Fall 2021 Lecture 2

$$\log_b\left(a^{\log_b x}\right) \quad \log_b\left(x^{\log_b a}\right)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\log_b x \log_b a = \log_b a \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$\left(\left(a^{\log_a b}\right)^{\log_b x}\right) = \left(a^{\log_a x}\right)$$

$$b \qquad\qquad x = x$$

$$x + x^2 + x^3 + \cdots x^{n+1} = S \cdot x$$

$S + x^{n+1} - 1 = S \cdot x$

$$S = \frac{x^{n+1}-1}{x-1}$$

$\leq 1 + \ln n$

$\ln(n+1) \leq 1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n} = ? \approx \ln n$

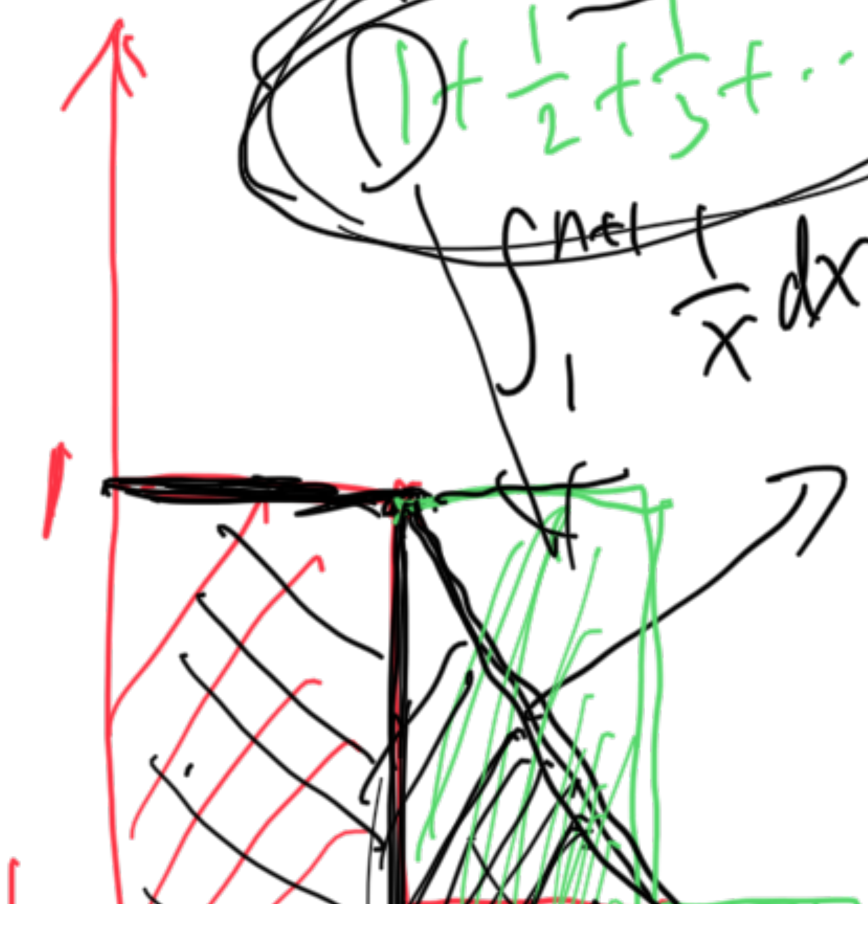
$1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}$ $\ln n$

