Universidad Autonoma del Estado de Hidalgo

Calculo Integral en una Variable

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1. Tarea 1

1. Definir las funciones $f(x) = sen(x)$ y $f(x) = \tan(x)$ de manera que sean biyectivas. De este modo existirá la función inversa de cada una. ¿Que se necesita para que la función inversa sea diferenciable?. Encuentre las derivadas de las funciones inversas

   sea $f^{-1}(x) = arc\operatorname{sen} x$ y $f(x) = \operatorname{sen} x$

   $f(f^{-1}) = \operatorname{sen}(arc\operatorname{sen} x) = x$

   $\operatorname{sen}(f^{-1}(x)) = x$

   $\cos(f^{-1}(x))\dfrac{df^{-1}}{dx} = 1$

   $\pm\sqrt{1 + sen^2(f^{-1}(x))}\dfrac{df^{-1}}{dx} = 1$

   $\pm\sqrt{1 + sen^2(f^{-1}(x))}\dfrac{df^{-1}}{dx} = \dfrac{1}{\pm\sqrt{1 + sen^2(f^{-1}(x))}}$

   en $[-\pi/2, \pi/2]$ el coseno es positivo, entonces

   $\operatorname{sen}^2(f^{-1}) = \operatorname{sen}^2(arc\operatorname{sen}(x))$

   $\operatorname{sen}^2(f^{-1}) = [\operatorname{sen}(arc\operatorname{sen}(x))][\operatorname{sen}(arc\operatorname{sen}(x))] = x$

   $\dfrac{d(arc\operatorname{sen}(x))}{dx} = \dfrac{1}{\sqrt{1 - x^2}}$

   sea $f^{-1}(x) = arc\tan x$ y $f(x) = \tan x$

   $\tan(f^{-1}(x)) = x$

   $\tan(f^{-1}(x))\dfrac{df^{-1}}{dx} = 1$

   $\dfrac{d\tan(g^{-1}(x))}{dx} = 1$

   $sec^2(g^{-1}(x))\dfrac{df^{-1}}{dx} = 1$

   $\dfrac{d(g^{-1}(x))}{dx} = \dfrac{1}{\sec^2(g^{-1}))}$

   $\dfrac{d(g^{-1}(x))}{dx} = \dfrac{1}{1 + \tan^2(g^{-1}))}$

   tenemos que:

   $\tan^2(g^{-1}(x)) = [tan(arc\tan(x))][\tan(arc\tan(x))]$

   $\tan^2(g^{-1}(x)) = x^2$

   $\dfrac{d(g^{-1}(x))}{dx} = \dfrac{1}{1 + x^2}$

   $\dfrac{d[arc\tan(x)]}{dx} = \dfrac{1}{1 + x^2}$

   en $[-\pi/2, \pi/2]$ el coseno es positivo, entonces

   $\operatorname{sen}^2(f^{-1}) = \operatorname{sen}^2(arc\operatorname{sen}(x))$

   $\operatorname{sen}^2(f^{-1}) = [\operatorname{sen}(arc\operatorname{sen}(x))][\operatorname{sen}(arc\operatorname{sen}(x))] = x$

   $\dfrac{d(arc\operatorname{sen}(x))}{dx} = \dfrac{1}{\sqrt{1 - x^2}}$

2. Trazar, en un mismo sistema de coordenadas, las gráficas de las siguientes funciones y mencionar similitudes y diferencias.

   a) $f(x) = \text{sen}(x)$ $\quad f(x) = \text{sen}^{-1}(x)$ $\quad f(x) = \csc(x)$

   b) $f(x) = \tan(x)$ $\quad f(x) = \arctan(x)$ $\quad f(x) = \cot(x)$

   En los ejercicios 3 - 11, hallar $L(f, P)$ y $U(f, P)$

3. $f(x) = 2x, x \in [0, 1], P = \{0, 1/4, 1/2, 1\}$

   $$L_f(P) = 0\left(\frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{4}\right) + 1\left(\frac{1}{2}\right) = \frac{5}{8} \qquad U_f(P) = \frac{1}{2}\left(\frac{1}{4}\right) + 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{2}\right) = \frac{11}{8}$$

4. $f(x) = 1 - x, x \in [0, 2], P = \{0, 1/3, 3/4, 1, 2\}$

   $$L_f(P)\frac{2}{3}\left(\frac{1}{3}\right)+\frac{1}{4}\left(\frac{5}{12}\right)+0\left(\frac{1}{4}\right)-1(1) = -\frac{97}{144} \qquad U_f(P) = 1\left(\frac{1}{3}\right)+\frac{2}{3}\left(\frac{5}{12}\right)+\frac{1}{4}\left(\frac{1}{4}\right) = \frac{97}{144}$$

5. $f(x) = x^2, x \in [-1, 0], P = \{-1, -1/2, -1/4, 0\}$

   $$L_f(P) = \frac{1}{4}\left(1 - \frac{1}{2}\right) + \frac{1}{16}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{9}{64}$$

   $$U_f(P) = 1\left(-\frac{1}{2} + 1\right) + \frac{1}{4}\left(-\frac{1}{4} + \frac{1}{2}\right) + \frac{1}{16}\left(\frac{1}{4}\right) = \frac{37}{64}$$

6. $f(x) = 1 - x^2, x \in [0, 1], P = \{0, 1/4, 1/2, 1\}$

   $$L_f(P) = \frac{15}{16}\left(\frac{1}{4} - 0\right) + \frac{3}{4}\left(-\frac{1}{2} - \frac{1}{4}\right) = \frac{27}{64}$$

   $$U_f(P) = 1\left(\frac{1}{4} - 0\right) + \frac{15}{16}\left(\frac{1}{2} - \frac{1}{4}\right) + \frac{3}{4}\left(1 - \frac{1}{2}\right) = \frac{55}{64}$$

7. $f(x) = 1 + x^3, x \in [0, 1], P = \{0, 1/2, 1\}$

   $$L_f(P) = \frac{1}{2} + \frac{9}{8}\left(\frac{1}{2}\right) = \frac{17}{16} \qquad U_f(P) = \frac{9}{8}\frac{1}{2} + 2\left(\frac{1}{2}\right) = \frac{25}{16}$$

8. $f(x) = \sqrt{x}, x \in [0, 1], P = \{0, 1/25, 4/25, 9/25, 16/25, 1\}$

   $$L_f(P) = \frac{1}{5}\left(\frac{3}{25}\right) + \frac{2}{5}\left(\frac{5}{25}\right) + \frac{3}{5}\left(\frac{7}{25}\right) + \frac{4}{5}\left(\frac{9}{25}\right) = \frac{14}{25}$$

   $$U_f(P) = \frac{1}{5}\left(\frac{1}{25}\right) + \frac{2}{5}\left(\frac{3}{25}\right) + \frac{3}{5}\left(\frac{5}{25}\right) + \frac{4}{5}\left(\frac{7}{25}\right) + 1\left(\frac{9}{25}\right) = \frac{19}{25}$$

9. $f(x) = \text{sen}(x), x \in [0, 7], P = \{0, 1/6\pi, 1/2\pi, \pi\}$

   $$L_f(P) = 0\left(\frac{1}{6}\pi\right) + \frac{1}{2}\left(\frac{1}{3}\pi\right) + 0\left(\frac{1}{2}\pi\right) = \frac{\pi}{6} \qquad U_f(P) = \frac{1}{2}\left(\frac{1}{6}\pi\right) + 1\left(\frac{1}{3}\pi\right) + \left(\frac{1}{2}\pi\right) = \frac{11}{12}\pi$$

10. $f(x) = \cos(x), x \in [0, 7], P = \{0, 1/3\pi, 1/2\pi, \pi\}$

    $$L_f(P) = \frac{1}{2}\left(\frac{\pi}{2} - \frac{\pi}{3}\right) - 1\left(1 - \frac{\pi}{2}\right) = -\frac{\pi}{3}$$

    $$U_f(P) = 1\left(\frac{\pi}{3}\right) + \frac{1}{2}\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \frac{5\pi}{12}$$

11. Usando la definición de integral con una partición regular, calcule las integrales desde a hasta b de las funciones $f(x) = k$, $f(x) = x^n$, $n = 1, 2, 3$.

$f(x) = k$
Dado una partición P y k una constante
$P\{t_{0,1}, t_2 \ldots t_n\}$ tenemos:
$L(f,p) = \sum_{i=1}^{n} m_i(t_i - t_{i-1})$ pero como es una constante
$a = t_0 < t_1 < t_2 < t_3 < t_4 < t_5 < \ldots < t_n < b$
$m_i = k \forall x \in [a,b]$ Dado esto, tenemos:
$k\sum_{i=1}^{n}(t_i - t_{i-1} = k(b-a)$
$P\{t_{0,1}, t_2 \ldots t_n\}$ tenemos:
$U(f,p)\sum_{i=1}^{n} M_i(t_i - t_{i-1})$ pero como es una constante
$a = t_0 < t_1 < t_2 < t_3 < t_4 < t_5 < \ldots < t_n < b$
$M_i = k \forall x \in [a,b]$ Dado esto, tenemos:
$k\sum_{i=1}^{n}(t_i - t_{i-1} = k(b-a)$
por lo tanto $\int_a^b k = k(b-a)$

para $f(x) = x$
tenemos $L(f,p)$ y $U(f,p)$ como:
$L(f,p) = t_0^2(t_1 - t_0) + t_1^2(t_2 - t_1) + \ldots + t_{n-1}^2(t_n - t_{n-1})$
$U(f,p) = t_1^2(t_1 - t_0) + t_2^2(t_2 - t_1) + \ldots + t_n^2(t_n - t_{n-1})$
$x_{j-1} \leq \frac{(x_j - x_{j-1}}{2} \leq x_j$
$x_{j-1}(x_j - x_{j-1}) \leq \frac{x_j^2 - x_{j-1}^2}{2} \leq x_j(x_j - x_{j-1})$
sumando desde $i = 1$ hasta $i = n$
$L(f,p) \leq \frac{(x_1^2 - x_0^2) + (x_2^2 - x_1^2) + (x_3^2 - x_2^2) + (x_4^2 - x_3^2) + \ldots + (x_n^2 - x_{n-1}^2}{2} \leq U(f,p)$
simplificado la suma tenemos
$L(f,p) \leq \frac{x_n^2 - x_0^2}{2} \leq U(f,p)$
$L(f,p) \leq \frac{b^2 - a^2}{2} \leq U(f,p)$
por lo tanto:
$\int_a^b x dx = \frac{b^2 - a^2}{2}$

si $f(x) = x^2$
tenemos $L(f,p)$ y $U(f,p)$ como:
$L(f,p) = t_0^2(t_1 - t_0) + t_1^2(t_2 - t_1) + \ldots + t_{n-1}^2(t_n - t_{n-1})$
$U(f,p) = t_1^2(t_1 - t_0) + t_2^2(t_2 - t_1) + \ldots + t_n^2(t_n - t_{n-1})$
dado a $L(f,p)$ y $U(f,p)$
$t_{i-1} \leq \frac{t_{i-1}^2 + t_{i-1}t_i + t_i^2}{3} \leq t_i$
multiplicando por $t_1 - t_{i-1}$
$t_{i-1}(t_1 - t_{i-1}) \leq \left(\frac{t_{i-1}^2 + t_{i-1}t_i + t_i^2}{3}\right)(t_1 - t_{i-1}) \leq t_i(t_1 - t_{i-1})$
$t_{i-1}(t_1 - t_{i-1}) \leq t_{i-1}^2 t_1 + t_{i-1}t_i^2 + t_i^3 - t_{i-1}^3 - t_{i-1}^2 t_i - t_{i-1}t_i^2 \leq t_i(t_i - t_{i-1})$
agrupando en una suma
$\sum_{i=1}^{n} t_{i-1}(t_i - t_{i-1}) \leq \sum_{i=1}^{n} \frac{t_i^3 - t_{i-1}^3}{3} \leq \sum_{i=1}^{n} t_i(t_i - t_{i-1})$

$L(f,p) \leq \dfrac{b^3-a^3}{3} \leq U(f,p)$
finalmente Obtenemos
$\int_a^b x dx = \dfrac{b^3-a^3}{3}$

12. Dada $P = t_0, t_1, \ldots, t_n$ una partición arbitraria de $[a,b]$ hallar $L(f,P)$ y $U(f,P)$ para $f(x) = x+3$, usar estas respuestas para evaluar $\displaystyle\int_a^b f(x)dx$.

$L_f(P) = (x_0+3)(x_1-x_0) + (x_1+3)(x_2-x_1) + (x_2-3)(x_3-x_2) + \ldots + (x_{n-1}-3)(x_n-x_{n-1})$
$U_f(P) = (x_1+3)(x_1-x_0) + (x_2+3)(x_2-x_1) + (x_3-3)(x_3-x_2) + \ldots + (x_n-3)(x_n-x_{n-1})$
$f(x) = x+3$
$x_{j-1}+3 \leq \displaystyle\int_a^b \frac{1}{2}(x_{j-1}-x_j)+3 \leq x_j+3$
Multiplcamos esta desigualdad por $x_j - x_{j-1}$
$(x_{j-1}+3)(x_{j-1}-x_j) \leq \displaystyle\int_a^b \frac{1}{2}(x_j^2+x_{j-1}^2)+3(x_j-x_{j-1}) \leq (x_j+3)(x_{j-1}-x_j)$
$L_f(P) \leq \dfrac{1}{2}(x_1-x_0)+3(x_1-x_0)+\ldots+\dfrac{1}{2}(x_n-x_{n-1})+3(x_n-x_{n-1}) \leq U_f(p)$
quedando solo
$\dfrac{1}{2}\left(x_n^2-x_0^2\right)+3(x_n-x_0) = \dfrac{1}{2}\left(b^2-a^2\right)+3(b-a)$
en conclusión
$\displaystyle\int_b^a f(x)dx = \frac{1}{2}\left(b^2-a^2\right)+3(b-a)$

13. Demuestre por inducción: $1\cdot 2+2\cdot 3+3\cdot 4+4\cdot 5+\ldots+n(n+1) = \dfrac{n(n+1)(n+2)}{3}$

Demostración

Sea $f(n)$ $12+23+34+45...n(n+1) = \dfrac{n(n+1)(n+2)}{3}$

Entonces para $f(1)$ $\dfrac{(1)(2)(3)}{3} = 2$ vemos que cumple

Hipótesis inductiva $f(k)$ es verdadera, es decir

$(1)(2)+(2)(3)+(3)(4)+....k(k+1) = \dfrac{n(n+1)(n+2)}{3}$

Sumando $(k+1)(k+2)$ a la hipótesis inductiva tenemos

$(1)(2)+(2)(3)+(3)(4)+....k(k+1) = \dfrac{n(n+1)(n+2)}{3}$
$\dfrac{k(k+1)(k+2)}{3}+(k+1)(k+2) = \dfrac{(k+1)(k+2)(k+3)}{3}$
Factorizando por $(k+1)(k+2)$ se tiene
$(k+1)(k+2)\left(1+\dfrac{k}{3}\right) = \dfrac{(k+1)(k+2)(k+3)}{3}$
queda desmostrado y lo que prueba que $p(n)$ es verdadera para todo n

2. Tarea 2

Graficar las funciones F, cuando sea posible, en un dispositivo electrónico

1. Suponiendo que

$$\int_1^4 f(x)dx = 5, \int_3^4 f(x)dx = 7, \int_1^8 f(x)dx = 11$$

hallar

a. $\int_4^8 f(x)dx = \int_1^8 f(x)dx - \int_1^4 f(x)dx = 11 - 5 = 6$

b. $\int_4^3 f(x)dx = -\int_4^3 f(x)dx = -7$

c. $\int_1^3 f(x)dx = \int_1^4 f(x)dx - \int_3^4 f(x)dx = 5 - 7 = -2$

d. $\int_3^8 f(x)dx = \int_1^8 f(x)dx - \left[\int_1^4 f(x)dx - \int_3^4 f(x)dx\right] = 11 - (-2) = 13$

e. $\int_8^4 f(x)dx = -\left[\int_1^8 f(x)dx - \int_1^4 f(x)dx\right] = -6$

f. $\int_4^4 f(x)dx = 0$

2. Usar sumas superiores e inferiores para demostrar que

$$0{,}6 < \int_0^1 \frac{dx}{1+x^2} < 1$$

sea $p = \left\{0, \frac{1}{2}, 1\right\}$ y $g(x) = \frac{1}{1+x^2}, x \in [0,1]$

$L(g,p) = \frac{4}{5}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right) = \frac{2}{5} + \frac{1}{4} = \frac{20}{13} \qquad \frac{20}{13} > \frac{1}{2}$

$U(g,p) = \frac{4}{5}\left(\frac{1}{2}\right) = \frac{2}{5} < 1$

dado:

$0{,}6 < L(g,p) \leq \int_0^1 \frac{dx}{1+x^2} \leq U(g,p) < 1$

3. Sea $F(x) = \int_\pi^x t \operatorname{sen} t dt$.

a. $F(\pi) \qquad F(\pi) = \int_\pi^x t \operatorname{sen} t dt = 0.$

b. $F'(x)$
$\frac{dF(x)}{dx} \int f(x)dx = f(x)$

c. $F'(\pi/2)$
por el inciso anterior podemos decir
$F'(\frac{\pi}{2}) = \int_\pi^x t \operatorname{sen} t dt = t \operatorname{sen} t = \frac{\pi}{2} \operatorname{sen}\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

4. Sea $F(x) = \int_0^x t(t-3)^2 dt$.

   a. Hallar los puntos críticos de F, determinar los intervalos de monotonía (donde F es creciente y decreciente) y los máximos y mínimos.

   b. Determinar la concavidad de la gráfica de F y hallar los puntos de inflexión, si existen.

   c. Bosquejar la gráfica de F

$F'(x) = x(x-3)^2$

puntos de inflexión $x = 3$ y $x = 1$

$-\infty, 0$	$0, 3$	$3, \infty$
$-$	$+$	$+$

minimo en 0

Observamos que es decreciente en $(-\infty, 0]$ y creciente en $[0, \infty)$

$F(x)'' = (x-3)^2 + 2x(x-3)$

$F(x)'' = x^2 - 6x + 9 + 2x^2 - 6x$

$F(x)'' = 3x^2 - 12x + 9$

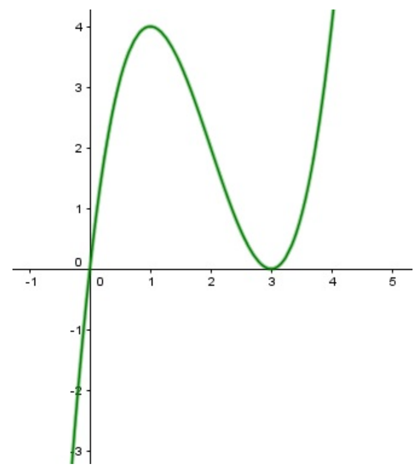
$F(x)'' = 3(x^2 - 4x + 3)$

$F(x)'' = 3(x-3)(x-1)$

	$-\infty, 1$	$1, 3$	$3, \infty$
$x-1$	$-$	$+$	$+$
$x-3$	$-$	$-$	$+$
	$+$	$-$	$+$

es cóncava hacia arriba de $(-\infty, 1) \cup (3, \infty)$

es cóncava hacia abajo en $(1, 3)$



5. Para la función dada, calcular $F'(-1)$, $F'(0)$, $F'(1/2), F''(x)$

   a. $F(x) = \int_0^x \frac{dt}{t^2+9}$

      $F'(x) = \int_0^x \frac{dt}{t^2+9}$ $\qquad F'(x) = \frac{dt}{t^2+9}$ entonces

      $F'(-1) = \frac{1}{10}$ $\qquad F'(0) = \frac{1}{9}$ $\qquad F'(1/2) = \frac{1}{\frac{1}{4}+9} = \frac{4}{37}$ $\qquad F''(x) = \frac{2t}{(t^2+9)^2}$

   b. $F(x) = \int_1^x \cos \pi t dt$

      $F'(x) = \int_1^x \cos \pi t = \cos \pi t$ entonces

      $F'(-1) = -1$ $\qquad F'(0) = \cos(0) = 1$ $\qquad F'(1/2) = \cos(\frac{\pi}{2}) = 0$ $\qquad F''(x) = -\pi \operatorname{sen} \pi t$

6. Hallar la derivada de F

   a. $F(x) = \int_0^{x^3} t \cos t dt$.

      Sug. Sea $u = x^3$ y utilizar la regla de la cadena

      Dado que $u = x^3$ entonces $\frac{du}{dx} = 3x^2$ y dado del ejercicio **3b** Tenemos

      $\frac{dF}{dx} = \frac{dF}{du}\frac{du}{dx}$

      $\frac{dF}{dx} = (u \cos u)(3x^2)$

$$\frac{dF}{dx} = 3x^5 \cos x^3$$

b. $F(x) = \int_1^{\cos x} \sqrt{1-t^2}dt.$

Haciendo $u = cosx \quad \frac{du}{dx} = -\operatorname{sen} x$

$$\frac{dF}{dx} = \sqrt{1-\cos^2 x}\cdot(-\operatorname{sen} x) = \frac{dF}{dx} = -(\operatorname{sen} x)^2$$

c. $F(x) = \int_{x^2}^{1} t - \operatorname{sen}^2 tdt.$

Sug. $-F(x) = \int_1^{x^2}(t - \operatorname{sen}^2 tdt)$

$F(x) = -\int_1^{x^2}(t - \operatorname{sen}^2 tdt)$ y dado que $u = x^2 \quad \frac{du}{dx} = 2x$

$$F'(x) - \frac{d}{dx}\int_1^{x^2}(u - \operatorname{sen}^2 u)du = 2x\left(sen^2(x^2) - x^2\right)$$

d. $F(x) = \int_0^{\sqrt{x}} \frac{t^2}{1+t^4}dt$

sea $u = \sqrt{x} \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$$\frac{df}{dx} = \frac{d}{dx}\int_0^u \frac{u^2}{1+u^4} = \frac{u^2}{1+u^4}du = \frac{x}{2\sqrt{x}(1-x^2)}$$

7. Sea $F(x) = 2x + \int_0^x \frac{\operatorname{sen} 2t}{1+t^2}dt$. Determinar $F(0),\ F'(0),\ F''(0)$

$F(0) = 0$

$$F'(x) = \frac{d}{dx}(2x) + \frac{d}{dx}\int_0^x \frac{\operatorname{sen} 2t}{1+t^2}dt = 2 + \frac{\operatorname{sen} 2x}{1+x^2}$$

$$F'(0) = 2 + \frac{\operatorname{sen} 0}{1} = 2$$

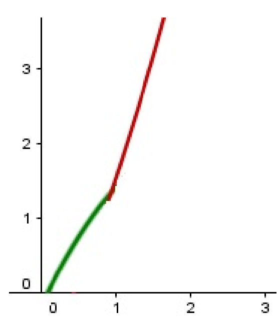
$$F''(t) = \frac{d}{dx}\left(2 + \frac{\operatorname{sen} 2x}{1+x^2}\right) = \frac{2(1+x^2)(\cos 2x) - 2x(\operatorname{sen} 2t)}{(1+x^2)^2} = \frac{2(\cos 2x)}{1+x^2} - \frac{2x(\operatorname{sen} 2x)}{(1+xt^2)} = \frac{2\cos 0}{1} - 0 = 2$$

8. Bosquejar la gráfica de la función $f(x) = \begin{cases} 2-x, & si \quad 0 \le x < 1; \\ 2+x, & si \quad 1 < x \le 3. \end{cases}$

Hallar la función $F(x) = \int_0^x f(t)dt$ con $x \in [0,3]$ y bosquejar su gráfica. ¿Qué se puede decir sobre f y F en $x = 1$?

