# Problema 1:

$$|\psi\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$= \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{2}|00\rangle - \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle\right)$$

$$= \frac{1}{\sqrt{8}}|000\rangle - \frac{1}{\sqrt{8}}|001\rangle + \frac{1}{\sqrt{8}}|010\rangle - \frac{1}{\sqrt{8}}|011\rangle + \frac{1}{\sqrt{8}}|100\rangle - \frac{1}{\sqrt{8}}|101\rangle + \frac{1}{\sqrt{8}}|110\rangle - \frac{1}{\sqrt{8}}|111\rangle$$

Probabilidad:

$$= \frac{1}{\sqrt{8}}|000\rangle - \frac{1}{\sqrt{8}}|001\rangle + \frac{1}{\sqrt{8}}|100\rangle - \frac{1}{\sqrt{8}}|101\rangle + \frac{1}{\sqrt{8}}|010\rangle - \frac{1}{\sqrt{8}}|011\rangle + \frac{1}{\sqrt{8}}|110\rangle - \frac{1}{\sqrt{8}}|111\rangle$$

$$= \left(\frac{1}{\sqrt{8}}|00\rangle - \frac{1}{\sqrt{8}}|01\rangle + \frac{1}{\sqrt{8}}|10\rangle - \frac{1}{\sqrt{8}}|11\rangle\right) \otimes |0\rangle$$

$$+ \left(\frac{1}{\sqrt{8}}|00\rangle - \frac{1}{\sqrt{8}}|01\rangle + \frac{1}{\sqrt{8}}|10\rangle - \frac{1}{\sqrt{8}}|11\rangle\right) \otimes |1\rangle$$

$$\Rightarrow P_0 = \left|\frac{1}{\sqrt{8}}\right|^2 + \left|-\frac{1}{\sqrt{8}}\right|^2 + \left|\frac{1}{\sqrt{8}}\right|^2 + \left|\frac{-1}{\sqrt{8}}\right|^2 = \frac{1}{2}$$

$$P_1 = P_0$$

## Problema 3

$$U3 = \begin{bmatrix} \cos\theta/2 & -e^{i\lambda}\sin\theta/2 \\ e^{i\phi}\sin\theta/2 & e^{i(\phi+\lambda)}\cos\theta/2 \end{bmatrix}$$

$$U3\,|0\rangle = \begin{bmatrix} \cos\theta/2 & -e^{i\lambda}\sin\theta/2 \\ e^{i\phi}\sin\theta/2 & e^{i(\phi+\lambda)}\cos\theta/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta/2 \\ e^{i\phi}\sin\theta/2 \end{bmatrix} = \cos\theta/2\,|0\rangle + e^{i\phi}\sin\theta/2\,|1\rangle$$

$$U3|1\rangle = \begin{bmatrix} \cos\theta/2 & -e^{i\lambda}\sin\theta/2 \\ e^{i\phi}\sin\theta/2 & e^{i(\phi+\lambda)}\cos\theta/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -e^{i\lambda}\sin\theta/2 \\ e^{i(\phi+\lambda)}\cos\theta/2 \end{bmatrix} = -e^{i\lambda}\sin\theta/2\,|0\rangle + e^{i(\phi+\lambda)}\cos\theta/2\,|1\rangle$$

$$\Rightarrow U3 = |0\rangle\left[\cos\theta/2\langle 0| + e^{i\phi}\sin\theta/2\langle 1|\right]$$

$$+ |1\rangle\left[-e^{i\lambda}\sin\theta/2\langle 0| + e^{i(\phi+\lambda)}\cos\theta/2\langle 1|\right]$$

$$= \cos\theta/2\,|0\rangle\langle 0| + e^{i\phi}\sin\theta/2\,|0\rangle\langle 1|$$

$$-e^{i\lambda}\sin\theta/2\,|1\rangle\langle 0| + e^{i(\phi+\lambda)}\cos\theta/2\,|1\rangle\langle 1|$$

$\#$

Problema 4

Si $\sum x_i z_i$ es par:

- $\sum x_i z_i \mod 2 = 0$

y existe un número par de operaciones $1 \oplus 1 = 0$

$1 \oplus 1 \oplus 1 \oplus 1 = 0$

etc.

como la cadena de $\oplus$ para pares de 1 siempre termina en 0, los $\oplus 0$ consecutivos terminarán en 0.

Si $\sum x_i z_i$ es impar

- $\sum x_i z_i \mod 2 = 1$

y existe un número impar de operaciones $1 \oplus 1 \oplus 1 = 1$

$1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 = 1$

etc.

como la cadena de $\oplus$ para impares de 1 siempre termina en 1, los $\oplus$O consecutivos terminarán en 1.

Por lo tanto,

$$\sum_{i=1}^{n} x_i z_i \pmod 2) = x_1 z_1 \oplus x_2 z_2 \oplus \cdots \oplus x_n z_n$$

## Problema 8

$$|\psi_3\rangle = U_{\omega}|\psi_2\rangle$$

$$= (2|s\rangle\langle s| - \mathbb{I})\left(-\underbrace{\sqrt{\frac{a}{2^n}}}_{\alpha}|A\rangle + \underbrace{\sqrt{\frac{b}{2^n}}}_{\beta}|B\rangle\right)$$

se calculará primero

$$|s\rangle\langle s|\left(\alpha|A\rangle + \beta|B\rangle\right) \cdots \star$$

con $|s\rangle\langle s| = \alpha^2|A\rangle\langle A| + \beta^2|B\rangle\langle B|$

$$+\alpha\beta(|A\rangle\langle B| + |B\rangle\langle A|)$$

$$* = \alpha^3|A\rangle + \alpha^2\beta|B\rangle + \beta^3|B\rangle + \alpha\beta^2|A\rangle$$

$$= (\alpha^3 + \alpha\beta^2)|A\rangle + (\beta^3 + \alpha^2\beta)|B\rangle$$

$$\Rightarrow |\psi_3\rangle = 2(\alpha^3 + \alpha\beta^2)|A\rangle + 2(\beta^3 + \alpha^2\beta)|B\rangle$$
$$- \alpha|A\rangle - \beta|B\rangle$$

$$= (2\alpha^3 + 2\alpha\beta^2 - \alpha)|A\rangle + (2\beta^3 + 2\alpha^2\beta - \beta)|B\rangle$$

$$= (-2\sin^3\theta + 2\sin\theta\cos^2\theta + \sin\theta)|A\rangle$$
$$+ (2\cos^3\theta - 2\sin^2\theta\cos\theta - \cos\theta)|B\rangle$$

$$= (-2\sin^3\theta + 2\sin\theta - 2\sin^3\theta + \sin\theta)|A\rangle$$
$$+ (2\cos^3\theta - 2\cos\theta + 2\cos^3\theta - \cos\theta)|B\rangle$$

$$= (3\sin\theta - 4\sin^3\theta)|A\rangle$$

$$+ (4\cos^3\theta - 3\cos\theta)|B\rangle$$

$$= \sin 3\theta \; |A\rangle + \cos 3\theta |B\rangle$$