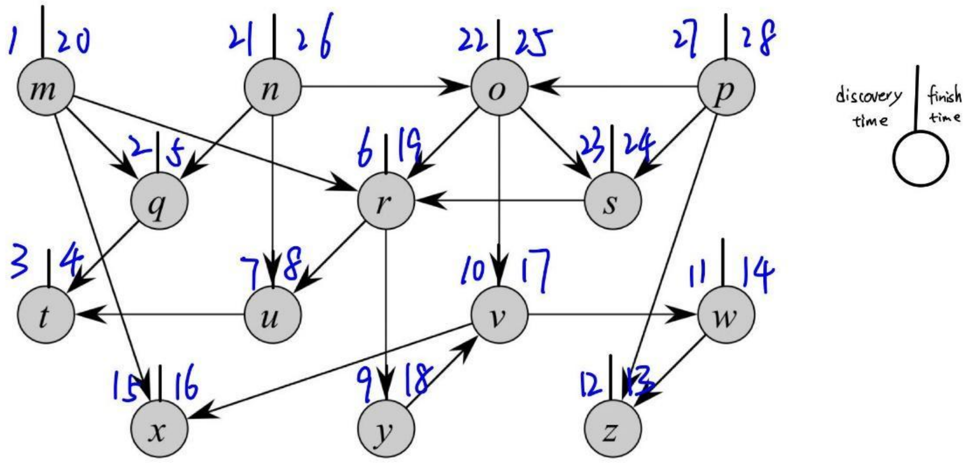
Fall 2021 Homework 9 Solution

1.

(a).

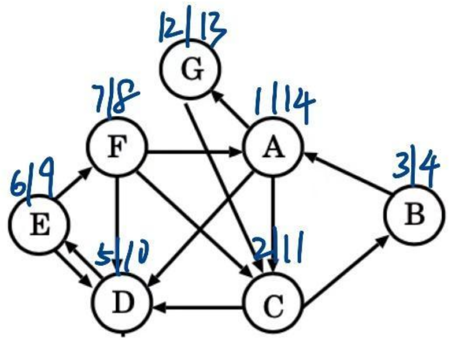


(b)

p n o s m r y v x w z u q t

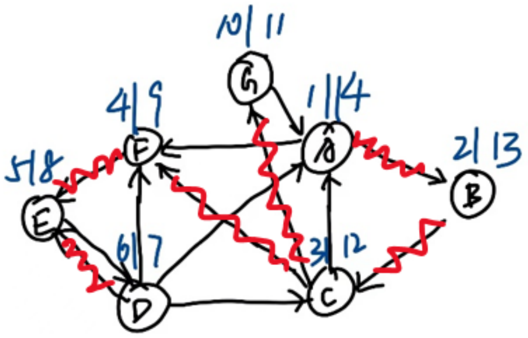
2.

Step 1: show DFS results on *G*:



order of decreasing f(u): A, G, C, D, E, F, B

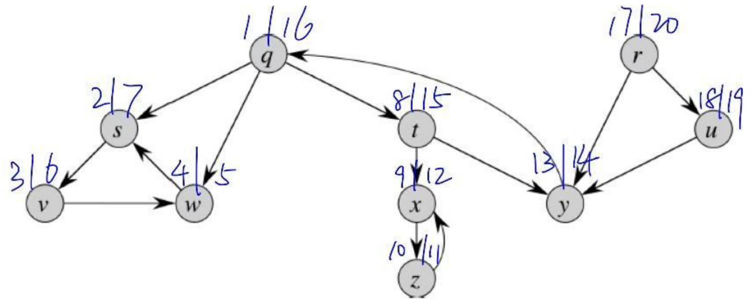
Step 2: then show DFS result on G^T, note: based on the order we got from the step 1



Only one strongly connected component: {A, B, C, D, E, F, G}, the DFS forest is marked in red.