Por una parte Tenemos

$F(x) = \int (2-x)dx = 2x - \frac{x^2}{2}$ y $F(x) = \int (2+x)dx = 2x + \frac{x^2}{2}$



es continua pero no diferenciable

9. Demostrar el Primer Teorema del valor medio para Integrales. Si f es continua en $[a,b]$, entonces existe al menos un número c en (a,b) tal que

$$\int_a^b f(x)dx = f(c)(b-a)$$

Sug. Aplicar el Teorema del valor medio a la función $F(x) = \int_a^x f(t)dt$ en $[a,b]$

$f(c+h) - f(c) = \int_a^{c+h} f(t)dt - \int_a^c f(t)dt$

$f(x+h)\int_a^c f(t)dt + \int_c^{c+h} f(t)dt - \int_a^c f(t)dt$

$f(c+h) - f(c) = \int_c^{c+h} f(t)dt$

Si dividimos entre h

$\dfrac{f(c+h) - f(c)}{h} = \dfrac{1}{h}\int_c^{c+h} f(t)dt$

por otra parte sabemos que: $f(U,P) \leq \dfrac{1}{h}\int_c^{c+h} f(t)dt \leq f(L,P)$

y por lo anterior tenemos

$f(U,P) \leq \dfrac{f(c+h) - f(c)}{h} \leq f(L,P)$

y como f esta entre $[c, c+h]$ cuando $h \to 0$ y cuando esto sucede $f(U,P) = f(L,P) = f(c)$, Calculado el limite, tenemos

$\lim\limits_{h\to 0} f(I,P) \leq \lim\limits_{h\to 0} \dfrac{f(c+h) - f(c)}{h} \leq \lim\limits_{h\to 0} f(L,P)$

$f(c) \leq F'(c) \leq f(c)$

por lo que queda demostrado que $F'(c) = f(c)$

10. Sea f continua y F definida por

$$F(x) = \int_c^x [t\int_1^t f(u)du]dt$$

Hallar $F'(x)$, $F'(1)$, $F''(x), F''(1)$

$F'(x) = \dfrac{d}{dx}\left[\int_c^x \left(x\int_1^t f(u)du\right) dt\right] = x\int_1^x f(u)du$

$F'(1) = x\int_1^1 f(u)du = 0$

$F''(x) = \dfrac{d}{dx}\left[x\int_1^x f(u)du\right] = x \cdot f(x) + \int_1^x f(u)du$

$F''(1) = f(x) + 0 = f(x)$

3. Tarea 3

En los ejercicios 1-3 determine el número c del Teorema del valor medio para la integral que se indica.

1. $\displaystyle\int_1^3 x^2dx$
   $f(c)(b-a)$
   $\displaystyle\int_1^3 x^2dx = c^2(3-1)$
   $\dfrac{3^3}{3} - \dfrac{1}{3} = 2c^2$
   $\dfrac{26}{3} = 2c^2$
   $c = \sqrt{\dfrac{26}{6}}$

2. $\displaystyle\int_0^3 (x^2+1)dx$
   $f(c)(b-a)$
   $\displaystyle\int_0^3 (x^2+1)dx = (c^2+1)(3-0)$
   $\dfrac{(3)^3}{3} + (3)dx = 3c^2 + 3 \ c = \sqrt{\dfrac{8}{3}}$

3. $\displaystyle\int_0^2 x^3dx$
   $f(c)(b-a)$
   $\displaystyle\int_0^2 x^3dx = c^3(2-0)$
   $\dfrac{(2)^4}{4} = 2c^3$
   $c = \sqrt[3]{2}$

En los ejercicios $4-22$ calcule la integral que se indica.

4. $\displaystyle\int_0^5 (x^3-2x^2+x-2)dx$
   $\displaystyle\int_0^5 (x^3-2x^2+x-2)dx = \int_0^5 x^3dx - 2\int_0^5 x^2dx + \int_0^5 xdx - 2\int_0^5 dx = \frac{x^4}{4}\Big|_0^5 - \frac{2x^3}{3}\Big|_0^5 + \frac{x^2}{2}\,|_0^5 - 2x|_0^5$
   $= \dfrac{625}{4} + \dfrac{250}{3} + \dfrac{25}{2} + 10 = \dfrac{787}{3}$

5. $\displaystyle\int_{-1}^2 (2x+x^2+x^3)dx$
   $\displaystyle\int_{-1}^2 (2x+x^2+x^3)dx = 2\int_{-1}^2 xdx + \int_{-1}^2 x^2dx + \int_{-1}^2 x^3dx = x^2|_{-1}^2 + \frac{x^3}{3} + \frac{x^4}{4}$
   $(2)^2 - (-1)^2 + \dfrac{(2)^3}{3} - \dfrac{(-1)^3}{3} + \dfrac{(2)^4}{4} - \dfrac{(-1)^4}{4} = \dfrac{39}{4}$

6. $\displaystyle\int_2^3 \left(1-\frac{3x}{2}\right)^2 dx$
   $\displaystyle\int_2^3 \left(1-\frac{3x}{2}\right)^2 dx = \int_2^3 \left(1-3x+\frac{9x^2}{4}\right) dx = \int_2^3 dx - 3\int_2^3 xdx + \frac{9}{4}\int_2^3 x^2dx = x|_2^3 - \frac{3x^2}{2}\Big|_2^3 + \frac{3x^3}{4}\,|_2^3$
   $= 3 - 2 - \left(\dfrac{3(3)^2}{2} - \dfrac{3(2)^2}{2}\right) + \left(\dfrac{3(3)^3}{4} - \dfrac{3(2)^3}{4}\right) = \dfrac{31}{4}$

7. $\displaystyle\int_1^5 \frac{x^3-1}{x^3}dx$
$$\int_1^5 \frac{x^3-1}{x^3}dx = \int_1^5 dx - \int_1^5 \frac{1}{x^3}dx = x|_1^5 + \frac{1}{2x^2}\bigg|_1^5 = 5-1+\frac{1}{2(5)^2}-\frac{1}{2(1)^2} = \frac{88}{25}$$

8. $\displaystyle\int_1^3 \frac{(x+2)^2}{x^5}dx$
$$\int_1^3 \frac{(x+2)^2}{x^5}dx = \int_1^3 \frac{x^2+2x+4}{x^5}dx = \int_1^3 \frac{1}{x^3}dx + 2\int_1^3 \frac{1}{x^4}dx + 4\int_1^3 \frac{1}{x^5}dx$$
$$= -\frac{1}{2x^2}\bigg|_1^3 - \frac{2}{3x^3}\bigg|_1^3 - \frac{1}{x^4}\bigg|_1^3 = \left(\frac{1}{2(3)^2}-\frac{1}{2(1)^2}\right) - \left(\frac{2}{3(3)^3}-\frac{2}{3(1)^3}\right) - \left(\frac{1}{(3)^4}-\frac{1}{(1)^4}\right) = \frac{4}{9}+\frac{52}{81}+\frac{80}{81} = \frac{56}{27}$$

9. $\displaystyle\int_2^4 \frac{\sqrt{x}-2}{\sqrt[3]{x^2}}dx$
$$\int_2^4 \frac{\sqrt{x}-2}{\sqrt[3]{x^2}}dx = \int_2^4 \frac{dx}{x^{1/6}} - 2\int_2^4 \frac{dx}{x^{2/3}} = \frac{6}{5\sqrt[6]{x^5}}\bigg|_2^4 - \frac{3}{\sqrt[3]{x}}\,|_2^4$$
$$= \left(\frac{6}{5\sqrt[6]{(4)^5}} - \frac{6}{5\sqrt[6]{(2)^5}}\right) - \left(\frac{3}{\sqrt[3]{2}} - \frac{3}{\sqrt[3]{2}}\right) = 0{,}1957190890928276257$$

10 $\displaystyle\int_0^4 \left(x^{\frac{2}{3}} - x^{\frac{3}{2}}\right)dx \int_0^4 x^{2/3}dx - \int_0^4 x^{3/2}dx = \frac{3x^{5/3}}{5}\bigg|_0^4 - \frac{2x^{5/2}}{5}\bigg|_0^4 = \frac{3(4)^{5/3}}{5} - \frac{2(4)^{5/2}}{5} = -6{,}752378960$

11. $\int_{-4}^0 |x+2|dx$

12. $\displaystyle\int_1^2 \frac{t^3-2t-1}{\sqrt{t}}dt$
$$\int_1^2 t^{5/2} - 2\int_1^2 t^{1/2} - \int_1^2 \frac{1}{\sqrt{t}}dt\frac{2}{7}x^{7/2}\bigg|_1^2 = -\frac{4}{3}x^{3/2}\,|_1^2 - 2\sqrt{x}|_1^2 = 3{,}3372$$

13. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\cos^2 v - \operatorname{sen}^2 v)dv$
$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\cos^2 v - \operatorname{sen}^2 v)dv = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\cos 2v)dv = \frac{1}{2}\operatorname{sen} 2v\bigg|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{2}\operatorname{sen}(\tfrac{\pi}{2}) - \left[-\frac{1}{2}\operatorname{sen}(\tfrac{\pi}{2})\right] = 1$$

14. $\displaystyle\int_0^4 \sqrt{x}(1-x)dx \int_0^4 \sqrt{x}(1-x)dx = \int_0^4 \sqrt{x}dx - \int_0^4 x^{3/2} = \frac{2x^{3/2}}{3}\bigg|_0^4 - \frac{2x^{5/2}}{5}\,|_0^4$
$$= \left(\frac{2(4)^{3/2}}{3}\right) - \left(\frac{2(4)^{5/2}}{5}\right) = \frac{16}{3} - \frac{64}{5} = -\frac{112}{15}$$

15. $\displaystyle\int_0^{\frac{\pi}{4}} \sec^2 x dx$
$$\int_0^{\frac{\pi}{4}} \sec^2 x dx = \tan x|_0^{\frac{\pi}{4}} = \tan\left(\frac{\pi}{4}\right) = 1$$

16. $\displaystyle\int_{-1}^0 2z\sqrt{1-z^2}dz$
$$\int_{-1}^0 2z\sqrt{1-z^2}dz = -\frac{2}{3}(1-x^2)^{3/2}\bigg|_{-1}^0 - \left[-\frac{2}{3}(1-x^2)^{3/2}\right] = -\frac{2}{3}(1-0^2)^{3/2} - \left(-\frac{2}{3}(1-0^2)^{3/2}\right)1+0 = 1$$

17. $\displaystyle\int_0^1 \sqrt{x+1}dx$
$$\int_0^1 \sqrt{x+1}dx = \frac{3}{2}(x+1)^{3/2}\bigg|_0^1 = \frac{2}{3}(2)^{3/2} - \frac{2}{3} = \frac{4\sqrt{2}}{3} - \frac{2}{3} \approx 1{,}22ss$$

18. $\displaystyle\int \frac{dt}{(2+t)^2} = \frac{-1}{2+t} + c$

19. $\displaystyle\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}$
$\displaystyle\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x} = \int_0^{\frac{\pi}{4}} \sec^2 x = \tan x|_0^{\pi/4} = \tan(\frac{\pi}{4}) = 1$

20. $\displaystyle\int_{-1}^{1} \frac{du}{1+u^2}$
$\displaystyle\int_{-1}^{1} \frac{du}{1+u^2} = arc\tan x|_{-1}^{1} = arc\tan(1) - arc\tan(-1) = \frac{\pi}{2}$

21. $\displaystyle\int_2^4 \frac{dx}{1-\cos x}$

$\displaystyle\int_2^4 \frac{dx}{1-\cos x}\left(\frac{1-\cos x}{1-\cos x}\right) = \int_2^4 \frac{1-\cos x}{\operatorname{sen}^2 x} = \int_2^4 \frac{1}{\operatorname{sen}^2 x} - \int_2^4 \frac{\cos x}{\operatorname{sen}^2 x}$
$\displaystyle = \frac{\cos x}{\operatorname{sen} x}\Bigg|_2^4 - \frac{1}{\operatorname{sen} x}|_2^4 = 3{,}74244$

22. Demuestre que si f es integrable en $[a,b]$, entonces

$$\left|\int_a^b f(t)dt\right| \leq \int_a^b |f(t)|dt$$

puesto a la propiedad $|f(t)| \leq f(t) \to -f(t) \leq f(t) \leq f(t)$ se sigue cumpliendo si, $-|f(t)| \leq f(t) \leq |f(t)|$ y también para $\displaystyle -\int_a^b |f(t)| \leq \int_a^b f(t) \leq \int_a^b |f(t)|$ por lo tanto $\displaystyle\left|\int_a^b f(t)dt\right| \leq \int_a^b |f(t)|dt$

23. Halle $F'(x)$ si

$$F(x) = \int_0^x xf(t)dt$$

Sug La respuesta no es $xf(x)$, debe realizarse una manipulación obvia en la integral antes de intentar calcular F'

esta derivada la podemos ver, como la del producto. Entonces tenemos

$\displaystyle F'(x) = x\int_0^x f(t)dt$
$\displaystyle F(x) = xf(x) + \int_0^x f(t)dt$

24. Demuestre que si f es continua, entonces

$$\int_0^x f(u)(x-u)du = \int_0^x \left(\int_0^u f(t)dt\right) du$$

Sug Derive ambos lados de la igualdad y use el problema anterior.

Tenemos $\displaystyle\int_0^x f(u)(x-u)du$ se puede escribir como $\displaystyle x\int_0^x f(u)du - \int_0^x uf(u)du$

La primera integral se realiza como el ejercicio anterior, quedando

$$x\int_0^x f(u)du = xf(x) + \int_0^x f(u)du$$

y la segunda integral :

$$-\int_0^x uf(u)du = -xf(x)$$

juntando los resultados de las integrales anteriores

$$xf(x) + \int_0^x f(u)du - xf(x)$$

$$\int_0^x f(u)du = c$$

25. Demuestre que si h es continua, f y g son diferenciables, y

$$F(x) = \int_{f(x)}^{g(x)} h(t)dt,$$

entonces $F'(x) = h(g(x)) \cdot g'(x) - h(f(x)) \cdot f'(x)$. **Sug** Intente reducir el problema a los dos casos ya conocidos, con una constante como límite de integración inferior o superior.

Caso 1
si $F(x) = \int_0^{g(x)} h(t)dt$,
entonces
$F'(x) = h(g(x)) \cdot g'(x)$

Caso 2
$F(x) = \displaystyle\int_{f(x)}^{0} h(t)dt$, si cambiamos los limites de integración
$F(x) = -\displaystyle\int_0^{f(x)} h(t)dt$,
$F'(x) = -h(f(x))f'(x)$

En conclusión obtenemos dado a $F(x) = \displaystyle\int_0^{g(x)} h(t)dt + \int_{f(x)}^{0} h(t)dt$

$F'(x) = h(g(x)) \cdot g'(x) - h(f(x)) \cdot f'(x)$

En los ejercicios $26 - 35$ calcule la derivada que se indica.

26. $\displaystyle\frac{d}{dx}\int_0^x t \operatorname{sen} t \, dt = x \operatorname{sen} x$

27. $\displaystyle\frac{d}{dx}\int_1^{\frac{1}{x}} \sqrt{t} \, dt = \sqrt{\frac{1}{x}} (\ln x)$

28. $\displaystyle\frac{d}{dx}\int_{\sqrt{x}}^{\sqrt[3]{x}} \operatorname{sen} t^6 \, dt = \frac{d}{dx}\left(\int_0^{\sqrt[3]{x}} \operatorname{sen} t^6 \, dt - \int_0^{\sqrt{x}} \operatorname{sen} t^6 \, dt\right) \operatorname{sen} t^6 \, dt = 6(\sqrt[3]{x})^5 \operatorname{sen}(\sqrt[3]{x})^6 - 6\sqrt{x}^5 \operatorname{sen}(\sqrt{x})^6$

29. $\displaystyle\frac{d}{dx}\int_0^{x^2} \cos\sqrt{t} \, dt = \cos\sqrt{t} \cdot 2x$

30. $\displaystyle\frac{d}{dx}\int_0^{x^3+1} \sqrt{1+t^2} \, dt_{=} \sqrt{1+(x^3+1)^2} \cdot 3x^2$

31. $\displaystyle\frac{d}{dx}\int_1^{\operatorname{arc\,sen} x} \sqrt{1-\operatorname{sen} t} \, dt \sqrt{1-\operatorname{sen}(\operatorname{arc\,sen} x)}\left(\frac{1}{\sqrt{1-x^2}}\right)$

32. $\displaystyle\frac{d}{dx}\int_1^{(x^2+1)} \cos 2x \frac{dt}{t} = 2x \cos 2(x^2+1)$

33. $\displaystyle\frac{d}{dx}\int_2^{\csc^2 x} \frac{dt}{1+t^2} = \frac{-2\csc^2 x \cot x}{1+\csc^4 x}$

34. $\dfrac{d}{dx}\displaystyle\int_{x}^{x^2} \operatorname{sen} t^2\, dt$

$\dfrac{d}{dx}\displaystyle\int_{0}^{x^2} \operatorname{sen} t^2 dx - \int_{0}^{x} \operatorname{sen} t^2 dt = \operatorname{sen} t^2\, dt = 2x \operatorname{sen} x^4 - \operatorname{sen} x^2$

35. $\dfrac{d}{dx}\displaystyle\int_{x \operatorname{sen} x}^{1} \frac{\operatorname{sen}^3 \sqrt{t+1}}{t^2}\, dt$

$-\dfrac{d}{dx}\displaystyle\int_{1}^{x \operatorname{sen} x} \frac{\operatorname{sen}^3 \sqrt{t+1}}{t^2}\, dt = \frac{\operatorname{sen}^3 \sqrt{(x \operatorname{sen} x) + 1}}{(x \operatorname{sen} x)^2} (\operatorname{sen} x + x \cos x)$

4. Tarea 4

En los ejercicios $1-3$, hallar el área comprendida entre la gráfica de f y el eje x

1. $f(x) = 2 + x^3,\ x \in [0, 1]$
   $A = \displaystyle\int_0^1 (2x+3)dx = \int_0^1 2dx + \int_0^1 x^3 dx = \left[2x + \frac{x^4}{4}\right]_0^1 = 2 + \frac{1}{4} = \frac{9}{4}$
2. $f(x) = x^2 - 4,\ x \in [1, 2]$
   $\displaystyle\int_1^2 (4 - x^2)dx = 4\int_1^2 dx - \int_1^2 x^2 = \left[4x - \frac{x^3}{3}\right]_1^2 = \left(8 - \frac{8}{3}\right) - \left(4 - \frac{1}{3}\right) = \frac{5}{3}$
3. $f(x) = \operatorname{sen} x,\ x \in [\pi/3, \pi/2]$
   $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{sen} x dx = [-\cos x]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = 0 - (-\frac{1}{2}) = \frac{1}{2}$

En los ejercicios $4-11$, dibujar la región limitada por las curvas y calcular su área.

4. $y = \sqrt{x},\ y = x^2\ \int_0^1 (\sqrt{x} - x^2)dx = \int_0^1 \sqrt{x}dx - \int_0^1 x^2 dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{2}x^3\right]_0^1 = \frac{1}{3}$
5. $y = 5 - x^2,\ y = 3 - x$
   $5 - x^2 = 3 - x$
   $x^2 - x + 2 = 0 \quad x = -1 \quad x = 2$
   $A = \displaystyle\int_{-1}^2 [(5 - x^2) - (3 - x)]dx = +\int_{-1}^2 x^2 dx + \int_{-1}^2 xdx2\int_{-1}^2 dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^2 = \left(-\frac{8}{3} - \frac{1}{3}\right) + \left(2 - \frac{1}{2}\right) + (4 - (-2)) = \frac{9}{2}$
6. $y = 8 - x^2,\ y = x^2$
   $8 - x^2 = x^2\ x^2 = 4\ x = \pm 2\ \displaystyle\int_{-2}^2 [(8 - x^2) - x^2]dx = 8\int_{-2}^2 dx - 2\int_{-2}^2 x^2 = \left[8x - \frac{3}{2}x^3\right]_{-2}^2 = (16 - (-16)) - \left(\frac{16}{3} + \frac{16}{3}\right) = 32 - \frac{32}{3} = \frac{64}{3}$
7. $x^3 - 10y^2 = 0,\ x - y = 0$
   Tenemos que las funciones son : $y = x$ y $y = \dfrac{x^{3/2}}{\sqrt{10}}$
   los puntos de intersección :
   $x = \dfrac{x^{3/2}}{\sqrt{10}}$
   $\sqrt{10} = \sqrt{x}$
   $x = 10 \qquad x = 0$
   $A = \displaystyle\int_0^{10} \left(x - \frac{x^{3/2}}{\sqrt{10}}\right) dx = \int_0^{10} xdx - \frac{1}{\sqrt{10}}\int_0^{10} x^{3/2}dx = \left[\frac{x^2}{2} - \frac{2}{5\sqrt{10}x^{5/2}}\right]_0^{10} = 50 - \frac{2}{5\sqrt{10}(10)^{5/2}} = 10$
8. $x - y^2 + 3 = 0,\ x - 2y = 0$
   las funciones son: $y = \dfrac{x}{2} \qquad y = \pm\sqrt{x+3} \in [-3, 2]$
   la intersección de estas funciónes es:
   $\dfrac{x}{2} = \sqrt{x+3} \qquad x = 6$
   $\dfrac{x}{2} = -\sqrt{x+3} \qquad x = -2$
   $\sqrt{x+3} = -\sqrt{x+3} \qquad x = -3$

   $A = \displaystyle\int_{-3}^{-2} (x+3)^{1/2}dx - \int_{-3}^{-2} -(x+3)^{1/2}dx + \left[int_{-2}^6 (x+3)^{1/2} - \frac{1}{2}\int_{-2}^6 xdx\right]$
   $A = 2\displaystyle\int_{-3}^{-2} (x+3)^{1/2}dx + \left[\int_{-2}^6 (x+3)^{1/2} - \frac{1}{2}\int_{-2}^6 xdx\right]$
   $A = \left[\dfrac{4}{3}(x+3)^{3/2}\right]_{-3}^{-2} + \left[\dfrac{2}{3}(x+3)^{3/2}\right]_{-2}^6 - \left[\dfrac{x^2}{4}\right]_{-2}^6 = \dfrac{4}{3} + \left(9 + \dfrac{1}{3}\right) = \dfrac{32}{3}$

9. $y_0 = x,\ y_1 = 2x,\ y_2 = 4$ en $x = 2 \to y_1 = y_2$ en $x = 4 \to y_0 = y_2$
$A_1 = \int_0^2 (2x - x)dx = \left[x^2 - \frac{x^2}{2}\right]_0^2 = 2$ $A_2 = \int_2^4 (4 - x)dx = \left[4x - \frac{x^2}{2}\right]_2^4 = 2$
$A = A_1 + A_2 = 4$

10. $y = \cos x,\ y = 4x^2 - \pi^2$
¿cuando es cero la función? $\cos x = 0 \to x = \frac{\pi}{2}$ $4x^2 - \pi^2 = 0 \to x = \frac{\pi}{2}$ $A = \int_{\pi/2}^{\pi/2} [\cos x - (4x^2 - \pi^2]dx = \left[senx - \frac{3}{4}x^3] + \pi^2 x\right]_{-\pi/2}^{\pi/2} = (1 - (-1)) - \left(\frac{\pi^3}{6} + \frac{\pi^3}{6} + \left(\frac{\pi^3}{2} + \frac{\pi^3}{3}\right) = 2 + \frac{2\pi^3}{3}$

En los ejercicios $11 - 16$, calcular las integrales indefinidas

11. $\int \frac{dx}{\sqrt{1+x}}$
$\int \frac{dx}{\sqrt{1+x}} = 2\sqrt{1+x} + c$

12. $\int g(x)g'(x)dx$ $\int g(x)g'(x)dx = \frac{[g(x)]^2}{2} + c$

13. $\int \tan x \sec^2 x dx$
$\int \sec x(\tan x \sec x)dx = \int \sec x(\tan x \sec x)'dx = \frac{\sec^2 x}{2} + c$

