# ABBAS's MATH 5300 BLOG

**Mathematics is the mother of all sciences.**

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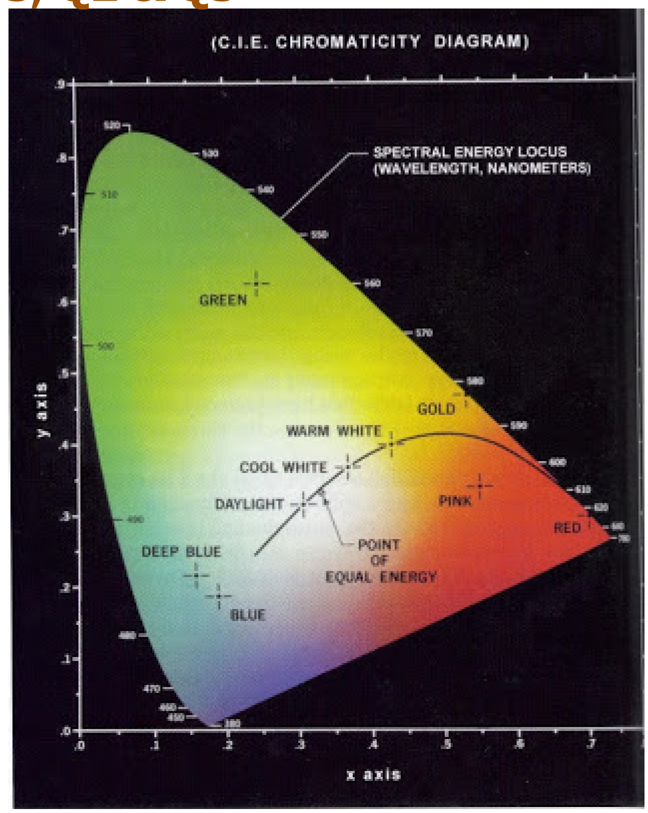
# Assignment #3, Q2 & Q3




Problem 2: (book Page 457) Consider any two valid colors $C_1$ and $C_2$ with coordinates

$(x_1, y_1)$ and $(x_2, y_2)$in the chromaticity diagram. Derive the necessary general expressions for computing the relative percentages of colors $C_1$ and $C_2$ composing a given color that is known to lie on the straight line joining these two colors.

Solution:

Let C(x, y) be any point (color) on the

line joining the two given point (colors) $C_1(x_1, y_1)$ and $C_2(x_2, y_2)$. The distance between the two given colors $C_1$ and $C_2$ is given by the distance formula in calculus:

$d\ (c_1, c_2) = \surd(x_1 - x_2)^2 + (y_1 - y_2)^2$

Distance between $C_1(x_1, y_1)$ and C(x, y) is $d_1\ (c_1, c) = = \text{sqrt}\{(x_1 - x)^2 + (y_1 - y)^2\}$ . And distance between $C_2(x_2, y_2)$ and (x, y) is

$d_2\ (c_2, c) = \text{sqrt}\{(x_2 - x)^2 + (y_2 - y)^2\}$.

Let $P_1$ = percentage of color $C_1(x_1, y_1)$ in C(x, y)

and $P_2$ = percentage of color $C_2(x_2, y_2)$ in C(x, y)

then $P_1= [\{d\ (c_1, c_2) - d_1\ (c_1, c)\}/\ d\ (c_1, c_2)\ ]^* 100\ \%$ .....(i)

and $P_2 = \{d\ (c_1, c_2) - d_2\ (c_2, c)\ \}/\ d\ (c_1, c_2) = (100 - P_1)\ \%$ ......(ii)

Special Cases:

(1) When $C = C_1$ then

$P_1$ = [{d ($c_1$, $c_2$) - 0} / d ($c_1$, $c_2$) ]* 100 % = 1 * 100 % = 100 % and

$P_2$ = {d ($c_1$, $c_2$) - $d_2$ ($c_2$, $c_1$)}/ d ($c_1$, $c_2$) = 0% = (100 – $P_1$) % = (100 – 100)% = 0%

(2)When C = $C_2$ then

$P_1$=[{d ($c_1$, $c_2$) - $d_1$ ($c_1$, $c_2$)}/ d ($c_1$, $c_2$) ]* 100 % = 0% and

$P_2$ = {d ($c_1$, $c_2$) - $d_2$ ($c_2$, $c_2$)}/ d ($c_1$, $c_2$) ]*100 % = 1*100% = 100%

= (100 – $P_1$) % = (100 – 0) % = 100%

Note:

Percentage of the colors $C_1(x_1, y_1)$ and $C_2(x_2, y_2)$ in any given point (color) between $C_1$ and $C_2$ can be calculated by using the equations (i) and (ii) above.

