Chapter-2 Polynomials

★ Notes :

• Degree of Polynomial :

(1) Linear → Degree 1 → $5x+3$
(2) Quadratic → Degree 2 → $6x^2+2x+1$
(3) Cubic → Degree 3 → x^3+x^2+x+1

• Important terms :

linear polynomial → straight
Quadratic poly → curve

(1) $3ab + 5ab^2$ → Binomial
(2) $5ab^2 + 6ab$ → Binomial
(3) $6x^2$ → Monomial
(4) $7abc^2$ → Monomial

• Three terms in a Polynomials :

(1) variable
(2) co-efficient
(3) Constant

• zero of a Polynomial

» zero of a Polynomial is that particular value which will result the whole polynomial into zero by subtituting that value in the polynomial.

» The maximum number of zeros what a polynomial can have is always equal to

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→ Ex. 2.1

(1) Find no. of zeros :

(i) No. of zeros : 0
(ii) No. of zeros : 1
(iii) No. of zeros : 3
(iv) No. of zeros : 2
(v) No. of zeros : 4
(vi) No. of zeros : 3

Middle-term-splitting (extra questions)

(i) $4x^2 - 4x + 1$
(ii) $x^2 - 2x - 8$
(iii) $2x^2 + 8x + 8$
(iv) $4x^2 + 12x + 5$
(v) $x^2 + x - 182$

→ Answers :

(i) $4x^2 - 4x + 1$
$= 4x^2 - 2x - 2x + 1$
$= 2x(2x - 1) - 1(2x - 1)$
$= (2x - 1)(2x - 1)$

$= x^2 - 2x - 8$
$= x^2 - 4x + 2x - 8$
$= x(x-4) + 2(x-4)$
$= (x+2)(x-4)$

(iii) $2x^2 + 8x + 8$
$= 2x^2 + 4x + 4x + 8$
$= 2x(x+2)\ 4(x+2)$
$= (2x+4)(x+2)$

(iv) $4x^2 + 12x + 5$
$= \cancel{4x^2 + 5x + 2x + 5}$
$= \cancel{x(4x + x5)}$
$= 4x^2 + 10x + 2x + 5$
$= 2x(2x+5) + 1(2x+5)$
$= (2x+1)(2x+5)$

(v) $x^2 + x - 182$
$= x^2 + 14x - 13x - 182$
$= x(x+14) - 13(x+14)$
$= (x-13)(x+14)$

→ Ex 2.2

(1) Verify the relationship:

(i) $x^2 - 2x - 8$

» Solving the polynomial by factorization we get,

$$x^2 - 2x - 8$$
$$= x^2 - 4x + 2x - 8$$
$$= x(x-4)\ 2(x-4)$$
$$= (x+2)(x-\cancel{2}\,4)$$

so $x = -2$ \$ $x = 4$

= we have : $\alpha = -2$ \& $\beta = 4$

= Now,

Sum of zeroes $= \alpha + \beta$
$= -2 + 4$
$= 2$ ---- (i)

Product of zeroes $= \alpha\beta$
$= -2 \times 4$
$= 8$ ----- (ii)

= Now, according to the formula,

= Sum of zeroes $= \dfrac{-b}{a} = \dfrac{-(-2)}{1} = +2$ ---- (iii)

= Product of zeroes $= \dfrac{c}{a} = \dfrac{-8}{1} = -8$ ---- (iv)

Hence 1 = 3 and
2 = 4

$\therefore$ It's verified

(ii) $4s^2 - 4s + 1$

=> Solving this polynomial by factorization we get,

$= 4s^2 - 4s + 1$
$= 4s^2 - 2s - 2s + 1$
$= 2s(2s-1) - 1(2s-1)$
$= (2s-1)(2s-1)$

so $s = \frac{1}{2}$ & $s = \frac{1}{2}$

so we have $\alpha = \frac{1}{2}$ & $\beta = \frac{1}{2}$

= Now,

= Sum of zeroes $= \alpha + \beta$
$= \frac{1}{2} + \frac{1}{2} = 1$ --- (i)