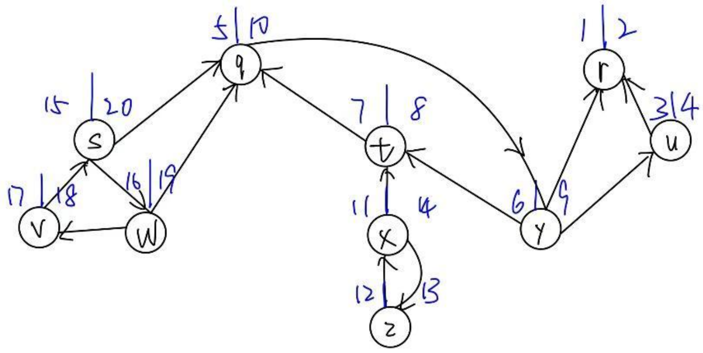
3.

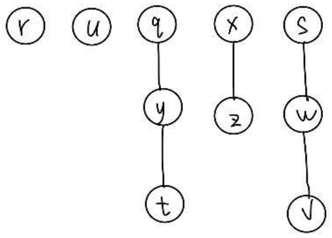
(a)



(b)

Run DFS on G^T. The number inside each node is the order computed from (a). Discover time and finish time lie on two sides of each node, the left is the discover time, the right is the finish time. As we can see, there are 5 distinct trees, which means the G^T has 5 strongly connected components.

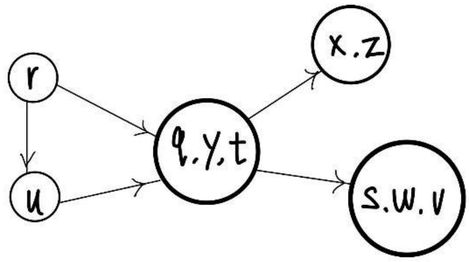




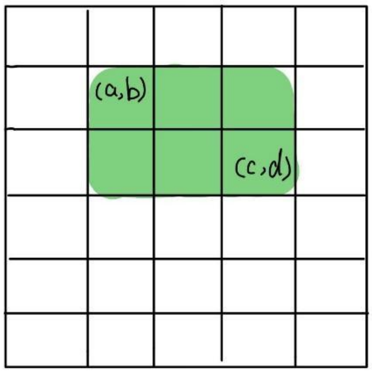
(c)

SCCs: {r}, {u}, {q, y, t}, {x, z}, {s, w, v}

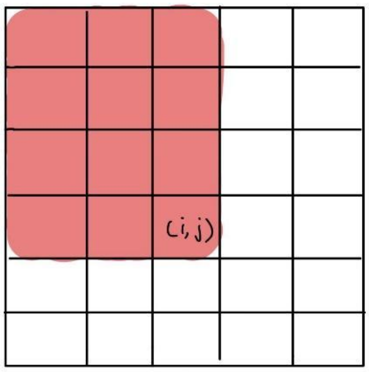
Component DAG:



4.



In order to get the sum of the “green area submatrix”, we need to know the sum of any submatrix beginning from (0, 0) to (i, j).



“red area submatrix”:
sum(i, j) = -sum(i-1, j-1) + sum(i-1, j) + sum(i, j-1) + D(i, j)
Optimal structure is obvious, therefore, we can use sum(i, j) to represent “green area submatrix”:
Sum_green area = sum(c, d) - sum(a-1, d) - sum(c, b-1) + sum(a-1, b-1)

```
SUBMATRIX-SUM(D, a, b, c, d)
    SUM(0,0) = D(0,0)
    for i=1 to a  # pre-process first column
        SUM(i,0) = SUM(i-1,0) + D(i,0)
    for j=1 to b  # pre-process first row
        SUM(0,j) = SUM(0,j-1) + D(0,j)

    for i=1 to a # pre-process rest of matrix
        for j =1 to b
            SUM(i,j) = SUM(i-1,j) + SUM(i,j-1) - SUM(i-1,j-1) + D(i,j)

    FINAL-SUM = SUM(c,d)
    # We have to take care of the cases where a=0 or/and b=0
    if a > 0
        FINAL-SUM = FINAL-SUM - SUM(a-1,d)
    if b > 0
        FINAL-SUM = FINAL-SUM -  SUM(c,b-1)
    if a > 0 and b > 0
        FINAL-SUM = FINAL-SUM + SUM(a-1,b-1)
    return FINAL-SUM
```

Analysis:
The total time complexity is $O(M \times N)$