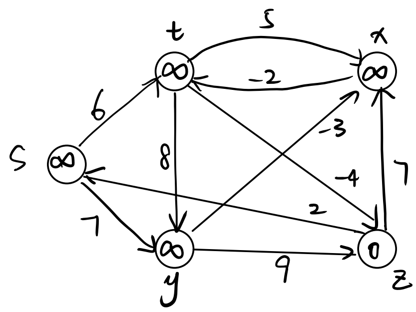
NetID: byz159

24.1-1

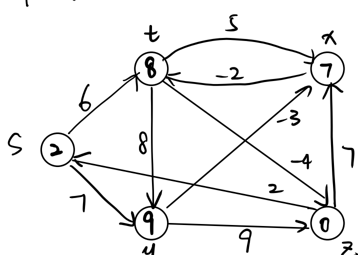
Run the Bellman-Ford algorithm on the directed graph of Figure 24.4, using vertex z as the source. In each pass, relax edges in the same order as in the figure, and show the d and π values after each pass. Now, change the weight of edge (z, x) to 4 and run the algorithm again, using s as the source.

①



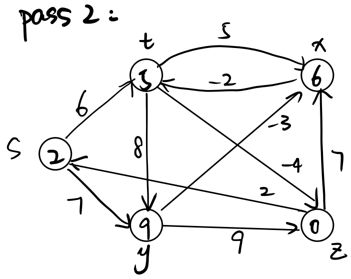
relax order: $(t, x), (t, y), (t, z), (x, t), (y, x), (y, z), (z, x), (z, s), (s, t), (s, y)$.

pass 1 :



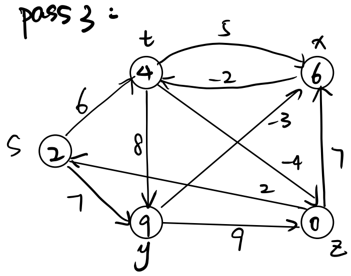
for (z, x)
$x.d = z.d + w(z,x) = 0+7 = 7$
$x.\pi = z$
for (z,s)
$s.d = z.d + w(z,s) = 0+2 = 2$
$s.\pi = z$
for (s,t)
$t.d = s.d + w(s,t) = 2+6 = 8$
$t.\pi = s$
for (s,y)
$y.d = s.d + w(s,y) = 2+7 = 9$
$y.\pi = s$

pass 2 :



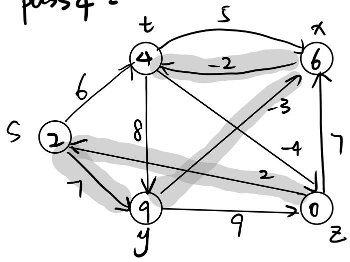
for (x,t)
$t.d = x.d + w(x,t) = 7 + (-2) = 5$
$t.\pi = x$
for (y,x)
$x.d = y.d + w(y,x) = 9 + (-3) = 6$
$x.\pi = y$

pass 3 :



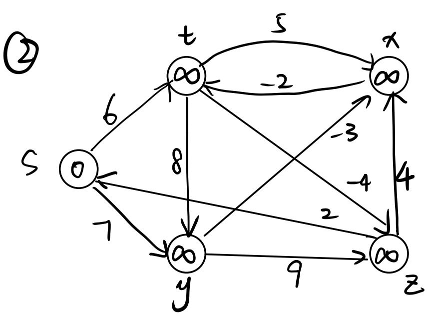
for (x,t)
$t.d = x.d + w(x,t) = 6 + (-2) = 4$
$t.\pi = x$

pass 4 :