14. $\int \frac{g'(x)}{[g(x)]^2}dx$
$\int \frac{g'(x)}{[g(x)]^2}dx = \frac{-1}{g(x)} + c$

15. $\int \frac{4}{(4x+1)^2}dx$
$\int \frac{4}{(4x+1)^2}dx = -\frac{1}{4x+1} + c$

16. $\int \frac{3x^2}{(x^3+1)^2}dx$
$\int \frac{3x^2}{(x^3+1)^2}dx = -\frac{1}{x^3+1} + c$

En los ejercicios $17 - 21$, hallar f a partir de la información dada

17. $f'(x) = 2x - 1,\ f(3) = 4$ $f(x) = \int 2xdx - \int dx = x^2 - x + C$
$f(3) = (3)^2 - 3 + C = 4$ $C = -2$
$f = x^2 - x - 2$

18. $f'(x) = \operatorname{sen} x,\ f(0) = 2$
$f(x)\int \operatorname{sen} xdx = -\cos x + C$
$f(0) = -\cos(0) + C = 2$ $C = 3$
$f(x) = 3 - \cos x$

19. $f''(x) = x^2 - x,\ f'(1) = 0,\ f(1) = 2$
$f'(x) = \int x^2dx - \int xdx = \frac{x^3}{3} - \frac{x^2}{2} + C_1$
$f'(0) = \frac{1}{3} - \frac{1}{2} + C = 2$ $C_1\frac{13}{6}$ $f(x) = \int \frac{x^3}{3} - \int \frac{x^2}{2}dx + \frac{13}{6}\int dx$

20. $f''(x) = \cos x,\ f'(0) = 1,\ f(0) = 2\ f'(x) = \displaystyle\int \cos x dx = \operatorname{sen} x + C_1$

$f'(0) = \operatorname{sen}(0) + C_1 = 1 \qquad C_1 = 1 \qquad f'(x) = \operatorname{sen} x + 1$

$f(x) = \int \operatorname{sen} x dx + \displaystyle\int dx = -\cos x + x + C_2$

$f(0) = -\cos(0) + 0 + C_2 = 2 \qquad C_2 = 3 \qquad -\cos x + x + 3$

21. $f''(x) = 2x - 3,\ f(2) = -1,\ f(0) = 3$

$f'(x) = 2\displaystyle\int x dx - 3\int dx = x^2 - 3x + C_1$

$f(x) = \displaystyle\int x^2 dx - 3\int x dx + C_1 \int dx = \frac{x^3}{3} - 3\frac{x^2}{2} + C_1 x + C_2$

$f(2) = \dfrac{(2)^3}{3} - \dfrac{3(2)^2}{2} + 2C_1 + C_2 = -1 \qquad \dfrac{8}{3} - 6 + 2C_1 + C_2 = 1$

$f(0) \quad k = 3$

22. Comparar

$$\frac{d}{dx}\left[\int f(x)dx\right],\ con\ \int \frac{d}{dx}[f(x)]dx$$

por una parte tenemos

$\dfrac{d}{dx}\left[\displaystyle\int f(x)dx\right] = f(x)$

y la siguiente es:

$\displaystyle\int \frac{d}{dx}[f(x)]dx = f(x) + C$

por lo que observamos que $\dfrac{d}{dx}\left[\displaystyle\int f(x)dx\right] \neq \displaystyle\int \frac{d}{dx}[f(x)]dx$

23. Calcular

$$\int [f(x)g''(x) - g(x)f''(x)]dx$$

Sug. Calcule la derivada de $f(x)g'(x) - f'(x)g(x)$

Tenemos :

$\displaystyle\int [f(x)g''(x) - g(x)f''(x)]dx$

le sumamos cero de tal forma $f'(x)g'(x) - f'(x)g'(x)$

$\displaystyle\int [f(x)g''(x) + f'(x)g'(x) - f'(x)g'(x) - g(x)f''(x)]dx$

rescribiendolo obtenemos:

$\displaystyle\int \left[\frac{d}{dx}[f(x)g'(x)] - \frac{d}{dx}[f'(x)g(x)]\right] dx = f(x)g'(x) - f'(x)g(x) + C$

24. Un objeto se mueve a lo largo de un eje coordenado con velocidad $v(t) = 6t^2 - 6$ unidades por segundo. Su posición inicial (t=0) es 2 unidades a la izquierda del origen. (a) Hallar la posición del objeto 3 segundos mas tarde. (b) Hallar la distancia total que viajo el objeto durante estos 3 segundos.

tenemos que la distancia esta dada por:

$x(t) = \displaystyle\int 6t^2 dt - 6\int dt = 2t^3 - 6t + C$

$x(0) = 2(0) - 6(0) + C = -2 \qquad C = -2 \qquad x(t) = 2t^3 - 6t - 2$

$x(3) = 2(3)^3 - 6(3) - 2 = 34 \rightarrow$ distancia total

25. Un objeto se mueve a lo largo de una linea con aceleración $a(t) = (t+1)^{-1/2}$ unidades por segundo cada segundo. (a) Hallar la función velocidad dado que la velocidad inicial es de 1 unidad por segundo. (b) Hallar la función de posición dado que la velocidad inicial es 1 unidad por segundo y la posición inicial es el origen.

tenemos que la aceleración esta descrita por:

$a(x) = \int (t+1)^{-1/2} dt$

$a(t) = 2(t+1)^{1/2} + C_1$
$v(0) = 2(0+1)^{1/2} + C_1 = 1 \qquad C_1 = -1$
por lo que la velocidad es:
$v(t) = 2(t+1)^{1/2} - 1$

$x(t) = 2\int (t+1)^{1/2} dt - \int dt$
$x(t) = \frac{4}{3}(t+1)^{3/2} - t + C_2$
$x(0) = \frac{4}{3}(0+1)^{3/2} + C_2 = \frac{4}{3} + C_2 = 0 \qquad C_2 = -\frac{4}{3}$
entonces:
$x(t) = \frac{4}{3}(t+1)^{3/2} - t - \frac{4}{3}$

26. Un carro viaja a 100 Km/h desacelera a razón de 6m por segundo en cada segundo. (a) ¿ Cuánto tardará el carro en detenerse por completo? (b) ¿ Qué distancia es requerida para que el auto se detenga por completo?

$x = vt$
$\frac{d}{dt}x = \frac{d}{dt}(vt)$
$v = at + v_0 \qquad t = \frac{v - v_0}{a} = \frac{-100km/h}{-6m/s^2} = \frac{-27m/s}{-6m/s^2} = 4{,}6s$

$\int vdt = \int (v_0 + at)dt$
$x = v_0 t + \frac{at^2}{2} = (27{,}7m/s)(4{,}6s) + \frac{6m/s^2)(4{,}65s)^2}{2} = \frac{3197}{50}m = 63{,}94m$

27. Un objeto que se mueve a lo largo del eje x con aceleración constante a. Comprobar que

$$[v(t)]^2 = v_0^2 + 2a[x(t) - x_0]$$

tenemos :
$v(t) = \int adt = at + v_0$ sea v_0 una constante
entonces:
$[v(t)]^2 = (at + v_0)^2 = a^2t^2 + 2atv_0 + v_0^2$
$[V(t)]^2 = a(at^2 + 2tv_0) + v_0^2$ $[V(t)]^2 = 2a(\frac{1}{2}at^2 + tv_0) + v_0^2$ Por otro lado tenemos que
$x(t) = \int (at + v_0)dt = \frac{1}{2}at^2 + v_0 t + x_0$
sumando cero:
$[V(t)]^2 = 2a(\frac{1}{2}at^2 + tv_0) + v_0^2 + x_0 - x_0$
y por lo anterior tenemos:
$[v(t)]^2 = v_0^2 + 2a(x - x_0)$

28. Conforme una partícula se va moviendo por el plano, su coordenada x varía a razón de $t^2 - 5$ unidades por segundo y su ordenada y varía a razón de $3t$ unidades por segundo. Si la partícula se encuentra en el punto $(4, 2)$ en el instante $t = 2$, ¿dónde se encontrará 4 segundos más tarde?

$x'(t) = t^2 - 5 \rightarrow$x(t)=$\int (t^2 - 5)dt = \frac{t^3}{3} - 5t + c_1$
$x(2) = \frac{8}{3} - 10 + c_1 = 4 \qquad x = \frac{34}{3}$
$x(t) = \frac{t^3}{3} - 5t + \frac{34}{3}$ $y'(t) = 3t \rightarrow$y(t)=$\int 3tdt = \frac{3t^2}{2} + C_2$
$y(2) = \frac{3(2)^2}{2} + C_2 \qquad C_2 = -4$ $y(6) = \frac{3(6)^2}{2} - 4 = 50 \qquad x(6) = \frac{(6)^3}{3} - 5(6) + \frac{34}{3} = \frac{160}{3}$

despues de 4 segundos se encuentra en el punto : $\left(\frac{160}{3}, 50\right)$

5. Tarea 5

Use un dispositivo electrónico para gráficar y aproximar donde se requiera

En los ejercicios $1-15$, calcule las integrales

1. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 t\, dt$

   $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 t\, dt = \int \cos^2 t dt = \int (1-\operatorname{sen} t)\cos t dt$

   sea $u = \operatorname{sen} t \rightarrow$du=$\cos t dt$

   $\displaystyle\int (1-u^2)du = \int du - \int u^2 du = u - \frac{u^3}{3} = \operatorname{sen} x|_0^{\frac{\pi}{2}} - \frac{\operatorname{sen}^3}{3}\bigg|_0^{\frac{\pi}{2}} = \frac{2}{3}$

2. $\displaystyle\int_0^{\pi} \operatorname{sen}^2 w\, dw$

   $\displaystyle\int_0^{\pi} \operatorname{sen}^2 w\, dw = \frac{1}{2}\int_0^{\pi} dx - \frac{1}{2}\int_0^{\pi} \cos(2x)dx = \frac{1}{2}\int_0^{\pi} dx - \frac{1}{4}\int_0^{\pi} 2\cos(2x)dx = \frac{x}{2}\bigg|_0^{\pi} - \frac{\operatorname{sen}(2x)}{4}\bigg|_0^{\pi} = \frac{\pi}{2}$

3. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\operatorname{sen}^3 x}\, dx$

   $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\operatorname{sen}^3 x}\, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x \csc^2 x dx = -\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} u du = -\frac{1}{2}u^2 = -\frac{1}{2}\csc^2\bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{2\operatorname{sen}^2(x)}\bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{2\operatorname{sen}^2(\frac{\pi}{2})} - \frac{1}{2\operatorname{sen}^2(\frac{\pi}{4})}$

   $\displaystyle-\left(\frac{1}{2} - \frac{1}{2(\frac{1}{2})}\right) = -\frac{1}{2}$

4. $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\operatorname{sen} x}{\sqrt{\cos x}}\, dx$

   Realizando un cambio de variable

   sea $u = \cos x$ y $du = \operatorname{sen} x dx$

   $\displaystyle\int_0^{\frac{\pi}{4}} \frac{du}{u^{1/2}} = -2\sqrt{u}|_0^{\frac{\pi}{4}} = 2\sqrt{\cos x}|_0^{\frac{\pi}{4}} = 2 - 2^{3/2}$

5. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \operatorname{sen}^3 u\, du$

   $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1-\cos^2 x)\operatorname{sen} u du) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \operatorname{sen} u du - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 u(-\operatorname{sen} u)du = -\frac{\cos^3 u}{3}\bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \cos u\bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$

6. $\displaystyle\int_0^{\frac{\pi}{4}} \sec^{\frac{3}{2}} x \tan x\, dx$

   $\displaystyle\int_0^{\frac{\pi}{4}} \sec^{\frac{3}{2}} x \tan x\, dx = \int_0^{\frac{\pi}{4}} (\sec x)^{1/2} \sec x \tan x dx$

   haciendo un cambio de variable $u = \sec x$ y $du = \sec x \tan x dx$

   $\displaystyle\int u^{1/2} du = \frac{2}{3}u^{3/2}$

   regresando a la variable

   $\displaystyle\frac{2}{3}(\sec x)^{3/2}\bigg|_0^{\frac{\pi}{4}} = \left(\frac{2}{3}\right)\left(\frac{2^{1/4}}{2}\right) = \frac{\sqrt[4]{2}}{3}$

7. $\displaystyle\int_0^1 \frac{x}{(x^2+2)^{\frac{3}{2}}}\, dx$

   $\displaystyle\int_0^1 \frac{x}{(x^2+2)^{\frac{3}{2}}}\, dx = \frac{1}{2}\int_0^1 \frac{2x}{(x^2+2)^{\frac{3}{2}}}\, dx$ dado a un cambio de variable

   $\displaystyle\frac{1}{2}\int \frac{du}{u^{\frac{3}{2}}} = -\frac{1}{2}\left(\frac{-2}{\sqrt{u}}\right)$

regresando a la variable

$$-\frac{1}{\sqrt{x^2+2}}\bigg|_0^1 = \frac{1}{3}$$

8. $\displaystyle\int_0^1 \frac{t}{\sqrt{1-t}}\,dt$

cambio de variable $u = 1-t$ $du = -dx$ $t = 1-u$ $\displaystyle -\int_0^1 \frac{(1-u)}{\sqrt{u}}du = -\int_0^1 \frac{du}{\sqrt{u}} + \int_0^1 \frac{udu}{\sqrt{u}} =$

$$-\int_0^1 \frac{du}{\sqrt{u}} + \int_0^1 \frac{udu}{\sqrt{u}}$$

$$-\int \frac{du}{\sqrt{u}} + \int \frac{udu}{\sqrt{u}} = -2\sqrt{u} + \frac{2u^{3/2}}{3}$$

$$-2\sqrt{1-t}|_0^1 + \frac{2(1-t)^{3/2}}{3}\bigg|_0^1 = -[0-2] + [0-\tfrac{2}{3}] = \frac{4}{3}$$

9. $\displaystyle\int_2^3 \frac{zdz}{(1+z)^{\frac{3}{4}}}$

realizando el cambio de variable $u = 1+z$ $du = dx$

$$\int \frac{udu}{u^{3/4}}du - \int \frac{du}{u^{3/4}} = \int u^{1/4}du - \int u^{-3/4}du = \frac{4}{5}u^{5/4} - 4u^{1/4}$$

$$\frac{4}{5}(1+z)^{5/4}\bigg|_2^3 - 4(1+z)^{1/4}|_2^3 = \frac{4}{5}(1+3)^{5/4} - \frac{4}{5}(1+2)^{5/4} - \big(4(1+3)^{1/4} - 4(1+2)^{1/4}\big)$$

$$= \frac{16(4)^{1/4}}{5} - \frac{12(3)^{1/4}}{5} - \big(4(4^{1/4} - 4(3)^{1/4}\big) \approx 1{,}3668 - 0{,}39255 \approx 0{,}9743$$

10. $\displaystyle\int_0^1 \frac{3x^2+2x}{\sqrt[5]{x^3+x^2}}dx$

por un cambio de variable $u = x^3+x^2$ $du = (3x^2+2x)dx$

$$\int \frac{du}{u^{\frac{1}{5}}} = \frac{5u^{4/5}}{4}$$

$$\frac{5(x^3+x^2)}{4}\bigg|_0^1 = \frac{5}{2}$$

11. $\displaystyle\int_{-1}^1 x\sqrt{x^2+1}\,dx$

sea $u = x^2+1$ $du = 2xdx$

$$\frac{1}{2}\int u^{1/2}du = \frac{u^{3/2}}{3}$$

$$\frac{(x^2+1)^{3/2}}{3}\bigg|_{-1}^1 = \frac{((1)^2+1)^{3/2)}}{3} - \frac{((-1)^2+1)^{3/2})}{3} = 0$$

12. $\displaystyle\int \frac{4x-2}{(x^2-x+1)^{\frac{1}{3}}}\,dx$

$$\int \frac{4x-2}{(x^2-x+1)^{\frac{1}{3}}}\,dx = 2\int \frac{2x-1}{(x^2-x+1)^{\frac{1}{3}}}\,dx$$

sea $u = x^2-x$ $du = (2x-1)dx$

$$2\int \frac{du}{u^{1/3}} = 2\cdot\frac{3u^{2/3}}{2} = 3u^{2/3} = 3(x^2-x)^{2/3}$$

13. $\int \operatorname{sen}^5 x\, dx$

$\int (\cos^2 x - 1)^2 \operatorname{sen} x dx$

$\int \cos^4 x \operatorname{sen} x dx - 2\int \cos^2 x \operatorname{sen} x dx + \int \operatorname{sen} x dx$

$-\frac{1}{5}\cos^5 x + \frac{2\cos^3 x}{3} - \cos x$

14. $\int (x^2 - 4x + 4)^{\frac{2}{3}}\, dx$

$\int (x^2 - 4x + 4)^{\frac{2}{3}}\, dx = \int ((x-2)^2)^{\frac{2}{3}}\, dx = \int ((x-2))^{\frac{4}{3}}\, dx$

dado $u = x - 2 \qquad du = dx$

$\int u^{4/3} du = \frac{3u^{7/3}}{7} = \frac{3(x-2)^{7/3}}{7}$

15. $\int \frac{x}{1+x^4}\, dx$

$\int \frac{x}{1+x^4} = \frac{1}{2}\int \frac{2x}{1+(x^2)^2} = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}ArcTanu = \frac{1}{2}ArcTanx^2$

16. Sin realizar ningún cálculo, hallar

a $\int_{-1}^{1} x^3\sqrt{1-x^2}dx$

sea $u = 1 - x^2$ observe cuando remplacemos u y evaluemos en 1, y -1 sera 0

$1-(1)^2 = 0 \qquad 1-(-1)^2 = 0$ Por lo tanto :

$\int_{-1}^{1} x^3\sqrt{1-x^2}dx = 0$

b $\int_{-1}^{1} (x^5+3)\sqrt{1-x^2}dx$

17. demuestre que los valores de la siguiente expresión no depende de x

$$\int_{-\cos x}^{\operatorname{sen} x} \frac{dt}{\sqrt{1-t^2}},\ x \in (0, \pi/2)$$

$f = \int_{-\cos x}^{\operatorname{sen} x} \frac{dt}{\sqrt{1-t^2}}$

$f' = \frac{dt}{\sqrt{1-\operatorname{sen}^2 x}} - \frac{-\operatorname{sen} x}{\sqrt{1-\cos^2 x}} = \frac{\cos x}{\sqrt{\cos^2 x}} + \frac{\operatorname{sen} x}{\sqrt{\operatorname{sen}^2 x}} = \frac{\cos x}{\cos x} + \frac{\operatorname{sen} x}{\operatorname{sen} x} = 1 + 1 = 2$

18. Encuentre una función g tal que

a. $\int_0^x tg(t)dt = x + x^2$ **b.** $\int_0^{x^2} tg(t)dt = x + x^2$

Sug. Observe que g no tiene que ser continua en 0.

$\int_0^x t\cdot g(t) = x + x^2$

$x \cdot g(x) = 1 + 2x$

$g(x) = \frac{1}{x} + 2$

g esta definida en $\mathbb{R}/\ \{0\}$

$\int_0^{x^2} t\cdot g(t) = x + x^2$

$x^2 \cdot g(x) = 1 + 2x$

$g(x) = \frac{1}{x} + \frac{2}{x}$

g esta definida en $\mathbb{R}/\ \{0\}$

En los ejercicios $19-22$, supongamos que f es continua en $[a,b]$ y que $\int_a^b f(x)dx = 0$

19. ¿Se deduce necesariamente que $f(x)=0,\ \forall x \in [a,b]$? No, se observa que si se toma un intervalo $(-c,c)$ y la función es impar.

20. ¿Se deduce necesariamente que $\exists x_0 \in [a,b]$ tal que $f(x_0)=0$? si
dado $f(x_0) \neq 0$ para todo $x \in [a,b]$ si es continua $f(x_0) > 0$ para todo $x \in [a,b]$ o también $f(x_0) < 0$ en cualquier caso $\displaystyle\int_a^b f(x)dx \neq 0$

21. ¿Se deduce necesariamente que $\displaystyle\int_a^b |f(x)|dx = 0$?

    No, si $a=-b$ tenemos:
    $\displaystyle\int_{-b}^b xdx = 0$ por otra parte $\displaystyle\int_a^b |x|dx \neq 0$

22. Se deduce necesariamente que $\left|\int_a^b f(x)dx\right| = 0$?
    si, Dado a que $f(x)=0 \rightarrow \left|\displaystyle\int_a^b f(x)\right| = |0| = 0$

23. Una varilla de longitud L está colocada sobre el eje x desde $x=0$ hasta $x=L$. Hallar la masa de la varilla y su centro de masa si la densidad de la varilla varía de manera directamente proporcional (a) a la raíz cuadrada de la distancia a $x=0$ y (b) al cuadrado de la distancia a $x=L$
    su masa es $M = \displaystyle\int_0^L k\sqrt{x}dx = k\left[\frac{2}{3}x^{3/2}\right]_0^L = \frac{2kL^{3/2}}{3}$

    y el centro de masa esta dado por:
    $$x_m = \frac{1}{M}\int_0^L x(k\sqrt{x})dx = \frac{1}{\frac{2kL^{3/2}}{3}}\int_0^L kx^{3/2}dx = \frac{1}{\frac{2kL^{3/2}}{3}}\left[\frac{2kx^{5/2}}{5}\right]_0^L$$
    $$x_M = \frac{\frac{2}{5}kL^{5/2}}{\frac{2}{3}kx^{3/2}} = \frac{1}{4}L$$

24. Una varilla de masa M, que se extiende desde $x=0$ hasta $x=L$, está formada por dos partes de masas M_1 y M_2. Sabiendo que el centro de masa de la varilla completa está situado en $x=L/4$ y que el centro de masa de la primera parte está situado en $x=L/8$, determinar el centro de masa de la segunda parte.
    $$\frac{1}{4}LM = \frac{1}{8}LM_1 + x_{M_2}M_2$$
    $$x_{M_2} = \frac{1}{M_2}\left(\frac{1}{4}LM - \frac{1}{8}LM_1\right) = \frac{L(2M - M_1}{8M_2}$$

6. Tarea 6

En los ejercicios del $1-12$ calcule las derivadas de las funciónes dadas

1. $f(x)=ln(\sqrt{3x-1}) \qquad f'(x)\dfrac{1}{\sqrt{3x-1}}(\sqrt{3x-1})'=\dfrac{\frac{3}{2\sqrt{3x-1}}}{\sqrt{3x-1}}=\dfrac{3}{6x-2}$

2. $f(t)=t\ln(t^2) \qquad f'(t)=t\dfrac{2t}{t^2}+\ln(t^2)=2+\ln(t^2)$

3. $f(x)=\ln\left(\dfrac{\operatorname{sen}x}{x}\right) \qquad f'(x)=\dfrac{1}{\frac{\operatorname{sen}x}{x}}\left(\dfrac{x\cos x-\operatorname{sen}x}{x^2}\right)=\dfrac{\cos x}{\operatorname{sen}x}-\dfrac{1}{x^2}=\cot x-\dfrac{1}{x^2}$

4. $h(t)=t^2\ln(\cos t) \qquad h'(t)=2t\ln(\cos t)-t^2\dfrac{sent}{\cos t}=2t\ln(\cos t)-t^2\tan t$

5. $g(t)=t(\ln t)^2 \qquad g'(t)=(\ln t)^2+t\ln t\left(\dfrac{1}{t}\right)=\ln t(\ln t+1)$

6. $f(x)=\ln\left(\dfrac{x+1}{x-1}\right) \qquad \dfrac{1}{\frac{x+1}{x-1}}\left(\dfrac{x+1}{x-1}\right)'=\dfrac{1}{\frac{x+1}{x-1}}\left(\dfrac{(x-1)-(x+1)}{(x-1)^2}\right)=\dfrac{\frac{-2}{(x-1)^2}}{\frac{x+1}{x-1}}=\dfrac{2}{(x-1)(x+1)}=$
$\dfrac{2}{x^2-1}$

