Barrier functions

In essence, barrier methods also use the notion of using proxies for the constraints in the objective function, so that an unconstrained optimisation problem can be solved instead. However, the concept of barrier functions is different than penalty functions in that they are defined to prevent the solution search method from leaving the feasible region, which is why some of these methods are also called interior point methods.

Consider the primal problem P being defined as flalign* (P) : f(x)

We define the barrier problem BP as -3pt flalign* (BP) : $\inf_{\mu}\ \theta(\mu)$

where $\theta(\mu) = \inf_x f(x) + \mu B(x) : g(x) < 0, x \in X$ and $B(x)$ is a barrier function. The barrier function is such that its value approaches $+\infty$ as the boundary of the region $x : g(x) \leq 0$ is approached from its interior. Notice that, in practice, it means that the constraint $g(x) < 0$ can be dropped, as they are automatically enforced by the barrier function.

The barrier function $B :^m \rightarrow$ is such that equation B(x) = $\sum_{i=1}^m \phi(g_i(x)), wherecases \phi(y) \geq 0, if y < 0;$

Perhaps the most important barrier function is the Frisch's log barrier function, used in the highly successful primal-dual interior point methods. We will describe its use later. The log barrier is defined as

$$B(x) = -\sum_{i=1}^{m} \ln(-g_i(x)).$$

Figure fig:different$_m uillustratesthebehaviourofthebarrierfunction.Ideally, thebarrierfunction$B(x)$hastheroleofa$
$\in x : g(x) < 0$ but assume infinite value if a solution is at the boundary $g(x) = 0$ or outside the feasible region. This is illustrated in the dashed line in Figure fig:different$_m u.Thebarrierfunctionsfordifferentvaluesofbarrierterm\mu$ illustrate how the log barrier mimics this behaviour, becoming more and more pronounced as μ decreases.

figure [width=]part$_2/chapter_1 0/figures/different_m u.pdf Thebarrierfunctionfordifferentvaluesof\mu$
fig:different$_m u$