# Advanced Studies In Mathematics Exercise

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1. Assume a linear model $y = \beta_1^T x + \beta_0 + \epsilon$. Suppose the error distribution follows the Laplace distribution $Laplace(0, b)$. Derive the corresponding loss function.

2. Assume a linear model Assume a linear model $y = \beta_1^T x + \beta_0 + \epsilon$. Suppose the error distribution follows the Gaussian distribution and set the Laplace prior on $(\beta_1, \beta_0)$. Derive the loss function to perform MAP estimation.

3. Derive the normal equation with $L^2-$ regularization term. Does the equation attain a unique solution?

4. (Python) Write a python program that performs linear regression.
(1) Define a target function $y = \beta_1^T x + \beta_0$, where $\beta_1, x \in \mathbb{R}^d, \beta_0 \in \mathbb{R}$ with arbitrary $\beta_1, \beta_0$.
(2) Make a dataset $\{(x_i, y_i)\}_{i=1}^N$, where $y_i = \beta_1^T x + \beta_0 + \epsilon$ where $\epsilon \sim \mathcal{N}(0, \sigma^2)$.
(3) Compute $\beta_1, \beta_0$ using normal equation.
(4) Compute $\beta_1, \beta_0$ using gradient descent.
(5) Compare the results of (3) and (4) with the true values in (1).
(6) Add an $L^2-$regularization term to the loss function and compare the convergence of the gradient descent.