Response to Reviewers

Haran Jackson

January 31, 2019

Reviewer 1

(amendments highlighted in yellow in the manuscript)

1) The definition of Frobenius norm in (3) should be given.

This has been done.

2) The distortion is not a tensor (it's columns are vectors !), that is why the word "tensor" should be deleted on p.3 as well on pages 4, 16, 17, 18 and 23.

This has been done.

3) The strain tensor G introduced on p. 3 is called the Finger tensor and it would be useful to mention it in order to make it clear for people from the solid mechanics community.

This has been done.

4) On p.4 it is necessary to give a reference on the paper or book on non-Newtonian fluids, in which the constitutive relations (12), (13) are presented.

This has been done.

5) In my understanding formula (44) for differential of the singular value decomposition is valid for constant orthogonal matrices V, U. Does it mean that ODE solver for distortion is applied for constant V, U? Please explain.

U, V are not necessarily constant. A derivation of the stated result is given in section 3.2 of the provided reference. This has been made more clear in my paper.

6) Section 4.1 (page 16). In the sentence "Take initial data used by Barton..." the reference is needed.

This has been done.

7) Each reference on Figure in the text has a link to the page number. Please delete these links because the page numbers are unknown.

These page numbers are automatically added by the LaTeX typesetting system, and thus are always correct. They will only appear in the final version of the manuscript if the JCP typesetting system chooses to add them in.

8) As it is noted on p. 4 the solution to the governing PDEs must satisfy to the algebraic constraint $\rho = \rho_0 detA$. Is this constraint is treated in the proposed numerical method? If it is not implemented numerically then it would be interesting to see the difference between computed ρ and $\rho_0 detA$ at least for some numerical test problems.

This constraint is enforced by rescaling the singular values of the distortion all by the same factor at each time step. A clarification of this has been added to the end of the introduction to section 3.

Reviewer 3

(amendments highlighted in blue in the manuscript)

1) In the "Significance and Novelty" it is written "there are many potential benefit to this..." as a reference to the fact that the formulation is able to describe both fluids and solids within the same hyperbolic system. However, these benefits are not explicitly commented into the paper. Please list in "Significance and Novelty" what these benefits are, and of course evidence these asseverations into the paper.

The benefits of the new formulation have been added to the “Significance and Novelty” document. They are also included in the background section of the paper.

2) In the equation (22a), please clarify if $det(A) = |A|$. Is has to be so, to have the equation (43).

This is in deed the case, and it has been clarified in the text.

3) In eq (29), Is there some reason to include the sign "$+$" in front the term "$\tilde{B}$"?

This was a typo, and has been corrected.

4) If one wants to use a numerical method for solving the ODE’s (instead of analytical solutions). Do you recommend the same strategy described by the equation (23), In such a case, Does the solver of (20a) become the most efficient?

Further discussion on this point as been added to the conclusions section. Whilst I suggest that using the same strategy would be effective, I have discussed the potential existence of more efficient methods than the WENO method used to solve (20a).

5) As written in the conclusions, "temporal splitting suffer from a lack of spatial resolution...". As pointed out by the Leveque and Yee paper [15] the smearing of discontinuities caused by the numerical diffusion of the solver for the conservation law, may affect the correct wave propagation even if an exact ODE solution is used for the ODE sub-system. This can be more evident if the ODE has non-zero equilibrium equations, which seems to be the case in the tests presented in the paper (this is due to the deviatoric terms). Should be interesting the investigation about the structure of the ODE system and if equilibrium states can be identified.

The only equilibrium state of the ODE system is $a_1, a_2, a_3 = \sqrt[3]{\rho/\rho_0}$. This is the state that the system tends towards, given any initial condition. An explanation of this has been added to section 5 (conclusions).