7. $g(t)=\ln\left(\dfrac{t^2}{t^2+1}\right)$
$g(t)=\ln\left(\dfrac{t^2}{t^2+1}\right) \qquad g'(t)=\dfrac{1}{\frac{t^2}{t^2+1}}\left[\dfrac{2t(t^2+1)-t^2(2t)}{(t^2+1)^2}\right]=\dfrac{2}{t(t^2+1)}$

8. $y=x^x \qquad \dfrac{dy}{dx}=x^{x-1}$

9. $f(x)=\ln(x\sqrt{x^2+1}) \qquad f'(x)=\dfrac{1}{x\sqrt{x^2+1}}\left(x\sqrt{x^2+1}\right)'=\dfrac{1}{x\sqrt{x^2+1}}\left(\sqrt{x^2+1}+x\dfrac{x}{\sqrt{x^2+1}}\right)=$
$\dfrac{\sqrt{x^2+1}+\frac{x^2}{\sqrt{x^2+1}}}{x\sqrt{x^2+1}}=\dfrac{\sqrt{x^2+1}\left(1+\frac{x^2}{x^2+1}\right)}{x\sqrt{x^2+1}}=\dfrac{\frac{2x^2+1}{x^2+1}}{x}=\dfrac{2x^2+1}{x(x^2+1)}$

10. $y=(1+\cos x)^{\operatorname{sen}x}$
$\ln y=\ln(1+\cos x)^{\operatorname{sen}x}$
$\ln y=\operatorname{sen}x\ln(1+\cos x)$
$e^{\ln y}=e^{\operatorname{sen}x\ln(1+\cos x)}$
$\dfrac{dy}{dx}=e^{\operatorname{sen}x\ln(1+\cos x)}\left[\cos x\left(\dfrac{senx}{1+\cos x}\right)+\ln(1+\cos x)\cos x\right]$

11. $y=(1+x)^{\frac{1}{x}}$
$\ln y=\ln(1+x)^{\frac{1}{x}}$
$\ln y=\left(\dfrac{1}{x}\right)\ln(1+x)$
$e^{\ln y}=e^{\left(\frac{\ln(1+x)}{x}\right.}$
$\dfrac{dy}{dx}=e^{\left(\frac{\ln(1+x)}{x}\right)}\left(\dfrac{\frac{x}{1+x}-\ln(1+x)}{x^2}\right)$

12. $y=\left(1+\dfrac{1}{x}\right)^x$
$\ln y=\ln\left(1+\dfrac{1}{x}\right)^x$
$\ln y=x\ln\left(1+\dfrac{1}{x}\right)$

$$y = e^{x\ln\left(1+\frac{1}{x}\right)}$$

$$\frac{dy}{dx} = e^{x\ln\left(1+\frac{1}{x}\right)}\left[\ln\left(1+\frac{1}{x}\right) - \frac{1}{\frac{1}{x}+1}\right]$$

En los ejercicios del $13-29$ evalué las integrales indefinidas

13. $\displaystyle\int \frac{dx}{3x+2}$

$$\int \frac{dx}{3x+2} = \frac{1}{3}\int \frac{3dx}{3x+2} = \ln(3x+2)+C$$

14. $\displaystyle\int \frac{dx}{1+3x^2}$

$$\int \frac{x}{1+3x^2}dx = \frac{1}{6}\int \frac{6xdx}{1+3x^2} = \frac{1}{6}\ln(1+3x^2)+C$$

15. $\displaystyle\int \frac{x^2}{4-x^3}$

$$\int \frac{x^2}{4-x^3} = -\frac{1}{3}\int \frac{-3x^2}{4-x^3} = -\frac{1}{3}\ln(4-x^3)+C$$

16. $\displaystyle\int \frac{(1+x)dx}{2x^2+4x+1}$

$$\int \frac{(1+x)dx}{2x^2+4x+1} = \frac{1}{4}\int \frac{(4x+4)dx}{2x^2+4x+1} = \frac{1}{4}\ln(2x^2+4x+1)*C$$

17. $\displaystyle\int \frac{\cos x dx}{1+\operatorname{sen} x}$

$$\int \frac{\cos x dx}{1+\operatorname{sen} x} = \int \frac{\cos x dx}{1+\operatorname{sen} x} = \ln(1+\operatorname{sen} x)+C$$

18. $\displaystyle\int \frac{(\ln x)^2}{x} \quad \int \frac{(\ln x)^2}{x} = \int (\ln x)^2\left(\frac{1}{x}\right)dx = \frac{1}{3}(\ln x)^3+C$

19. $\displaystyle\int \frac{dx}{x\ln x}$

$$\int \frac{dx}{x\ln x} = \int \frac{du}{u} = \ln u + C = \ln(\ln x)+C$$

20. $\displaystyle\int \frac{1}{1+x}dx$

$$\int \frac{1}{1+x}dx = \ln(1+x)+C$$

21. $\displaystyle\int \frac{x}{1-x^2}dx$

$$\int \frac{x}{1-x^2}dx = -\frac{1}{2}\int \frac{-2x}{1-x^2}dx = -\frac{1}{2}\ln(1-x^2)+C$$

22. $\displaystyle\int \frac{2x+1}{x^2+x+1} \quad \int \frac{2x+1}{x^2+x+1} = \ln(x^2+x+1)+C$

23. $\displaystyle\int \frac{x+1}{x^2+2x+3} \quad \int \frac{x+1}{x^2+2x+3} = \frac{1}{2}\int \frac{2x+2}{x^2+2x+3} = \frac{1}{2}\ln(x^2+2x+3)+C$

24. $\displaystyle\int \frac{\ln x}{x}dx \quad \int \frac{\ln x}{x}dx = \int \ln x \cdot \frac{1}{x}dx = \int udu = \frac{1}{2}u^2+C = \frac{1}{2}(\ln x)^2+C$

25. $\displaystyle\int \frac{\ln(x^3)dx}{x}$

$$\int \frac{\ln(x^3)dx}{x} = 3\int \frac{\ln(x)dx}{x} = 3\int \ln x \cdot \frac{dx}{x} = \frac{3}{2}(\ln x)^2+C$$

26. $\displaystyle\int \frac{\operatorname{sen}(2x)}{1+\cos(2x)}dx// \int \frac{\operatorname{sen}(2x)}{1+\cos(2x)}dx = -\frac{1}{2}\int \frac{-2\operatorname{sen}(2x)dx}{1+\cos(2x)} = -\frac{1}{2}\ln(1+\cos(2x))+C$

27. $\displaystyle\int \frac{1}{x(\ln x)^2}dx$

$$\int \frac{1}{x(\ln x)^2}dx = \int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\ln x} + C$$

28. $\displaystyle\int \frac{x^2-2x}{x^3-3x^2+1}dx$

$$\int \frac{x^2-2x}{x^3-3x^2+1}dx = \frac{1}{3}\int \frac{3x^2-6x}{x^3-3x^2+1}dx = \frac{1}{3}\ln(x^3-3x^2+1) + C$$

29 $\displaystyle\int \frac{dx}{x^{1/2}(1+x^{1/2})}$

$$\int \frac{dx}{x^{1/2}(1+x^{1/2})} = 2\int \frac{1}{(1+x^{1/2})}\left(\frac{dx}{2\sqrt{x}}\right) = 2\int \frac{du}{u} = 2\ln u + C = 2\ln(1+x^{1/2}) + C$$

En los ejercicios del $30-35$ Calcule los limites

30. $\displaystyle\lim_{x\to\infty} \frac{\ln(x^{1/2})}{x} \ \frac{1}{2}\lim_{x\to\infty}\frac{\ln x}{x} = \frac{1}{2}\lim_{x\to\infty}\frac{1}{x} = 0$

31. $\displaystyle\lim_{x\to\infty} \frac{\ln(x^3)}{x^2} \ \frac{3}{2}\lim_{x\to\infty}\frac{1}{x^2} = 0$

32. $\displaystyle\lim_{x\to\infty} \frac{\ln x}{x^{1/2}}$ sug. $x = u^2$ $\displaystyle\lim_{x\to\infty}\frac{\ln u^2}{u} = 2\lim_{x\to\infty}\frac{\ln u}{u} = 2\lim_{x\to\infty}\frac{1}{u} = 2\lim_{x\to\infty}\frac{1}{x^{1/2}} = 0$

33. $\displaystyle\lim_{x\to\infty} \frac{(\ln x)^2}{x}$

$$\lim_{x\to\infty}\frac{(\ln x)^2}{x} = 2\lim_{x\to\infty}\frac{\ln x}{x} = 2\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{x}} = 2\lim_{x\to\infty}\frac{1}{x^2} = 0$$

34. $\displaystyle\lim_{x\to 0^+} x^{1/2}\ln x$

$$\lim_{x\to 0^+}\frac{\ln x}{x^{-1/2}} = -\frac{\frac{1}{x}}{\frac{3x^{-3/2}}{2}} = -\lim_{x\to 0^+}\frac{2}{x^{-1/2}} = -2\lim_{x\to 0^+}\sqrt{x} = 0$$

35. $\displaystyle\lim_{x\to 0^+} x\ln x$ sug: $x = \dfrac{1}{u}$

$$\lim_{x\to 0^+}\frac{\ln(\frac{1}{u})}{u} = u = 0$$

7. Tarea 7

1. $f(x) = (e^x)^{\tan\frac{1}{x}}$
   $e^{x\tan\frac{1}{x}}\left[\tan\dfrac{1}{x} + x\sec^2\left(\dfrac{1}{x}\right)\left(\dfrac{1}{x^2}\right)\right] = e^{x\tan\frac{1}{x}}\left[\tan\dfrac{1}{x} + \dfrac{1}{x}\sec^2\left(\dfrac{1}{x}\right)\right]$

2. $y = e^{2x^2+\ln\sqrt{x}}$
   $$y' = e^{2x^2+\ln\sqrt{x}}\left[4x + \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}}\right] = e^{2x^2+\ln\sqrt{x}}\left[4x + \frac{1}{2x}\right] = (8x^2+1)e^{2x^2+\ln\sqrt{x}}$$

3. $f(x) = e^{x\,\mathrm{sen}\,3x}$
   $f'(x) = e^{x\,\mathrm{sen}\,3x}\,(3x\cos 3x + \mathrm{sen}\,3x)$

4. $f(x) = \displaystyle\int_1^{e^{\mathrm{sen}\,x}} \ln t\,dt$
   $f'(x) = \ln(e^{\mathrm{sen}\,x})\cdot e^{\mathrm{sen}\,x}\cos x = e^{\mathrm{sen}\,x}\,\mathrm{sen}\,x\cos x$

5. $f(x) = \exp\left(\displaystyle\int_1^{x\sqrt{1+x^2}} \ln t\,dt\right)$
   $$f'(x) = \exp\left(\int_1^{x\sqrt{1+x^2}} \ln t\,dt\right)\left[\int_1^{x\sqrt{1+x^2}} \ln t\,dt\right]'$$
   $$f'(x) = \exp\left(\int_1^{x\sqrt{1+x^2}} \ln t\,dt\right)\left[\ln(x\sqrt{1+x^2})\left(\sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}}\right)\right]$$
   $$f'(x) = \exp\left(\int_1^{x\sqrt{1+x^2}} \ln t\,dt\right)\left[\ln(x\sqrt{1+x^2})\left(\frac{1+2x^2}{\sqrt{1+x^2}}\right)\right]$$

6. $f(x) = e^x\cos e^x$
   $f'(x) = e^x\cos e^x - e^{2x}\,\mathrm{sen}\,e^x = e^x(\cos x - e^x\,\mathrm{sen}\,e^x)$

7. $f(x) = \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$
   $$f'(x) = \frac{(e^x-e^{-x})(e^x-e^{-x}) - (e^x-e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2} = -\frac{e^{2x}-e^{-2x}}{(e^x-e^{-x})^2}$$

8. $f(x) = \ln\left(\dfrac{e^x-e^{-x}}{2}\right)$
   $f'(x) = \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$

9. $f(x) = e^{\mathrm{sen}\,x\ln x}$
   $f'(x) = e^{\mathrm{sen}\,x\ln x}\left(\dfrac{\mathrm{sen}\,x}{x} + \cos x\ln x\right)$

En los ejercicios $10-21$, evalúe las integrales indefinidas.

10. $\displaystyle\int e^{2+x}\,dx \int e^{2+x}\,dx = e^{2+x} + C$

11. $\displaystyle\int e^{2x+1}\,dx\ \frac{1}{2}\int e^{2x+1}2\,dx = \frac{e^{2x+1}}{2} + C$

12 $\displaystyle\int \mathrm{sen}\,2xe^{\mathrm{sen}^2 x}\,dx$
   $$\int \mathrm{sen}\,2xe^{\frac{1-\cos 2x}{2}}\,dx = \frac{1}{4}\int 2\,\mathrm{sen}\,2xe^{\frac{1-\cos 2x}{2}}\,dx = \frac{1}{4}\int e^u\,du = \frac{e^u}{4} + C = \frac{e^{\frac{1-\cos 2x}{2}}}{4} + C = \frac{e^{sen^2 x}}{4} + C$$

13. $\displaystyle\int (1+\tan^2 x)e^{\tan x}\,dx$
   $$\int e^{\tan x}\sec^2 x\,dx = e^{\tan x} + C$$

14. $\int x^2 e^{3x^3+1}\,dx$

$\frac{1}{3}\int 3x^2 e^{3x^3+1}\,dx = e^{3x^3+1} + C$

15. $\int \frac{e^{2x}}{1+e^{2x}}\,dx$

$\int \frac{e^{2x}}{1+e^{2x}}\,dx = \frac{1}{2}\int \frac{2e^{2x}dx}{1+e^{2x}} = \frac{1}{2}\int \frac{du}{1+u} = \frac{1}{2}\ln|1+u| + C = \frac{1}{2}\ln|1+e^{2x}| + C$

16. $\int (\operatorname{sen} 2x)e^{1-\cos 2x}\,dx$

$\int (\operatorname{sen} 2x)e^{1-\cos 2x}\,dx = \frac{1}{2}\int 2(\operatorname{sen} 2x)e^{1-\cos 2x}\,dx$

sea $u = 1 - \cos 2x$ $\rightarrow$du=2sen $2x dx$

$\frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{1-\cos 2x} + C$

17. $\int \frac{x+e^{2x}}{x^2+e^{2x}}\,dx$

sea $u = x^2 + e^{2x}$ $\rightarrow$du=(2x+2e^{2x})dx

$\int \frac{x+e^{2x}}{x^2+e^{2x}}\,dx = \frac{1}{2}\int \frac{(2x+2e^{2x})dx}{x^2+e^{2x}} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + C = \frac{1}{2}\ln|x^2+e^{2x}| + c$

18. $\int te^{-t^2/2}\,dt$

sea $u = -\frac{t^2}{2} \quad \rightarrow \quad du = -tdt$

$-\int -te^{-t^2/2}\,dt = -\int e^u du = -e^u + C = -e^{-t^2/2}$

19. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

sea $u = \sqrt{x}$ $\rightarrow$du= $\frac{dx}{2\sqrt{x}}$

$2\int e^u du = 2e^u + C = 2e^{\sqrt{x}} + C$

20. $\int \frac{e^x}{1+e^x}\,dx$

$\int \frac{e^x}{1+e^x}\,dx = \ln e^x + C$

21. $\int \sqrt{x}\,e^{-\sqrt{x^3}}\,dx$

$\int \sqrt{x}e^{-x^{3/2}\ln(e)}dx$

$2\int u^2 e^{-u^3 \ln e}du$

$\frac{2}{3}\int e^{-\ln e u_1}du_1$

$\frac{2}{3\ ln(e)}\int e^{u_2}du_2$

$\frac{2}{3\ ln(e)}e^{u_2}$

$\frac{2}{3\ ln(e)}e^{-\ln(e)u_1}$

$\frac{2}{3\ ln(e)}e^{-u^3\ln(e)}$

$\frac{2}{3\ ln(e)}e^{-x^{3/2}\ln(e)}$

En los ejercicios $22-29$, calcule los límites

22. $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}$

por L'Hopital obtenemos

$\lim\limits_{x\to 0}e^x=e^0=1$

23. $\lim\limits_{x\to 0}\dfrac{e^{8x}-e^{5x}}{x}$

$\lim\limits_{x\to 0}\dfrac{8e^{8x}-5e^{5x}}{1}=8e^0-5e^0=3$

24. $\lim\limits_{x\to\infty}\dfrac{e^x}{x}$

$\lim\limits_{x\to\infty}\dfrac{e^x}{x}=\lim\limits_{x\to\infty}=e^x=e^\infty=\infty$

25. $\lim\limits_{x\to\infty}\dfrac{e^x}{x^{1/2}}$

$\lim\limits_{x\to\infty}2e^x\sqrt{x}$

$2\lim\limits_{x\to\infty}e^x\lim\limits_{x\to\infty}\sqrt{x}=\infty$

26. $\lim\limits_{x\to\infty}\dfrac{e^{x^{1/2}}}{x}$

$\lim\limits_{x\to\infty}\dfrac{1}{2\sqrt{x}}\cdot e^{\sqrt{x}}\qquad\dfrac{1}{2}\lim\limits_{x\to\infty}\dfrac{e^{\sqrt{x}}}{\sqrt{x}}=\infty$

27. $\lim\limits_{x\to\infty}x^2e^{-x}$

$2\lim\limits_{x\to\infty}xe^{-x}\qquad 2\lim\limits_{x\to\infty}e^{-x}=0$

28. $\lim\limits_{x\to\infty}\dfrac{e^x}{x^{12}+16x^8-3x^3}$

$\lim\limits_{x\to\infty}\dfrac{e^x}{x^{12}+16x^8-3x^3}=\lim\limits_{x\to\infty}\dfrac{e^x}{k}=\infty$ sea k Una constante

29. $\lim\limits_{x\to\infty}\dfrac{x^{15}-\sqrt{2}x^{11}+3x^3-x}{e^x}$

$\lim\limits_{x\to\infty}\dfrac{x^{15}-\sqrt{2}x^{11}+3x^3-x}{e^x}\lim\limits_{x\to\infty}\dfrac{15x^{14}-11\sqrt{2}x^{10}+9x^2-1}{e^x}=\lim\limits_{x\to\infty}\dfrac{k}{e^x}=0$ sea k una contaste

En los ejercicios $30-45$ encuentre la derivada de la función:

30. $y=10^x$

$\ln y=x\ln 10\qquad\dfrac{1}{y}=\ln 10\qquad y=\dfrac{1}{\ln 10}$

31. $y=2^{1/x^2}$

$\ln y=\dfrac{1}{x^2}\ln 2\qquad\dfrac{1}{y}=-\dfrac{1}{x^3}\ln 2\qquad y=-\dfrac{x^3}{\ln 2}$

32. $y=\dfrac{3^x}{4^x}$

$\ln y=\ln 3^x-\ln 4^x\qquad\ln y=x\ln 3-x\ln 4\qquad\dfrac{1}{y}=\ln 3-\ln 4$

$y=\dfrac{\ln 4}{\ln x}$

33. $y=7^{\cos x}$

$\ln y=\cos x\ln 7\qquad\dfrac{1}{y}=-\ln 7\,\mathrm{sen}\,x$

34. $y=2^{x\sqrt{x}}$

$\ln y=x\sqrt{x}\ln 2\qquad\dfrac{1}{y}=\ln 2\left(\dfrac{x}{2\sqrt{x}}+\sqrt{x}\right)\qquad y\dfrac{2\sqrt{x}}{3x\ln 2}$

35. $y=2^{\ln x}$

$\ln y=\ln 2\ln x\qquad\dfrac{1}{y}=\dfrac{\ln 2}{x}\qquad y=\dfrac{x}{\ln 2}$

36. $y = 17^x$

$\ln y = x \ln 17 \qquad \dfrac{1}{y} = \dfrac{\ln 17}{x} \qquad y = \dfrac{x}{\ln 17}$

37. $y = 10^{1/x}$ $\ln y = \dfrac{1}{x} \ln 10 \qquad \dfrac{1}{y} = -\dfrac{\ln 10}{x^2} \qquad y = -\dfrac{x^2}{\ln 10}$

38. $y = 2^{2^x}$ $\ln y = 2^x \ln 2 \qquad \ln(\ln y) = x \ln 2 \ln(\ln 2) \qquad \ln y = e^{x \ln 2} \ln 2$

$\dfrac{1}{y} = \ln^2 2e^{x \ln 2} \qquad y = \dfrac{1}{\ln^2 2e^{x \ln 2}}$

39. $y = \log_3 \sqrt{x^2+4}$

$y' = \dfrac{\ln 3}{\sqrt{x^2+4}} \left(\dfrac{x}{\sqrt{x^2+4}} \right) = \dfrac{x \ln 3}{x^2+4}$

40. $y = \log_3(2^x)$

$y = x \log_3 2 \qquad y' = \log_3 2$

41. $y = \log_2(\log_3 x)$

$y' = \dfrac{\ln 2}{\log_3 x} \left(\dfrac{\ln 3}{x} \right)$

42. $y = \exp(\log_{10} x)$

$y' = \exp(\log_{10} x) \left(\dfrac{\ln 10}{x} \right)$

43. $y = \log_{10}(\log_{10} x)$

$y' = \dfrac{\ln 10}{\log_{10} x} \left(\dfrac{\ln 10}{x} \right)$

44. $y = \pi^x + x^\pi + \pi^\pi$

$\dfrac{d}{dx} e^{x \ln(\pi)} + \pi x^{\pi-1}$

$e^{x \ln(\pi)} \ln(\pi) + \pi x^{\pi-1}$

45. $y = \pi^{x^3}$

$\ln y = x^3 \ln \pi$

$\dfrac{1}{y} = 3 \ln(\pi) x^2$

$y = \dfrac{1}{3 \ln(\pi) x^2}$

En los ejercicios $46 - 53$ evalúe las integrales dadas

46. $\displaystyle\int 3^{2x}\, dx$

$\ln 3 \displaystyle\int 2x = \ln 3x^2 + C$

47. $\displaystyle\int x(10^{-x^2})\, dx \quad \int x e^{-x^2 \ln(10)} dx = \frac{1}{2} \int 2x e^{-\ln(10) x^2} dx = \frac{1}{2} \int e^{-\ln(10) u} du = \frac{1}{2 \ln 10} \int e^u du = \frac{e^u}{2} \ln 10 =$ $\dfrac{e^{x^2 \ln(10)}}{2 \ln 10} + C$

48. $\displaystyle\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx$

$\displaystyle\int \frac{e^{\sqrt{x} \ln(2)}}{\sqrt{x}} dx = 2 \int \frac{e^{\sqrt{x} \ln(2)}}{2\sqrt{x}} dx = 2 \int e^{u \ln(2)} du = \frac{2}{\ln 2} \int e^{u_1} du_1 = \frac{2e^{u_1}}{\ln 2} = \frac{2e^{u \ln(2)}}{\ln 2} = \frac{2e^{\sqrt{x} \ln(2)}}{\ln 2} + C$

49. $\displaystyle\int \frac{10^{1/x}}{x^2}\, dx \quad \int \frac{e^{\frac{\ln 10}{x}}}{x^2}\, dx = -\int e^{\ln 10 u} du = -\frac{1}{\ln 10} \int e^{u_1} du_1 = -\frac{e^{u_1}}{\ln 10} = -\frac{e^{\ln 10 u}}{\ln 10} = -\frac{e^{\frac{\ln 10}{x}}}{\ln 10} + C$

50. $\displaystyle\int x^2 7^{x^3+1}\, dx$

$\displaystyle\int x^2 e^{(x^3+1) \ln 7}\, dx = \frac{7}{3} \int e^{\ln 7 u}\, du = \frac{7}{3 \ln(7)} \int e^{u_1} du_1 = \frac{7e^{u_1}}{3 \ln(7)} = \frac{7e^{u \ln(7)}}{3 \ln(7)} = \frac{7e^{x^3 \ln(7)}}{3 \ln(7)} + C$