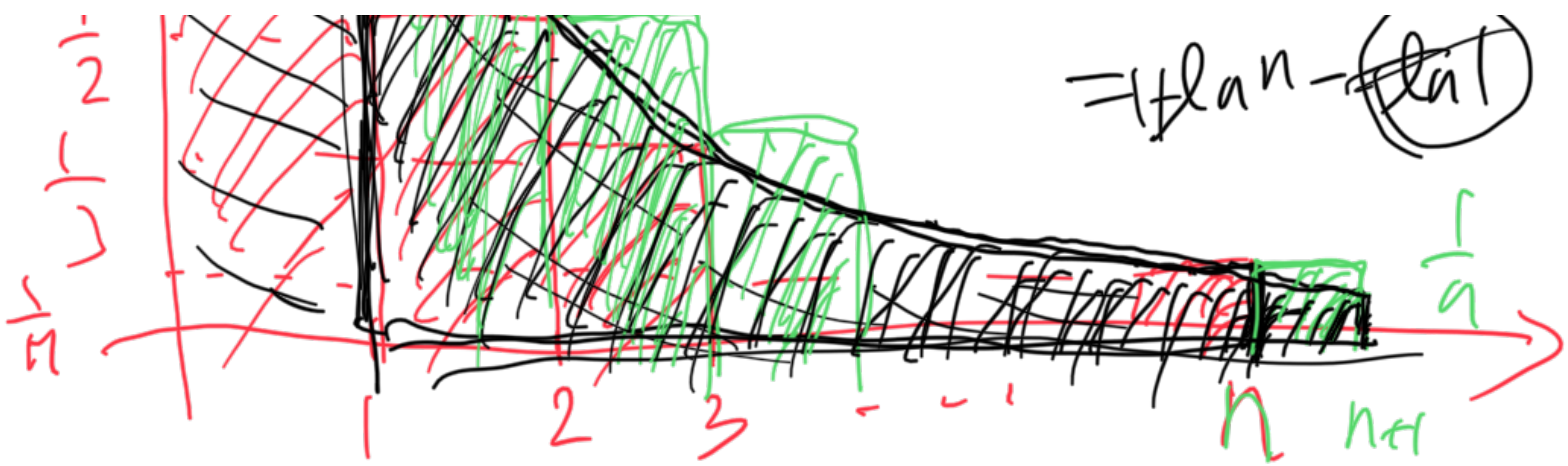
$\int_1^{n+1} \frac{1}{x}\,dx = \ln(n+1)$

