Final Project Team 6

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May 2021

Problem

Let f be a three times differentiable function (defined on $\mathbb{R}$ and real-valued) such that f has at least five distinct real zeros. Prove that $f + 6f' + 12f'' + 8f'''$ has at least two distinct real zeros.

Polynomial Function

A polynomial is generally represented as $P(x)$. The highest power of the variable of $P(x)$ is known as its degree. Degree of a polynomial function is very important as it tells us about the behaviour of the function $P(x)$ when x becomes very large, and also helps us to know the number of roots that we can have in a function. The domain of a polynomial function is entire real numbers $\mathbb{R}$.

Rolle's Thorem

Let f be a continuous function on $[a, b]$ and differentiable on $]a, b[$ such that $f(a) = f(b)$. Then there exists $c \in]a, b[$ such that $f'(c) = 0$.

This theorem will be admitted because its proof requires results which are not seen in this course. If f is the constant function, the result is obvious. Otherwise, since f is continuous over $[a, b]$, f is bounded over $[a, b]$ and reaches its bounds. This means that there exists $m \in [a, b]$ and $M \in [a, b]$ such that $\forall x \in [a, b]$, $f(m) \leqslant f(x) \leqslant f(M)(f(m) \neq f(M)$ because f is not a constant function). As $f(a) = f(b)$, then we have the following cases:

1. if $m = a$ or $m = b$, then $M \in]a, b[$, hence we have $c = M$;
2. if $M = a$ or $M = b$, then $m \in]a, b[$, hence we have $c = m$;
3. $m \in]a, b[$ and $M \in]a, b[$, hence we have $c = m$ or $c = M$.

So in all cases f admits a local extremum at a point c of $]a, b[$ and is differentiable on $]a, b[$, hence, according to the proposition 22, $f'(c) = 0$.

Hint

Use $g : x \to e^{\alpha x}$