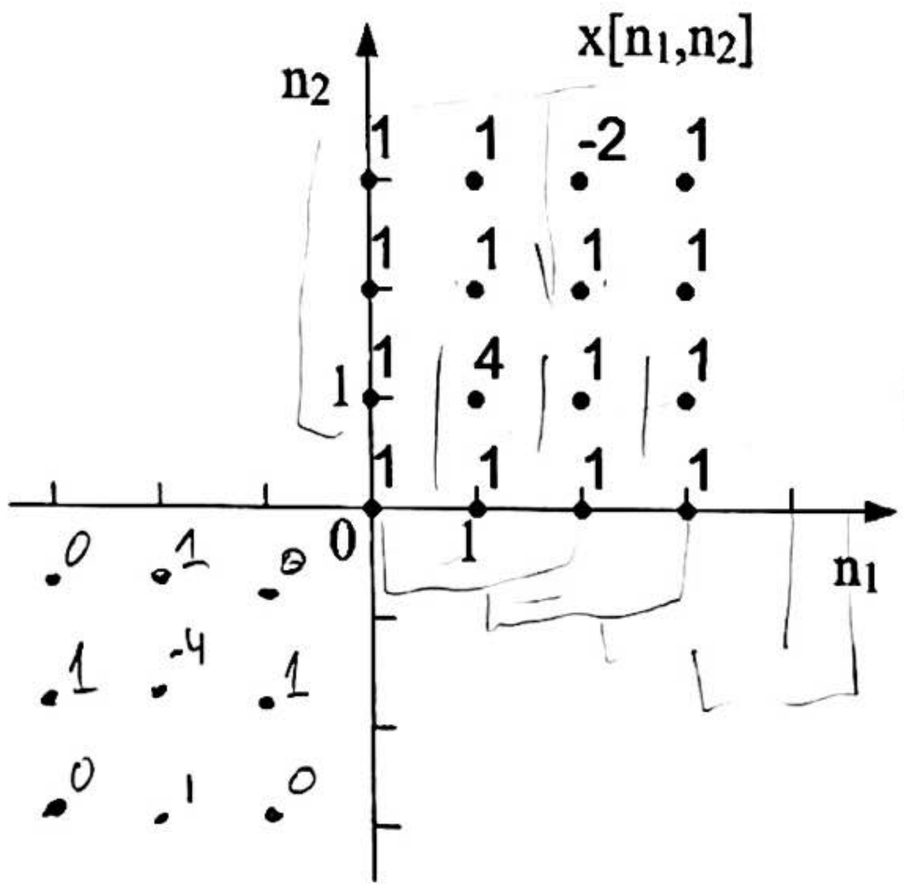
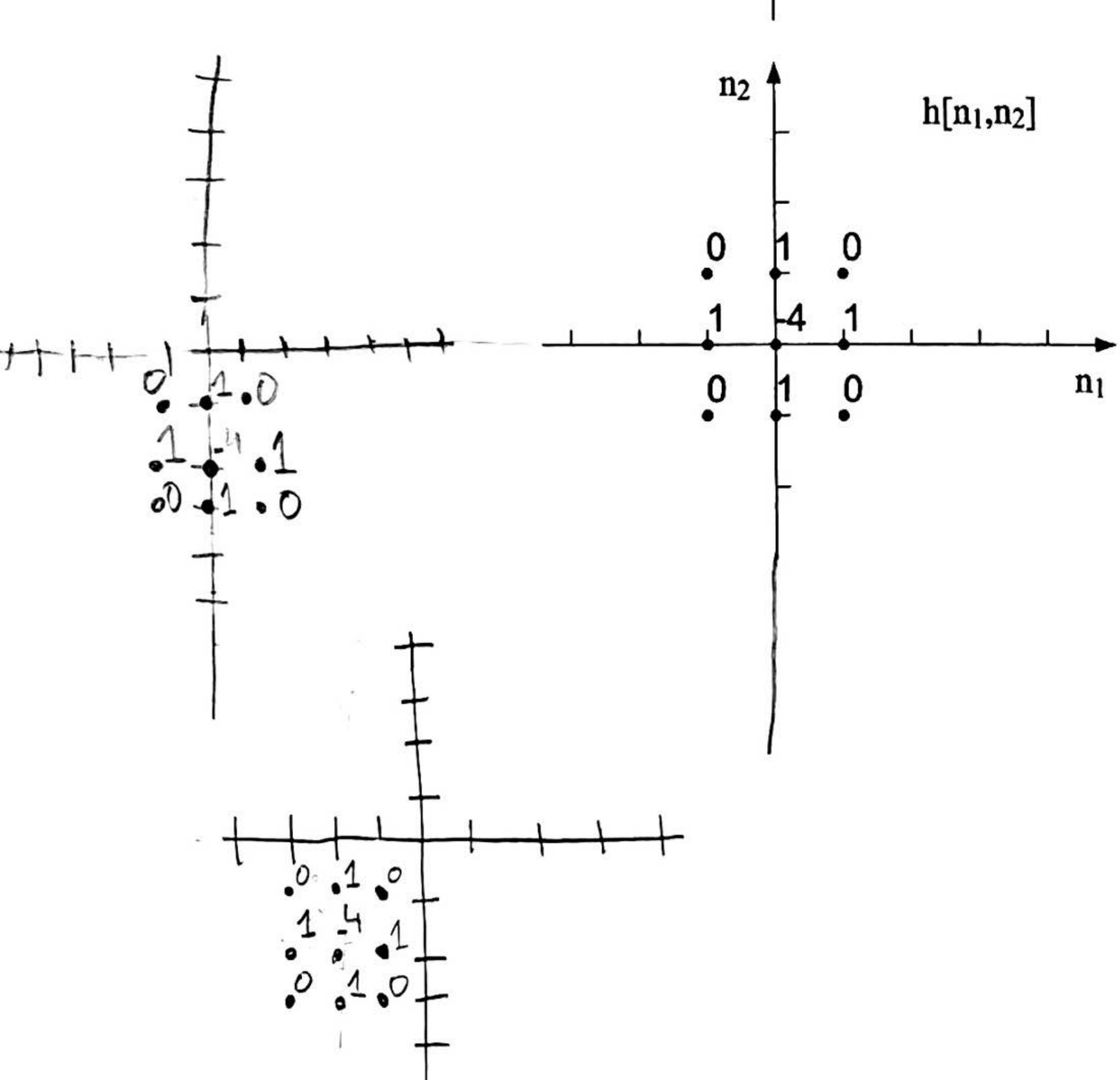
Stevens Institute of Technology
Department of Electrical and Computer Engineering

