BC$

$$\cancel{\frac{AP}{PB}} = \frac{AQ}{QC} \quad \text{By BPT}$$

$$1 = \frac{AQ}{QC}$$

$\therefore AQ = QC$



Q8 Given - $AP = PB$
$AQ = QC$

P.T. - $PQ \parallel BC$

$AP = PB$

$$\therefore \frac{AP}{PB} = 1 \quad \text{①}$$

$AQ = QC$ —— (2)

$\therefore \frac{AQ}{QC} = 1$

By (1) and (2),

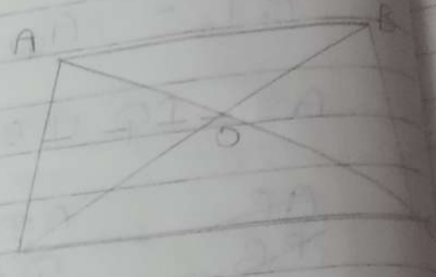
$\frac{AP}{PB} = \frac{AQ}{QC}$

By CBPT,

$PQ \parallel BC$.

Q9 Given - $AB \parallel CD$

P.T. - $\frac{AO}{BO} = \frac{CO}{DO}$



Const. - $PO \parallel AB \parallel CD$

→ In ΔABD,

$PO \parallel AB$

$\frac{AP}{PD} = \frac{BO}{OD}$ —— (1) (By BPT)

→ In ΔADC,

$PO \parallel CD$

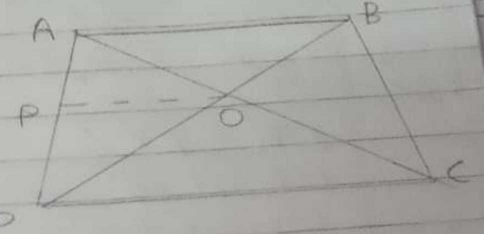
$\frac{AP}{PD} = \frac{AO}{CO}$ —— (2) (By BPT)

By ① and ②,

$$\frac{BO}{DO} = \frac{AO}{CO}$$

$$\therefore \frac{AO}{BO} = \frac{CO}{DO}$$

Q10 Given - $\frac{AO}{BO} = \frac{CO}{DO}$



P.T. - $AB \parallel CD$

Const. - $PO \parallel AB$

$$\therefore \frac{AO}{CO} = \frac{BO}{DO} \quad \text{--- ①}$$

In $\triangle ABD$,

$$\frac{AP}{PD} = \frac{BO}{DO} \quad \text{--- ②}$$

By ① and ②,

$$\frac{AO}{CO} = \frac{AP}{PD}$$

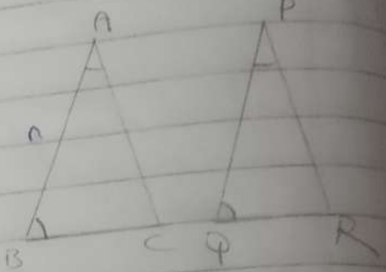
By CBPT, $AB \parallel CD$.

7/7/22

☆ Criteria for similarity of triangles →

→ AAA Similarity :-

~~Between~~ If in 2 triangles, the corresponding angles are equal, then the corresponding sides are in same ratio. Hence the 2 triangles are similar.



$\angle A = \angle P$
$\angle B = \angle Q$
$\angle C = \angle R$

$\therefore \triangle ABC \sim \triangle PQR$ (AA)

→ SSS similarity :-

If in 2 triangles, sides of 1 triangle are proportional to sides of another triangle then there corresponding angles are equal. Hence the 2 $\triangle$s are equal.

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

$\therefore \triangle ABC \sim \triangle PQR$ (SSS)

→ SAS criteria :-

If 1 angle of a $\triangle$ is eq to the 1 angle of a other $\triangle$ and the sides including this angles are proportional then the 2 $\triangle$s are similar.

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\angle A = \angle P$

$\triangle ABC \sim \triangle PQR$ (SAS)

Ex 6.3

Q1 i) $\angle A = \angle P$

$\angle C = \angle R$

$\therefore \triangle ABC \sim \triangle PQR$ (AA)

ii) $\frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ}$

$\therefore \triangle ABC \sim \triangle QRP$ (SSS)

iii) Not similar.

iv) $\frac{MN}{PQ} = \frac{ML}{QR}$

$\angle M = \angle Q$

$\therefore \triangle MNL \sim \triangle PQR$ (AA)

v) Not similar

vi) $\angle E = \angle Q$

$\angle F = \angle R$

$\therefore \triangle DEF \sim \triangle PQR$ (AA)