= Product of zeroes $= \alpha\beta$
$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ---- (ii)

= Now, according to the formula,

= Sum of zeroes $= \frac{-b}{a} = \frac{-(-4)}{4} = \frac{4}{4} = 1$ --- (iii)

= Product of zeroes $= \frac{c}{a} = \frac{1}{4}$ --- (iv)

Hence ① = ③ and
② = ④

∴ It's verified

(iii) $6x^2 - 3 - 7x$
$6x^2 - 7x - 3$

>> Solving this polynomial by factorization we get,

$= 6x^2 - 7x - 3$
$= 6x^2 - 9x + 2x - 3$
$= 3x(2x-3) + 1(2x-3)$
$= (3x+1)(2x-3)$

so $x = \frac{-1}{3}$ & $x = \frac{3}{2}$

so $\alpha = \frac{-1}{3}$ & $\beta = \frac{3}{2}$

= Now,

= Sum of zeroes $= \alpha + \beta$

$= \frac{-1 \times 2}{3 \times 2} + \frac{3 \times 3}{2 \times 3}$

$= \frac{-2}{6} + \frac{9}{6}$

$= \frac{7}{6}$ --- (i)

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= Product of zeroes = $\alpha\beta$

$= \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2}$ ... (ii)

⇒ Now, according to the formula,

= Sum of zeroes $= \frac{-b}{a} = \frac{+7}{6}$ --- (iii)

= Product of zeroes $= \frac{c}{a} = \frac{-3}{6} = \frac{-1}{2}$ ---(iv)

Hence (1) = (3)

(2) = (4)

∴ It's verified

(iv) $4u^2 + 8u$

$= 4u(u+2)$

∴ The 2 factors are $4u$ & $(u+2)$

$= \alpha = 4u = 0$ & $\beta = u+2 = 0$

$u = 0$ $u = -2$

= Now,

= Sum of zeroes $= \alpha + \beta$

$= 0 + (-2)$

$= -2$ --- (i)

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= Product of zeroes $= \alpha\beta$
$= 0 \times (-2)$
$= 0$ --- (ii)

>> Now according to the formula

= Sum of zeroes $= \frac{-b}{a} = \frac{-8}{4} = -2$ --- (iii)

= Product of zeroes $= \frac{c}{a} = \frac{0}{4} = 0$ --- (iv)

Hence (1) = (3)
(2) = (4)

$\therefore$ It's verified

(v) $t^2 - 15$

>> Solving the polynomial by factorization we get,

= $t^2 - 15 = 0$
= $t^2 = 15$
$t = \pm\sqrt{15}$

so $\alpha = +\sqrt{15}$ & $\beta = -\sqrt{15}$

= Now,

>> Sum of zeroes $= \alpha + \beta$
$= +\sqrt{15} - \sqrt{15} = 0$

∘ Product of zeroes = $\alpha\beta$

$= +\sqrt{15} \times (-\sqrt{15}) = -\sqrt{15}$ --- (ii

>> Now, according to the formula

= Sum of zeroes = $\frac{-b}{a} = \frac{0}{1} = 0$ --- (iii

= Product of zeroes = $\frac{c}{a} = \frac{-15}{1} = -15$ --- (i

Hence ① = ③

② = ④

∴ It's verified

(vi) $3x^2 - x - 4$

>> Solving the polynomial by factorization we get,

= $3x^2 - x - 4$

= $3x^2 - 4x + 3x - 4$

= $x(3x-4) + 1(3x-4)$

= $(x+1)(3x-4)$

⇒ $\alpha = x+1 = 0$ & $\beta = 3x-4 = 0$

$= x = -1$ $\quad$ $x = 4/3$

• Now,

∴ Sum of zeroes = $\alpha + \beta$

$= \frac{-1\times3}{1\times3} + \frac{4\times1}{3\times1}$

$= \frac{-3}{3} + \frac{4}{3}$

$= \frac{1}{3}$ --- (i)