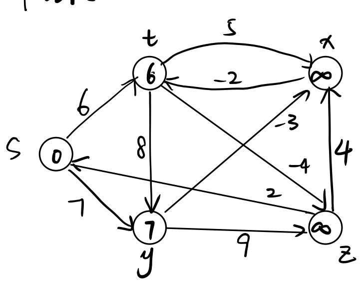
no update

②



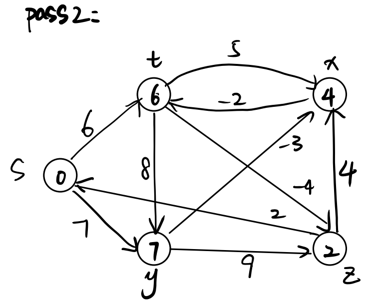
relax order: $(t, x), (t, y), (t, z), (x, t), (y, x), (y, z), (z, x), (z, s), (s, t), (s, y)$.

pass 1 :



for (s,t)
$t.d = s.d + w(s,t) = 0+6 = 6$
$t.\pi = s$
for (s,y)
$y.d = s.d + w(s,y) = 0+7 = 7$
$y.\pi = s$

pass2:



for (t,x)

$x.d = t.d + w(t,x) = 6+5 = 11$

$x.\pi = t$

for (t,z)

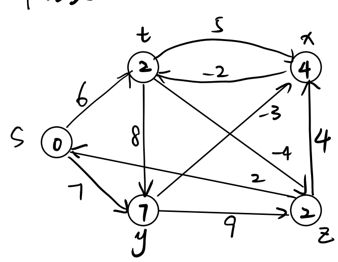
$z.d = t.d + w(t,z) = 6+(-4) = 2$

$z.\pi = t$

for (y,x)

$x.d = y.d + w(y,x) = 7+(-3) = 4$

pass3:

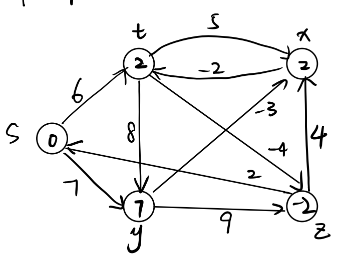


for (x,t)

$t.d = x.d + w(x,t) = 4+(-2) = 2$

$t.\pi = x$

pass4:



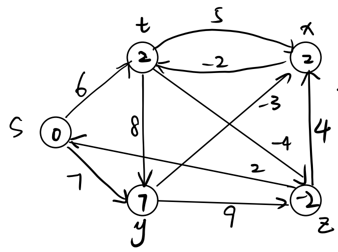
for (t,z)

$z.d = t.d + w(t,z) = 2+(-4) = -2$

for (z,x)

$x.d = z.d + w(z,x) = -2+4 = 2$

Check :



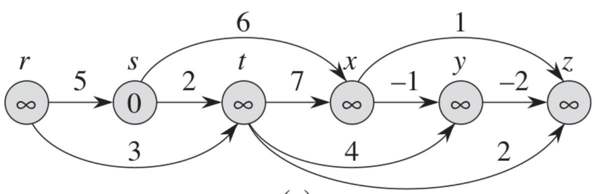
for (x,t)

$t.d = 2 > x.d + w(x,t) = 2+(-2) = 0$

$\therefore$ Return false

24.2-1

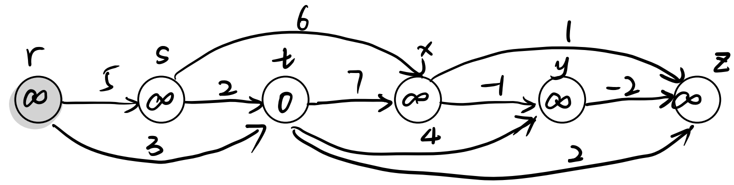
Run DAG-SHORTEST-PATHS on the directed graph of Figure 24.5, using vertex t as the source.

