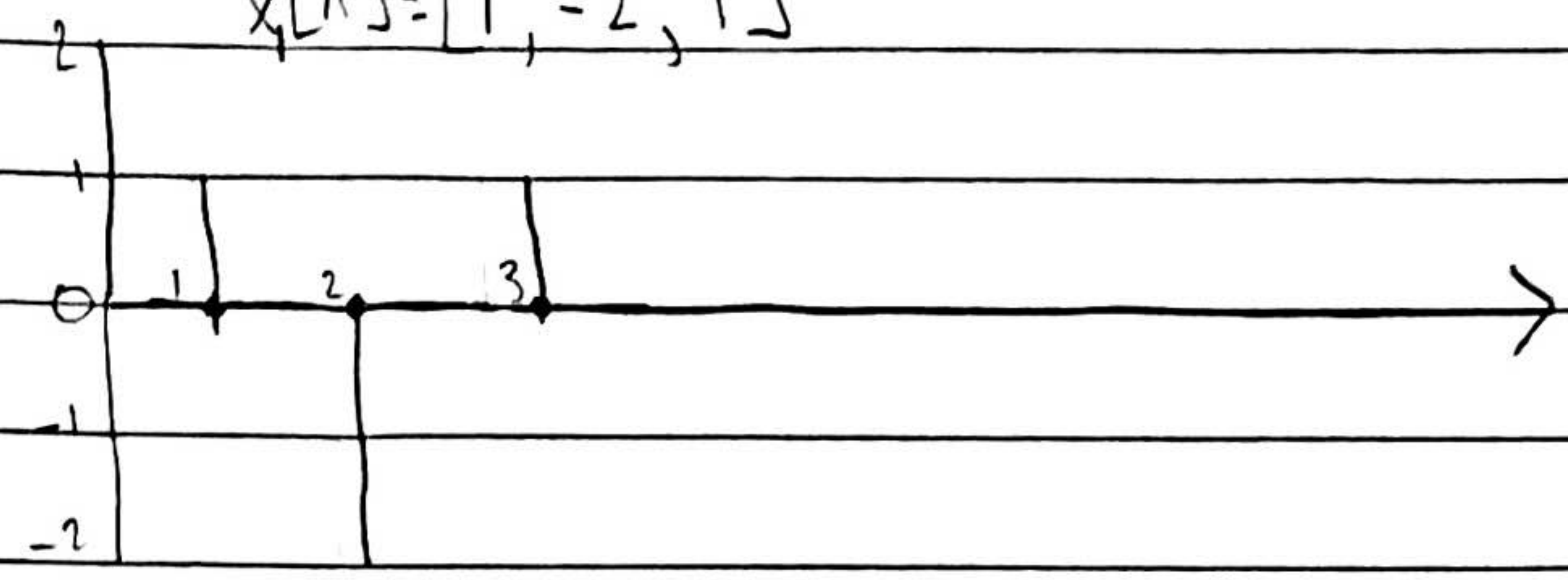
Midterm

a) $x_1[n]=\delta[n]-2\delta[n-1]+\delta[n-2]$, sketch

$x_1[n]=[1,-2,1]$



b) Given $X_2[n_1,n_2]$, express signal as sum of weight

$n_1=2$
$n_2=2$

$$x[n_1,n_2] = -2\delta[1,1]+\delta[1,2]+\delta[2,1]$$

c) Calculate DTFT of $x_1[n]$ and $x_2[n_1,n_2]$

$X_1[n]=[1\ -2\ 1]$ DTFT $=\sum_{n=-\infty} x[n]e^{-j\Omega n}$

$X_1[0]=(1\cdot1)+(-2\cdot1)+(1\cdot1)=0$

$X_2[1]=1\cdot1+(-2\cdot -j)+(1\cdot -1)=2j \rightarrow [0,2j,4]$

$X_3[2]=1\cdot1+(-2\cdot -1)+(1\cdot1)=4$

$$X_2[n_1,n_2] = \begin{bmatrix}1&0\\-2&1\end{bmatrix}$$

$X_2[n,0]=(1\cdot1)+(0\cdot1)=1 \rightarrow \begin{bmatrix}1,1\\2,1-j\end{bmatrix}$

