

# Topics in Applied Mathematics Exercise

Hwijae Son

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1. (a) Write a program to find the root of the function  $f(x) = x^2 - 1$  using Newton's method. The initial value is  $x_0 = 10$ .  
(b) Write a program to find the minimum of the function  $f(x) = x^2$  using Gradient Descent. The initial value is  $x_0 = 10$ .
2. (a) The power method is an iterative algorithm to find the dominant eigenvalue. The algorithm can be summarized as follows:  
For  $A \in \mathbb{R}^{n \times n}$ ,  $b_0 \in \mathbb{R}^n$ , if we assume  $A$  has an eigenvalue that is strictly greater in magnitude than its other eigenvalues and the starting vector  $b_0$  has a nonzero component in the direction of an eigenvector associated with the dominant eigenvalue, then a sequence  $b_{k+1} = \frac{Ab_k}{\|Ab_k\|}$  converges to an eigenvector associated with the dominant eigenvalue.  
Make a random matrix and find its dominant eigenvalue by using the power method.  
(b) Write a function that takes a matrix and a number and returns the closest eigenvalue of the matrix from a given number using the inverse shifted power method.
3. (a) Create a random matrix and check whether it is invertible.  
(b) If an inverse exists, compute the QR decomposition of the matrix.  
(c) Check the identities for the orthogonal matrix  $Q$ 
  - i.  $\det(Q) = \pm 1$
  - ii. If  $Qx = \lambda x$ , then  $|\lambda| = 1$ .