$1 + \int_1^n \frac{1}{x}\,dx$

$\frac{1}{x} \approx \ln n$

$\int_1^n \frac{1}{x}\,dx$ $\ln n - \ln 1$ $=0$

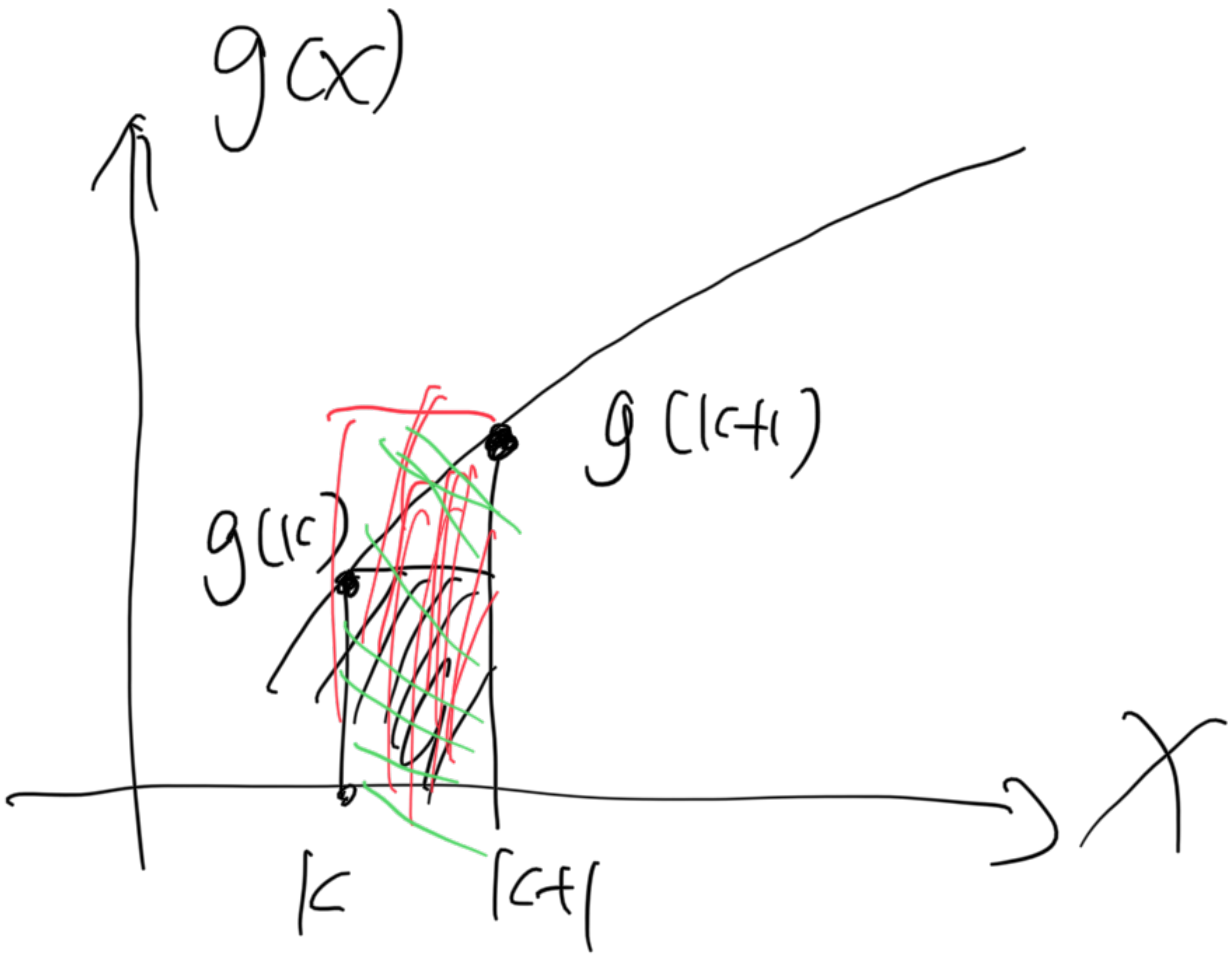




$= 1 + \ln n - \cancel{\ln 1}$

$$\int_1^{n+1} \frac{1}{x}\,dx$$

$$= \ln(n+1)$$