51. $\displaystyle\int \frac{dx}{x \log_{10} x}$

$\displaystyle\int \frac{\ln(10)}{x \ln x} = \ln(10) \frac{1}{u} du = \ln(10) \ln u = \ln(10) \ln(\ln x) + C$

52. $\int \frac{\log_2 x}{x}\, dx$

$\int \frac{\ln x}{\ln(2)x} dx = \frac{1}{\ln(2)} \int u du = \frac{\ln(x)^2}{2\ln(2)} + C$

53. $\int (2^x) 3^{(2^x)}\, dx$

$\int e^{\ln(2)x} e^{e^{\ln(2)x}\ln(3)} dx = \int e^u e^{e^u \ln(3)} du = \frac{1}{\ln(2)} \int e^{u_1 \ln(3)} du = \frac{1}{\ln(2)\ln(3)} e^{u_2} du_2 = \frac{e^{u_2}}{\ln(2)\ln(3)} + C =$

En los ejercicios $54 - 66$ demuestre las afirmaciones

54. $\cosh^2 x - \operatorname{senh}^2 x = 1$

Demostración:

$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} - \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} = \frac{e^{2x} + 2 + e^{-2x} - (e^{2x} - 2 + e^{-2x})}{4} = \frac{4}{4} = 1$

55. $1 - \tanh^2 x = \sec h^2 x$

Demostración:

$\frac{e^x + e^{-x}}{e^x + e^{-x}} - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2 = \frac{e^{2x} + 2 + e^{-2x} - (e^{2x} - 2 + e^{-x})}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2} = \left(\frac{2}{e^x + e^{-x}}\right)^2 = \sec h^2 x$

56. $\coth^2 x - 1 = \csc h^2 x$

Demostración:

$\left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right)^2 - \frac{e^x - e^{-x}}{e^x - e^{-x}} = \frac{e^{2x} + 2 + e^{-2x} - (e^{2x} - 2 + e^{-x})}{(e^x - e^{-x})^2} = \frac{4}{(e^x - e^{-x})^2} = \left(\frac{2}{e^x - e^{-x}}\right)^2 = \sec h^2 x$

57. $\operatorname{senh} 2x = 2 \operatorname{senh} x \cosh x$

Demostración:

En $\operatorname{senh}(x + x) = \operatorname{senh} x \cosh x + \operatorname{senh} x \cosh x = 2 \operatorname{senh} x \cosh x$

58. $\cosh 2x = \cosh^2 x + \operatorname{senh}^2 x$

Demostración:

$\left(\frac{e^x + e^{-x}}{2}\right)^2 + \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x} + e^{2x} - 2 + e^{-2x}}{4} = \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x$

59. $\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$

Demostración:

$\left(\frac{e^x + e^x}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4} = \frac{1}{2}\left(\frac{e^{2x} + e^{2x}}{2} + 1\right) = \frac{1}{2}(\cosh 2x + 1)$

60. $\operatorname{senh}^2 x = \frac{1}{2}(\cosh 2x - 1)$

Demostración:

$\left(\frac{e^x - e^x}{2}\right)^2 = \frac{e^{2x} - 2 + e^{-2x}}{4} = \frac{1}{2}\left(\frac{e^{2x} + e^{2x}}{2} - 1\right) = \frac{1}{2}(\cosh 2x - 1)$

61. $\operatorname{senh}(x + y) = \operatorname{senh} x \cosh y + \cosh x \operatorname{senh} y$

Demostración:

$\left(\frac{e^x - e^{-x}x}{2}\right)\left(\frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^y - e^{-y}}{2}\right)$
$= \frac{e^{x+y} + e^{x-y} - e^{y-x} - e^{-x-y} + e^{x+y} - e^{x-y} + e^{y-x} - e^{-x-y}}{4} = \frac{2(e^{x+y} - e^{-(x+y)})}{4} = \frac{e^{x+y} - e^{-(x+y)}}{2} =$
$\operatorname{sen} h(x + y)$

62. $\cosh(x + y) = \cosh x \cosh y + \operatorname{senh} x \operatorname{senh} y$

Demostración:

$\left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^y - e^{-y}}{2}\right)$
$= \frac{e^{x+y} + e^{x-y} + e^{y-x} + e^{-x-y} + e^{x+y} - e^{x-y} - e^{y-x} + e^{-x-y}}{4} = \frac{2(e^{x+y} + e^{-(x+y)})}{4} = \frac{e^{x+y} + e^{-(x+y)}}{2} =$
$\cosh(x + y)$

63. $D_x \cosh x = \operatorname{senh} x$

Demostración:

tenemos que el coseno hiperbolico en terminos de la exponencial es: $\frac{e^x + e^{-x}}{2}$

entonces:
$\dfrac{d}{dx}\left(\dfrac{e^x+e^{-x}}{2}\right)=\dfrac{e^x-e^{-x}}{2}$

y sabemos que el seno hiperbolico en terminos de la exponencial es $\dfrac{e^x-e^{-x}}{2}=\operatorname{senh} x$

64. $D_x \tanh x = \sec h^2 x$
Demostración:
Sabemos que la tangente hiperbólica lo podemos expresar en términos de exponencial
$\dfrac{d}{dx}\left(\dfrac{e^x-e^{-x}}{e^x+e^{-x}}\right)=\dfrac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}=\dfrac{e^{2x}+2+e^{-2x}-(e^{2x}-2+e^{-2x})}{(e^x+e^{-x})^2}=$
$\dfrac{4}{(e^x+e^{-x})^2}=\left(\dfrac{2}{e^x+e^{-x}}\right)^2=\sec h^2 x$

65. $D_x \coth x = -\csc h^2 x$
Demostración:
Sabemos que la cotangente hiperbólica lo podemos expresar en términos de exponencial
$\dfrac{d}{dx}\left(\dfrac{e^x+e^{-x}}{e^x-e^{-x}}\right)=\dfrac{(e^x-e^{-x})(e^x-e^{-x})-(e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}=\dfrac{e^{2x}-2+e^{-2x}-(e^{2x}+2+e^{-2x})}{(e^x-e^{-x})^2}=$
$-\dfrac{4}{(e^x-e^{-x})^2}=-\left(\dfrac{2}{e^x-e^{-x}}\right)^2=\cot h^2 x$

66. $D_x \sec hx = -\sec hx \tanh x$ $\dfrac{d}{dx}\left(\dfrac{2}{e^x+e^{-x}}\right)=-2\left(\dfrac{e^x-e^{-x}}{(e^x+e^{-x})2}\right)=\left(\dfrac{-2}{e^x+e^{-x}}\right)\left(\dfrac{e^x-e^{-x}}{e^x+e^{-x}}\right)=-\sec hx \tanh x$

En los ejercicios $67-74$, hallar las derivadas de las funciones

67. $y=\cosh(3x-2)$
$y'=3\operatorname{senh}(3x-2)$

68. $y=\operatorname{senh}\sqrt{x}$
$y'=\dfrac{\cosh(\sqrt{x})}{2\sqrt{x}}$

69. $y=x^2\tanh\dfrac{1}{x}$
$y'=2x\tanh\dfrac{1}{x}+sec^2\dfrac{1}{x}$

70. $y=\coth^3 4x$
$y'=12\coth^2 4x(\csc h^2 4x)$

71. $y=e^{\csc hx}$
$y'=e^{\csc hx}(-\csc hx \coth x)$

72. $y=\operatorname{sen}(\operatorname{senh} x)$
$y'=\cos(\operatorname{senh} x)\cosh x$

73. $y=\operatorname{senh} x^4$
$y'=4x^3\cosh x^4$

74. $y=\dfrac{1}{x+\tanh x}$
$y'=\dfrac{1+\sec hx}{(x+\tanh x)^2}$

En los ejercicios $75-85$ hallar las integrales

76. $\displaystyle\int x\operatorname{senh} x^2\,dx$
$\displaystyle\frac{1}{2}\int 2x\operatorname{senh} x^2\,dx=\frac{1}{2}\cosh x^2+C$

77. $\displaystyle\int \cosh^2 3u\,du \quad \frac{1}{3}\int 3\cosh^2 3u\frac{1}{3}\frac{\cosh^3 3u}{3}+C$

78. $\displaystyle\int\frac{\operatorname{senh} x}{\cosh^3 x}dx \quad \int\frac{\operatorname{senh} x}{\cosh^3 x}dx=\int\frac{du}{u^3}=-\frac{1}{2u^{-2}}+C=-\frac{1}{2\cosh^2 x}+C$

79. $\int \operatorname{senh}^4 x\, dx$

$\int \left(\frac{\cosh(2x)-1}{2}\right)^2 dx$

$= \frac{1}{4}\int \cosh^2(2x)dx - \frac{1}{4}\int \cosh(2x)dx + \frac{1}{4}\int dx$

$= \frac{1}{8}\int \cosh^2(u)du - \frac{1}{16}\int du - \frac{1}{8}\cosh(u)du + \frac{1}{4}\int dx$

$\frac{1}{16}\int \cosh(2u)du - \frac{1}{16}\int du - \frac{1}{8}\operatorname{senh}(2x) + \frac{1}{4}x$

$\frac{1}{32}\operatorname{senh}(2u) + \frac{1}{8}x - \frac{1}{8}\operatorname{senh}(2x) + \frac{1}{4}x + C$

$\frac{1}{32}\operatorname{sen}(4x) + \frac{3}{8}x - \frac{1}{8}\operatorname{senh}(2x) + C$

80. $\int \cot hx \csc h^2 x\, dx$

$\int \csc hx(-\cot hx \csc hx\, dx) = -\int udu = \frac{u^2}{2} + C = \frac{(\csc hx)^2}{2} + C$

81. $\int \sec hx\, dx$

$\int \frac{\sec h^2 x + \sec hx \tanh x}{\sec hx + \tanh x} dx = \int \frac{du}{u} = \ln u + C = \ln(\sec hx + \tanh x) + C$

82. $\int \frac{\operatorname{senh} x}{1+\cosh x}\, dx$

$\int \frac{\operatorname{senh} x dx}{1+\cosh x} = \int \frac{du}{u} = \ln u + C = \ln(1+\cosh x) + C$

83. $\int \frac{\operatorname{senh} \ln x}{x}\, dx$ $\int \operatorname{senh} u du = \cosh u + C = \cosh(\ln x) + C$

84. $\int \frac{1}{(e^x + e^{-x})^2}\, dx$

$\frac{1}{4}\int \frac{4}{(e^x+e^{-x})^2}\, dx = \frac{1}{4}\int \frac{1}{\left(\frac{e^x+e^{-x}}{2}\right)^2}\, dx = \frac{1}{4}\int \frac{1}{(\cosh x)^2} = \frac{1}{4}\int \sec h^2 x dx = \tanh x + C$

85. $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\, dx$

$\int \sec hx\, dx$

$\int \frac{\sec h^2 x + \sec hx \tanh x}{\sec hx + \tanh x} dx = \int \frac{du}{u} = \ln u + C = \ln(\sec hx + \tanh x) + C$

En los ejercicios $86 - 91$ hallar los límites.

86. $\lim_{x\to\infty} \operatorname{senh} x$

$\lim_{x\to\infty} (e^x - e^{-x}) \lim_{x\to\infty} e^x - \lim_{x\to\infty} \frac{1}{e^x} = \lim_{x\to\infty} e^x = \infty$

87. $\lim_{x\to-\infty} \operatorname{senh} x$ $\lim_{x\to-\infty} (e^x - e^{-x}) = \lim_{x\to-\infty} e^x - \lim_{x\to-\infty} \frac{1}{e^x} = 0 - \infty = -\infty$

88. $\lim_{x\to\infty} \cosh x$ $\lim_{x\to\infty} (e^x + e^{-x}) \lim_{x\to\infty} e^x + \lim_{x\to\infty} \frac{1}{e^x} = \lim_{x\to\infty} e^x = \infty$

89. $\lim_{x\to-\infty} \cosh x$ $\lim_{x\to-\infty} (e^x + e^{-x}) = \lim_{x\to-\infty} e^x + \lim_{x\to-\infty} \frac{1}{e^x} = 0 + \infty = +\infty$

90. $\lim_{x\to\infty} \tanh x$

91. $\lim_{x\to-\infty} \tanh x$

8. Tarea 8

1. $\int \operatorname{sen}^3 x\, dx$

   $-\int \cos^2 x \operatorname{sen} x\, dx + \int \operatorname{sen} x\, dx = \int u^2\, du + \int \operatorname{sen} x\, dx = \dfrac{\cos^3 x}{3} - \cos x + C$

2. $\int \operatorname{sen}^2 x \cos^3 x\, dx$

   $= \int \operatorname{sen}^2 (1 - \operatorname{sen}^2 x) \cos x dx$

   $= \int \operatorname{sen}^2 x \cos x dx - \int \operatorname{sen}^4 x \cos x dx$

   $= \dfrac{\operatorname{sen}^3 x}{3} - \dfrac{\operatorname{sen}^5 x}{5} + C$

3. $\int \cos^5 x\, dx = \int (\operatorname{sen}^2 - 1)^2 \cos x dx$

   $= \int \operatorname{sen}^4 x \cos x dx - 2 \int \operatorname{sen}^2 x \cos x + \int \cos x dx$

   $= \dfrac{\operatorname{sen}^5 x}{5} - \dfrac{2 \operatorname{sen}^3 x}{3} + \operatorname{sen} x + C$

4. $\int \dfrac{\operatorname{sen}^3 x}{\sqrt{\cos x}}\, dx$

   $\int \dfrac{(\cos^2 - 1)(\operatorname{sen} x dx)}{\sqrt{\cos x}} \qquad u = \cos x$

   $\int \dfrac{u^2 - 1}{\sqrt{u}} du \qquad v = \sqrt{u}$

   $2 \int v^4 dv - 2 \int dv$

   $\dfrac{2v^5}{5} - 2v + C$

   $\dfrac{2}{5u^{5/2}} - 2\sqrt{u} + C$

   $\dfrac{2 \cos^{5/2} x}{5} - 2\sqrt{\cos x} + C$

5. $\int \operatorname{sen}^5 2z \cos^2 2z\, dz$

   $= \dfrac{1}{2} \int \operatorname{sen}^5 u \cos^2 u du$

   $= \dfrac{1}{2} \int \cos^2 u (\cos^4 u - 2 \cos^2 u + 1) \operatorname{sen} u du$

   $= \dfrac{1}{2} \int \cos^6 u \operatorname{sen} u du - \int \cos^4 u \operatorname{sen} u du + \dfrac{1}{2} \int \cos^2 u \operatorname{sen} u du$

   $= -\dfrac{1}{2} \cdot \dfrac{\cos^7}{7} + \dfrac{\cos^5 u}{5} - \dfrac{1}{2} \cdot \dfrac{\cos^3 x}{3} + C$

   $= -\dfrac{\cos^7(2x)}{14} + \dfrac{\cos^5(2x)}{5} - \dfrac{\cos^3(2x)}{6} + C$

6. $\int \dfrac{\operatorname{sen}^3 4x}{\cos^2 4x}\, dx$

   $= \dfrac{1}{4} \int \dfrac{\operatorname{sen}^3 u du}{\cos^2 u}$

   $= \dfrac{1}{4} \int \dfrac{(\cos^2 u - 1) \operatorname{sen} u du}{\cos^2 u}$

   $= \dfrac{1}{4} \int \operatorname{sen} u du - \dfrac{1}{4} \int \dfrac{\operatorname{sen} u du}{\cos^2 u}$

   $= \dfrac{\cos u}{4} + \dfrac{1}{4 \cos u}$

   $= \dfrac{\cos(4x)}{4} + \dfrac{\sec(4x)}{4} + C$

7. $\int \sec^4 t\, dt$
$= \int \sec^2 x + \int \sec^2 x \tan^2 x dx$
$= \tan x + \dfrac{\tan^3 x}{3} + C$

8. $\int \cot^3 2x\, dx$
$= \dfrac{1}{2}\int \csc^2 u \cot u du - \dfrac{1}{2}\int \cot u du$
$= -\dfrac{1}{2}\int - \csc^2 \cot u du + \dfrac{1}{2}\int \dfrac{\cos u}{\operatorname{sen} u}$
$= \dfrac{1}{4}\csc^2 u - \dfrac{\ln(\operatorname{sen} u)}{2}$
$= \dfrac{\csc^2(2x)}{4} - \dfrac{\ln(\operatorname{sen}(2x))}{2} + C$

9. $\int \tan^5 2x \sec^2 2x\, dx$
$= \dfrac{1}{2}\int 2\tan^5 2x \sec^2 2x dx = \dfrac{1}{12}\tan^6(2x) + C$

10. $\int \csc^6 2t\, dt$
$\dfrac{1}{2}\int (1+^2)^2 \csc^2 u du = \dfrac{1}{2}\int \csc^2 u du + \int \csc^2 u \cot^2 u du + \dfrac{1}{2}\int \cot^4 u \csc^2 u du = -\dfrac{\cot u}{2} - \dfrac{\cot^3 u}{3} - \dfrac{1}{2}\cdot$
$\dfrac{\cot^5 u}{5} + C = -\dfrac{\cot(2x)}{2} - \dfrac{\cot^3(2x)}{3} - \dfrac{\cot^5(2x)}{5} + C$

11. $\int \dfrac{\tan^3\theta}{\sec^4\theta}\, d\theta$
$\int \operatorname{sen}^3\theta \cos\theta d\theta = \dfrac{\operatorname{sen}^4\theta}{4} + C$

12. $\int \dfrac{\tan^3 t}{\sqrt{\sec t}}\, dt$
$\int \dfrac{\operatorname{sen}^3 x}{\cos^{5/2} x} = \int \dfrac{(\cos^2 x - 1)\operatorname{sen} x dx}{cos^{5/2}} = \int \dfrac{u^2-1}{u^{5/2}} du = \int u^{-1/2} du - \int u^{-5/2} du = 2\sqrt{u} + \dfrac{2}{3u^{3/2}} + C =$
$2\sqrt{\cos x} + \dfrac{2\sqrt{\sec^3 x}}{3}$

13. $\int \dfrac{\cot\theta}{\csc^3\theta}\, d\theta$
$\int \dfrac{\frac{\cos x}{\operatorname{sen} x}}{\frac{1}{\operatorname{sen}^3 x}} = \int \operatorname{sen}^2 x \cos x dx = \dfrac{\operatorname{sen}^3 x}{3} + C$

14. $\int \cos^3 5t\, dt$
$-\dfrac{1}{5}\int \operatorname{sen}^2 u \cos u + \dfrac{1}{5}\int \cos u du = -\dfrac{\operatorname{sen}^3 u}{15} + \operatorname{sen} u + C = -\dfrac{\operatorname{sen}^3(5t}{15} + \operatorname{sen}(5t) + C$

15. $\int \cot^4 3t\, dt$
$\dfrac{1}{3}\int \cot^4 u du$
$\dfrac{1}{3}\int \csc^4 u - \dfrac{2}{3}\int \csc^2 u du + \dfrac{1}{3}\int du$
$\dfrac{1}{3}\int \csc^2 u du + \dfrac{1}{3}\int \csc^2 u \cot^2 u) du - \dfrac{2}{3}\int \csc^2 u du + \dfrac{1}{3}\int du$
$\dfrac{-\cot u}{3} - \dfrac{\cot^3 u}{9} + \dfrac{2\cot u}{3} + \dfrac{1}{3}u$
$\dfrac{\cot(3t)}{3} - \dfrac{\cot^3(3t)}{9} + x + C$

16. $\displaystyle\int \operatorname{sen}^5 2t \cos^{3/2} 2t\, dt$

17. $\displaystyle\int \operatorname{sen}^{3/2} x \cos^3 x\, dx \int \operatorname{sen}^{3/2} x(1{-}\operatorname{sen}^2 x) \cos x dx = -\int \operatorname{sen}^{7/2} x \cos x dx {+} \int \operatorname{sen}^{3/2} \cos x dx = -\frac{2 \operatorname{sen}^{9/2}}{9} +$
$\displaystyle\frac{2 \operatorname{sen}^{5/2}}{5} + c$

18. $\displaystyle\int \frac{\sec^4 t}{\tan^2 t}\, dt$

19. $\displaystyle\int \frac{\cot^3 \theta}{\csc^2 \theta}\, d\theta \int \frac{\frac{\cos^3 \theta}{\operatorname{sen}^3 \theta}}{\frac{1}{\operatorname{sen}^2 \theta}} d\theta = \int \frac{\cos^3 \theta}{\operatorname{sen} \theta} = \int \frac{(1 - \operatorname{sen}^2 \theta) \cos \theta d\theta}{\operatorname{sen} \theta} = \int \frac{\cos \theta d\theta}{\operatorname{sen} \theta} - \int \operatorname{sen} \theta \cos \theta d\theta = \ln |\operatorname{sen} \theta -$
$\displaystyle\frac{\operatorname{sen}^2 \theta}{2} + C$

20. $\displaystyle\int \cot^3 t \csc^{3/2} t\, dt$
$\displaystyle\int \left(\frac{\cos^3 t}{\operatorname{sen}^3 t}\right) \bigg(\frac{1}{\operatorname{sen}^{3/2} t} = \int \frac{\cos^3 t}{\operatorname{sen}^{9/2} t} dt = \int \cot^3 t \csc^{3/2} t = \int \csc^{3/2} (\csc^2 t - 1) \cot t dt = \int \csc^{7/2} t \cot t dt - \int \csc^{3/2} t \text{ c}$

21. $\displaystyle\int \frac{\tan x + \operatorname{sen} x}{\sec x}\, dx$
$\displaystyle\int \frac{\frac{\operatorname{sen} x + \cos x \operatorname{sen} x}{\cos x}}{\frac{1}{\cos x}} dx = \int \operatorname{sen} x (\cos x + 1) dx = \int \cos \operatorname{sen} x dx + \int \operatorname{sen} x dx = \frac{\cos^2 dx}{2} + \cos x + C$

22. $\displaystyle\int \frac{\cot x + \csc x}{\operatorname{sen} x}\, dt$
$\displaystyle\int \frac{\frac{\cos x}{\operatorname{sen} x} + \frac{1}{\operatorname{sen} x}}{\operatorname{sen} x} dx = \int \frac{1 + \cos x}{\operatorname{sen}^2 x} = \int \csc x \cot x dx + \csc^2 x dx = -\csc x - \cot x$

23. $\displaystyle\int \frac{\cot x + \csc^2 x}{1 - \cos^2 x}\, dx$
$\displaystyle\int \frac{\frac{cosx \operatorname{sen} x + 1}{\operatorname{sen}^2 x}}{\operatorname{sen}^2 x} dx = \int \frac{\cos x}{\operatorname{sen}^3 x} dx + \int \csc^4 x dx = \int \cot x \csc^2 x dx + \int \csc^2 (1 + \cot^2 x) dx = -\frac{\csc^2 x}{2} +$
$\displaystyle\int \csc^2 x dx + \int \cot^2 x \csc x dx = \frac{\cot^3 x}{3} - \frac{\csc^2 x}{2} + \cot x + C$

24. $\displaystyle\int \tan^2 2t \sec^4 2t\, dt$
$\displaystyle\frac{1}{2} \int \tan^2 u \sec^4 u du = \frac{1}{2} \int \tan^2 u (1 + \tan^2 u) \sec^2 u du = \frac{1}{2} \int \tan^4 u \sec^2 u du + \frac{1}{2} \int \tan^2 u \sec^2 u du =$
$\displaystyle\frac{\tan^5 u}{10} + \frac{\sec^3 u}{3} + C = \frac{\tan^5 (2t)}{10} + \frac{\sec^3 (2t)}{3} + C$

25. Deduzca una fórmula de reducción para $\displaystyle\int \cot^n x\, dx$

En los ejercicios $26 - 29$, use las identidades
$\operatorname{sen} A \operatorname{sen} B = \frac{1}{2} (\cos(A - B) - \cos(A + B))$
$\cos A \cos B = \frac{1}{2} (\cos(A - B) + \cos(A + B))$
$\operatorname{sen} A \cos B = \frac{1}{2} (\operatorname{sen}(A - B) + \operatorname{sen}(A + B))$