Q2 Given - $\triangle ODC \sim \triangle OBA$

$\angle BOC = 125^\circ$

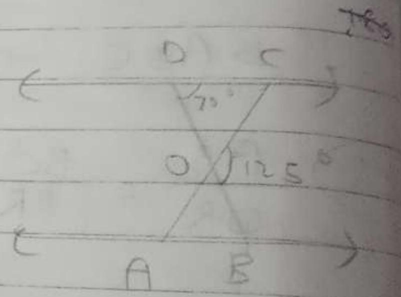
$\angle CDO = 70^\circ$

$\angle 125^\circ + \angle DOC = 180^\circ$ (Linear Pair)

$\angle DOC = 55^\circ$

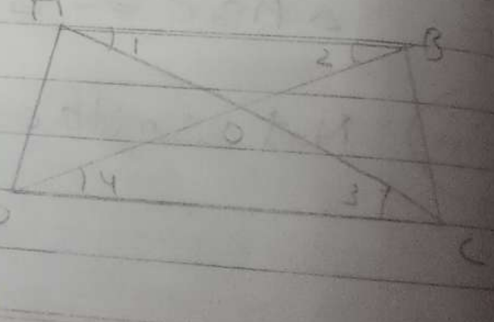
$\therefore \angle OCD = 180^\circ - 125^\circ$

$= 55^\circ$



$\angle C = \angle A = 55^\circ$

Q3 P.T. - $\frac{OA}{OC} = \frac{OB}{OD}$



1. In $\triangle AOB$ and $\triangle COD$,

$\angle 1 = \angle 3$ (Alt. angle)

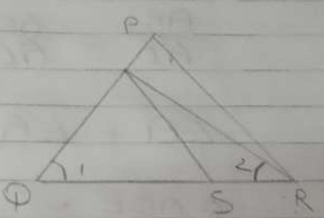
$\angle AOB = \angle COD$ (VOA)

$\therefore \triangle AOB \sim \triangle COD$ (AA)

$\frac{OA}{OC} = \frac{OB}{OD}$

Q4 $\angle 1 = \angle 2$

$\frac{QR}{QS} = \frac{QT}{PR}$



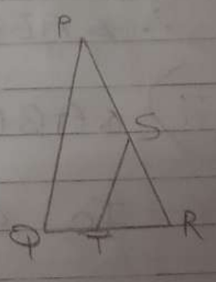
P.T. - $\triangle PQS \sim \triangle TQR$

Sol. In $\triangle PQR$,

$\angle 1 = \angle 2$

$PR = PQ$ (Sides opp. to eq $\angle$s are eq)

$= \frac{QR}{QS} = \frac{QT}{PQ}$



Q5 P.T. - $\triangle RPQ \sim \triangle RTS$

In RPQ and $\triangle RTS$,

$\angle R = \angle R$ (common)

$\angle P = \angle T$ (Given)

$\therefore \triangle RPQ \sim \triangle RTS$ (AA)

Q6 Given - $\triangle ABE \cong \triangle ACD$
P.T. - $\triangle ADE \sim \triangle ABC$

$AB = AC$ —— (1)
$AD = AE$ —— (2)

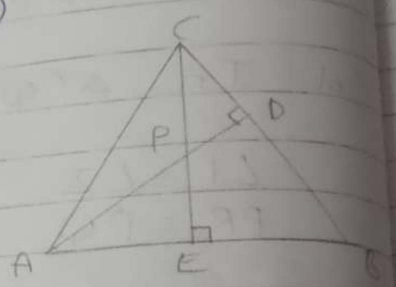
Dividing (1) and (2),

$\frac{AB}{AD} = \frac{AC}{AE}$

$\angle A = \angle A$ (common)

$\therefore \triangle ADE \sim \triangle ABC$ (SAS)

Q7 i) $\triangle AEP \sim \triangle CDP$



In $\triangle AEP$ and $\triangle CDP$

$\angle AEP = \angle CDP$ (Each 90°)
$\angle APE = \angle CPD$ (VOA)

$\therefore \triangle AEP \sim \triangle CDP$ (AA)

ii) $\triangle ABD \sim \triangle CBE$

In $\triangle ABD$ and $\triangle CBE$,

$\angle B = \angle B$ (common)
$\angle ADB = \angle CEB$ (Each 90°)

$\therefore \triangle ABD \sim \triangle CBE$ (AA)

iii) $\triangle AEP \sim \triangle ADB$

$\angle A = \angle A$ (common)
$\angle AEP = \angle ADB$ (Each 90°)

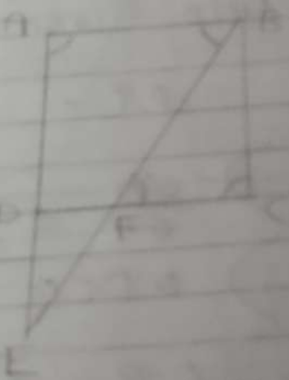
$\therefore \triangle AEP \sim \triangle ADB$ (AA)

iv) $\triangle PDC \sim \triangle BEC$

In $\triangle PDC$ and $\triangle BEC$,

$\angle C = \angle C$ (common)
$\angle PDC = \angle BEC$ (Each 90°)