CpE 645 Image Processing

Homework 1

1.1 Calculate the 2-D convolution $\mathbf{x[n_1,n_2]}$ ** $\mathbf{h[n_1,n_2]}$, provide necessary intermediate steps.





HW-1

1.2) Given $x_1[n] = \delta[n] - 2\delta[n-1]$

$x_2[n] = 2\delta[n] + \delta[n-1] - \delta[n-2]$

$x_1[n] = [1, -2, 0, 0]$

$x_2[n] = [2, 1, -1, 0]$

$X[0] = 1$

$X[1] = -2 - j$

$X[2] = -1$

$X[3] = -2 + j$

1) Linear Convolution $x[n] * x_2[n]$

	1	-2
2	2	-4
1	1	-2
-1	-1	2

$y[n] = [2, -3, -3, 2]$

2) Compute 4 point DFT $X_1[k] = DFT\{x_1[n]\}$ and $X_2[k] = DFT\{x_2[n]\}$

X_1

$X_1[0] = (1 \times 1) + (-2 \times 1) + (0 \times 1) + (0 \times 1) = -1$

$X_1[1] = (1 \times 1) + (-2 \times -j) + (0 \times -1) + (0 \times j) = 1 + 2j$

$X_1[2] = (1 \times 1) + (-2 \times -1) + (0 \times 1) + (0 \times -1) = 3$

$X_1[3] = (1 \times 1) + (-2 \times j) + (0 \times -1) + (0 \times -j) = 1 - 2j$

$X_1[DFT] = [-1, 1+2j, 3, 1-2j]$

X_2

$X[0] = (2 \times 1) + (1 \times 1) + (-1 \times 1) + (0 \times 1) = 2$

$X[1] = (2 \times 1) + (1 \times -j) + (-1 \times -1) + (0 \times j) = 3 - j$

$X[2] = (2 \times 1) + (1 \times -1) + (-1 \times 1) + (0 \times -1) = 0$

$X[3] = (2 \times 1) + (1 \times j) + (-1 \times -1) + (0 \times -j) = 3 + j$

$X_2[DFT] = [2, 3-j, 0, 3+j]$

HW-1

1.2) 3) Compute 4 point DFT $Y[k] = DFT\{y[n]\}$

$$y[n] = [2, -3, -3, 2]$$

$$Y[0] = (2\times1) + (-3\times1) + (-3\times1) + (2\times1) = -2$$

$$Y[1] = (2\times1) + (-3\times -j) + (-3\times -1) + (2\times j) = -1 + 5j$$

$$Y[2] = (2\times1) + (-3\times -1) + (-3\times 1) + (2\times -1) = 0$$

$$Y[3] = (2\times1) + (-3\times j) + (-3\times -1) + (2\times -j) = -1 - 5j$$

4) Show that $Y[k] = X_1[k]\,X_2[k]$

$$Y[k] = [-2, -1+5j, -6, -1-5j]$$

$$\begin{cases} X_1[k] = [-1, 1+2j, 3, 1-2j] \\ X_2[k] = [2, 3-j, 0, 3+j] \end{cases}$$

$$[-2, 3+6j-j-2j^2, 0, 3-6j+j-2j^2]$$

$$[-2, -2j^2+5j+3, 0, -2j^2-5j+3]$$

$$[-2, -1+5j, -6, -1-5j] = [-2, -1+5j, -6, -1-5j]$$

1.3) Find Impulse Response for the following 2D signals

1) $H(z_1, z_2) = 1 - a_1 z_1^{-1} - a_2 z_2^{-1} - a_3 z_1^{-1} z_2^{-1} - a_4 z_1^{-1} z_2^{-1}$

2) $H(w_1, w_2) = 1 - 2\cos(w_1) - 2\cos(w_2)$

1) $x[n_1, n_2] = \delta[n_1, n_2] - \delta[n_1 - 1, n_2] - \delta[n_1, n_2 - 1] - \delta[n_1 - 1\ n_2 - 1] - \delta[n_1, n_2 - 1]$

2) $x[n_1, n_2] = \frac{1}{2\pi^2} \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} X(w_1, w_2)\, e^{j(w_1 n_1 + w_2 n_2)}\, 2\cos(w_1)\cdot 2\cos(w_2)$