∴ Product of zeroes = $\alpha\beta$

$= -1 \times \frac{4}{3} = \frac{-4}{3}$ --- (ii)

» Now according to the formula, we get

∴ Sum of zeroes = $\frac{-b}{a} = \frac{+1}{3}$ --- (iii)

∴ Product of zeroes = $\frac{c}{a} = \frac{-4}{3}$ --- (iv)

Hence 1 = 3
2 = 4

∴ It's verified

(2) Find a quadratic polynomial :

(i) $\frac{1}{4}$, -1 (For 1 mark)

: Sum of zeroes = $\frac{1}{4}$

= Product of zeroes = -1

: ~~Form~~ From general form,

= $x^2 - (\alpha+\beta)x + \alpha\beta$

= $x^2 - \left(\frac{1}{4}\right)x + (-1)$

= $x^2 - \frac{1x}{4} + -1$ (L.C.M) = 4

= $\frac{x^2 \times 4}{1 \times 4} \; \frac{1x \times 1}{4 \times 1} + \frac{-1 \times 4}{1 \times 4}$

= $\frac{4x^2 - x - 4}{4} = 0$

= $\frac{4x^2 - x - 4}{-}$

(i) $\frac{1}{4}$, -1 (For 3 marks)

· Sum of zeroes = $\frac{1}{4}$

: product of zeroes = -1

= we know,

= sum of zeroes = $\frac{-b}{a}$

= $\frac{1k}{4k} = \frac{-b}{a}$ (k is constant)

= $b = -1k$ & $a = 4k$

= Product of zeroes = $\frac{c}{a}$

= $\frac{-1}{1} = \frac{c}{a}$

∴ subitituting the value we get,

= $-1 = \frac{c}{4k}$

∴ $c = -4k$

= Now from standard form we get,

$ax^2 + bx + c$

= $4kx^2 - 1kx - 4k$

= $k(4x^2 - x - 4)$

(ii) $\sqrt{2}, \frac{1}{3}$ (1 mark)

= Sum of zeroes : $\sqrt{2}$

= Product of zeroes : $\frac{1}{3}$

= From general form:

= $x^2 - (\alpha+\beta)x + (\alpha\beta)$

= $x^2 - (\sqrt{2})x + (\frac{1}{3})$

= $x^2 - \sqrt{2}x + \frac{1}{3}$

= $\frac{x^2 \times 3}{1\times 3} - \frac{\sqrt{2}x \times 3}{1 \times 3} + \frac{1\times 1}{3\times 1}$ (L.C.M = 3)

= $\frac{3x^2 - 3\sqrt{2}x + 1}{3} = 0$

= $3x^2 - 3\sqrt{2}x + 1 = 0$

(ii) $\sqrt{2}, \frac{1}{3}$ (For 3 mark)

= Sum of zeroes = $\sqrt{2}$ ~~$+ \frac{1}{3}$~~

= Product of zeros = $\frac{1}{3}$

= we know,

= Sum of zeroes $= \frac{-b}{a}$

= $\frac{\sqrt{2}k}{k} = \frac{-b}{a}$ (k be constant)

= Product of zeroes $= \frac{c}{a}$

= $\frac{1}{3} = \frac{c}{a}$

= $\frac{1}{3} = \frac{c}{k}$

= $c = \frac{k}{3}$

= Now, ~~fo~~ from standard form we get

= $ax^2 + bx + c$

$kx^2 - \sqrt{2}kx + \frac{k}{3}$

= $k\left(x^2 - \sqrt{2}x + \frac{1}{3}\right)$

= $\frac{3x^2 - 3\sqrt{2}x + 1}{3} = 0$ $\therefore 3x^2 - 3\sqrt{2}x + 1$

(iii) $0, \sqrt{5}$ (1 marks)