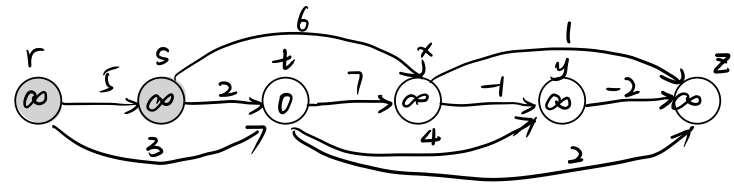


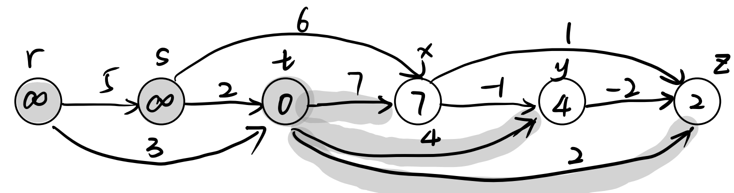
(a)

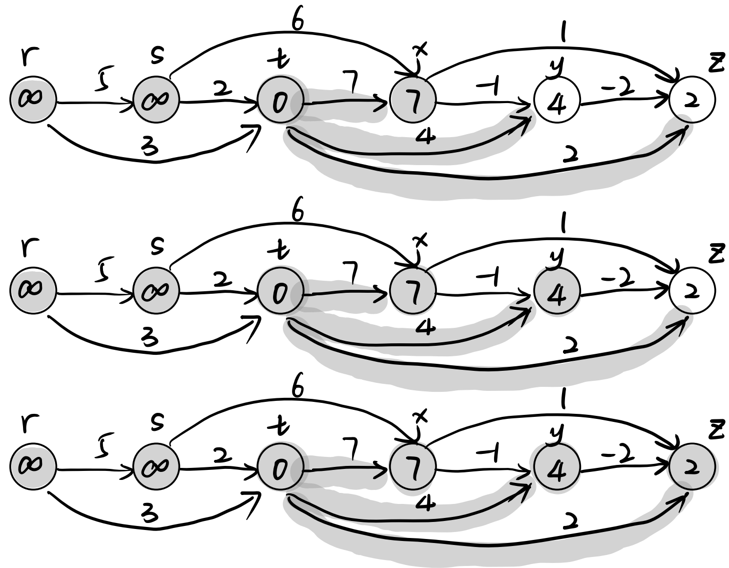
DAG-SHORTEST-PATHS(G, w, s)

```
topologically sort the vertices of G
INITIALIZE-SINGLE-SOURCE(G, s)
for each vertex u, taken in topologically sorted order
    for each vertex v ∈ G.Adj[u]
        RELAX(u, v, w)
```



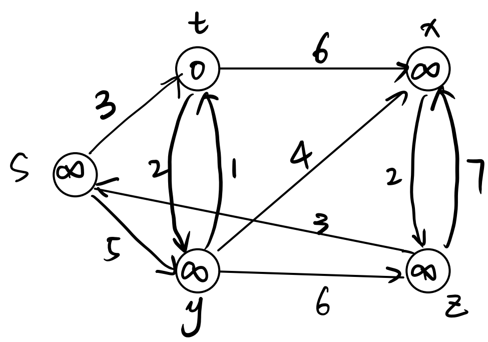






24.3-1

Run Dijkstra's algorithm on the directed graph of Figure 24.2, first using vertex s as the source and then using vertex z as the source. In the style of Figure 24.6, show the d and π values and the vertices in set S after each iteration of the **while** loop. (use only t as source)

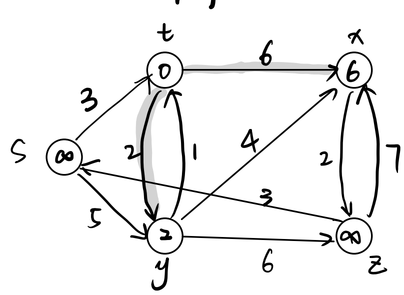


```
DIJKSTRA(G, w, s)
INITIALIZE-SINGLE-SOURCE(G, s)
S = ∅
Q = G.V
while Q ≠ ∅
    u = EXTRACT-MIN(Q)
    S = S ∪ {u}
    for each vertex v ∈ G.Adj[u]
        RELAX(u, v, w)
```

① $Q = \{s, t, x, y, z\} \neq \emptyset$

$u = t$

$S = \{t\}$



for (t,x)

$x.d = t.d + w(t,x)$
$= 0+6 = 6$

$x.\pi = t$

for (t,y)

