Response to Reviewers

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NOTE: Additions have been highlighted in blue in the manuscript, with removals given as footnotes.

Reviewer 1

0) In the abstract: it is better to relax the line "...this study represents the first publicly-available method to handle multimaterial problems under this model" because there are papers at least by Angelo Iollo et al (e.g. see references in the introduction of Peshkov et al, 2019, JCP, "Theoretical and numerical comparison of hyperelastic and hypoelastic formulations for Eulerian non-linear elastoplasticity", https://doi.org/10.1016/j.jcp.2019.02.039)) who use a very similar PDE system (for the distortion A but without the heat conduction) and the level set method for multimaterial modeling. However, it seems that they are not aware that these PDEs can be applied to the modeling of viscous flows as well. The authors could also mention the papers by Illo et al, e.g. refs. [25],[26] from the mentioned paper by Peshkov et al. I leave it up to the authors.

The line has been relaxed to "this study represents a novel approach for handling multimaterial problems under this model" (page 1). The references mentioned here are now discussed in 1.1 Multimaterial Models (page 2).

1) The line before eq. (1a), in references "...(see [33], [12], [8])": it is worth to add the mentioned above paper by Peshkov et al in JCP because it contains a review of Eulerian approaches to nonlinear elastoplasticity including the GPR model.

This additional reference has been added (page 2).

2) Even though the non-Newtonian fluids are not considered here, I suggest adding an expression for \tau_1 in (8a) for non-Newtonian fluids from authors' recent paper, Ref.[22] (recently accepted in JCP, https://doi.org/10.1016/j.jcp.2019.02.025). This will emphasize even more the multimaterial capabilities of the approach.

This has been added (page 5).

3) I believe, the quasilinear form from the footnote on p.12 should be placed right at the beginning of Sec.2.1. Otherwise, the meaning of M matrix in eq.(38) is not clear.

This has been done (page 14).

4) In eq.(29), say what the matrix L is. Should be the left eigenvectors but still, the authors should explicitly mention this. Later in the text, it is indeed named the left eigenvectors.

Before (29), it is already stated: "[...] analysis yields the left eigenvectors as the rows of (29)". This statement has been made more prominent on the page (page 16).

5) Wherever the dots appear in the matrices, like in (37), (40b), an so on, it should be declared what these dots stand for.

This notation has been removed by defining new temporary variables (pages 24,25).

6) Sec.2.2.2. Please give a short explanation for the intention of this section.

This has been added (page 23).

7) A few lines have to be devoted to the scheme which is used outside of the interface, e.g. the order of the scheme, the type, etc.

A comprehensive description of the scheme is now given in section 1.4 (pages 9-13).

8) In Sec.3.3, it is not clear to which of the previously mentioned EOS "the Godunov-Romenski EOS" is referenced to.

All four EOS used in this study are now stated explicitly in section 1.2 (pages 4,5).

9) In Sec.3.8, it is better to reference the line "One can clearly see the separation between the elastic precursor wave and the trailing plastic wave..." to Fig.17 but not to Fig.16.

This change has been made (page 36).

Reviewer 2

1. Background: The authors need to give more informations about the GPR model. A self consistent paper is always easier to read. Moreover, the GPR model is far from being a well established model. The authors should also explain in what extent this model encompasses both fluid and solid modellings. They have also to answer the crucial question of materials characterized by tabulated equations of state. Another question is related to the choice of entropy variable to express the energies E1 and E2. The choice of entropy renders quite difficult the simulation of materials response whose parameters, such as the shear modulus, depend on temperature. The reviewers asks the authors to address these questions. Finally, the reviewer wonders why the authors are not aware of the works of Gavrilyuk and Favrie about the numerical modelling of multimaterial flows.

A more thorough explanation of the GPR model has been given in section 1.2, and a description of the way in which it handles both fluid and solid modeling has been provided (pages 3-5). The question of tabulated equations of state has been addressed in this section (page 5), and the ability to use other variables for E1 and E2 - rather than entropy - has been discussed (page 5). The work of Gavrilyuk and Favrie is now rightfully noted and discussed in section 1.1 (page 2).

