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EL9343 Homework 8

(Due Nov 15th, 2021)

No late submission accepted

All problem/exercise numbers are for the third edition of CLRS text book

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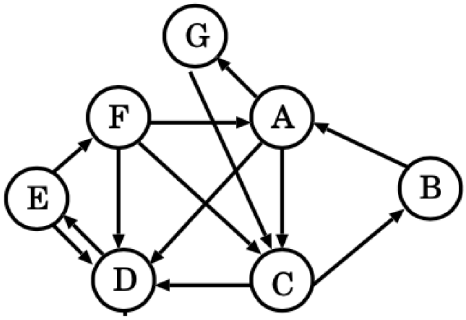
1. What is the running time of DFS if the graph is given as an adjacency list and adjacency matrix? Justify your running time.
2. Run DFS on the graph below, assume that DFS considers vertices in alphabetical order and the adjacency lists are also alphabetical order. Show the discover times and finishing times of each vertex in the graph.



3. Write a method that takes any two nodes u and v in a tree T, and quickly determines if the node u in the tree is a $descendant$ or $ancestor$ of node v
4. Draw the parenthesis structure of the dfs of Figure 1 (start from u, assume that DFS considers vertices in alphabetical order) and see the example parenthesis structure as Figure 2.

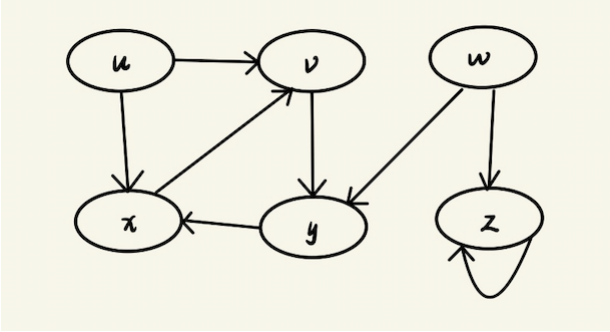


Figure 1

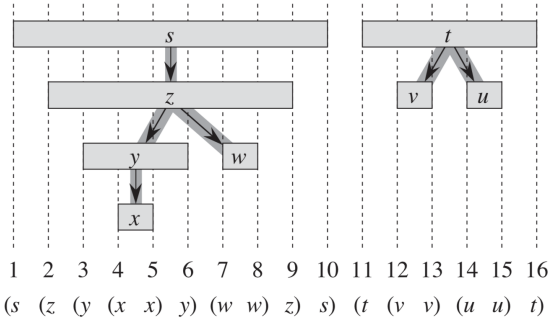


Figure 2

Q1.

If an adjacency list is used, DFS has complexity $O(V+E)$

If an adjacency matrix is used, DFS has complexity $O(|V|^2)$

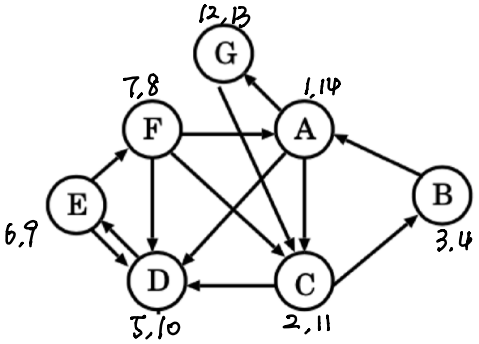
prove:

In DFS, the algorithm will visit all the adjacent vertex and do DFS. For adjacency list, it's $O(V+E)$ since find adjacent vertex is quick. However, in adjacency matrix, every element in the matrix will be visited. The complexity of this step is $O(V^2)$.

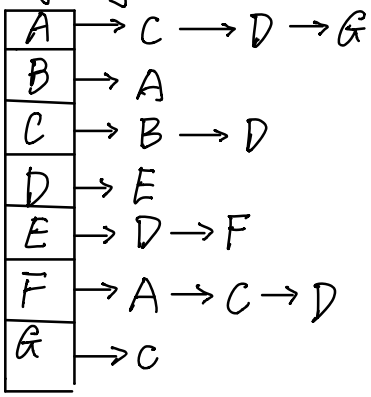
Thus: Running time of adjacency list = $O(V+E)$

Running time of adjacency matrix = $O(V^2)$

Q2.



The adjacency list:



Q3.

IDEA 1:

use DFS at the given tree

Case1: if $d[u]<d[v]$ and $f[v]<f[u]$ ⇒ u is the ancestor of v

Case2: if $d[v]<d[u]$ and $f[u]<f[v]$ ⇒ u is the descendant of v

IDEA 2:

We can use white-path theorem to solve this problem.

Start a new DFS search at the given tree.

Case 1: After u was discovered, if vertex v can be reached from u vertex along a path consisting all of white vertices : u is the ancestor of v

Case2: After v was discovered, if vertex u can be reached from v vertex along a path consisting all of white vertices : u is the descendant of v.

Case3: For other case, u is neither ancestor nor descendant of v

Q4.