= Sum of zeroes = 0

= Product of zeroes = $\sqrt{5}$

= From general form we get,

$= x^2 - (\alpha+\beta)x + \alpha\beta$

$= x^2 - (0)x + (\sqrt{5})$

$= x^2 + \sqrt{5}$

(iii) $0, \sqrt{5}$ (3 marks)

= Sum of zeroes = 0

= Product of zeroes = $\sqrt{5}$

= Sum of zeroes = $\frac{-b}{a}$

$= \frac{0k}{1k} = \frac{-b}{a}$

$\therefore$ $a = 1k$, $b = 0k$

= Product of zeroes = $\frac{c}{a}$

$= \sqrt{5} = \frac{c}{1k}$ $\therefore$ $c = \sqrt{5}k$

= From standard form we get,

= $ax^2 + bx + c$

= $1kx^2 + 0kx + \sqrt{5}k$

= $k(x^2 + 0 + \sqrt{5})$

= $k(x^2 + \sqrt{5})$

$\therefore x^2 + \sqrt{5}$

(iv) $\underline{1}, \underline{1}$ (1 mark)

= Sum of zeroes = $\underline{1}$

= Product of zeroes = $\underline{1}$

= From general form we get

= $x^2 - (\alpha + \beta)x + \alpha\beta$

= $x^2 - (1)x + \underline{1}$

= $x^2 - x + 1$

(iv) $\underline{1}, \underline{1}$ (3 marks)

= Sum of zeroes = $\underline{1}$

= Product of zeroes = $\underline{1}$

= Sum of zeroes = $\frac{-b}{a}$

= $\frac{1k}{1k} = \frac{-b}{a}$

$\therefore$ $a = 1k$

$b = -1k$

= Product of zeros $= \frac{c}{a}$

$= \frac{1}{1} = \frac{c}{1k}$

$= c = 1k$

= Now from the general formula we get

$= ax^2 + bx + c$

$= 1kx^2 - 1kx + 1k$

$= k(x^2 - x + 1)$ k is constant

(v) Sum of zeros $= -1/4$ (1 marks)

Product of zeros $= 1/4$

= From the formula we get

$= x^2 - (\alpha+\beta)x + (\alpha\beta)$

$= x^2 - \left(\frac{-1}{4}x\right) + \left(\frac{1}{4}\right)$

$= x^2 + \frac{1x}{4} + \frac{1}{4}$

$\therefore$ $x^2 + \frac{1x}{4} + \frac{1}{4}$

(v) Sum of zeros $= \frac{-b}{a}$ (3 marks)

$= \frac{-1k}{4k} = \frac{-b}{a}$

$= b = 1k$
$= a = 4k$

= Product of zeros $= \frac{c}{a}$

$= \frac{1k}{4k} = \frac{c}{4k} = c = 1k$

= from general formula we get
$= ax^2 + bx + c$

$= 4x^2 + 1kx + 1k$
$= \underline{4x^2 + x + 1}$

(vi) Sum of zeros $= 4$ (1 marks)
Product of zeros $= 1$

= using $= k\,(x^2 - (\alpha+\beta) + \alpha\beta)$
$= \underline{k(x^2 - 4x + 1)}$

= Sum of zeros $= \frac{-b}{a}$

$\frac{4k}{1k} = \frac{-b}{a} = a = 1k$
$b = -4k$

$\therefore$ Product of zeros $= \frac{c}{a}$

$= \frac{1k}{1k} = \frac{c}{1k} \quad \therefore c = 1k$

So using the general formula we get,

$= 1kx^2 - 4kx + 1$

$= k\ (x^2 - 4x + 1)$

→ Exercise 2·3

(1) Divide :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$
$g(x) = x^2 - 2$

$$
= \begin{array}{r|l}
 & x - 3 \\
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & -x^3 + 2x \\
 & -3x^2 + 7x \\
 & +3x^2 - 6 \\
 & 7x - 9
\end{array}
$$

