

Topics in Applied Mathematics Exercise

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1. Write a python function that takes three numbers as input and returns the maximum, minimum, and average values.
2. Write a python function that takes an arbitrary number of inputs and returns the maximum, minimum, and average values.
3. Write a python function that takes a single string literal and returns the number of vowels in the string.
4. Write a python function that takes a single string literal and returns the number of words (space separated).
5. Write a python function that returns p^n for given a float p and an integer n . (Hint: Use recursive function)
6. Write a python function that returns True if a given n is a prime number and False unless.
7. The bisection method is a root finding algorithm that applied to any continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$. The algorithm can be summarized as follows.

Given a continuous function f , an interval $[a, b]$, and the function value $f(a)$ and $f(b)$, where $f(a)f(b) < 0$. Repeat the following:

1. Compute $c = \frac{a+b}{2}$
2. Compute $f(c)$
3. If $|f(c)|$ is sufficiently small, stop the iteration
4. Examine the sign of $f(c)$, and replace either $(a, f(a))$ or $(b, f(b))$ with $(c, f(c))$ so that there is a zero crossing within the new interval $[a, c]$ or $[c, b]$.

Write a python program that computes the root of $f(x) = x^2 - x - 1$ in an interval $[1, 2]$.

8. Newton's method is a root-finding algorithm which produces successively better approximations to the roots of a real-valued function. It is well known that $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuously differentiable and $f(\alpha) = 0$, then an iteration

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

with an initial guess $x_0 \in B(\alpha, r)$, for some $r > 0$, converges to α . Write a python program that computes the root of $f(x) = x^2 - x - 1$, with $x_0 = 10$, and $x_0 = -10$.