HW 2

2.1) Compute 1D inverse DFT of Fourier coefficients for $N=4$

$$X[k] = \{10, -2+2j, -2, -2-2j\}$$

$$W_N^* = e^{j\frac{2\pi}{N}} \; ; \; W_N = e^{-j\frac{2\pi}{N}}$$

$$\underset{4\times1}{x_N} = \frac{1}{N} \underset{4\times4}{W_N^*} \underset{4\times1}{X_N}$$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & W_4^{*0} & W_4^{*0} & W_4^{*0} & W_4^{*0} \\ 1 & W_4^{*0} & W_4^{*1} & W_4^{*2} & W_4^{*3} \\ 2 & W_4^{*0} & W_4^{*2} & W_4^{*4} & W_4^{*6} \\ 3 & W_4^{*0} & W_4^{*3} & W_4^{*6} & W_4^{*9} \end{array} \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}$$

$$W_4^{*0} = 1$$
$$W_4^{*1} = e^{j\frac{2\pi}{4}\cdot 1} = \cos\left(\frac{2\pi}{4}\right) + j\sin\left(\frac{2\pi}{4}\right) = j$$
$$W_4^{*2} = e^{j\frac{2\pi}{4}\cdot 2} = \cos(\pi) + j\sin(\pi) = -1$$
$$W_4^{*3} = e^{j\frac{2\pi}{4}\cdot 3} = -j$$
$$W_4^{*4} = e^{j\frac{2\pi}{4}\cdot 4} = 1$$
$$W_4^{*6} = e^{j\frac{2\pi}{4}\cdot 6} = -1$$
$$W_4^{*9} = e^{j\frac{2\pi}{4}\cdot 9} = j$$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 8 \\ 16 \end{bmatrix}$$

$$x(n) = \{4, 8, 8, 16\}$$

HW-2

2.2) Assume Image Block

$$
\begin{array}{cc}
 & n \\
x[n_1 n_2] = & \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 3 \\ 3 & 4 & 3 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix} k
\end{array}
$$

A) Generate transform matrix A for 1-D 4 point DCT

$$y[k] = a[k] \sum_{n=0}^{N-1} x[n] \cos \left[\frac{\pi(2n+1)k}{2N} \right]$$

$$a[0] = \sqrt{\tfrac{1}{N}} \text{ else } a[k] = \sqrt{\tfrac{2}{N}} \rightarrow a[0] = \sqrt{\tfrac{1}{4}} = \tfrac{1}{2} \Big\} \; a[k] = \sqrt{\tfrac{2}{4}} \; \sqrt{\tfrac{1}{2}}$$

$$U_{k,n} = a(k) \cos\left(\frac{\pi k}{2N}(2n+1)\right)$$

$$A = \begin{bmatrix}
\frac{1}{2}\cos\left(\frac{0}{8}(1)\right) & \frac{1}{2}\cos\left(\frac{0}{8}(3)\right) & \frac{1}{2}\cos\left(\frac{0}{8}(5)\right) & \frac{1}{2}\cos\left(\frac{0}{8}(7)\right) \\
\sqrt{\frac{2}{4}}\cos\left(\frac{\pi}{8}(1)\right) & \sqrt{\frac{2}{4}}\cos\left(\frac{\pi}{8}(3)\right) & \sqrt{\frac{2}{4}}\cos\left(\frac{\pi}{8}(5)\right) & \sqrt{\frac{2}{4}}\cos\left(\frac{\pi}{8}(7)\right) \\
\sqrt{\frac{2}{4}}\cos\left(\frac{2\pi}{8}(1)\right) & \sqrt{\frac{2}{4}}\cos\left(\frac{2\pi}{8}(3)\right) & \sqrt{\frac{2}{4}}\cos\left(\frac{2\pi}{8}(5)\right) & \sqrt{\frac{2}{4}}\cos\left(\frac{2\pi}{8}(7)\right) \\
\sqrt{\frac{2}{4}}\cos\left(\frac{3\pi}{8}(1)\right) & \sqrt{\frac{2}{4}}\cos\left(\frac{3\pi}{8}(3)\right) & \sqrt{\frac{2}{4}}\cos\left(\frac{3\pi}{8}(5)\right) & \sqrt{\frac{2}{4}}\cos\left(\frac{3\pi}{8}(7)\right)
\end{bmatrix}$$

$$A = \begin{bmatrix}
.5 & .5 & .5 & .5 \\
.70709 & .70696 & .70669 & .70629 \\
.70704 & .70650 & .70545 & .70385 \\
.70695 & .70576 & .70337 & .69980
\end{bmatrix}$$