For Example:

Let us consider 380 nm and 520 nm wavelengths. 380 nm wavelength has x and y coordinates as $C_1$ (0.175, 0.003) and 520nm wavelength has x and y coordinated as $C_2$ (0.055, 0.840). We take any point (color) e.g. C(x, y) with x and y coordinates say C (0.115, 0.4215) on the line joining $C_1$ and $C_2$. We can calculate percentage of $C_1$ and $C_2$ in C as follows:

Using (i) above: Percentage of 380nm wavelength in C(x, y) = $P_1$, so

$P_1$= {d ($c_1$, $c_2$) - $d_1$ ($c_1$, c)}/ d ($c_1$, $c_2$) * 100 %

= {√ (0 .175 – 0.055)$^2$ + (0.003 – 0.840)$^2$ - √ (0.175 - 0.115)$^2$ + (0.003 – 0.4215)$^2$} / {√ (0 .175 – 0.055)$^2$ + (0.003 – 0.840)$^2$} * 100 %

= {√0.0144 + 0.700569 - √0.0036 + 0.17514225} / {√0.0144 + 0.700569} * 100 %

= (0.845558395 – 0.422779197)/ (0.845558395) * 100 %

= (0.422779198) / (0.845558395) * 100 %

= 0.5 * 100 %

= 50%

Therefore, Percentage of 380nm wavelength [$C_1$ (0.175, 0.003] in C(x, y) = C (0.115, 0.4215) is 50 %

Hence, percentage of 520nm wavelength [$C_2$ (0.055, 0.840)] in C(x, y)
$P_2$= (100 – $P_1$) % = (100 - 50)% = 50% ------------------------------------------------------------------------------------------

## **Problem #3** ( Problem 6.3 p 457)

Consider any three valid colors now c1, c2 and c3 with coordinates (x1,y1), (x2, y2), and
(x3,y3) in the chromacity diagram of Fig 6.5. Derive the necessary general expressions for computing the relative percentages of c1, c2 and c3 composing a given color that is known to lie with in the triangle whose vertices are the coordinates of c1, c2 and c3.

Solution:

Generalizing the result from above for 3 input colours:
We will assume that our given colour $(x, y)$ is composed of a fraction, $f_1$ of colour $c_1(x_1, y_1)$ and a fraction $f_2$ of colour $c_2(x_2, y_2)$ and therefore a fraction $f_3 = 1 - f_1 - f_2$ of colour $c_3(x_3, y_3)$.

Therefore, for the point $(x, y)$:

$$x = f_1x_1 + f_2x_2 + (1 - f_1 - f_2)x_3 \quad (1)$$

$$y = f_1y_1 + f_2y_2 + (1 - f_1 - f_2)y_3 \quad (2)$$

Solving equation (1) for $f_1$:

$$\begin{aligned} x &= f_1x_1 + f_2x_2 + (1 - f_1 - f_2)x_3 \\ &= f_1x_1 + f_2x_2 + x_3 - f_1x_3 - f_2x_3 \\ &= f_1(x_1 - x_3) + f_2(x_2 - x_3) + x_3 \end{aligned}$$

$$\therefore f_1 = \frac{x - x_3 - f_2(x_2 - x_3)}{x_1 - x_3} \quad (3)$$

Substituting this into equation (2), and solving for $f_2$ we obtain (omitting the messy algebra which Maple did for me anyhow):

$$f_2 = \frac{yx_1 - yx_3 - xy_1 + xy_3 + x_3y_1 - x_3y_3 - y_3x_1 + y_3x_3}{-x_2y_1 + x_2y_3 + x_3y_1 + y_2x_1 - y_2x_3 - y_3x_1} \quad (4)$$

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So, to compute the relative percentages, use equation (4) to get $f_2$, then equation (3) to determine $f_1$ and finally, subtract $1 - f_1 - f_2$ to determine $f_3$.

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$$\begin{cases} a_1 + a_2 + a_3 = 1 \\ x_1a_1 + x_2a_2 + x_3a_3 = x \\ y_1a_1 + y_2a_2 + y_3a_3 = y \end{cases}$$

Solving this system is the same as solving the below matrix equation:

$$\begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$ Then $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$

$$|A| = (x_2y_3 + x_3y_1 + x_1y_2) - (x_2y_1 + x_3y_2 + x_1y_3)$$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} x_2y_3 - x_3y_2 & x_1y_3 - x_3y_1 & x_1y_2 - x_2y_1 \\ y_3 - y_2 & y_3 - y_1 & y_2 - y_1 \\ x_3 - x_2 & x_3 - x_1 & x_2 - x_1 \end{bmatrix}$$

Posted by ABBAS at 7:54:00 PM

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