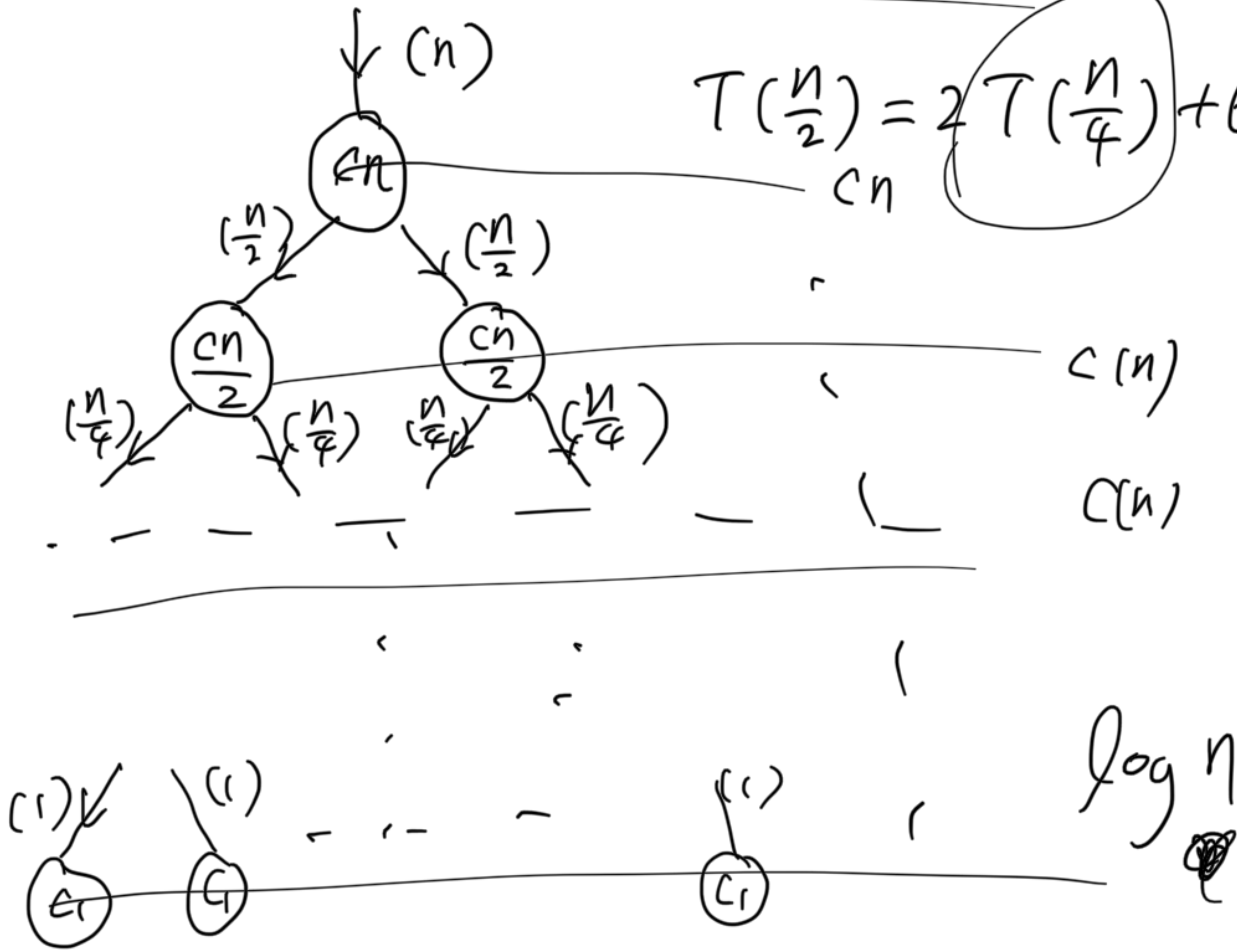
$$\frac{\int_1^{n} g(x)\,dx}{\sum_{k=1}^{n} g(k)}$$

$$\int^{k+1}$$

$$g(k) \le \int_{k}^{k+1} g(x)\,dx \le g(k+1)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + \underline{\underline{\theta(n)}} \rightarrow cn$$