$\therefore$ $q(x) = x - 3$
$r(x) = 7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$
$g(x) = x^2 - x + 1$

$$
\begin{array}{r}
x^2+x-3 \\
= x^2-x+1 \,\big)\, \overline{x^4-3x^2+4x+5} \\
\underline{-x^4+x^3-x^2} \\
x^3-4x^2+4x \\
\underline{-x^3+x^2-x} \\
-3x^2+3x-5 \\
\underline{+3x^2-3x+3} \\
8
\end{array}
$$

$= g(x) = x^2+x-3 \quad \& \quad r(x) = 8$

(iii) $p(x) = x^4-5x+6$

$g(x) = -x^2+2$

$$
\begin{array}{r}
-x^2-2 \\
= -x^2+2 \,\big)\, \overline{x^4-5x+6} \\
\underline{+x^4+2x^2} \\
2x^2-5x \\
\underline{\mp 2x^2+4} \\
-5x+10
\end{array}
$$

$\therefore \quad q(x) = -x^2-2$

$r(x) = -5x+10$

(2) verify the factor :

(i) $p(x) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$
$g(x) = t^2 - 3$

$$
\begin{array}{r}
2t^2 + 3t + 4 \\
t^2 - 3 \,\big)\, \overline{2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
\underline{-2t^4 + 6t^2} \\
3t^3 + 4t^2 \\
\underline{-3t^3 + 9t} \\
4t^2 - 12 \\
\underline{-4t^2 + 12} \\
0
\end{array}
$$

$\therefore$ $t^2 - 3 = 0$
so we can say that it is a factor.

(ii) $p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$
$g(x) = x^2 + 3x + 1$

$$
\begin{array}{r}
3x^2 - 4x + 2 \\
x^2 + 3x + 1 \,\big)\, \overline{3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
\underline{-3x^4 - 9x^3 - 3x^2} \\
-4x^3 - 10x^2 + 2x \\
\underline{+4x^3 + 12x^2 + 4x} \\
2x^2 + 6x + 2 \\
\underline{-2x^2 - 6x - 2} \\
0
\end{array}
$$

$\therefore$ $x^2 + 3x + 1 = 0$
so we can say that it is a factor

(iii) $p(x) = x^5 - 4x^3 + x^2 + 3x + 1$
$g(x) = x^3 - 3x + 1$

$$= \begin{array}{r|l} & x^2 - 1 \\ \hline x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\ & -x^5 + 3x^3 - x^2 \\ \hline & -x^3 + 3x + 1 \\ & +x^3 - 3x + 1 \\ \hline & 2 \end{array}$$

$\therefore\ x^2 - 1 = 2$

so we can say that it is not a factor

(3) $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$
roots $= \sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$

= Sum of zeroes $= \sqrt{\frac{5}{3}} - \sqrt{\frac{5}{3}} = 0$

= Product of zeroes $= \sqrt{\frac{5}{3}} \times -\sqrt{\frac{5}{3}} = \frac{-5}{3}$

= From the standard formula we get

$= x^2 - (\alpha + \beta)x + \alpha\beta$
$= x^2 - (0)x + \left(\frac{-5}{3}\right)$

$= x^2 - \frac{5}{3} = 0$

$= \frac{3x^2 - 5}{3} = 0$

$=$ $\underline{\underline{3x^2 - 5}}$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$

$$
\begin{array}{r|l}
 & \cancel{3}x^2 + 2x + 1 \\
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & -3x^4 + 5x^2 \\
 & \quad 6x^3 + 3x^2 \\
 & \quad -6x^3 + 10x \\
 & \qquad 3x^2 - 5 \\
 & \qquad -3x^2 + 5 \\
 & \qquad\quad 0
\end{array}
$$