$\therefore \triangle PDC \sim \triangle BEC$ (AA)



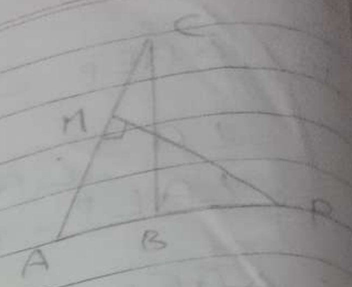
28 P.T. - $\triangle ABE \sim \triangle CFB$

In $\triangle ABE$ and $\triangle CFB$

$\angle A = \angle C$ (opp. $\angle$s of a llgm are equal)

$\angle ABE = \angle CFB$ (Alt. interior angle)

$\therefore \triangle ABE \sim \triangle CFB$ (AA)



Q 9.) PT:- $\triangle ABC \sim \triangle AMP$

In $\triangle ABC$ and $\triangle AMP$,

$\angle A = \angle A$ (common)

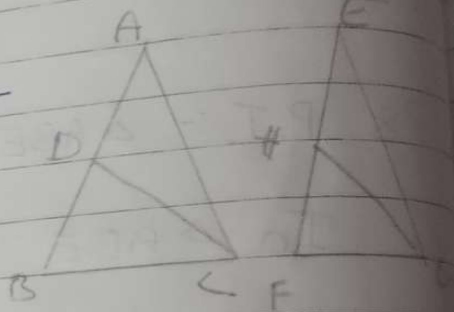
$\angle B = \angle M$ (90° each)

$\therefore \triangle ABC \sim \triangle AMP$ (AA)

ii) $\frac{CA}{PA} = \frac{BC}{MP}$

$\therefore$ As $\triangle ABC \sim \triangle AMP$

$\frac{CA}{PA} = \frac{BC}{MP}$



Q10 Given - $\triangle ABC \sim \triangle FEG$

P.T -

~~i)~~ $\cancel{\frac{CD}{GH} = \frac{AC}{FG}}$

i) $\triangle DCB \sim \triangle HGE$

$\angle B = \angle E$ (Given)

$\angle C = \angle G$

$\frac{1}{2} \times \angle C = \frac{1}{2} \times \angle G$

$\therefore \angle BCD = \angle HGE$

$\therefore \triangle DCB \sim \triangle HGE$ (AA)

iii) $\triangle DCA \sim \triangle HGF$

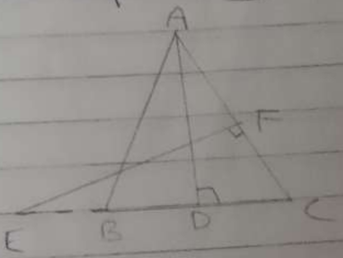
$\angle A = \angle F$ (Given)
$\angle C = \angle G$ (Given)
$\frac{1}{2} \times \angle C = \frac{1}{2} \times \angle G$

$\therefore \angle DCA = \angle HGE$

$\therefore \triangle DCA \sim \triangle HGF$ (AA)

$\therefore \frac{CD}{GH} = \frac{AC}{FG}$

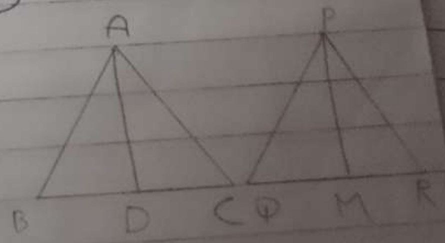
Q11 Given :- $AB = AC$
$\angle ABC = \angle ACB$ ($\angle$s opp to eq sides)



P.T. :- $\triangle ABD \sim \triangle ECF$

$\angle ADB = \angle EFC$ ($90°$ each)
$\angle ABD = \angle ECF$ (P.A.)

$\triangle ABD \sim \triangle ECF$ (AA)



Q12 Given :- $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

P.T. :- $\triangle ABC \sim \triangle PQR$

In $\triangle ABD$ and $\triangle PQM$,
$\frac{BC \times \frac{1}{2}}{QR \times \frac{1}{2}} = \frac{BD}{QM}$

$\therefore \frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$

$\therefore \triangle ABD \sim \triangle PQM$ (SSS)

$\angle B = \angle Q$ (CPST)

In $\triangle ABC$ and $\triangle PQR$,

$\angle B = \angle Q$ (P.A.)

$\frac{AB}{PQ} = \frac{BC}{QR}$ (Given)

$\therefore \triangle ABC \sim \triangle PQR$ (SAS)