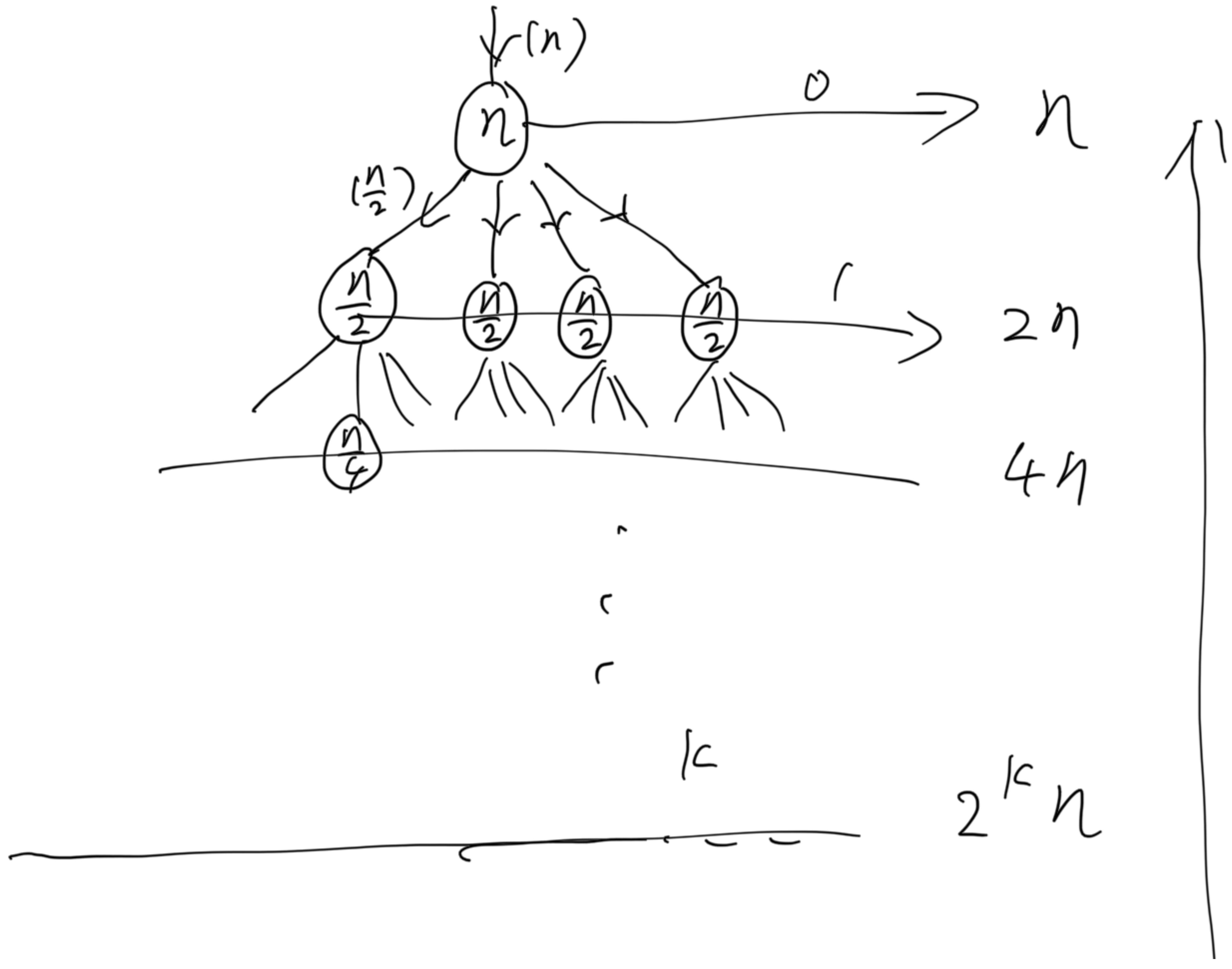
$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \theta\left(\frac{n}{2}\right)$$



$c(n \log n)$

$\Theta(n \log n)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n$$



$k = \log n$

$n + 2n + 4n + \cdots \quad 2^{(k=\log n)} \cdot n$

$= n(1 + 2 + 4 \cdots + n)$

$n \times \frac{2n-1}{2-1} = \theta(n^2)$

$T(n) = \theta(n^2) = \theta(n^2 - n)$

$\Leftrightarrow C_2 n^2 \le T(n) \le C_1 n^2$

$T(n) = 4T(\frac{n}{2}) + n \le c_1(n^2 - n)$

$T(k) \le c_1 k^2, \ \forall k < n$

$$\le 4c_1\left(\left(\frac{n}{2}\right)^2 - \frac{n}{2}\right) + n = \cancel{c_1 n^2} - c_1 n + n$$

