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Q1:

Proof:
if $f(n)=O(g(n)) \Rightarrow \exists c, n_0$; for $\forall n \geq n_0$, $f(n) \leq c g(n)$, $g(n) \geq \frac{f(n)}{c}$

We set $c_0 = \frac{1}{c}$ so that:

$\exists c_0, n_0$; for $\forall n \geq n_0$, $g(n) \geq c_0 f(n) = \frac{f(n)}{c}$

Then: $g(n) = \Omega(f(n))$ is proved according to the defination.

Q2:

(a) proof:

① Firstly, we are going to prove when $k=d$, $p(n)=O(n^d)$

We assume $c = a_d + c_0$ so that we should prove:

$$\sum_{i=0}^{d} a_i n^i = a_0 n^0 + a_0 n^0 + \cdots + a_{d-1} + a_d \leq c n^{d_0}$$

$$\frac{a_0}{n^d} + \frac{a_1}{n^{d-1}} + \cdots + \frac{a_{d-1}}{n} + a_d \leq a_d + c_0$$

$$\frac{a_0}{n^d} + \frac{a_1}{n^{d-1}} + \cdots + \frac{a_{d-1}}{n} \leq c_0$$

$f(n) \frac{a_0}{n^d} + \cdots + \frac{a_{d-1}}{n}$ is decreasing, this inequation will hold if n_0:

$$\frac{a_0}{n_0^d} + \frac{a_1}{n_0^{d-1}} + \cdots + \frac{a_{d-1}}{n_0} \leq c_0.$$

And n_0 exist since $f(n)$ is decreasing and $\lim_{n\to\infty} f(n) = 0$

Hence, $p(n)=O(n^d)$ holds.

② Secondly, since $k=d$ $p(n)=O(n^d)$ is proved, for $k>d$: $n^k > n^d$.

Thus, $p(n)=O(n^k)$ holds for $k \geq d$.

(a) is proved.

(b) proof: ① Firstly, we are going to prove when $k=d$, $p(n)=\Omega(n^d)$. We assume $c = a_0 - c_0$, so that:

similarly:
$$0 \leq \frac{a_0}{n_0^d} + \frac{a_1}{n_0^{d-1}} + \cdots + \frac{a_{d-1}}{n_0} + c_0 \text{, should be proved}$$

since $\lim_{n\to\infty} f(n) = \frac{a_0}{n^d} + \frac{a_1}{n_0^{d-1}} + \cdots + \frac{a_{d-1}}{n_0} = 0$, $f(n) > 0$, $c_0 > 0$.

Hence $p(n) = \Omega(n^d)$

② Secondly, since $k=d$, $p(n)=\Omega(n^d)$ is proved, for $k<d$: $n^k < n^d$. Thus $p(n)=O(n^k)$ holds for $k \leq d$

(b) is proved

(c) proof: Since (a) and (b) are proved, (c) is proved

Q2

(d) According to (a)②, (d) is proved.
(e) According to (b)②, (e) is proved

Q3

	A	B	Big O	o	Ω	ω	Θ
line 1:	$\lg^k n$	n^c	Yes	Yes	No	No	No
line 2:	n^k	c^n	Yes	Yes	No	No	No
line 3:	$\sqrt{n}$	$n^{\sin n}$	No	No	No	No	No
line 4:	2^n	$2^{n/2}$	No	No	Yes	Yes	No
line 5:	$n^{\lg c}$	$c^{\lg c}$	Yes	No	Yes	No	Yes
line 6:	$\lg(n!)$	$\lg(n^n)$	Yes	No	Yes	No	Yes

Q4:

(a) A_2; A_3; A_4; A_5
(b) A_3; A_4
(c) A_1; A_3; A_4; A_5
(d) A_4; A_5
(e) A_3
(f) A_3; A_4
(g) A_3; A_4

Q5:

Print: (8, 10)
(6, 10)
(2, 4)
(4, 10)