26. $\displaystyle\int \operatorname{sen} 3x \cos 5x\, dx$

27. $\displaystyle\int \operatorname{sen} 2x \operatorname{sen} 4x\, dx$

28. $\displaystyle\int \cos x \cos 4x\, dx$

29. $\displaystyle\int \operatorname{sen} 20x \cos 15x\, dx$

En los ejercicios $30 - 47$, use integración por partes para encontrar las integrales

30. $\displaystyle\int xe^{2x}\, dx$

sea $u = x \to du = dx \qquad dv = e^{2x}dx \to v = \dfrac{e^{2x}}{2}$

$= \dfrac{xe^{2x}}{2} - \dfrac{1}{2}\displaystyle\int e^{2x}dx = \dfrac{xe^{2x}}{2} - \dfrac{e^{2x}}{4} + C = \dfrac{e^{2x}}{2}\left(x - \dfrac{1}{2}\right) + C$

31. $\displaystyle\int t \operatorname{sen} t\, dt$

sea $u = t \to du = dt \qquad dv = sentdt \to v = -\cos t$

$= -x\cos t + \displaystyle\int \cos t dt = -x\cos t + \operatorname{sen} t + C$

32. $\displaystyle\int x\cos 3x\, dx$

sea: $u = x \to du = dx \qquad dv = \cos(3x)dx \to v = \dfrac{\operatorname{sen}(3t)}{3}$

$\dfrac{x\operatorname{sen}(3x)}{3} + \dfrac{1}{3}\displaystyle\int \operatorname{sen}(3x)dx = \dfrac{x\operatorname{sen}(3x)}{3} + \dfrac{\cos(3x)}{9}dt$

33. $\displaystyle\int x^3 \ln x\, dx$

$u = \ln x \to \dfrac{dx}{x} \qquad dv = x^3 \to v = \dfrac{x^4}{4}$

$= \dfrac{x^4 \ln x}{4} - \dfrac{1}{4}\displaystyle\int x^3 dx = \dfrac{x^4 \ln x}{4} - \dfrac{x^4}{16}$

34. $\displaystyle\int \tan^{-1} x\, dx$

$u = \tan^{-1} x \to du = \dfrac{dx}{1+x^2} \qquad dv = x \to v = \dfrac{x^2}{2}$

$\dfrac{x^2}{2}\tan^{-1} x - \dfrac{1}{4}\displaystyle\int \dfrac{2x}{1+x^2}$

$\dfrac{x^2}{2}\tan^{-1} x - \dfrac{1}{4}\ln|1+x^2| + C$

35. $\displaystyle\int y^{1/2} \ln y\, dy$ sea $u = \ln x \to du = \dfrac{1}{x}dx \qquad dv = \sqrt{x}dx \to v = \dfrac{2}{3}x^{3/2}$

$\dfrac{2}{3}x^{3/2}\ln x - \dfrac{2}{3}\displaystyle\int \sqrt{x}dx = \dfrac{2}{3}x^{3/2}\ln x - \dfrac{4}{9}$

36. $\displaystyle\int (\ln t)^2\, dt$

sea $u = \ln^2 x \to du = 2\ln(x)\dfrac{1}{x} \qquad dv = dx \to v = 1$

$x\ln^2 x - 2\displaystyle\int \ln x\left(\dfrac{1}{x}\right)xdx = x\ln^2 x - 2\displaystyle\int \ln x dx$

sea $u_1 = \ln x \to du = \dfrac{1}{x}dx \qquad dv = dx \to v = x$

$x\ln^2 x - 2\left[x\ln x - \displaystyle\int dx\right] = x\ln^2 x - 2x\ln x + 2x + C$

37. $\displaystyle\int x\sqrt{x+3}\, dx$

sea $u = x \to du = dx \qquad dv = \sqrt{x+3} \to v = \dfrac{2}{3}(x+3)^{3/2}$

$\dfrac{2x}{3}(x+3)^{3/2} - \dfrac{3}{2}\displaystyle\int (x+1)^{3/2}dx = \dfrac{2x}{3}(x+3)^{3/2} - \dfrac{4}{15}(x+1)^{5/2}dx + C$

38. $\int x^5\sqrt{x^3+1}\,dx$

$\frac{2x^5}{3}(x^3+1)^{5/2}-\frac{10}{3}\int(x^3+1)^{3/2}x^4dx$

$\frac{2x^5}{3}(x^3+1)^{5/2}-\frac{10}{3}\left[\frac{2x^4}{5}(x^3+1)^{5/2}-\frac{8}{5}\int(x^3+1)^{5/2}x^3dx\right]$

$\frac{2x^5}{3}(x^3+1)^{5/2}-\frac{10}{3}\left[\frac{2x^4}{5}(x^3+1)^{5/2}-\frac{8}{5}\left[\frac{2x^3}{7}(x^3+1)^{7/2}-\frac{6}{7}\int(x^3+1)^{7/2}x^2dx\right]\right]$

$\frac{2x^5}{3}(x^3+1)^{5/2}-\frac{10}{3}\left[\frac{2x^4}{5}(x^3+1)^{5/2}-\frac{8}{5}\left[\frac{2x^3}{7}(x^3+1)^{7/2}-\frac{6}{7}\left[\frac{2x^2}{9}(x+1)^{9/2}-\frac{4}{9}\left[\int(x^3+1)^{9/2}x^2dx\right]\right]\right]\right]$

$\frac{2x^5}{3}(x^3+1)^{5/2}-\frac{10}{3}\left[\frac{2x^4}{5}(x^3+1)^{5/2}-\frac{8}{5}\left[\frac{2x^3}{7}(x^3+1)^{7/2}-\frac{6}{7}\left[\frac{2x^2}{9}(x+1)^{9/2}-\frac{4}{9}\left[\frac{2x}{10}(x+1)^{11/2}-\right.\right.\right.\right.\right.$
$\frac{11}{2}\int(x+1)^{11/2}dx$

$\frac{2x^5}{3}(x^3+1)^{5/2}-\frac{10}{3}\left[\frac{2x^4}{5}(x^3+1)^{5/2}-\frac{8}{5}\left[\frac{2x^3}{7}(x^3+1)^{7/2}-\frac{6}{7}\left[\frac{2x^2}{9}(x+1)^{9/2}-\frac{4}{9}\left[\frac{2x}{10}(x+1)^{11/2}-\frac{11}{2}\right.\right.\right.\right.\right.$
$\left(\frac{2}{13}(x^3+1)^{13/2}\right)+C$

39. $\int \csc^3 t\,dt\ \int \csc\csc^2 t\,dt=$

sea $u=\csc t\to du=-\csc\cot x\qquad dv=\csc^2 x dx\to v=-\cot x$

$-\csc t\cot-\int\cot^2\csc t dt$

$=-\csc t\cot-\int\csc t(\csc^2+1)dt$

$=-\csc t\cot t+\int\csc-\int\csc^3 t dt$

$\int\csc^3 t\,dt=-\csc t\cot t+\ln|\csc t-\cot t|-\int\csc^3 t dt$

$2\int\csc^3 t\,dt=-\csc t\cot t+\ln|\csc t-\cot t|+C$

$\int\csc^3 t\,dt=\frac{-\csc t\cot t}{2}+\frac{\ln|\csc t-\cot t|}{2}+C$

40. $\int x^2\arctan x\,dx$

sea $u=\arctan x\to du=\frac{dx}{1+x^2}\qquad dv=x^2\to v=\frac{x^3}{3}$

$=\frac{x^3}{3}\arctan x-\frac{1}{3}\int\frac{x^3}{1+x^2}$

$=\frac{x^3}{3}\arctan x-\frac{1}{3}\int\frac{x^3+x-x}{1+x^2}=\frac{x^3}{3}\arctan x-\frac{1}{3}\int\frac{x(1+x^2)}{1+x^2}dx+\frac{1}{3}\int\frac{x)}{1+x^2}dx$

$=\frac{x^3}{3}\arctan x-\frac{1}{3}\int xdx+\frac{1}{6}\int\frac{2x)}{1+x^2}dx$

$\frac{x^3}{3}\arctan x-\frac{x^2}{6}+\ln|1+x^2|+C$

41. $\int\sec^{-1}\sqrt{x}\,dx$

sea $u=\sec^{-1}(\sqrt{x})\to du=\frac{\frac{1}{2\sqrt{x}}dx}{\sqrt{x}\sqrt{x-1}}=\frac{dx}{2x\sqrt{x-1}}\qquad dv=dx\to v=x$

$x\sec^{-1}(\sqrt{x})-\frac{1}{2}\int\frac{xdx}{x\sqrt{x-1}}$

$x\sec^{-1}(\sqrt{x})-\frac{1}{2}\int\frac{dx}{\sqrt{x-1}}$

$x\sec^{-1}(\sqrt{x})-\sqrt{x-1}+C$

42. $\displaystyle\int \tan^{-1}\sqrt{x}\,dx$

$u = \tan^{-1}\sqrt{x} \to du = \dfrac{dx}{2\sqrt{x}1+x)} \qquad dv = dx \to v = x$

$x\tan^{-1}\sqrt{x} - \dfrac{1}{2}\displaystyle\int \dfrac{\sqrt{x}dx}{1+x}dx$

sea : $u^2 = x \to 2udu = dx$

$u^2\tan^{-1}u - \dfrac{1}{2}\displaystyle\int \dfrac{2u^2du}{1+u^2}$

$u^2\tan^{-1}u - \displaystyle\int \dfrac{1+u^2}{1+u^2}du + \int \dfrac{du}{1+u^2}$

$u^2\tan^{-1}u - u + \tan^{-1}u + C$

$x\tan^{-1}(\sqrt{x}) - \sqrt{x} + \tan^{-1}\sqrt{x} + C$

43. $\displaystyle\int x\csc^2 x\,dx$

sea $u = x \to du = dx \qquad dv = \csc^2 xdx \to v = -\cot xdx$

$-x\cot x + \displaystyle\int \cot xdx$

$-x\cot x + \ln|\operatorname{sen} x| + C$

44. $\displaystyle\int x^3\cos x^2\,dx$

sea $u = x^2 \to du = 2x \qquad dv = x\cos x^2$

$\displaystyle\int x\cos x^2dx = \dfrac{1}{2}\int 2x\cos x^2dx = \dfrac{1}{2}\operatorname{sen} x^2 = v$

$\dfrac{x^2}{2}\operatorname{sen} x^2 - \dfrac{1}{2}\displaystyle\int \operatorname{sen} x^2 2xdx$

$\dfrac{x^2}{2}\operatorname{sen} x^2 - \dfrac{\cos x^2}{2} + C$

9. Tarea 9

En los ejercicios $1-22$ use sustituciones trigonométricas para hallar las integrales

1. $\displaystyle\int \frac{\sqrt{1-x^2}}{x^2}$

$\operatorname{sen}\theta = x \qquad dx = \cos\theta d\theta \qquad \cos\theta = \sqrt{1-x^2}$

$$\int \frac{\cos^2\theta d\theta}{\operatorname{sen}^2\theta} = \int \csc^2\theta d\theta - \int d\theta = -\cot\theta - \theta + C = \frac{\cos\theta}{\operatorname{sen}\theta} - \theta + C = -\frac{\sqrt{1-x^2}}{x} - \operatorname{arc}\cos(\sqrt{1-x^2}) + C$$

2. $\displaystyle\int \frac{\sqrt{x^2-1}}{x^2}dx$

$x = \sec\theta \qquad dx = \tan\theta\sec\theta d\theta \qquad \tan\theta = \sqrt{x^2-1}$

$$\int \frac{tan^2\theta\sec\theta d\theta}{\sec^2\theta} = \int \frac{\tan^2\theta}{\sec\theta}d\theta = \int \sec\theta d\theta - \int \cos\theta d\theta = \ln|\sec\theta + \tan\theta| - \operatorname{sen}\theta + C = \ln\left|x + \sqrt{x^2-1}\right| - \frac{\sqrt{x^2-1}}{x} + C$$

3. $\displaystyle\int x^3\sqrt{9+4x^2}dx$

$3\sec\theta = \sqrt{9+4x^2} \qquad \tan\theta = \dfrac{2x}{3} \quad x = \dfrac{3}{2}\tan\theta \qquad dx = \dfrac{3}{2}\sec^2\theta d\theta$

$$\frac{3^5}{2^3}\int \tan^3\theta\sec^3\theta d\theta$$

$$= \frac{3^5}{2^3}\int \sec^3\theta(\sec^2\theta + 1)\tan\theta d\theta$$

$$= \frac{3^5}{2^3}\left[\int \sec^2\theta(\sec\theta\tan\theta d\theta) + \int \sec^4\theta(\sec\theta\tan\theta d\theta)\right]$$

$$= \frac{3^5}{2^3}\left[\frac{\sec^3\theta}{3} + \frac{\sec^3\theta}{3}\right] + C$$

$$= \frac{3^5}{2^3}\left[\frac{\left(\dfrac{\sqrt{9+4x^2}}{3}\right)^3}{3} + \frac{\left(\dfrac{\sqrt{9+4x^2}}{3}\right)^5}{5}\right] + C$$

$$\frac{3\left(\sqrt{9+4x^2}\right)^3}{8} + \frac{\left(\sqrt{9+4x^2}\right)^5}{40}$$

4. $\displaystyle\int \frac{(1-4x^2)^{1/2}}{x}dx$

$\operatorname{sen}\theta = 2x \qquad dx = \dfrac{1}{2}\cos\theta d\theta \qquad \cos\theta = \sqrt{1-4x^2}$

$$\int \frac{\cos^2\theta d\theta}{\operatorname{sen}\theta} = \int \csc\theta - \int \operatorname{sen}\theta d\theta = \ln|\csc\theta + \cot\theta| + \cos\theta + C$$

$$= \ln\left|\frac{1}{\operatorname{sen}\theta} + \frac{\cos\theta}{\operatorname{sen}\theta}\right| + \cos\theta + C$$

$$= \ln\left|\frac{1+\sqrt{1-4x^2}}{2x}\right| + \sqrt{1-4x^2} + C$$

5. $\displaystyle\int \frac{dx}{\sqrt{9+4x^2}}$

$3\sec\theta = \sqrt{9+4x^2} \qquad \tan\theta = \dfrac{2x}{3} \quad x = \dfrac{3}{2}\tan\theta \qquad dx = \dfrac{3}{2}\sec^2\theta d\theta$

$$\frac{1}{2}\int \sec\theta d\theta = \frac{1}{2}\ln|\sec\theta + \tan\theta| + C = \frac{1}{2}\ln\left|\frac{\sqrt{9+4x^2}}{3} + \frac{2x}{3}\right| + C$$

6. $\displaystyle\int \frac{x^2dx}{\sqrt{25-x^2}}$

$\operatorname{sen}\theta = \dfrac{x}{5}$ $\qquad \cos\theta = \dfrac{\sqrt{25-x^5}}{5}$ $\qquad dx = 5\cos\theta d\theta$

$25\displaystyle\int sen^2\theta d\theta = \frac{25}{2}\int d\theta - \frac{25}{4}\int \cos(2\theta)2d\theta = \frac{25}{2}\theta - \frac{25}{4}\operatorname{sen}2\theta + C = \frac{25}{2}\arcsin\left(\frac{x}{5}\right) - \frac{25}{4}\operatorname{sen}\left(2\arcsin\left(\frac{x}{5}\right)\right) + C$

7. $\displaystyle\int \frac{x^2dx}{\sqrt{1+x^2}}$

$\tan\theta = x$ $\qquad dx = \sec^2\theta$ $\qquad \sec\theta = \sqrt{1+x^2}$

$\displaystyle\int \frac{\sec^2\theta\tan\theta d\theta}{\sec\theta} = \int \tan^2\theta\sec\theta d\theta = \int \sec^3\theta d\theta + \int \sec\theta d\theta$

resolviendo la integral $\displaystyle\int \sec^3\theta d\theta$ por partes

$\displaystyle\int \sec\theta\sec^2\theta d\theta = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta + \int \sec\theta d\theta$

observación: $\displaystyle\int \sec\theta\tan^2$ es la integral de la cual partimos, entonces:

$I = \sec\theta\tan\theta - I - \displaystyle\int \sec\theta d\theta$

obteniendo como resultado

$I = \dfrac{\sec\theta\tan\theta}{2} + \dfrac{\ln|sec\theta + \tan\theta}{2}$

8. $\displaystyle\int \frac{x^2dx}{\sqrt{4+9x^2}}dx$

$\sec\theta = \dfrac{\sqrt{9+4x^2}}{3}$ $\qquad x = \dfrac{3\tan\theta}{2}$ $\qquad dx = \dfrac{3\sec^2\theta d\theta}{2}$

$\dfrac{9}{8}\displaystyle\int \frac{\tan^2\theta\sec^2\theta d\theta}{\sec\theta} = \int \sec^3\theta d\theta + \int \sec\theta d\theta$

resolviendo la integral $\displaystyle\int \sec^3\theta d\theta$ por partes

$\displaystyle\int \sec\theta\sec^2\theta d\theta = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta + \int \sec\theta d\theta$

observación: $\displaystyle\int \sec\theta\tan^2$ es la integral de la cual partimos, entonces:

$\dfrac{9}{8}I = \sec\theta\tan\theta - I - \displaystyle\int \sec\theta d\theta$

obteniendo como resultado

$I = \dfrac{8\sec\theta\tan\theta}{17} + \dfrac{8\ln|sec\theta + \tan\theta}{17}$

9. $\displaystyle\int \frac{dx}{(1+x^2)^{3/2}}$ $\quad \sec\theta = \sqrt{1+x^2}$ $\qquad \sec^3\theta = \sqrt{1+x^2}^3$ $\qquad \tan\theta = x$ $\qquad dx = \sec^2\theta d\theta$

$$\int \frac{\sec^2\theta d\theta}{\sec^3\theta} = \int \frac{d\theta}{\sec\theta} = \int \cos\theta d\theta = \operatorname{sen}\theta + C = \frac{x}{\sqrt{1+x^2}} + C$$

10. $\displaystyle\int (1-x^2)^{3/2}dx$

$\operatorname{sen}\theta = x$ $\qquad dx = \cos\theta d\theta$ $\qquad \sqrt{1-x^2} = \cos\theta$

$\cos^4\theta d\theta = \left(\dfrac{1}{2} + \dfrac{\cos 2\theta}{2}\right)^2 d\theta$

$= \dfrac{1}{4}\theta + \dfrac{1}{4}\cos 2\theta d\theta + \dfrac{1}{8}d\theta + \dfrac{1}{8}\cos 4\theta d\theta$

$\dfrac{3}{8}\theta + \dfrac{\operatorname{sen}\theta}{2} + \dfrac{1}{2}\operatorname{sen}\theta + C$

11. $\dfrac{dx}{(4-x^2)^2}dx$

sea $x = 4\operatorname{sen}\theta \qquad dx = 4\cos\theta d\theta \qquad \sqrt{4-x^2} = 4\cos\theta$

$\dfrac{4\cos\theta d\theta}{4^4\cos^4\theta}$

$= \dfrac{1}{64}\dfrac{d\theta}{\cos^3\theta}$

$= \dfrac{1}{64}\sec^3\theta d\theta$

$= \sec\theta\tan\theta - \sec\theta\tan^2\theta d\theta$

$= \sec\theta\tan\theta + \sec\theta d\theta - \sec^3 d\theta$

$\dfrac{64\sec\theta\tan\theta}{65} + \dfrac{64\ln(\sec\theta+\tan\theta)}{65} + C$

12. $\dfrac{dx}{(4-x^2)^3}$

$x = 2\operatorname{sen}\theta \qquad dx = 2\cos\theta d\theta \qquad \sqrt{4-x^2} = 2\cos\theta$

$\dfrac{2\cos\theta d\theta}{2^5\cos^5\theta}$

$= \dfrac{1}{16}\sec^4\theta$

$= \dfrac{1}{16}sec^2\theta(1+\tan^2)d\theta$

$= \dfrac{1}{16}\sec^2\theta d\theta + \dfrac{1}{16}\sec^2\theta\tan^2\theta$

$= \dfrac{\tan\theta}{16} + \dfrac{\tan^3\theta}{48}$

$= \dfrac{\sqrt{4-x^2}}{16x^2} + \dfrac{(4-x^2)^{3/2}}{48x^3} + C$

13. $\sqrt{9+16x^2}dx$

$\tan\theta = \dfrac{4x}{3} \qquad dx = \dfrac{3}{4}\sec^2\theta d\theta \qquad \sqrt{9+16x^2} = 3\sec\theta$

$I = \dfrac{9}{4}\sec^3\theta d\theta$

$= \sec\theta\tan\theta - \sec\theta\tan^2\theta d\theta$

$= \sec\theta\tan\theta + \sec\theta d\theta - \sec^3 d\theta$

$\dfrac{9}{4}I = \sec\theta\tan\theta + \sec\theta d\theta - I$

$= \dfrac{4\sec\theta\tan\theta}{13} + \dfrac{4\sec\theta}{13} + C$

14. $x^2\sqrt{x^2-1}dx$

$x = \csc\theta \qquad dx = -\csc\theta\cot\theta d\theta \qquad \cot\theta = \sqrt{x^2-1}$

$-\csc^3\theta\cot^2\theta d\theta$

$-\csc\theta(1+\cot\theta)\cot^2\theta d\theta$

$-\cot^4\theta\csc\theta d\theta - \cot^2\theta\csc\theta d\theta$

$-\cot^3\theta\cot\theta\csc\theta d\theta - \cot\theta\cot\theta\csc\theta d\theta$

$-\dfrac{\cot^4\theta}{4} - \dfrac{\cot^2\theta}{2} + C$

$\dfrac{1-x^2}{2} - \dfrac{(x^2-1)^2}{4}$

15. $\dfrac{dx}{(4x^2-1)^{3/2}}$
$x = \dfrac{\csc\theta}{2} \qquad dx = \dfrac{\csc\theta\cot\theta d\theta}{2} \qquad \sqrt{4x^2-1} = \cot\theta$
$\dfrac{1}{2}\dfrac{\csc\theta\cot\theta}{cot^3}d\theta$
$\dfrac{1}{2}\dfrac{\operatorname{sen}\theta}{\cos^2\theta}d\theta = \dfrac{1}{2\cos\theta} = \dfrac{1}{2}\sec\theta = \dfrac{x}{\sqrt{x^2-1}}$

16. $\dfrac{\sqrt{x^2-5}}{x^2}$
$x = \sqrt{5}\csc\theta \qquad dx = \sqrt{5}\csc\theta\cot\theta d\theta \qquad \sqrt{x^2-5} = \sqrt{5}\sqrt{x^2-5}$
$-\dfrac{\sqrt{5}\cot\theta\sqrt{5}\csc\theta\cot\theta d\theta}{5\csc^2}$
$\dfrac{\cot^2\theta d\theta}{\csc\theta}$
$\dfrac{\cos^2\theta}{\operatorname{sen}\theta}d\theta$
$\csc\theta d\theta - \operatorname{sen}\theta d\theta$
$\ln|\csc\theta - \cot\theta| - \cos\theta$
$\ln\left|\dfrac{x-\sqrt{x^2-5}}{\sqrt{5}}\right| - \dfrac{\sqrt{x^2-5}}{x} + C$

17. $\dfrac{\sqrt{9x^2-16}}{x}dx$
$x = \dfrac{4}{3}\csc\theta \qquad dx = \dfrac{4}{3}\csc\theta\cot\theta d\theta \qquad = \sqrt{9x^2-16} = 4\cot\theta$
$= \dfrac{4\cot\theta\left(\dfrac{4}{3}\csc\theta\cot\theta d\theta\right)}{\dfrac{4}{3}\csc\theta}$
$= 4\cot^2\theta d\theta$
$4\csc^2\theta d\theta - 4\theta$
$-4\cot\theta - 4\theta = -\sqrt{9x^2-16} - 4Arc\csc\left(\dfrac{3x}{4}\right) + C$