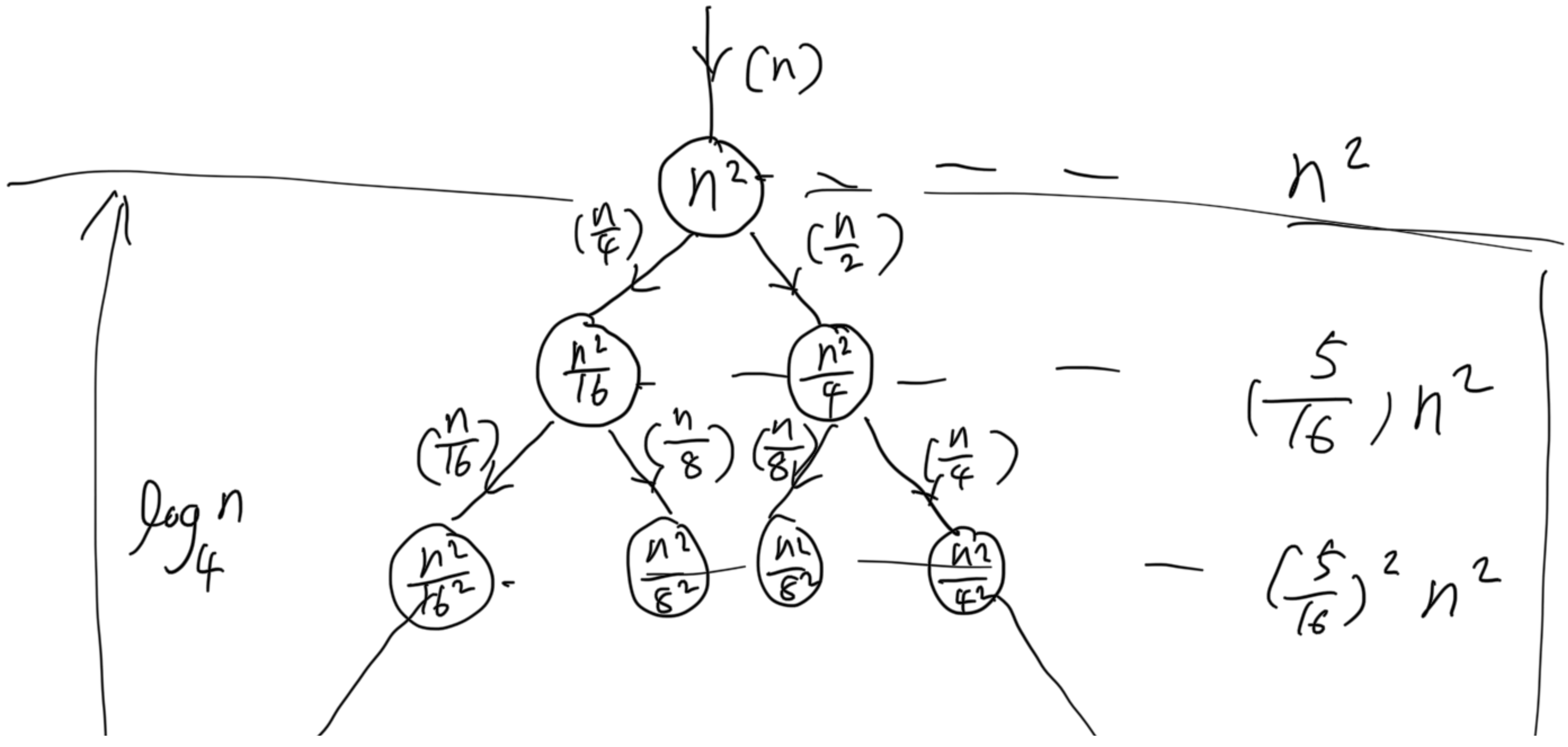
$$\le \cancel{c_1 n^2 - c_1 n}$$

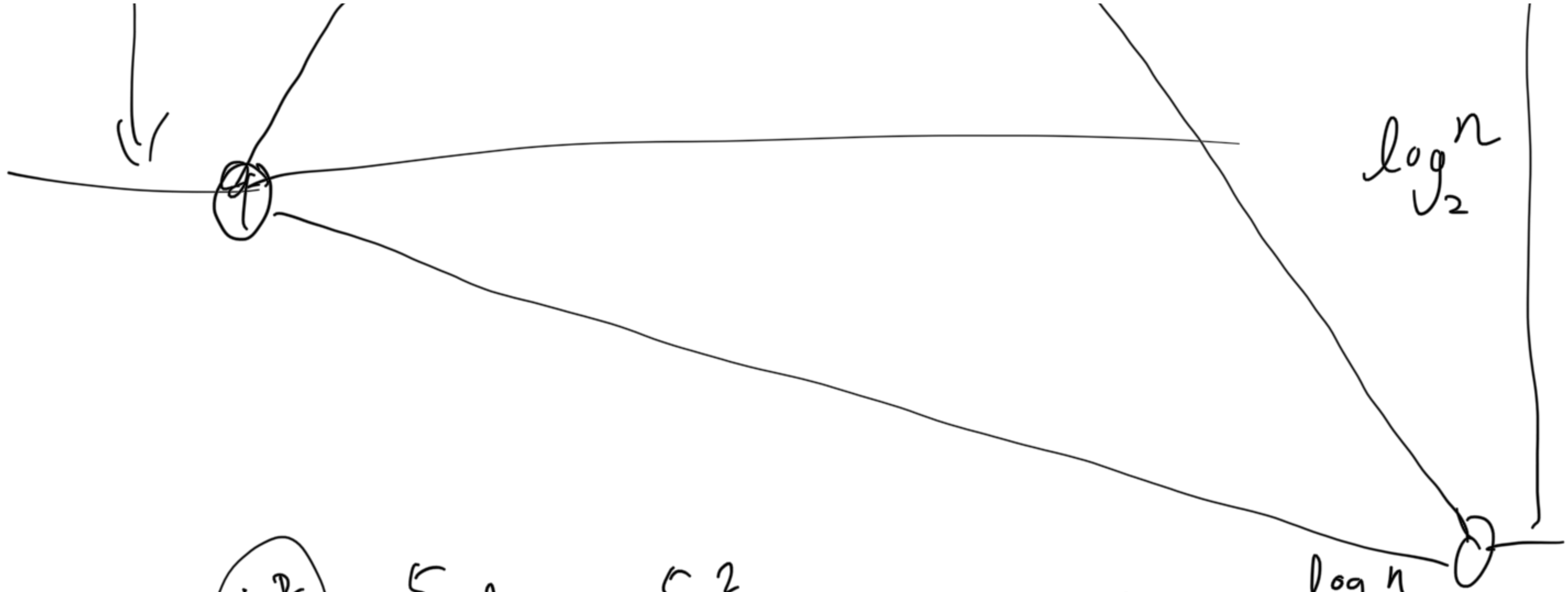
$$T(k) \le c_1(k^2 - k)$$

$$-c_1 n + n \le 0$$

$$\Rightarrow c_1 \ge 1 \quad \checkmark$$

$$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n^2$$





$$n^2 + \frac{5}{16}n^2 + \left(\frac{5}{16}\right)^2 n^2 \cdots \left(\frac{5}{16}\right)^{k} n^2$$