$X_2[n,1]=(1\cdot1)+(0\cdot -j)=1$

$X_2[n_2]=(1\cdot1)+(1\cdot1)=2$

$X_2[n_2]=(1\cdot1)+(1\cdot -j)=1-j$

2 5 -1 -2

	1	2
2	2	4
1	1	2
-1	-1	-2

	1	2	-1
2	2	4	-2
1	1	2	-1
-1	-1	-2	1

2) $x[n] = \delta[n+1] + 2\delta[n] - \delta[n-2]$

$h[n] = 2\delta[n] + \delta[n-1] - \delta[n-2]$

$$\begin{array}{cccc} n+1 & n & n-1 & n-2 \end{array}$$

$x[n] = [1 \;\; 2 \;\; 0 \;\; -1]$

$h[n] = [0 \;\; 2 \;\; 1 \;\; -1]$

	1	2	0	-1
0	0	0	0	0
2	2	4	0	-2
1	1	2	0	-1
-1	-1	-2	0	1

$y[n] = [0, 2, 5, 1, -4, -1, 1]$

$y[n] = 0\delta[n+1] + 2\delta[n] + 5\delta[n-1] + \delta[n-2] - 4\delta[n-3] - \delta[n-4] + \delta[n-5]$

$X(z) = z^{1} + 2 + 0 - z^{-2}$

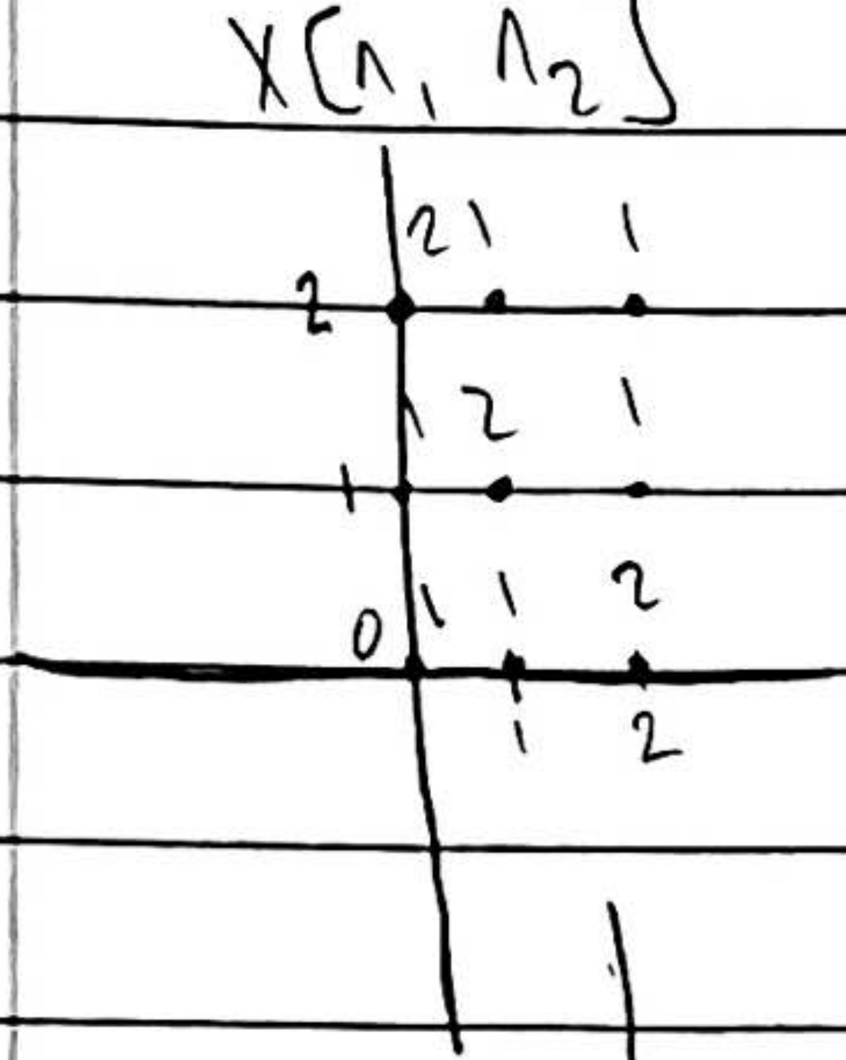
$h(z) = 0 + 2 + z^{-1} - z^{-2}$

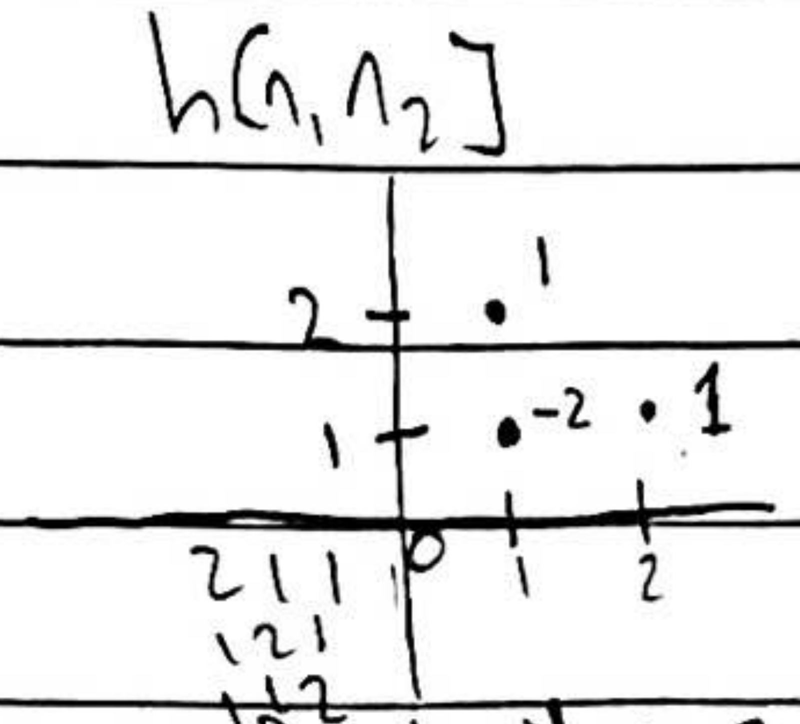
$y(z) = 0 + 2 + 5z^{-1} + z^{-2}$

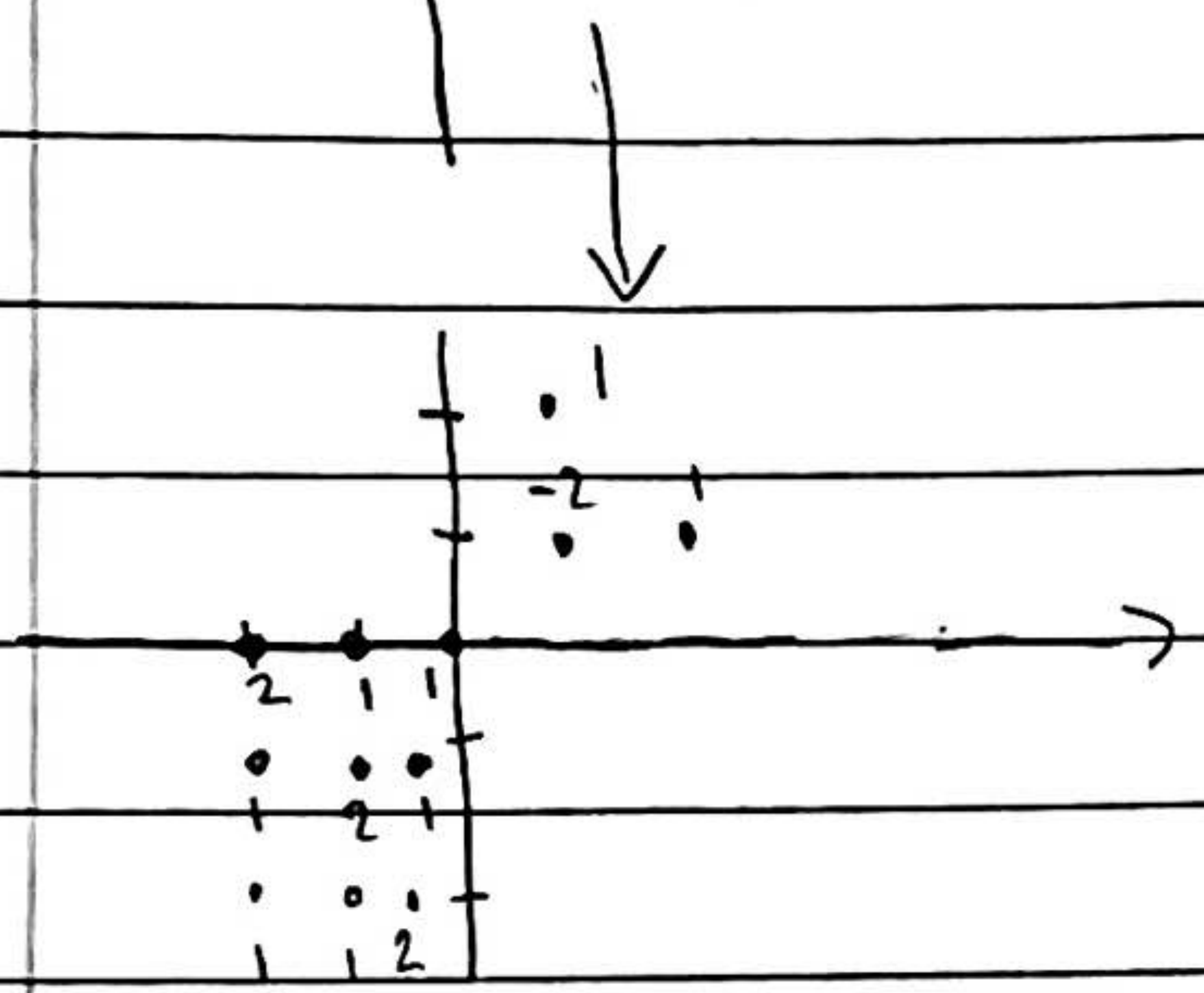
$X(z) \cdot h(z) = (z^{1} + 2 + 0 - z^{-2})(0 + 2 + z^{-1} - z^{-2})$

