Final Project Team 6

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Abstract

The hypothesis of this theorem is that we have a function F that is continuous in a closed interval $[a, b]$, differentiable in the open interval (a, b) and whose values in its extremes $F(a)$ and $F(b)$ match. The thesis of the theorem is that, in this case, the derived function vanishes at some point in the interval (a, b) Notice that since the interval is closed, it makes sense to talk about both $F(a)$ and $F(b)$. We will see that, intuitively, this statement is very simple. Rolle's theorem guarantees us that, under these conditions, there must be at least a certain value x of the interval (a, b) for which $F'(x) = 0$. But it only assures us that there has to be that value, not tells us nothing about its how to find it.

This will be necessary for the problem, because it has a great complexity and will make us develop different mathematical skills to understand how the application of integrals and derivatives works.

1 Problem

Let f be a three times differentiable function (defined on $\mathbb{R}$ and real-valued) such that f has at least five distinct real zeros. Prove that $f + 6f' + 12f'' + 8f'''$ has at least two distinct real zeros.

2 Polynomial Function

A polynomial is generally represented as $P(x)$. The highest power of the variable of $P(x)$ is known as its degree. Degree of a polynomial function is very important as it tells us about the behaviour of the function $P(x)$ when x becomes very large, and also helps us to know the number of roots that we can have in a function. The domain of a polynomial function is entire real numbers $\mathbb{R}$.

3 Rolle's Theorem

Suppose $f(x)$ is a function that satisfies all of the following. $f(x)$ is continuous on the closed interval $[a, b]$. $f(x)$ is differentiable on the open interval (a, b).

$$f(a) = f(b)$$

Then there is a number c such that $a < c < b$ and $f'(c) = 0$. Or, in other words $f(x)$ has a critical point in (a, b).

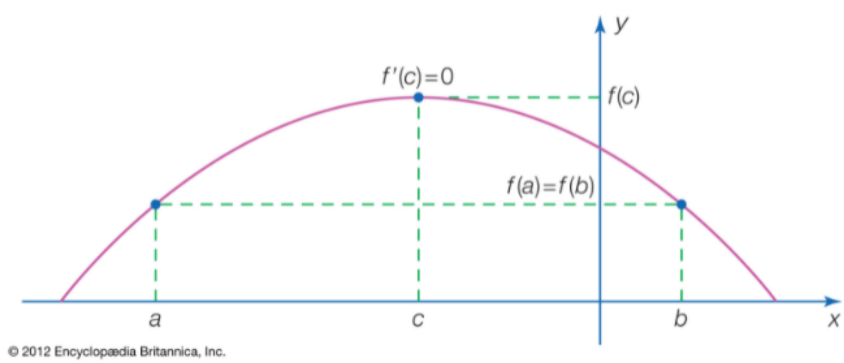

Figure 1: Graphic of Rolle's Theorem