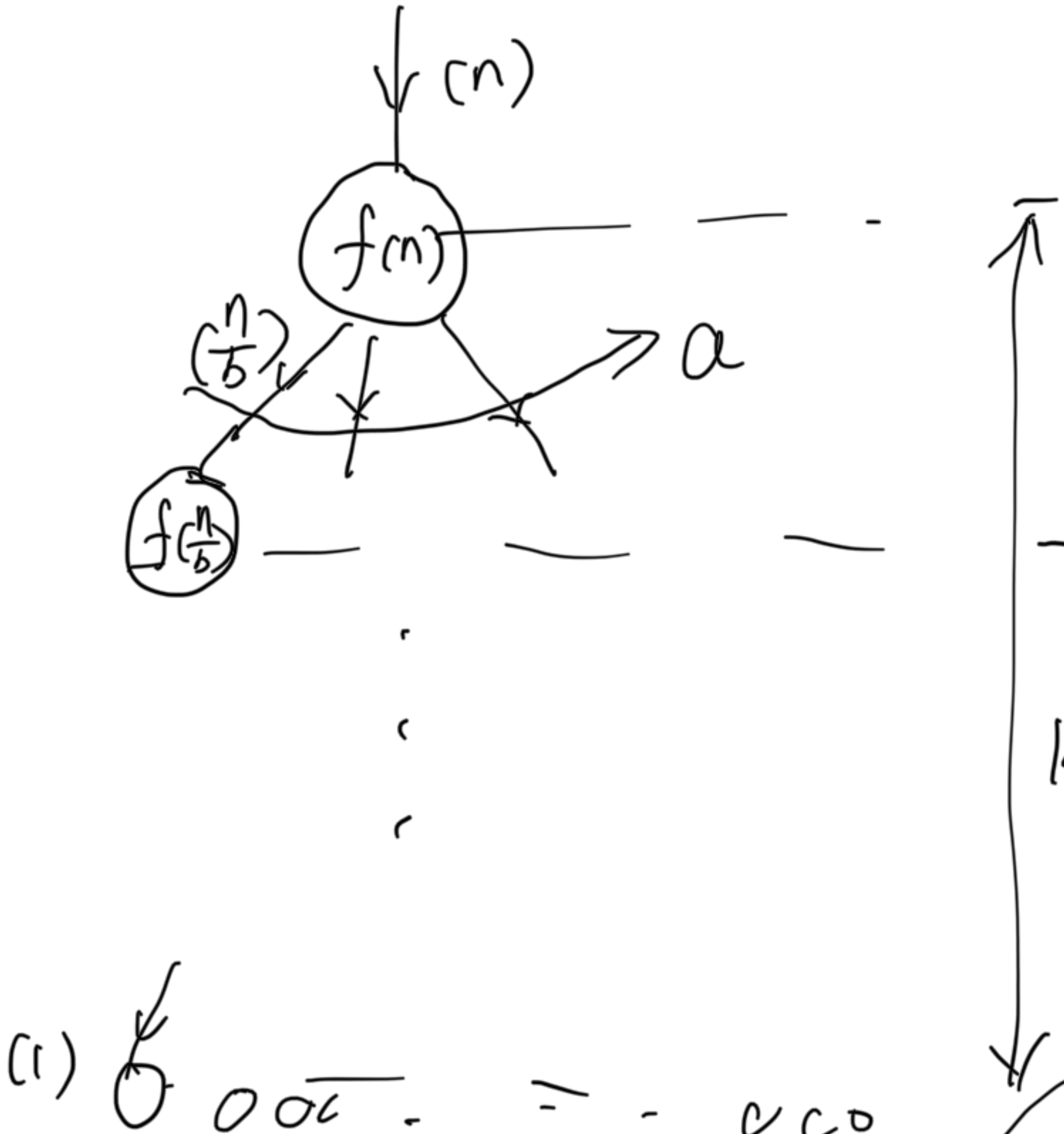
$\log_2 n$

$$n^2\left(1 + \frac{5}{16} + \cdots \left(\frac{5}{16}\right)^{\log_2 n}\right) \;?$$

$$\le n^2 \sum_{i=0}^{\infty} \left(\frac{5}{16}\right)^i = n^2 \frac{1}{1-\frac{5}{16}}$$

$$\Theta(n^2)$$

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$



$f(n)$

$af\left(\frac{n}{b}\right)$

$k = \underline{\log_b n}$

$a^{\log_b n} = n^{\log_b a}$

$$f(n) + af\left(\frac{n}{b}\right) + a^2 f\left(\frac{n}{b^2}\right) + \cdots\ a^{\log_b n} f(1)$$

$$n^{\log_b a}$$

$$\lim_{n \to \infty} \frac{n^b}{a^n} \to 0$$

$\forall \varepsilon > 0 \quad \exists N_0, \quad \text{s.t.} \quad n > N_0, \quad \frac{n^b}{a^n} < \varepsilon$

$$\lim_{\log n \to \infty} \frac{(\log n)^b}{a^{\log n}} \to 0$$

$(2^k)^{\log n}$

$\log n > N_0 \implies n > 2^{N_0}$