2. A Riemann ghost fluid…: The multimaterial treatment in the Eulerian framework proposed by the authors is the ghost fluid method. More precisely, they promote an improvement of this approach based on an approximate Riemann solver of the GPR model. This is the main contribution of the paper which is quite difficult to read and understand. The reviewer believes that the readability of the paper could be dramatically improved by rewriting this part. The authors have first to explain properly what they want to do: i) recalling the system of PDEs to be discretized ; ii) meshing the computational domain ; iii) discretizing the PDEs (space, time explicit/implicit???) ; iv) addressing the multimaterial treatment.

All of these suggestions have now been incorporated into the text, in section 1.4 (pages 9-13). The readability of section 2 has been improved by restructuring the text, adding extra explanations of variables and derivations where appropriate, and removing notation that is not entirely standardized (pages 14-25).

3. Results: The authors should explain how they have computed the analytical solutions. The results displayed for test 3.5 in Figure 12 for the heat flux are not correct. At the material interface, the heat flux has to be continuous, at least its normal component. The authors should check whether or not their numerical method ensures the continuity of the normal heat flux at material interfaces. A majority of 1D test cases is provided and 2 2D planar test cases are given which is not enough.

An explanation for the derivation of each analytical solution has been added (applicable to sections 3.1-3.5) (pages 26,29). The aberrations in the heat flux around the interface present in Figure 12 have been noted in section 3.5 (page 33) and discussed at some length in section 4.2 (page 44). Three more multidimensional test cases (including more material pairings than before, and reactive species) have been provided in 3.8-3.10 (pages 36, 40-43), and a convergence study has been provided in section 3.11 (pages 40,41).

Reviewer 4

Please give a definitions for all variables used in the GPR model (1a)-(1e). Even though some quantities are standard (like density and velocity), the distortion tensor A is definitely a non-classical object, which is of crucial important in the model description. Therefore, I suggest to properly address all quantities appearing in (1a)-(1e).

This has been done (page 3).

Please give also explicit formulae for the equation of state used in Section 3. It is not clear what the parameters contained in Table 1 refer to.

All three EOS used in this study are now stated explicitly in section 1.2 (pages 4,5).

What is tensor \Sigma^*? It seems to appear for the first time in equation (47a)

This has been clarified (page 20).

Could you please give an explicit formula of the discretization of the governing equations? I suppose you use a finite volume scheme, so please write the numerical scheme for all equations. How do you compute the time step? Nothing is said about it.

A comprehensive description of the scheme is now given in section 1.4 (pages 9-13).

Figure 10, density panel: there is a lowering in the density distribution at the right boundary. Do you have any explanation? How do you impose physical boundary conditions (not interface BCs)? Some comments should be made.

This effect is discussed in the text. The lowering in the density corresponds to the "heating errors" discussed in Barton & Drikakis 2010. It is reassuring to see that under the GPR system with heat conduction, this effect disappears, as explained in the text. Artificial physical BCs - such as those used in the aforementioned paper - are unnecessary, as explained. This discussion has been made more prominent in the paper (page 29).

Figures 10-11: these results refer to Section 3.4, where the physical domain is taken to be [0,1]. Why do you plot all figures in the range [0,0.65-0.7]?

The region [0.65-1] is empty, as it is occupied by vacuum. The plots are made over [0,0.7] to give greater resolution to the region of interest. This has been clarified in the text (page 29).

Could you provide a convergence study of your scheme? I assume it is first order in space and time, but nothing is said about it.

A convergence study has been added to section 3.11 (pages 40,41).

One bracket is not needed at line 2 of the Introduction "Lagrangian and Arbitrary-Lagrangian-Eulerian)"

Corrected (page 1).

Conclusions: "the implementation OF a mixture mode"

Corrected (page 44).