① if $f(n) = O(g(n))$:

$\exists c, n_0;$ for $\forall n \geq n_0 \Rightarrow f(n) \leq c\,g(n)$

$g(n) \geq \frac{f(n)}{c}$

We Set $c_0 = \frac{1}{c}$:

$\exists c_0, n_0;$ for $\forall n \geq n_0 = g(n) \geq c_0 f(n)$

Then: $g(n) = \Omega(f(n))$

②

EL9343 Homework 1

(Due September 20th, 2021)

No late assignments accepted

All problem/exercise numbers are for the third edition of CLRS text book

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1. Prove the *Transpose Symmetry* property, i.e., f(n) = O(g(n)) if and only if g(n) = Ω(f(n))

2. Problem 3-1 in CLRS Text book.

3. Problem 3-2 in CLRS Text book.

Small o

4. You have 5 algorithms, A1 took $O(n)$ steps, A2 took $\Theta(n\ log\ n)$ steps, and A3 took $\Omega(n)$ steps, A4 took $O(n^3)$ steps, A5 took $o(n^2)$ steps. You had been given the exact running time of each algorithm, but unfortunately you lost the record. In your messy desk you found the following formulas:

(a) $3n\log_2 n + \log_2 \log_2 n$ $A_2 - \Theta(n \log n)$ / $A_3 - \Omega(n)$ / $A_4\ O(n^3)$ / $A_5\ O(n^2)$

$3n^2+5n+Const =$ (b) $3(2^{2\log_2 n}) + 5n + 1234567$ $A_4 - O(n^3)$ / $\cancel{A_5 - o(n^2)}$ / $A_3 - \Omega(n)$

(c) $\frac{2^{\log_4 n}}{3} + n + 9$ $A_1 - O(n)$ / $A_3 - \Omega(n)$ / $A_4 - O(n^3)$ / $A_5 - o(n^2)$

(d) $(\log_2 n)^2 + 5$ $\cancel{A_3 - \Omega(n)}$ / $A_4 - O(n^3)$ / $A_5 - o(n^2)$ / $\cancel{A_1 - O(n)}$

(e) $3n!$ $A_3 - \Omega(n)$

n^3 (f) $2^{3\log_2 n}$ $A_3 - \Omega(n)$ / $A_4 - O(n^3)$

(g) $2^{2\log_2 n}$ $A_3 - \Omega(n)$ / $A_4 - O(n^3)$ / $\cancel{A_5 - o(n^2)}$

n^2 //

For each algorithm write down all the possible formulas that could be associated with it.

5. For the following algorithm: Show what is printed by the following algorithm when called with MAXIMUM(A, 1, 5) where A = [10, 8, 6, 4, 2]? Where the function PRINT simple prints its arguments in some appropriate manner.

```
MAXIMUM(A, l, r)
if (r − l == 0)
    return A[r]

lmax = MAXIMUM(A, l, ⌊(l + r)/2⌋)
rmax = MAXIMUM(A, ⌊(l + r)/2⌋ + 1, r)
PRINT(rmax, lmax)
if rmax < lmax
    return lmax
else
    return rmax
```

$lmax_2 = MAX(A,1,1)$ $A[1] = 10$

$lmax_1 = MAX(A,1,2) = 10$ $rmax_2 = MAX(A,2,2)$ $A[2] = 8$

$lmax_0 = MAX(A,1,3) = 10$ $rmax_1 = MAX(A,3,3) = 6$

$rmax_0 = MAX(A,4,5) = 4$ $lmax_1 = MAX(A,4,4)$ $A[4] = 4$

$rmax_1 = MAX(A,5,5)$ $A[5] = 2$

(8, 10)

(6, 10)

(2, 4)

(4, 10)