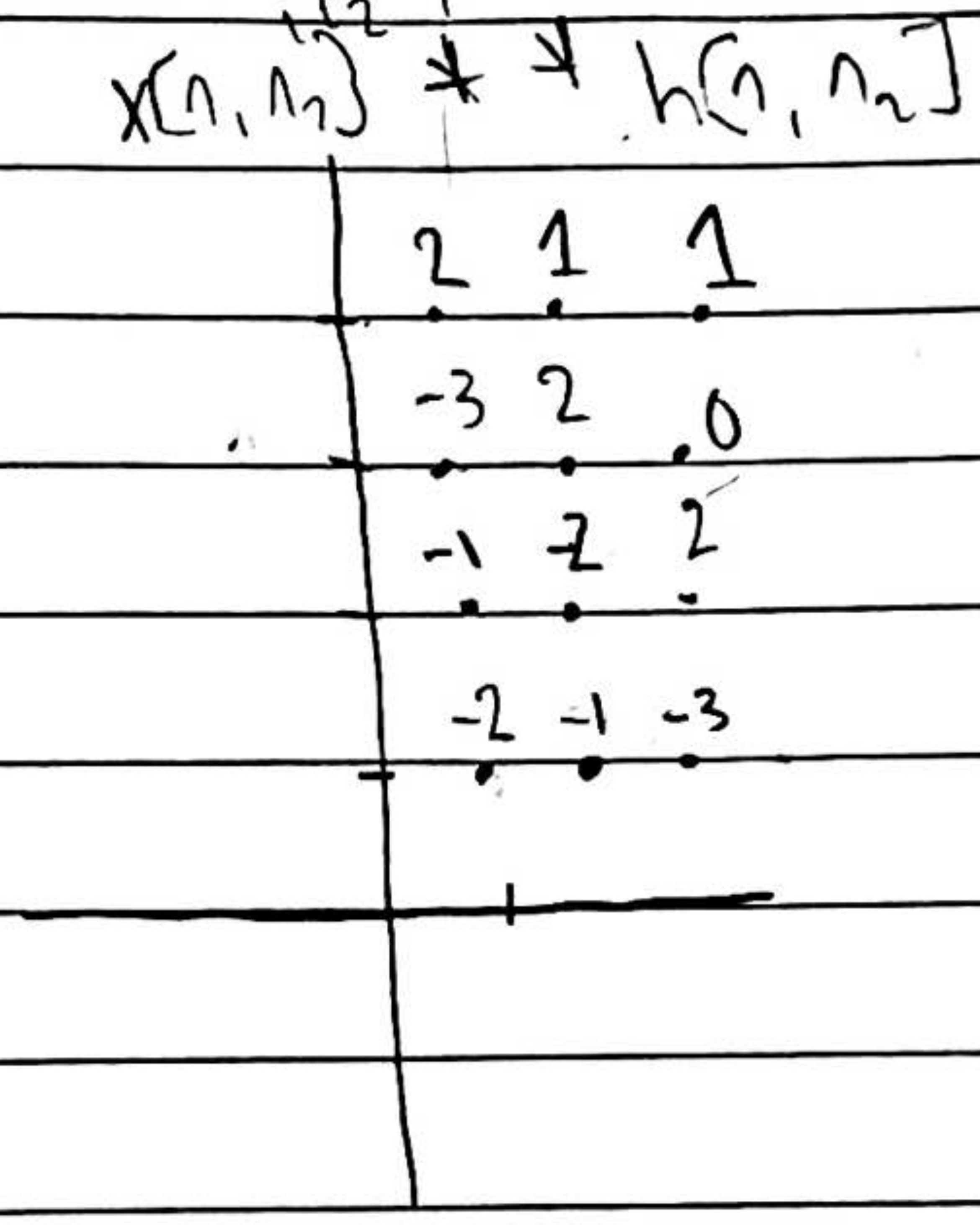
$X(z) \cdot h(z) = 2 + 5z^{-1} + z^{-2} - 4z^{-3} - z^{-4} + z^{-5}$

3) Calculate 2D convolution $y[n_1, n_2] = x[n_1, n_2] ** h[n_1, n_2]$









4) 1D Discrete Sine Transform (DST)

$$y[k] = \sum_{n=0}^{N-1} x[n] \sin\left[\frac{\pi(2n+1)(k+1)}{2N}\right] \quad 0 \le k \le N-1$$

a) Generate transform matrix for 4-point DST

$$A = \begin{bmatrix} \sin\left(\frac{\pi}{8}\right), & \sin\left(\frac{3\pi}{8}\right), & \sin\left(\frac{5\pi}{8}\right), & \sin\left(\frac{7\pi}{8}\right) \\ \sin\left(\frac{2\pi}{8}\right), & \sin\left(\frac{6\pi}{8}\right), & \sin\left(\frac{10\pi}{8}\right), & \sin\left(\frac{14\pi}{8}\right) \\ \sin\left(\frac{3\pi}{8}\right), & \sin\left(\frac{4\pi}{8}\right), & \sin\left(\frac{15\pi}{8}\right), & \sin\left(\frac{21\pi}{8}\right) \\ \sin\left(\frac{4\pi}{8}\right), & \sin\left(\frac{12\pi}{8}\right), & \sin\left(\frac{20\pi}{8}\right), & \sin\left(\frac{28\pi}{8}\right) \end{bmatrix}$$

