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Q1. According to following Theorem (in page 274, Introduction to algorithm. CLRS)

**Theorem 11.6**

Given an open-address hash table with load factor  $\alpha = n/m < 1$ , the expected number of probes in an unsuccessful search is at most  $1/(1-\alpha)$ , assuming uniform hashing.

**Corollary 11.7**

Inserting an element into an open-address hash table with load factor  $\alpha$  requires at most  $1/(1-\alpha)$  probes on average, assuming uniform hashing.

**Theorem 11.8**

Given an open-address hash table with load factor  $\alpha < 1$ , the expected number of probes in a successful search is at most

$$\frac{1}{\alpha} \ln \frac{1}{1-\alpha},$$

assuming uniform hashing and assuming that each key in the table is equally likely to be searched for.

Proves of above theorem are given at books at page 274~275

According to above theorem, in most situations, the insertion and search complexity is  $O(1)$ . The expected cost is  $O(1)$  when load factor  $\ll 1$  (much smaller than 1). However, when load-factor approaches 1, we need to expand the hash table and copy all the element. In this case, the cost is no longer  $O(1)$  and would be linear in the number of elements stored instead.

Q2. Pseudocode:

```
1 Alg:RIGHT-ROTATE(tree, y):
2   x = y.left
3   y.left = x.right
4   if (x.right != tree.nil):
5       x.right.parent = y
6   x.parent = y.parent
7   if (y.parent == tree.nil):
8       tree.root = x
9   elif (y == y.parent.right):
10      y.parent.right = x
11   else:
12      y.parent.left = x
13   x.right = y
14   y.parent = x
15
```

Q3.  $\lambda(5)=5$   $\lambda(28)=1$   $\lambda(17)=1$   $\lambda(15)=6$   $\lambda(20)=2$   
 $\lambda(33)=6$   $\lambda(12)=3$   $\lambda(117)=8$   $\lambda(10)=1$

0			
1	10	→	19
2	20		→
3	12		
4			
5	5		
6	33	→	15
7			
8	17		

Q4. After Single Rotation:  
 Final Result=