B) Calculate forward 2D transform $Y = AXA^T$

$$Y = \begin{bmatrix}
.5 & .5 & .5 & .5 \\
.70709 & .70696 & .70669 & .70629 \\
.70704 & .70650 & .70545 & .70385 \\
.70695 & .70576 & .70337 & .69980
\end{bmatrix}
\begin{bmatrix}
1 & 2 & 3 & 4 \\
2 & 3 & 4 & 3 \\
3 & 4 & 3 & 2 \\
4 & 3 & 2 & 1
\end{bmatrix}
\begin{bmatrix}
.70695 & .70704 & .70709 & .5 \\
.70576 & .70650 & .70696 & .5 \\
.70337 & .70545 & .70669 & .5 \\
.69980 & .70385 & .70629 & .5
\end{bmatrix}$$

$$Y = \begin{bmatrix}
5 & 6 & 6 & 5 \\
7.1 & 8.5 & 8.5 & 7.1 \\
7.1 & 8.5 & 8.5 & 7.1 \\
7.0 & 8.4 & 8.4 & 7.0
\end{bmatrix}
\begin{bmatrix}
.70695 & .70704 & .70709 & .5 \\
.70576 & .70650 & .70646 & .5 \\
.20337 & .70545 & .70664 & .5 \\
.69980 & .70385 & .70629 & .5
\end{bmatrix}$$

$$Y = \begin{bmatrix}
12.5 & 15.5 & 15.5 & 11 \\
17.7 & 22.0 & 22.0 & 15.6 \\
17.7 & 22.0 & 22.0 & 15.6 \\
17.5 & 21.7 & 21.8 & 15.4
\end{bmatrix}$$

HW-2

2.3) Period of sinusoid = .25mm

eye is 20mm in diameter w/ focal point of 3mm from the lens

A) What is the vertical spatial frequency of the projection when viewed at 25cm (250mm) away? In units of cycles/degree

$h_r = \frac{d_f h}{d_o} = h_r = \frac{d_f \cdot (.25)}{250mm}$

d_f = distance focal center + retina

$d_f = 20 - 3$

$d_f = 17mm$

$h_r = \frac{17mm \cdot (.25mm)}{250 mm}$

$h_r = .017 mm$

$\theta = \tan^{-1}(.017) = .97393$

$\frac{1}{\theta}$ cycles/degree $= \frac{1}{.97393} \approx \boxed{1.027 \text{ cycles/degree}}$

B) Determine Optimal viewing distance for this image (projection of the image has a spatial frequency b/w 3~10 cycles/degree on retina)

4 cycles/degree $= \frac{1}{\theta}$

$\theta = .25$

$\tan(\theta) = h_r$

$\tan(.25) = .0044$

$h_r = \frac{d_f h}{d_o}$

$.0044 = \frac{17mm \cdot (.25mm)}{x}$

$.0044 = \frac{4.25mm}{x}$

$4.25 = .0044x$

$x = 965.91mm = \boxed{96.591cm}$

HW-2

2.4) Consider 2 valid colors (c_1 c_2) w/ coordinates (x_1, y_1) and (x_2, y_2) in the chromaticity diagram. Derive the expression(s) for computing the relative percentages of color c_1 and c_2 composing a given color c that is known to lie of the straight line joining these colors

$c_1 = (x_1, y_1)$ $[X_1\ Y_1\ Z_1]$ $P_1 = \%$ of color c_1 in C

$c_2 = (x_2, y_2)$ $[X_2\ Y_2\ Z_2]$ $P_2 = \%$ of color c_2 in C

$C = (x\ y)$ $[X\ Y\ Z]$

$$d(c_1, c_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d(c_1, c) = \sqrt{(x_1 - x)^2 + (y_1 - y)^2}$$

$$d(c_2, c) = \sqrt{(x_2 - x)^2 + (y_2 - y)^2}$$

$$P_1 = \frac{d(c_1, c_2) - d(c_1, c)}{d(c_1, c_2)} \cdot 100 = \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2}}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} \times 100$$

$$P_2 = \frac{d(c_1, c_2) - d(c_2, c)}{d(c_1, c_2)} \cdot 100 = \frac{\sqrt{(x_1 + x_2)^2 + (y_1 - y_2)^2} - \sqrt{(x_2 - x)^2 + (y_2 - y)^2}}{\sqrt{(x_1 - y_2)^2 + (y_1 - y_2)^2}} \cdot 100$$

when $C = C_1 \rightarrow P_1 = 100\%$ and $P_2 = 0\%$

when $C = C_2 \rightarrow P_1 = 0\%$ and $P_2 = 100\%$