Advanced Studies In Mathematics Exercise

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1. Show that for any probability distributions P, Q, $D_{KL}(P\|Q) \geq 0$.

2. Compute the KL-divergence between two 1-d Gaussians $\mathcal{N}_1(x|\mu_1, \sigma_1^2)$, and $\mathcal{N}_2(x|\mu_2, \sigma_2^2)$.

3. Show that if $X = (X_1, \ldots, X_d)$ is a continuous random variable such that $X_1, \ldots, X_d$ are independent and $Y = (Y_1, \ldots, Y_d)$ is a continuous random variable such that $Y_1, \ldots, Y_d$ are independent, then

$$D_{KL}(X\|Y) = D_{KL}(X_1\|Y_1) + \cdots + D_{KL}(X_d\|Y_d).$$

4. (Python) Choose a simple dataset (e.g., MNIST digits) and implement VAE and GAN using PyTorch.