$=$ we have $g(x) = x^2 + 2x + 1$ which is a factor of $p(x)$

$= x^2 + 2x + 1$
$= x^2 + x + x + 1$
$= x(x+1) + 1(x+1)$
$= (x+1)(x+1)$

$\therefore$ The other 2 factors are $-1, -1$

so All four factors are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$

Extra → Find all the zeros of $2x^4 - 3x^3 - 3x^2 + \ldots$, if you know that 2 of the zeros are $\sqrt{2}$ & $-\sqrt{2}$. Find other zeros

Ans = Sum of zeros = $\sqrt{2} - \sqrt{2} = 0$

Product of zeros = $\sqrt{2} \times (-\sqrt{2}) = -2$

= From the formula we get,

$= x^2 - (\alpha + \beta)x + (\alpha\beta)$

$= x^2 - (0)x + (-2)$

$= \underline{\underline{x^2 - 2}}$

$\therefore$ $x^2 - 2$ is a factor of $p(x)$

so

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 1 \\
\hline
x^2 - 2 & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\
 & -2x^4 + 4x^2 \\
 & -3x^3 + x^2 \\
 & +3x^3 - 6x \\
 & x^2 - 2 \\
 & -x^2 + 2 \\
 & 0
\end{array}
$$

$= 2x^2 - 3x + 1$

$= 2x^2 - 2x - x + 1$

$= 2x(x-1) - 1(x-1)$

$= (2x-1)(x-1)$

So the 2 zeros are $\frac{1}{2}$, 1

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so All the four zeros are $\sqrt{2}, -\sqrt{2}, \frac{1}{2}, 1$

(4) $p(x) = x^3 - 3x^2 + x + 2$
$g(x) = x - 2$
$r(x) = -2x + 4$
$q(x) = ?$

$\therefore$ we know,

$p(x) = g(x) \times q(x) + r(x)$
$q(x) = \dfrac{p(x) - r(x)}{g(x)}$

$\Rightarrow q(x) = \dfrac{(x^3 - 3x^2 + x + 2) - (-2x + 4)}{(x - 2)}$

$\Rightarrow q(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x - 2}$

$\therefore$ By long division method

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & -x^3 + 2x^2 \\
 & \quad -x^2 + 3x \\
 & \quad +x^2 - 2x \\
 & \qquad x - 2 \\
 & \qquad -x + 2 \\
 & \qquad 0
\end{array}
$$

$\therefore q(x) = x^2 - x + 1$

(5) (i) $p(x) = 2x^2 - 2x + 14$
$g(x) = 2$
$q(x) = x^2 - x + 7$
$r(x) = 0$

(ii) $p(x) = x^3 + x^2 + x + 1$
$g(x) = x^2 - 1$
$q(x) = x + 1$
$r(x) = 2x + 2$

(iii) $p(x) = x^3 + 2x^2 - x + 2$
$g(x) = x^2 - 1$
$q(x) = x + 2$
$r(x) = 4$

→ Exercise 2.4

(1) verify & check :

(i) $2x^3 + x^2 - 5x + 2, \left(\frac{1}{2}, 1, -2\right)$

= ~~Sum of zeroes~~ $\cancel{= \frac{1}{2}}$

= $p(1) = 2x^3 + x^2 - 5x + 2$
= $p(1) = (2 \times (1)^3) + (1)^2 - (5 \times 1) + 2$
= $p(1) = 2 + 1 - 5 + 2$
= $p(1) = 0$

∴ $x - 1$ is a factor of $2x^3 + x^2 - 5x + 2$

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$$= \begin{array}{r|l} & 2x^2 + 3x - 2 \\ \hline x-1 & 2x^3 + x^2 - 5x + 2 \\ & -2x^3 + 2x^2 \\ \hline & 3x^2 - 5x \\ & -3x^2 + 3x \\ \hline & -2x + 2 \\ & +2x - 2 \\ \hline & 0 \end{array}$$

$= 2x^2 + 3x - 2$

$= 2x^2 + 4x - x - 2$

$= 2x(x+2) - 1(x+2)$

$= (2x-1)(x+2)$

$= x = \frac{1}{2}, x = -2$

$\therefore$ the 3 zeros are $\frac{1}{2}, -2, 1$

so Now:

$=$ Sum of zeroes $= \frac{1}{2} + \frac{-2\times 2}{1\times 2} + \frac{1\times 2}{1\times 2}$

$= \frac{1-4+2}{2} = \frac{-1}{2}$ — (i)

$=$ Product of zeros $= \frac{1}{\cancel{2}} \times \frac{-1}{\cancel{2}} \times 1 = -1$ — (ii)

= ~~Products~~ Products of zeros = $\left(\frac{-1}{2} \times -2\right) + ($

$+ \left(\frac{1}{2} \times 1\right)$

= Products of zeros = $\frac{-1 \times 4}{1 \times 2} + \frac{(-2) \times 2}{1 \times 2} + \frac{1}{2}$

= $\frac{-2 + -4 + 1}{2} = \frac{5}{2}$ — (iii)

= from the formula we can derive that:

= $\alpha + \beta + \gamma = \frac{-b}{a}$ (sum of zeros)

= $\alpha + \beta + \gamma = \frac{-1}{2}$ — (iv)

= $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{c}{a}$ (Product)

= $\frac{-5}{2}$ --- (v)

= $\alpha\beta\gamma = \frac{-d}{a}$ (Product at a time)

= $\frac{-2}{2} = -1$ --- (vi)

$\therefore$ (i) = (iv)
(ii) = (vi)
(iii) = (v)

$\therefore$ It is verified

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$ where: 2, 1, 1 (zeros

$= p(2) = (2)^3 - (4 \times (2)^2) + (5 \times 2) - 2$ (let $x = 2$)
$= p(2) = 8 - 16 + 10 - 2$
$= p(2) = 0$

$\therefore$ $x - 2$ is a factor of $p(x)$

so By long division:

$$
\begin{array}{r|l}
 & x^2 - 2x + 1 \\
\hline
x - 2 & x^3 - 4x^2 + 5x - 2 \\
 & -x^3 + 2x^2 \\
\hline
 & \quad -2x^2 + 5x \\
 & \quad +2x^2 - 4x \\
\hline
 & \qquad x - 2 \\
 & \qquad -x + 2 \\
\hline
 & \qquad 0
\end{array}
$$

$= x^2 - 2x + 1$
$= x^2 - x - x + 1$
$= x(x - 1) - 1(x - 1)$
$= (x - 1)(x - 1)$

$= x = 1,\ x = 1$

∴ The 3 factors of polynomial $p(x)$ is
2, 1, 1

Sum of zeros = 2+1+1
= 4 --- (i)

Product of zeros (once) = [(2×1)+(1×1)+(1×2)]
= 2+1+2
= 5 --- (ii)

Product at a time = 2×1×1
= 2 --- (iii)

From the formula we can say that:

Sum of zeros = $\frac{-b}{a}$

= $\frac{-(-4)}{1}$ = 4 --- (iv)

Product of zeros (once) = $\frac{c}{a}$

= $\frac{5}{1}$ = 5 --- (v)

Product at a time = $\frac{-d}{a}$

= $\frac{-(-2)}{1}$ = +2 --- (vi)

Hence (i) = (iv)
(ii) = (v)
(iii) = (vi)

$\therefore$ It's verified

(2) Sum of zeros = 2 (1 mark)
Sum of the Product = -7
Product of zeros = -14

= from general form we get :

$= x^3 - (\alpha+\beta+\gamma)x^2 + [(\alpha\beta)+(\beta\gamma)+(\gamma\alpha)]x - (\alpha\beta\gamma)$
$= x^3 - (2)x^2 + (-7)x - (-14)$
$= x^3 - 2x^2 - 7x + 14$

$\therefore$ Polynomial is $x^3 - 2x^2 - 7x + 14$

(2) Sum of zeros = 2 (3 marks)
Sum of product of zeros = -7
Product of zeros = -14

