

## Astrophysical Fluid Dynamics

## Example sheet 3

1. Show that for a strong shock (where the upstream Mach number  $\mathcal{M}_1$  is large), the downstream Mach number  $\mathcal{M}_2$  satisfies

$$\mathcal{M}_2^2 \simeq \frac{\gamma - 1}{2\gamma}.$$

Hence obtain an equation for the sound-speed ratio  $c_2/c_1$ .

A shock from a supernova travelling through the surrounding interstellar medium is observed to be travelling with speed 3000 km/s. What is the temperature immediately behind the shock?

[You may assume, if you wish, the surrounding interstellar medium to have temperature 100 K and density  $10^7$  particles m<sup>-3</sup>.] (From 1997 Examination)

2. A stellar wind is maintained at a temperature of  $T = 2 \times 10^6$  K by magnetic heating; calculate the radius at which it achieves the isothermal sound speed if the star from which it blows has mass  $M$ . You may assume the gas is made of fully ionised hydrogen. Evaluate your answer when  $M$  is the mass of the Sun  $M_\odot = 2 \times 10^{30}$  kg.
3. Isothermal gas of pressure  $\rho_0 c_l^2$  and density  $\rho_0$  at large distances from a star is steadily accreted by a star of mass  $M$  at the origin.

Calculate the accretion rate assuming that the gas remains isothermal. At what radius does the infalling gas achieve the sound velocity?

If  $c_l = 1$  km/s and  $n_\infty = 10^9$  hydrogen molecules per m<sup>3</sup> and the mass of the hydrogen atom is  $1.67 \times 10^{-27}$  kg evaluate this radius in terms of the solar radius  $R_\odot = 7 \times 10^5$  km assuming that the stellar mass is equal to the mass of the Sun ( $M_\odot = 2 \times 10^{30}$  kg).

Calculate the accretion rate in kg/s. How long will it take in seconds and in years to double the mass of the star whose initial mass is  $M_\odot$ ? How does this time depend on that initial mass?

4. Clusters of ionising stars, total luminosity  $L$ , sweep out cavities in the interstellar medium whose undisturbed density is  $\rho_0$ . Use the similarity solution method to determine the evolution of cavity radius  $r$  with time  $t$  – i.e. determine  $a, b, c$ , where  $r \propto L^a \rho_0^b t^c$ .

These bubbles stall when their expansion velocities become of order the sound speed in the interstellar medium. Show that the area occupied by a stalled bubble is proportional to  $L$  and hence comment on how the porosity of the interstellar medium [defined as the fractional area of the galaxy as seen from above the disk occupied by stalled bubbles] depends on how ionising stars (with given total luminosity) are organised into clusters.

If the disc of a galaxy can be approximated by a uniform density gas slab with a sharp edge at height  $z$ , comment on the different behaviours of clusters of small and large  $L$ .

5. A star is described by a polytropic equation of state, index  $n$ . For what values of  $n$  is such a star stable against convection? In the case that the star is convectively unstable, describe its structure and how energy is transported from its core to a distant point outside the star. [Assume that the star consists of fully ionised, monatomic hydrogen.]
6. A self-gravitating slab of gas is heated by cosmic rays (constant heating rate per unit mass) and cooled by optically thin thermal bremsstrahlung for which the cooling rate per unit mass is proportional to  $\rho T^{0.5}$ . Discuss the stability of the slab to (a) thermal and (b) convective instabilities.

7. Calculate the ratio of the free fall time to the sound wave crossing time for a uniform gaseous sphere containing one Jeans mass.

Show that if such a sphere contracts homogeneously (i.e. maintaining uniform density) and isothermally, the number of Jeans masses it contains rises as  $\mathfrak{R}^{3/2}$  where  $\mathfrak{R}$  is the collapse factor (i.e. the ratio of the initial value of the radius to its current value).

8. A thin disc of gas, rotating around an object mass  $M$ , is supported in the vertical ( $z$ ) direction by a balance between pressure forces and the  $z$ -component of the central object's mass. Write down the equation of hydrostatic equilibrium in the  $z$  direction. If the density distribution is of the form  $\rho(z) \propto (z_m^2 - z^2)^2$  deduce the pressure distribution and polytropic index,  $n$ , for the disc. Is the disc stable against convection if composed of a) monatomic b) diatomic gas?