18. $\dfrac{x^2}{\sqrt{4x^2-9}}dx$
$x = \dfrac{3}{2}\csc\theta \qquad dx = -\dfrac{3}{2}\csc\theta\cot\theta d\theta \qquad \sqrt{4x^2-9} = 3\cot\theta$
$\dfrac{\dfrac{9}{4}\csc^2\theta\left(\dfrac{3}{2}\csc\theta\cot\theta d\theta\right)}{3\cot\theta}$
$\dfrac{9}{8}\csc^3\theta d\theta$
$u = \csc\theta \qquad du = -\csc\theta\cot\theta d\theta \qquad dv = \csc^2\theta d\theta \qquad v = -\cot\theta$

$\dfrac{9}{8}\csc^2\theta\csc\theta d\theta$
$= -\csc\theta\cot\theta - \cot^2\theta\csc\theta d\theta$
$= -\csc\theta\cot\theta - \csc^3\theta d\theta + \csc\theta d\theta$
$\dfrac{9}{8}I = -\csc\theta\cot\theta + \ln|\csc\theta - \cot\theta| + I$
$= \dfrac{4\ln|\csc\theta - \cot\theta|}{9} - \dfrac{4\cot\theta\csc\theta}{9}$
$= \dfrac{4\ln\left|\dfrac{2x-\sqrt{4x^2-9}}{3}\right|}{9} - \dfrac{8x\sqrt{4x^2-9}}{3} + C$

19. $\dfrac{dx}{x^2\sqrt{4x^2-9}}$

$x = \dfrac{3}{2}\csc\theta \qquad dx = -\dfrac{3}{2}\csc\theta\cot\theta d\theta \qquad \sqrt{4x^2-9} = 3\cot\theta$

$-\dfrac{\dfrac{3}{2}\csc\theta\cot\theta d\theta}{\left(\dfrac{3}{2}\right)^2\csc^2(3\cot\theta)}$

$= -\dfrac{2}{9}\dfrac{d\theta}{\csc\theta}$

$= -\dfrac{2}{9}\operatorname{sen}\theta d\theta = \dfrac{2\cos\theta}{9} = \dfrac{\sqrt{4x^2-9}}{9x} + C$

En los siguientes ejercicios, integre por el método de Fracciones parciales

20. $\dfrac{x^2}{x-1}dx$

$= (x+1)dx + \dfrac{dx}{x-1}$

$= \dfrac{x^2}{2} + x + \ln|x-1| + C$

21. $\dfrac{x^2}{2x-1}dx$

$= \dfrac{x^2}{2}dx + \dfrac{1}{4}xdx + \dfrac{1}{8}dx + \dfrac{1}{16}\dfrac{2dx}{2x-1}$

$\dfrac{x^3}{6} + \dfrac{x^2}{8} + \dfrac{x}{8} + \dfrac{\ln|2x-1|}{16} + C$

22. $\dfrac{dx}{x^2-3x}$

$= \dfrac{dx}{x(x-3)} = \dfrac{A}{x}dx + \dfrac{B}{x-3}dx \ A(x+3) + Bx = 1 \qquad \text{si, } x = 0 \rightarrow A = \dfrac{1}{3} \qquad \text{si, } x = -3 \rightarrow B = -\dfrac{1}{3}$

$\dfrac{1}{3}\dfrac{dx}{x} - \dfrac{1}{3}\dfrac{dx}{x-3} = \dfrac{\ln x}{3} - \dfrac{\ln|x+3|}{3} + C$

23. $\dfrac{x}{x^2+4x}dx$

$= \dfrac{xdx}{x^2+4x} = \dfrac{xdx}{x(x+4)} = \dfrac{A}{x}dx + \dfrac{B}{x+4}dx$

$A(x+4) + Bx = x \qquad x = -4 \rightarrow B = 1 \qquad A = 0$

$\dfrac{dx}{x+4} = \ln|x+4| + C$

24. $\dfrac{dx}{x^2+x-6}$

$\dfrac{dx}{x^2+x-6} = \dfrac{A}{x+3} + \dfrac{B}{x-2}$

$A(x-2) + B(x+3) = 1 \qquad \text{si } x = 2 \rightarrow B = \dfrac{1}{5} \qquad \text{si } x = -3 \rightarrow A = -\dfrac{1}{5}$

$\dfrac{1}{3}\dfrac{dx}{x-2} - \dfrac{1}{5}\dfrac{dx}{x+3}$

$\dfrac{1}{5}\ln|x-2| - \dfrac{1}{5}\ln|x+3| + C$

25. $\dfrac{dx}{x^3+4x}$

$\dfrac{dx}{x(x(x^2+4)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+4}$

$A(x^2+4)+(Bx+C)x = 1 \qquad A = \dfrac{1}{4}$

si $x=1 \qquad 5A+B+C=1 \qquad B+C=-\dfrac{1}{4}$

$x=-1 \qquad 5A+B-C=-1 \qquad B-C=-\dfrac{1}{4}$

$2B=-\dfrac{1}{2} \qquad B=-\dfrac{1}{4} \qquad C=0$

$\dfrac{1}{4}\dfrac{dx}{x} - \dfrac{1}{4}\dfrac{xdx}{x^2+4}$

$\dfrac{1}{4}\dfrac{dx}{x} - \dfrac{1}{8}\dfrac{2x}{x^2+4}$

$\dfrac{\ln x}{4} - \dfrac{\ln|x^2+4|}{8} + C$

26. $\dfrac{dx}{(x+1)(x^2+1)}$

$\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1}$

$A(x^2+1)+(Bx+C)(x+1) = 1 \qquad$ si $x=-1 \to A = \dfrac{1}{2}$

si $x=0 \qquad A+C=1 \qquad C=-\dfrac{1}{2}$

$\dfrac{1}{2}\dfrac{dx}{x+1} - \dfrac{1}{2}\dfrac{x-1}{x^2+1}$

$= \dfrac{1}{2}\dfrac{dx}{x+1} - \dfrac{1}{4}\dfrac{2xdx}{x^2+1} + \dfrac{1}{2}\dfrac{dx}{x^2+1}$

$= \dfrac{\ln|x+1|}{2} - \dfrac{\ln|x^2+1|}{4} + Arc\tan x + C$

27. $\dfrac{x^4}{x^2+4}dx$

$(x^2-4)dx + 16\dfrac{dx}{x^2+4}$

$\dfrac{x^3}{3} - 4x + 8Arc\tan\left(\dfrac{x}{2}\right) + C$

28. $\dfrac{2x-4}{x^2-x}dx$

$\dfrac{2x-4}{x(x-1}dx = \dfrac{A}{x}dx + \dfrac{B}{x-1}dx$

$A(x-1)+Bx = 2x-4$

si $x=0 \to A=-4$

$x=-1 \to B=-2$

$4\dfrac{dx}{x} - 2\dfrac{dx}{x-1}$

$= 4\ln x - 2\ln(x-1) + C$

$\ln x^4 - \ln(x^2-1)^2 + C$

$\dfrac{\ln x^4}{\ln(x^2-1)^2} + C$

29. $\dfrac{dx}{(x^2+1)(x^2+4)}$

$\dfrac{Ax+B}{x^2+1}dx + \dfrac{Cx+D}{x^2+4}$

$Ax^3+4Ax+Bx^2+4B+Cx^3+Cx+Dx+D=1$

$A+C=0$

$4A+C=0$

$3A=0$

$A=0$

$c=0$

$B+C=0$

$4B+D=1$

$3B=1$

$B=\dfrac{1}{3}$

$D=-\dfrac{1}{3}$

$\dfrac{1}{3}\dfrac{dx}{x^2+1} - \dfrac{1}{3}\dfrac{dx}{x^2+4}$

$\dfrac{1}{3}Arc\tan x - \dfrac{1}{6}Arc\tan\left(\dfrac{x}{2}\right)+C$

30. $\dfrac{x^4dx}{x^2+4x+4}$ $x^2dx - 4xdx + 12dx - 16\dfrac{2x+3}{x^2+4x+4}dx$

$x^2dx - 4xdx + 12dx - 16\dfrac{A}{x+2}dx - 16\dfrac{B}{(x+2)^2}dx$

$A(x+2)^2 + B(x+2) = 2x+3$

Si $x=0$ $\quad 4A+2B=3 \quad 4A+2B=3$

si $x=-1$ $\quad A+B=1 \quad -2a-2B=-2$

$A=\dfrac{1}{2} \qquad B=\dfrac{1}{2}$

$= x^2dx - 4xdx + 12dx - 8\dfrac{dx}{x+2} - 8\dfrac{dx}{(x+2)^2}$

$\dfrac{x^3}{3} - 2x^2 + 12x - 8\ln|x+2| + \dfrac{8}{x+2} + C$

31. $\dfrac{dx}{x^2-4}$

$\dfrac{A}{x+2}dx + \dfrac{B}{x-2}dx$

$A(x-2)+B(x+2)=1 \qquad A=-\dfrac{1}{4} \qquad B=\dfrac{1}{4}$

$=\dfrac{1}{4}\dfrac{dx}{x-2} - \dfrac{1}{4}\dfrac{dx}{x+2}$

$\dfrac{\ln|x-2|}{4} - \dfrac{\ln|x+2|}{4} + C$

10. Tarea 10

En los ejercicios del $1-12$. Halla las integrales (sug. utilize sustitucióńes de racionalizacion)

1. $\int x^3\sqrt{3x-2}dx$

sea $u^2=3x-2 \qquad x=\dfrac{u^2+2}{3} \qquad dx=\dfrac{2udu}{3}$

$\int\left(\dfrac{u^2+2}{3}\right)^3\left(\dfrac{2udu}{3}\right)u=\dfrac{2}{81}\int(u^6+6u^4+12u^2+8)u^2du=\dfrac{2}{81}\left(\int u^8du+\int 6u^6du+\int 12u^4du+\int 8u^2du\right)=$
$\dfrac{2}{81}\left(\dfrac{u^9}{9}+\dfrac{6u^7}{7}+\dfrac{12u^5}{5}+\dfrac{8u^3}{3}\right)+C=\dfrac{2((3x-2)^{9/2}}{729}+\dfrac{4(3x-2)^{7/2}}{189}+\dfrac{8(3x-2)^{5/2}}{135}+\dfrac{16(3x-2)^{3/2}}{243}+C$

2. $\int x^3\sqrt[3]{x^2+1}dx$

sea $u=x^2+1 \qquad x=\sqrt{u-1} \qquad dx=\dfrac{du}{2\sqrt{u-1}}$

$\dfrac{(u-1)^{3/2}u^{1/3}du}{2(u-1)^{1/2}}=\dfrac{1}{2}(u-1)^3u^{1/3}du=\dfrac{1}{2}u^{4/3}du-\dfrac{1}{2}u^{1/3}du=\dfrac{1}{2}\left(\dfrac{3u^{7/3}}{7}-\dfrac{3}{4u^{4/3}}\right)+C=\dfrac{3(x^2+1)^{7/3}}{14}-$
$\dfrac{3(x^2+1)^{4/3}}{8}+C$

3. $\dfrac{dx}{1+\sqrt{x}}$ sea $u^2=x \qquad 2udu=dx$

$2\dfrac{udu}{1+u}=2du-2\dfrac{du}{1+u}=2u-2\ln|1+u|+C=2\sqrt{x}-2\ln|1+\sqrt{x}|+C$

4. $\dfrac{dx}{x^{1/2}-x^{1/4}}$

sea $u^4=x \qquad 4u^3=dx$
$4\dfrac{u^3du}{u^2-u}4\dfrac{u^3du}{u(u-1)}4\dfrac{u^2du}{u-1}=4(u+1)du+4\dfrac{du}{u-1}=2u^2+4u+\ln(u-1)^4+C=2\sqrt{x}+4\sqrt[4]{x}+4\ln(\sqrt[4]{x}-1)+C$

5. $\dfrac{x^3dx}{(x^2-1)^{4/3}}$

Sea $u=x^2-1 \qquad x=\sqrt{u+1} \qquad dx=\dfrac{du}{2\sqrt{u+1}}$

$\dfrac{(u+1)^{3/2}du}{2(u+1)^{1/2}u^{4/3}}=\dfrac{1}{2}\dfrac{(u+1)du}{u^{4/3}}=\dfrac{1}{2}u^{-1/3}du+\dfrac{1}{2}u^{-4/3}du=\dfrac{3u^{2/3}}{4}-\dfrac{3}{u^{1/3}+C}=\dfrac{3(x^2-1)^{2/3}}{4}-$
$\dfrac{3}{2(x^2-1)^{1/3}}+C$

6. $\dfrac{1-\sqrt{x}}{1+\sqrt{x}}dx$ sea $u^4=x \qquad 4u^3du=dx$

$4\dfrac{1-u^2}{1+u}u^3du=\dfrac{u^3-u^5}{1+u}du=4u^2-4udu+4du-4u^4du+4u^3du-4u^2du+4udu-4du=\dfrac{4u^3}{3}-2u^2+$
$4u-\dfrac{4u^5}{5}+u^4-\dfrac{4u^3}{3}+2u^2-4u+C=-\dfrac{4u^5}{5}+u^4+C=-\dfrac{4x^{5/4}}{5}+x+C$

7. $\dfrac{x^5}{\sqrt{x^3+1}}dx$

sea $u=x^3+1 \qquad x=\sqrt[3]{u-1} \qquad dx=\dfrac{du}{3(u-1)^{2/3}}$

$\dfrac{(u+1)^{5/3}du}{3(u-1)^{2/3}u^{1/2}}=\dfrac{1}{3}\dfrac{u-1}{u^{1/2}}=\dfrac{1}{3}u^{1/2}du-\dfrac{1}{3}u^{-1/2}du=\dfrac{2(x^3+1)^{3/2}}{9}-\dfrac{2(x+1)}{3}+C$

8. $\dfrac{dx}{1+x^{2/3}}$

Sea $u^3=x \qquad 3u^2du=dx$
$3\dfrac{u^2du}{1+u^2}=3du-3\dfrac{du}{1+u^2}=3u-3Arc\tan(u)+C=3x^{1/3}-3Arc\tan x^{1/3}+C$

9. $\dfrac{dx}{1+\sqrt{x+4}}$
sea $u = x+4 \qquad x = u-4 \qquad dx = du$
segundo cambio de variable
$\dfrac{2udu}{1+u^2} = \ln|1+u^2| + C = \ln|x+5| + C$

10. $\dfrac{d\theta}{1+\operatorname{sen}\theta}\ \dfrac{d\theta}{1+\operatorname{sen}\theta}\left(\dfrac{1-sen\theta}{1-\operatorname{sen}\theta}\right) = \dfrac{(1-\operatorname{sen}\theta)d\theta}{cos^2\theta} = \sec^2 - \tan\theta\sec\theta d\theta = \tan\theta - \sec\theta + C$

11. $\dfrac{d\theta}{\operatorname{sen}\theta+\cos\theta}\ \dfrac{d\theta}{\operatorname{sen}\theta+\cos\theta}$
$z = \tan\left(\dfrac{x}{2}\right) \qquad x = 2Arc\tan z \qquad dx = \dfrac{2dz}{z^2+1} \qquad \operatorname{sen}x = \dfrac{2z}{z^2+1} \qquad \cos x = \dfrac{z^2-1}{z^2+1}$

$$\frac{1}{\operatorname{sen}x+\cos x}dx = \left[\frac{1}{\dfrac{2z}{z^2+1}+\dfrac{z^2-1}{z^2+1}}\right]\left[\frac{2dz}{z^2+1}\right] = \frac{1}{\dfrac{z^2+2z-1}{z^2+1}}\left(\frac{2dz}{z^2+1}\right) = \frac{2}{z^2+2z+1-2}dz =$$
$$\frac{2}{(z+1)^2-\sqrt{z}^2}$$
sea $u = z+1 \qquad du = dz$
$$\frac{2}{u^2-(\sqrt{2})^2}du = \frac{2}{\sqrt{2}}Arc\tan\frac{u}{\sqrt{2}} + C = \frac{2}{\sqrt{2}}Arc\tan\frac{z+1}{\sqrt{2}} + C = \frac{2}{\sqrt{2}}Arc\tan\frac{\tan\left(\dfrac{x}{2}\right)+1}{\sqrt{2}} + C$$

12. $\dfrac{\operatorname{sen}\theta}{2+\cos\theta}d\theta$
sea $u = 2+\cos\theta \qquad du = \operatorname{sen}\theta d\theta$
$\dfrac{du}{u} = \ln u + C = \ln|2+\cos x| + C$

En los ejercicios $13-32$, Calcule la integral que se indica y diga si es convergente o divergente

13. $\displaystyle\int_{-\infty}^{-3} x^{-3}dx$
$$\lim_{b\to-\infty}\int_b^{-3} x^{-3}dx = \lim_{b\to-\infty}\left[\frac{-1}{2x^2}\right|_b^{-3} = \frac{-1\lim\limits_{b\to-\infty}}{2(-3)} - \frac{1}{\infty} = \frac{1}{6}$$
converge a $\dfrac{1}{6}$

14. $\displaystyle\int_0^5 \frac{xdx}{25-x^2}$
sea $u = 25-x^2 \qquad du = 2xdx$
$-\dfrac{1}{2}\displaystyle\int_0^5 \frac{-2xdx}{25-x^2}$
$\dfrac{1}{2}\displaystyle\int\frac{du}{u} = \frac{1}{2}\ln u$
$\dfrac{1}{2}\ln(25-x^2)|_0^5 = \dfrac{1}{2}\ln(25-25) - \dfrac{1}{2}\ln(25)$
como $\ln 0$ no existe,

15. $\displaystyle\int_1^{\infty} x^{-2/3}dx$
$$\lim_{b\to\infty}\int_1^b x^{-2/3}dx = \lim_{b\to\infty}\left[3x^{1/3}\right|_1^b = \lim_{b\to\infty}3b^{1/3} - 3 = \infty$$
diverge

16. $\displaystyle\int_0^{\infty} e^{-x}dx$
$$\lim_{b\to\infty}\int_0^b e^{-x}dx = \lim_{b\to\infty} -\frac{1}{e^x}\Big|_0^b = \lim_{b\to\infty} -\frac{1}{e^b} + \frac{1}{e^0} = 1$$
converge a 1

17. $\displaystyle\int_{-\infty}^{\infty} x^2 \lim_{a\to-\infty}\int_a^0 x^2 + \lim_{b\to\infty}\int_0^b x^2 = \lim_{a\to-\infty}\frac{x^3}{3}\Big|_a^0 + \lim_{b\to\infty}\frac{x^3}{3}\Big|_0^b = \frac{0}{3} - \frac{a^3}{3} + \frac{b^3}{3} - \frac{0}{3} = -\infty + \infty$

indeterminacion

18. $\displaystyle\int_{-\infty}^{0} xe^x dx$

$xe^x - \displaystyle\int e^x dx = e^x(x-1)$

$\lim b \to -\infty\, [xe^x - e^x]_b^0 = -1 - (be^b - e^b) = \infty$

diverge a ∞

19. $\displaystyle\int_{-\infty}^{0} \frac{dx}{\sqrt{1-x}}$

$\operatorname{sen}^2\theta \qquad dx = 2\cos\theta \qquad \cos\theta = \sqrt{1-x}$

$2\displaystyle\lim_{b\to-\infty}\int_b^0 \frac{\cos\theta d\theta}{\cos\theta} = 2\lim_{b\to-\infty}\int_b^0 d\theta = 2\lim_{b\to-\infty}\theta|_b^0 = 2\lim_{b\to-\infty} Arc\operatorname{sen}\sqrt{x}|_b^0 = 0 - \infty = -\infty$

Diverge a $-\infty$

20. $\displaystyle\int_{-\infty}^{-1} \ln\left(1 - \frac{1}{x}\right) dx$

$\displaystyle\int_{-\infty}^{-1} \ln\left(\frac{x-1}{x}\right) dx = \int_{-\infty}^{-1} \ln x dx - \int_{-\infty}^{-1} \ln(x-1) dx$

sea $u = \ln x \qquad du = \dfrac{1}{x} \qquad dv = dx \qquad v = x$

sea $u = \ln(x-1) \qquad du = \dfrac{1}{x-1} \qquad dv = dx \qquad v = x$

$x\ln x - \displaystyle\int_{-\infty}{}_{-1} dx - \left[x\ln(x-1) - \int_{-\infty}{}_{-1}\frac{xdx}{x-1}\right]$

$x(\ln x - 1) - \left[x\ln(x-1) - \displaystyle\int_{-\infty}{}_{-1}\frac{x-1}{x-1}dx + \int_{-\infty}{}_{-1}\frac{dx}{x-1}\right]$

$x(\ln x - 1) - [x\ln(x-1) - x + \ln(x-1)]$

$[x(\ln x - 1) - 2x\ln(1-x) + x]_{-\infty}^{-1}$

diverge

21. $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{|x|+1}$

$\displaystyle\lim_{a\to-\infty}\int_a^0 \frac{dx}{1-x} + \lim_{b\to\infty}\int_0^b \frac{dx}{x-1} = \lim_{a\to-\infty} \ln(1-x)|_a^0 + \lim_{b\to\infty} \ln(x-1)|_0^b =$

22. $\displaystyle\int_{e}^{\infty} \ln\left(\frac{1}{x}\right) dx$

$u = \ln\left(\dfrac{1}{x}\right) \qquad du = xdx \qquad dv = dx \qquad v = x$

$x\ln\left(\dfrac{1}{x}\right) - x^2 dx$

$\displaystyle\lim_{a\to-\infty}\left[x\ln\left(\frac{1}{x}\right) - \frac{x^3}{3}\right]_e^b$

$\left[b\ln\left(\dfrac{1}{b}\right) - \dfrac{b^3}{3}\right] - \left[x\ln\left(\dfrac{1}{x}\right) - \dfrac{x^3}{3}\right] =$ **Diverge**

23. $\displaystyle\int_{e}^{\infty} \ln(e^x) dx$

$u = \ln(e^x) \to du = \dfrac{e^x dx}{e^x} = dx \qquad dv = dx \to x = v$

$\displaystyle\int_{e}^{\infty} \ln(e^x)dx = x\ln(e^x) - xdx = \left[x\ln(e^x) - \frac{x^2}{2}\right]_e^{\infty} = [\infty - \infty] - [e\ln(e^e) - \frac{e^2}{2}]$ **Indeterminación**

24. $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$

$\displaystyle\lim_{a\to-\infty}\int_a^0 \frac{dx}{x^2+1} + \lim_{b\to\infty}\int_0^b \frac{dx}{x^2+1} = \lim_{a\to-\infty} Arc\tan x|_a^0 + \lim_{b\to\infty} Arc\tan x|_0^b = -\left(-\frac{\pi}{2}\right) + \frac{\pi}{2} = \pi$

Converge a π

25. $\displaystyle\int_{-\infty}^{\infty} \cosh x dx$

$\displaystyle\lim_{a\to-\infty}\int_a^0 \cosh x dx + \lim_{b\to\infty}\int_0^b \cosh x dx = \lim_{a\to-\infty} \operatorname{sen} hx + \lim_{b\to\infty} \operatorname{sen} hx = -(-\infty) + \infty = \infty$

Diverge a ∞

26. $\displaystyle\int_{-\infty}^{0} \frac{dx}{1-x}$

$\displaystyle\lim_{b\to-\infty}\int_b^0 \frac{dx}{1-x} = \lim_{b\to-\infty} \ln(x-1)|_b^0 = \lim_{b\to-\infty}[\ln(-1) - \ln(b-1)]$

No Existe!