b) Basis Vectors = [0 1 2 3]

c) 2D 4-point DST $Y = AXA^T$

$$Y = \begin{array}{cc} A & X \\ \begin{bmatrix} \sin\left(\frac{\pi}{8}\right), & \sin\left(\frac{3\pi}{8}\right), & \sin\left(\frac{5\pi}{8}\right), & \sin\left(\frac{7\pi}{8}\right) \\ \sin\left(\frac{2\pi}{8}\right), & \sin\left(\frac{6\pi}{8}\right), & \sin\left(\frac{10\pi}{8}\right), & \sin\left(\frac{14\pi}{8}\right) \\ \sin\left(\frac{3\pi}{8}\right), & \sin\left(\frac{4\pi}{8}\right), & \sin\left(\frac{15\pi}{8}\right), & \sin\left(\frac{21\pi}{8}\right) \\ \sin\left(\frac{4\pi}{8}\right), & \sin\left(\frac{12\pi}{8}\right), & \sin\left(\frac{20\pi}{8}\right), & \sin\left(\frac{28\pi}{8}\right) \end{bmatrix} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array} A^T$$

$$Y = \begin{bmatrix} .1046 & .1645 & .1645 & .1046 \\ .2191 & .3287 & .3287 & .2191 \\ .3282 & .4925 & .4925 & .3282 \\ .4369 & .6558 & .6558 & .4369 \end{bmatrix} \begin{bmatrix} \sin\left(\frac{4\pi}{8}\right), & \sin\left(\frac{3\pi}{8}\right), & \sin\left(\frac{2\pi}{8}\right), & \sin\left(\frac{\pi}{8}\right) \\ \sin\left(\frac{12\pi}{8}\right), & \sin\left(\frac{4\pi}{8}\right), & \sin\left(\frac{6\pi}{8}\right), & \sin\left(\frac{3\pi}{8}\right) \\ \sin\left(\frac{20\pi}{8}\right), & \sin\left(\frac{15\pi}{8}\right), & \sin\left(\frac{10\pi}{8}\right), & \sin\left(\frac{5\pi}{8}\right) \\ \sin\left(\frac{28\pi}{8}\right), & \sin\left(\frac{21\pi}{8}\right), & \sin\left(\frac{14\pi}{8}\right), & \sin\left(\frac{7\pi}{8}\right) \end{bmatrix}$$

$$Y = \begin{bmatrix} .0549 & .045 & .03 & .01503 \\ .1197 & .0894 & .06 & .03 \\ .1794 & .1347 & .0894 & .045 \\ .2388 & .1794 & .1197 & .0549 \end{bmatrix}$$

5) 256 level gray image contains flat background w/ amplitude of 200, and small foreground object w/ amplitude of 202

a) Is this object visible?

No because this image will have equal histograms within that range. Because the object is only 2 pixels different than the background, it will blend into the background and cannot be seen

b) Suggest and explain reasonable gamma value

Because these values are so close together, and appear on the right side of the spectrum (bright), a gamma value between 1 and 2 will suffice. By using this value, the image will become darker and will show edges within objects that were originally clustered as bright areas. This range is good for showing detail in areas that were originally bright

6) period of waveform $T = 2$ pixels
TV resolution 3840×2160 and $1439mm \times 809\ mm$
Calculate viewing distance such that the frequency in retina is 10 cycles/degree

$d_f = 10\ mm$

$$h_r = \frac{d_f\, h}{d_o}$$

$$\frac{3840}{1439} = \frac{2}{x} \qquad 3840x = 2878$$

$$x = .749479\ mm$$

$$h_r = \frac{(10mm)(.749479)}{x}$$

$$10\ \frac{\text{cycles}}{\text{degree}} = \frac{1}{\theta}$$

$$\theta = .10$$

$$\tan(.10) = .00175$$

$$.00175 = \frac{(10mm)(.749479)}{x}$$

$$.00175x = 7.49479$$

$$\boxed{x = 4294.195\ mm}$$