= we know that,

= Sum of zeros $= \frac{-b}{a}$

$= \frac{2k}{1k} = \frac{-b}{a}$ (k is constant)

so $b = -2k$ & $a = k$

= ~~Prod~~ Sum of product = $\frac{c}{a}$

= $\frac{-7\cancel{k}}{1\cancel{k}} = \frac{c}{k}$

= $c = -7k$

= Product of zeros = $\frac{-d}{a}$

= $-14 = \frac{-d}{k}$

= $d = 14k$

= Now from the standard form we get :

= $ax^3 + bx^2 + cx + d$

= $kx^3 + (-2k \times x^2) + (-7k \times x) + 14k$

= $kx^3 - 2kx^2 - 7kx + 14k$

= $k(x^3 - 2x^2 - 7x + 14) = 0$

$\therefore$ $x^3 - 2x^2 - 7x + 14$

(3) $p(x) = x^3 - 3x^2 + x + 1$

zeros = $a-b, a, a+b$

= Sum of zeros = $\frac{-b}{a} = \frac{-(-3)}{1} = \frac{3}{}$

= from formula we get:

= Sum of zeros = $(a-b+a+a+b)$

= $3a = 3$

$a = \underline{\underline{1}}$

= Product of zeros = $\frac{-d}{a}$

= Product = $\frac{-1}{1} = -1$

= ~~fro~~ from formula we get:

= $(a-b)(a)(a+b) = -1$

= $(a^2-b^2)(a) = -1$

= $(1^2-b^2)(1) = -1$

= $1-b^2 = -1$

= $-b^2 = -1-1$

= $+b^2 = +2$

= $b^2 = 2$

= $b = \sqrt{2}$

$\therefore$ $a = \underline{1}$ & $b = \sqrt{2}$

(4) $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

zeros = $(2+\sqrt{3})(2-\sqrt{3})$

= Sum of zeros = $2+\sqrt{3}-\sqrt{3}+2 = 4$

= Product of zeros = $(2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$

= from the formula we get:

= $x^2 - (\alpha+\beta)x + \alpha\beta$

= $x^2 - (4)x + (1)$

= $x^2 - 4x + 1$

$$
\begin{array}{r}
x^2 - 2x - 35 \\
x^2-4x+1 \overline{\big) x^4 - 6x^3 - 26x^2 + 138x - 35} \\
-x^4 + 4x^3 - x^2 \\
\hline
-2x^3 - 27x^2 + 138x \\
+2x^3 - 8x^2 + 2x \\
\hline
-35x^2 + 140x - 35 \\
+35x^2 - 140x + 35 \\
\hline
0
\end{array}
$$

= $x^2 - 2x - 35$

= $x^2 - 7x + 5x - 35$

= $x(x-7)\ 5(x-7)$

= $(x+5)(x-7)$

= $x = -5$ & $x = 7$

$\therefore$ All the zeros are $= -5, 7, 2+\sqrt{3}, 2-\sqrt{3}$

(5) $p(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$

$g(x) = x^2 - 2x + k$

$r(x) = x + a$

$k = ?$

$a = ?$

$$\begin{array}{r|l} & x^2 - 4x + (8-k) \\ \hline x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\ & \underline{-x^4 + 2x^3 - kx^2} \\ & -4x^3 + (16-k)x^2 - 25x \\ & \underline{+4x^3 - 8x^2 + 4kx} \\ & (8-k)x^2 + (-25 + 4k)x + 10 \\ & \underline{-(8-k)x^2 - (-16+2k)x - 8k - k^2} \\ & (-9+2k)x + 10 - 8k + k^2 \end{array}$$

= we have : $r(x) = x + a$

= $(-9+2k)x + 10 - 8k + k^2 = x + a$

= Comparing the eq :

= we get,

$(-9 + 2k) = 1$

$2k = 10$

$k = 5$

- Also :

$a = 10 - 8k + k^2$

$a = 10 - 40 + 25$

$a = -5$