27. $\displaystyle\int_{-\infty}^{0} Arc\tan x dx$

sea $u = Arc\tan x \qquad du = \dfrac{dx}{1+x^2} \qquad dv = dx \qquad v = x$

$xArc\tan x - \dfrac{1}{2}\dfrac{2xdx}{1+x^2} = xArc\tan x - \dfrac{1}{2}\ln|1+x^2|$

$\displaystyle\lim_{b\to-\infty}\left[xArc\tan x - \frac{1}{2}\ln|1+x^2|\right]_b^0 = 0 - (\infty - \infty)$

28. $\displaystyle\int_0^2 \frac{dx}{\sqrt{x}} \int_0^2 \frac{dx}{\sqrt{x}} = 2\sqrt{x}|_0^2 = 2\sqrt{2}$

converge a $2\sqrt{2}$

29. $\displaystyle\int_1^2 \frac{dx}{(1-x)^2} \lim_{b\to1}\int_b^2 \frac{dx}{(1-x)^2} \lim_{b\to1} = -\frac{1}{1-x}\Big|_1^2 = \frac{1}{1-b} - \left(-\frac{1}{1-2}\right) = -\infty - (-1)$

Diverge

30. $\displaystyle\int_{-1}^{0} \frac{dx}{(x+1)^3}$

$\displaystyle\int_{-1}^{0} \frac{dx}{(x+1)^3} = -\frac{1}{2(x+1)^2}\Big|_{-1}^{0} = -1 + \frac{1}{0} = \infty$

Diverge

31. $\displaystyle\int_0^{\frac{\pi}{2}} \sec x dx \qquad \int_0^{\frac{\pi}{2}} \sec x dx = [\ln|\sec x + \tan x|]_0^{\frac{\pi}{2}} = \infty - 0 = \infty$ **Diverge**

32. $\displaystyle\int_0^{\frac{\pi}{2}} \csc x dx \qquad \int_0^{\frac{\pi}{2}} \csc x dx = [\ln|\csc x - \cot|]_0^{\frac{\pi}{2}} = \ln|1-0| - \ln|\infty - \infty|$

11. Tarea 11

En los ejercicios del $1-6$ hallar las integrales impropias usando el criterio de comparación

1. $\displaystyle\int_1^\infty \frac{x}{\sqrt{1+x^5}}dx$

   Converge dado $\displaystyle\int_\infty \frac{dx}{x^{3/2}}$

2. $\displaystyle\int_1^\infty 2^{-x^2}dx$

   converge si $\displaystyle\int_1^\infty e^{-x}dx$

3. ${}_0^\infty(1+x^5)^{-1/6}dx$ diverge puesto que para x muy grande el integrando es muy grande dado a $\displaystyle\int_\infty \frac{dx}{x}$

4. $\displaystyle\int_\pi^\infty \frac{\operatorname{sen}^2 2x}{x^2}dx$

   Dado a $\displaystyle\int_\pi^\infty \frac{dx}{x^2}$ podemos decir que converge

5. $\displaystyle\int_1^\infty \frac{\ln x}{x^2}dx$

   Dado a $\displaystyle\int_1^\infty \frac{dx}{x^{3/2}}$ podemos decir que converge

6. $\displaystyle\int_e^\infty \frac{dx}{\sqrt{x+1}\ln x}dx$ Dado a $\displaystyle\int_e^\infty \frac{dx}{(x+1)\ln(x+1)}$ podemos decir que converge

En los ejercicios $7-16$ represente gráficamente el punto que se indica en coordenadas polares (r,θ)

7. $\left(3, \frac{5\pi}{6}\right)$ 8. $\left(2, -\frac{3\pi}{4}\right)$

9. $\left(-1, -\frac{4\pi}{3}\right)$ 10. $\left(-2, \frac{3\pi}{4}\right)$

11. $\left(-1, \frac{5\pi}{4}\right)$ 12. $\left(\sqrt{2}, \frac{2\pi}{3}\right)$

13. $\left(\sqrt{3}, -\frac{2\pi}{3}\right)$ 14. $\left(-\sqrt{2}, -\frac{5\pi}{6}\right)$

15. $\left(1, \frac{7\pi}{3}\right)$ 16. $\left(1-\sqrt{2}, -\frac{7\pi}{6}\right)$

En los ejercicios $17-24$, exprese la ecuación cartesiana dada, en coordenadas polares.

17. $x = 4 \rightarrow \text{r}\cos\theta = 4 \rightarrow \text{r}=4\sec\theta$
18. $x = 3y \rightarrow \text{r}\cos\theta = 3r\operatorname{sen}\theta$
19. $xy = 1 \rightarrow \text{r}^2\cos\theta\operatorname{sen}\theta = 1 \rightarrow \text{r}= \dfrac{1}{\cos\theta\operatorname{sen}\theta}$
20. $y^2+x^2 = 25 \rightarrow \text{r}^2(\operatorname{sen}^2\theta + \cos^2\theta = 25 \rightarrow \text{r}=5\text{x}^2 - y^2 = 1 \rightarrow \text{r}^2(sen^2\theta - \cos^2) = 1 \rightarrow \text{r}=\sqrt{\dfrac{1}{\cos 2\theta}}$
22. $y = x^2 \rightarrow \text{rsen}\,\theta = r^2\cos\theta \rightarrow \text{r}\cos^2\theta = \operatorname{sen}\theta \rightarrow \text{r}=\tan\theta\sec\theta$
23. $y + x = 4 \rightarrow \text{r}(\operatorname{sen}\theta + \cos\theta) = 4 \rightarrow \text{r}= \dfrac{4}{\operatorname{sen}\theta+\cos\theta}$
24. $y = 6 \rightarrow \text{r sen}\,\theta = 6 \rightarrow \text{r}=6\csc\theta$

En los ejercicios $25-32$, exprese en coordenadas cartesianas la ecuación polar dada.

25. $r = 3$ $\quad\sqrt{x^2+y^2} = 3$
26. $\theta = \dfrac{3\pi}{4}$ $\quad\tan^{-1}\left(\dfrac{x}{y}\right) = 1{,}17$ $\quad\tan\left[\tan^{-1}\left(\dfrac{x}{y}\right)\right] = \tan(1{,}17)$ $\quad\dfrac{x}{y} = 2{,}36$ $\quad x = 2{,}36y$

27. $r=-5\cos\theta = r^2 = 5r\cos\theta \qquad x^2+y^2=5x$

28. $r=\operatorname{sen}2\theta \quad r=2(\cos\theta\operatorname{sen}\theta) \quad \dfrac{r^2}{r\cos\theta}=2\dfrac{r\operatorname{sen}\theta}{r\cos\theta} \quad \dfrac{\sqrt{x^2+y^2}}{x}=\dfrac{2y}{x} \quad 2y=\sqrt{x^2+y^2}$

29. $r=1-\cos 2\theta \quad r=1-\cos^2\theta-\operatorname{sen}^2\theta \quad r=\operatorname{sen}^2\theta-\operatorname{sen}^2\theta \quad r=0 \quad \sqrt{x^2+y^2}=0$

30. $r=2+\operatorname{sen}\theta = r^2=2r-r\operatorname{sen}\theta \quad x^2+y^2=2\sqrt{x^2+y^2}-y$

32. $r=3\sec\theta \to r=\dfrac{3}{\cos\theta}\to r\cos\theta=3 \qquad x=3$

33. $r^2=\cos 2\theta \quad r^2+1=1+\cos 2\theta \quad r^2+1=1+\cos^2\theta-\operatorname{sen}^2\theta \quad r^2+1=2\cos^2\theta \quad x^2+y^2+1=2x^2$
$x^2-y^2=1$

En los ejercicios $33-39$, escriba la ecuación dada tanto en coordenadas polares como cartesianas y grafique en ambos sistemas de coordenadas CON y SIN dispositivo electrónico..

33. La recta vertical que pasa por $(2,0)$ $\qquad x=2 \qquad r\cos\theta=2$
34. La recta horizontal que pasa por $(1,3)$ $\qquad y=1 \qquad r\operatorname{sen}\theta=1$
35. La recta que pasa por $(2,-1)$ con pendiente -1.
$y-(-1)=-1(x-2) \qquad y=-x+1 \qquad r\operatorname{sen}\theta=-r\cos\theta+1$
36. La recta que pasa por $(1,3)$ y $(3,5)$.
$y-3=x-1 \qquad y=x+2 \qquad r\operatorname{sen}\theta=r\cos\theta+2$
37. La circunferencia con centro en $(0,-4)$ y que pasa por el origen.
$x^2+(y+4)^2=16 \qquad r^2\cos^2\theta+(r\operatorname{sen}\theta+4)^2=16 \qquad r^2\cos^2\theta+r^2\operatorname{sen}^2\theta+8\operatorname{sen}\theta+16=16$
$r^2(\operatorname{sen}^2\theta+\cos^2\theta)+2r\operatorname{sen}\theta=0 \qquad r^2+r\operatorname{sen}\theta=0$
38. La circunferencia con centro en $(3,4)$ y radio 5.
$(x-3)^2+(y-4)^2=25 \qquad x^2-6x+9+y^2-8y+16=25 \qquad r^2\cos^2\theta-6r\cos\theta+r^2\operatorname{sen}^2\theta-8r\operatorname{sen}\theta=0 \qquad r^2-6r\cos\theta-8r\operatorname{sen}\theta=0$
39. La circunferencia con centro en $(1,1)$ y que pasa por el origen.
$(x-1)^2+(y-1)^2=1 \qquad x^2-2x+1+y^2-2y+1=1 \qquad r^2(\operatorname{sen}^2\theta+\cos^2\theta)-2r(\cos\theta+\operatorname{sen}\theta)=-1$
$r^2+2r(\cos\theta+\operatorname{sen}\theta)=-1$

En los ejercicios $40-46$, dibuje las gráficas de las ecuaciones polares CON y SIN dispositivo electrónico.

40. $r=2\cos\theta$ 41. $r=2\operatorname{sen}\theta+2\cos\theta$ 42. $r=1+\cos\theta$

43. $r=2+4\cos\theta$ 44. $r=2\operatorname{sen}2\theta$ 45. $r=3\cos 3\theta$

46. $r=2\operatorname{sen}5\theta$

En los ejercicios $47-49$ encuentre los puntos de intersección de las curvas dadas.

47. $r=2,\ r=\cos\theta$
$\cos\theta=2$ dado que la función coseno para ningun angulo es 2 estas curvas no se interceptan
48. $r=\operatorname{sen}\theta,\ r=\cos 2\theta$ $\operatorname{sen}\theta=\cos 2\theta$
$\operatorname{sen}\theta=1-2\operatorname{sen}^2\theta$
$\operatorname{sen}\theta+2\operatorname{sen}^2\theta=1$
$\operatorname{sen}\theta(1+2\operatorname{sen}\theta)=1$
$\operatorname{sen}\theta=1\to\theta=\dfrac{\pi}{2}$
$\left(1,\dfrac{\pi}{2}\right)$
49. $r=1-\cos\theta,\ r^2=4\cos\theta$
$(1-\cos\theta)^2=4\cos\theta$
$1-2\cos\theta+\cos^2\theta=4\cos\theta$
$1=6\cos\theta-\cos^2\theta$
$1=\cos\theta(6-\cos\theta)$

$\cos\theta=1$

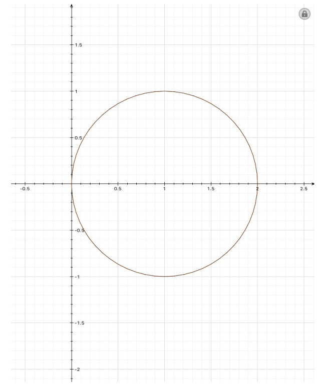
$r=\sqrt{4(1)} \qquad r=2$

se interceptan en:
$(2,0)$ y $(2,2\pi)$

12. Tarea 12

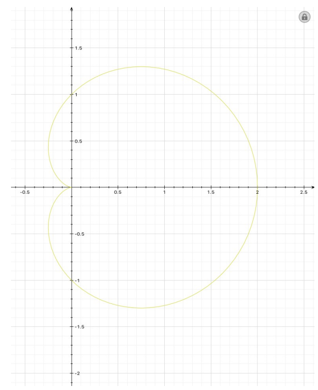
En los ejercicios $1-8$, encuentre el área limitada por la curva dada.

1. $r = 2\cos\theta$



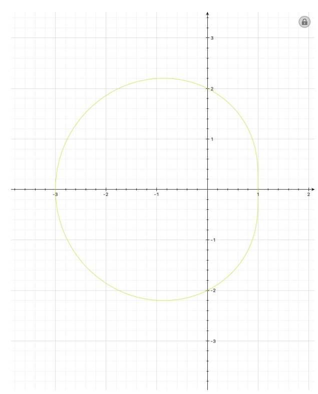
$4\int_0^{\pi} \cos^2\theta d\theta$
$= 2\int_0^{\pi} d\theta + \int_0^{\pi} 2\cos(2\theta)d\theta$
$= [2\theta + \operatorname{sen}(2\theta)]_0^{\pi}$
$A = 2\pi$

2. $r = 1 + \cos\theta$



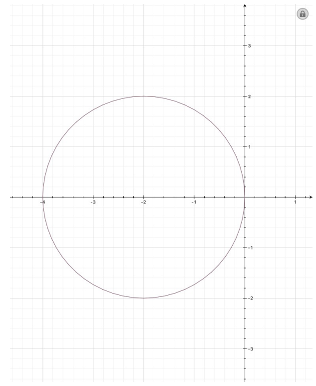
$\int_0^{\pi} (1+\cos\theta)d\theta$
$= \int_0^{\pi} d\theta + 2\int_0^{\pi} \cos\theta + \frac{1}{2}\int_0^{\theta} d\theta + \frac{1}{4}\int_0^{\pi} 2\cos 2\theta d\theta$
$= \left[\theta + 2\operatorname{sen}\theta + \frac{\theta}{2} + \operatorname{sen} 2\theta\right]_0^{\pi}$
$A = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

3. $r = 2 - \cos\theta$



$\int_0^{\pi} (2-\cos\theta)^2 d\theta$
$= 4\int_0^{\pi} d\theta - 4\int_0^{\pi} \cos\theta + \frac{1}{2}\int_0^{\pi} d\theta + \frac{1}{4}\int_0^{\pi} 2\cos 2\theta d\theta$
$= \left[4\theta - 4\operatorname{sen}\theta\frac{\theta}{2} + \operatorname{sen} 2\theta\right]_0^{\pi}$
$A = 4\pi + \frac{\pi}{2} = \frac{9\pi}{2}$

4. $r = -4\cos\theta$



$16\int_0^{\pi} \cos^2\theta d\theta$
$= 8\displaystyle\int_0^{\pi} d\theta + 4\int_0^{\pi} 2\cos 2\theta d\theta$
$= 8\theta + 4\operatorname{sen} 2\theta|_0^{\pi}$
$A = 8\pi$

5. $r = 5(1 + \operatorname{sen}\theta)$

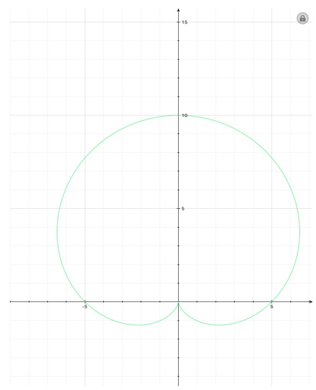
$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [5(1+\operatorname{sen}\theta)^2 d\theta$$

$$25\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta + 50\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}\theta d\theta + 25\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}^2\theta d\theta$$

$$25\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta + 50\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}\theta d\theta + \frac{25}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta - \frac{25}{4}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\cos 2\theta$$

$$= \left[25\theta + 50\cos\theta + \frac{25}{2}\theta - \operatorname{sen} 2\theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \left[\frac{75}{2}\left(\frac{3\pi}{2}\right)\right] - \left[\frac{75}{2}\left(\frac{\pi}{2}\right) - 1\right] = \frac{225\pi}{4} - \frac{75\pi}{4} + 1 = \frac{75}{2}\pi + 1$$

$$A = \frac{75\pi}{2} + 1$$



6. $r = 3 + 2\operatorname{sen}\theta$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (3 + 2\operatorname{sen}\theta)^2 d\theta$$

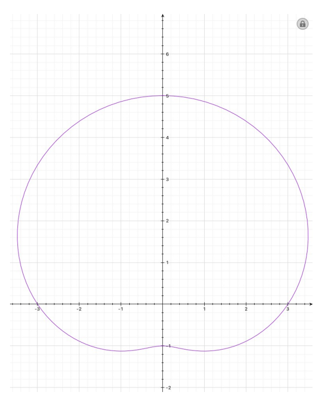
$$9\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta + 12\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}\theta d\theta + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}^2\theta d\theta$$

$$9\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta + 12\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}\theta d\theta + \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta - \frac{1}{4}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\cos 2\theta d\theta$$

$$\left[9\theta - 12\cos\theta + \frac{\theta}{2} + \operatorname{sen} 2\theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \left[\frac{27\pi}{2} + \frac{3\pi}{4}\right] - \left[\frac{9\pi}{2} + \frac{\pi}{4}\right] = \frac{57\pi}{4} - \frac{19\pi}{2} = \frac{13}{2}\pi$$

$$A = \frac{13}{2}\pi$$



7. $r = 2 + 3\operatorname{sen}\theta$

$$4\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta + 12\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}\theta d\theta + 9\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}^2\theta d\theta$$

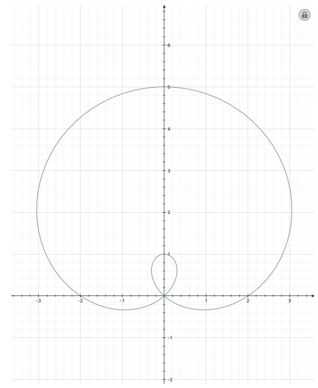
$$4\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta \ + \ 12\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}\theta d\theta \ + \ \frac{9}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta \ -$$

$$\frac{9}{4}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\cos 2\theta d\theta$$

$$\left[4\theta - 12\cos\theta + \frac{9\theta}{2} - \frac{9}{4}\operatorname{sen} 2\theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$6\pi + \frac{27\pi}{4} - 2\pi - \frac{9\pi}{4}$$

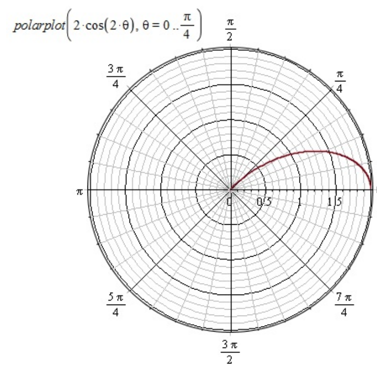
$$A = \frac{17}{2}\pi$$



8. $r = 3 + \operatorname{sen}\theta + \cos\theta$

En los ejercicios $9 - 14$, encuentre el área limitada por un rizo de la curva dada

9. $r = 2\cos 2\theta$



$$= \int_0^{\frac{\pi}{4}} (2\cos 2\theta)^2 d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} d\theta + \int_0^{\frac{\pi}{4}} 4\cos 4\theta d\theta$$

$$= [2\theta + \operatorname{sen} 4\theta]_0^{\frac{\pi}{4}} = \frac{\pi}{2}$$

$$A = \frac{\pi}{2}u^2$$

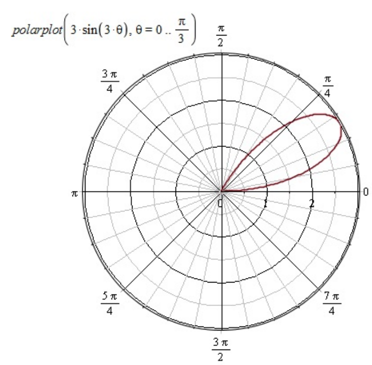
10. $r = 3\operatorname{sen} 3\theta$

$$\frac{1}{2}\int_0^{\frac{\pi}{3}} (3\operatorname{sen} 3\theta)^2 d\theta$$

$$\frac{9}{4}\int_0^{\frac{\pi}{3}} d\theta - \frac{1}{12}\int_0^{\frac{\pi}{3}} 6\cos 6\theta d\theta$$

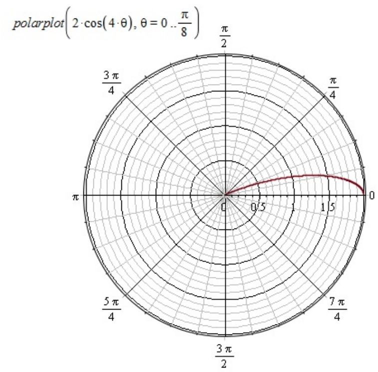
$$\left[\frac{9\theta}{4} - \operatorname{sen} 6\theta\right]_0^{\frac{\pi}{3}} = \frac{3\pi}{4}$$

$$A = \frac{3\pi}{4}u^2$$



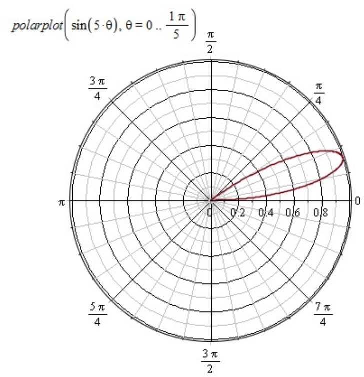
11. $r = 2\cos 4\theta$

$$\int_0^{\frac{\pi}{8}} (2\cos 4\theta)^2 d\theta$$
$$= 2\int_0^{\frac{\pi}{8}} d\theta + 4\frac{1}{8}\int_0^{\frac{\pi}{8}} \frac{8\cos 8\theta d\theta}{2}$$
$$2\int_0^{\frac{\pi}{8}} d\theta + \frac{1}{4}\int_0^{\frac{\pi}{8}} 8\cos 8\theta d\theta$$
$$\left[2\theta + \frac{\operatorname{sen} 8\theta}{4}\right]_0^{\frac{\pi}{8}} = \frac{\pi}{4}$$
$$A = \frac{\pi}{4}u^2$$



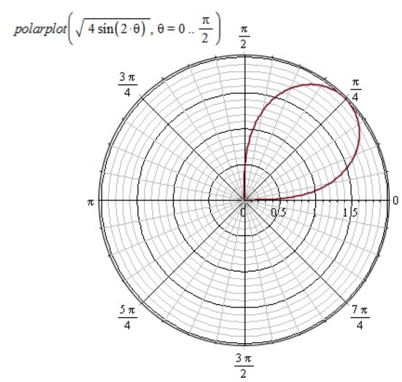
12. $r = \operatorname{sen} 5\theta$

$$\frac{1}{2}\int_0^{\frac{\pi}{5}} (\operatorname{sen} 5\theta)^2 d\theta$$
$$\frac{1}{4}\int_0^{\frac{\pi}{5}} d\theta - \frac{1}{200}\int_0^{\frac{\pi}{5}} 10\cos 10\theta$$
$$\left[\frac{1}{4}\theta - \frac{1}{200}\operatorname{sen} 10\theta\right]_0^{\frac{\pi}{5}} = \frac{\pi}{9}$$
$$A = \frac{\pi}{9}u^2$$



13. $r^2 = 4\operatorname{sen} 2\theta$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} (\sqrt{4\operatorname{sen} 2\theta})^2 d\theta$$
$$2\int_0^{\frac{\pi}{2}} \operatorname{sen} 2\theta d\theta$$
$$\int_0^{\frac{\pi}{2}} 2\operatorname{sen} 2\theta d\theta$$
$$-\cos 2\theta|_0^{\frac{\pi}{2}} 1 + 1 = 2$$
$$A = 2u^2$$



14. $r^2 = 4\operatorname{sen}\theta$

$$\frac{1}{2}\int_0^{\pi} (\sqrt{4\operatorname{sen}\theta})^2 d\theta$$
$$\frac{1}{2}\int_0^{\pi} 4\operatorname{sen}\theta d\theta \quad 2\int_0^{\pi} \operatorname{sen}\theta d\theta \quad [-2\cos\theta]_0^{\pi} \quad =$$
$$2 + 1 = 3$$
$$A = 3u^2$$