Penalty functions

The employment of penalty functions is a paradigm for solving constrained optimisation problems. The central idea of this paradigm is to convert the constrained optimisation problem into an unconstrained optimisation problem that is augmented with a penalty function, which penalises violations of the original constraints. The role of the penalty function is to allow steering the search towards feasible solutions in the search for optimal solutions.

Consider the problem  $(P): f(x): g(x) \leq 0, h(x) = 0, x \in X$ . A penalised version of P is given by

$$(P_{\mu}): f(x) + \mu \alpha(x): x \in X,$$

where  $\mu > 0$  is a penalty term and  $\alpha(x) : {}^{n} \mapsto$  is a penalty function of the form equation  $\alpha(x) = \sum_{i=1}^{m} \phi(g_{i}(x)) + \sum_{i=1}^{l} \psi(h_{i}(x)) \cdot eq : penalty_{f}unction$