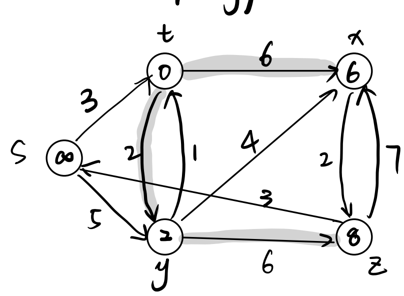
$y.d = t.d + w(t,y)$
$= 0+2 = 2$

$y.\pi = t$

② $Q = \{s, x, y, z\} \neq \emptyset$

$u = y$

$S = \{t, y\}$



for (y,z)

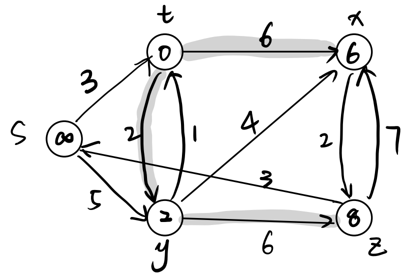
$z.d = y.d + w(y,z)$
$= 2+6 = 8$

$z.\pi = y$

③ $Q = \{ s, x, z \} \neq \phi$

$u = x$

$S = \{t, y, x\}$

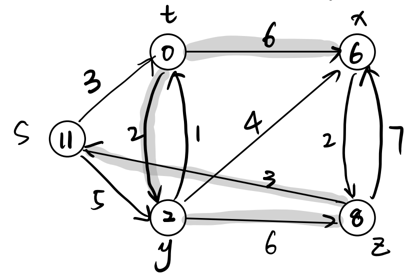


no update

④ $Q = \{ s, z \} \neq \phi$

$u = z$

$S = \{t, y, x, z\}$



for (z,s)

$s.d = z.d + w(z,s)$

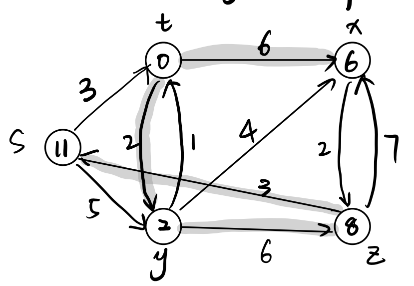
$= 8 + 3 = 11$

$s.\pi = z$

⑤ $Q = \{ s \} \neq \phi$

$u = s$

$S = \{t, y, x, z, s\}$

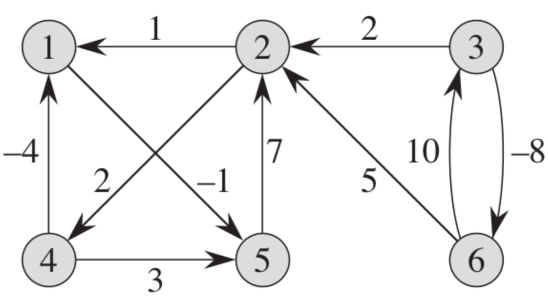


no update

⑥ $Q = \phi$

25.2-1

Run the Floyd-Warshall algorithm on the weighted, directed graph of Figure 25.2. Show the matrix $D^{(k)}$ that results for each iteration of the outer loop.



FLOYD-WARSHALL(W)

```
n = W.rows
D^(0) = W
for k = 1 to n
    let D^(k) = (d_ij^(k)) be a new n × n matrix
    for i = 1 to n
        for j = 1 to n
            d_ij^(k) = min(d_ij^(k-1), d_ik^(k-1) + d_kj^(k-1))
return D^(n)
```

$$W: \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & \infty & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & 3 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{pmatrix}$$

$$D^{(0)}: \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & \infty & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & 3 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{pmatrix}$$

$$D^{(1)}: \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{pmatrix}$$

$$D^{(2)}: \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{pmatrix}$$

$$D^{(3)}: \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{pmatrix}$$

$$D^{(4)}: \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{pmatrix}$$

$$D(5): \begin{pmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{pmatrix}$$

$$D^{(6)} \begin{pmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & 1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{pmatrix}$$