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CS 355 HW5

Problem 1) Pg 184, #1
4

$n \in [0,1]$

$y = g(n)$

$$g(n) = \begin{cases} 1 & ; n \le 1/3 \\ 2 & ; x > 1/3 \end{cases}$$

PMF;

$P(y=1) = P(n \le 1/3) = \boxed{1/3 = P(y=1)}$

$P(y=2) = P(n > 1/3) = \boxed{2/3 = P(y=2)}$

$E[y] = 1 \cdot P(y=1) + 2 \cdot (y=2)$

$= 1\left(\frac{1}{3}\right) + 2\left(\frac{2}{3}\right)$

$= 1\left(\frac{1}{3}\right) + 4\left(\frac{1}{3}\right) = \frac{5}{3}$

$\therefore$ Expected value of $\boxed{y = \frac{5}{3}}$

Problem 2

$P(0,0) = 0.6, \quad P(0,1) = 0.1$
$P(1,0) = 0.1 \quad P(1,1) = 0.2$

Hardward failures $\rightarrow X$
Soft ware " $\rightarrow y$

(a) $P(X=0) = P(0,0) + P(0,1) = 0.6 + 0.1 = 0.7$
$P(X=1) = P(1,0) + P(1,1) = 0.1 + 0.2 = 0.3$

For y

$P(y=0) = P(1,0) + P(0,0) = 0.1 + 0.6 = 0.7$
$P(y=1) = P(1,1) + P(0,1) = 0.2 + 0.1 = 0.3$

$\therefore$ $P(0,0) = 0.6$
But $P(x=0)\,P(y=0) = (0.7)(0.7) = 0.49$

$P(1,1) = 0.2$
But $P(x=1)\,P(y=1) = (0.3)(0.3) = 0.09$

Both are not equal which gives that X & y are not independent.

b) $E(x+y) = E(x) + E(y)$

$E(x) = 0 . P(x=0) + 1 . P(x=1)$

$E(y) = 0 . P(y=0) + 1 . P(y=1)$

$E(x) = 0 + 1(0.3) = 0.3.$

$E(y) = 0 + 1(0.3) = 0.3$

$E(x+y) = 0.3 + 0.3$

$= 0.6$

$\boxed{E(x+y) = 0.6}$

Problem 3] Pg 123, #20

Geometric distribution

$X \sim Geo(p)$

$\therefore E[x] = \frac{1}{p}$ (mean-Expected value)

$\Rightarrow var(x) = \frac{1-p}{p2}$ (variance)

Problem 4

$$f_X = \begin{cases} Cx^2, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Constant, C; $|x| \leq 1$

$\therefore -1 \leq x \leq 1$

$$\therefore \int_{-1}^{1} Cx^2 \, dx = 1$$

$$C \left[\frac{x^3}{3}\right]_{-1}^{1} = 1$$

$$\frac{C}{3}\left[1 - (-1)\right] = 1$$

$$C\left(\frac{2}{3}\right) = 1 \qquad \boxed{C = \frac{3}{2}}$$

(b) $$E[X] = \int_{-1}^{1} x f(x) \, dx = \int_{-1}^{1} x \, \frac{3}{2} x^2 \, dx$$

$$= \frac{3}{2} \int_{-1}^{1} x^3 \, dx = \frac{3}{2} \left[\frac{x^4}{4}\right]_{-1}^{1}$$

$$\frac{3}{2\times4}\left[1-(+1)\right] = 0$$

$$E[x] = 0$$

$$Var(x) = E[x^2] - (E[x])^2$$

$$= E[x^2] - 0$$

$$= E[x^2]$$

$$E[x^2] = \int_{-1}^{1} x^2 f(x)\,dx$$

$$= \int_{-1}^{1} x^2 \frac{3}{2} x^2\,dx$$

$$= \frac{3}{2}\int_{-1}^{1} x^4\,dx$$

$$= \frac{3}{2\times5}(x^5)_{-1}^{1}$$

$$= \frac{3}{10}(2) = \boxed{\frac{3}{5}}$$

c) $P(x \geq \frac{1}{2}) = \int_{1/2}^{1} f(x)\,dx$

$= \int_{1/2}^{1} \frac{3}{2} x^2\,dx$

$= \frac{\cancel{3}}{2} \left(\frac{x^3}{\cancel{3}}\right)_{1/2}^{1}$

$= \frac{1}{2}\left(1 - \frac{1}{(2)^3}\right)$

$= \frac{1}{2}\left(1 - \frac{1}{8}\right) = \frac{1}{2}\left(\frac{7}{8}\right) = \boxed{\frac{7}{16}}$

Problem 5] $f(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & \text{Otherwise} \end{cases}$

$P(x \leq 2/3 \mid x > 1/3)$

$= \frac{P(1/3 < x \leq 2/3)}{P(x > 1/3)}$

$$P\left(\frac{1}{3} < x \le \frac{2}{3}\right):$$

$$\int_{1/3}^{2/3} 4x^3\,dx = \frac{4}{4}\left(x^4\right)_{1/3}^{2/3}$$

$$= \left(\frac{2}{3}\right)^4 - \left(\frac{1}{3}\right)^4$$

$$= 2^4 \cdot \left(\frac{1}{3}\right)^4 - \left(\frac{1}{3}\right)^4$$

$$= (16-1)\left(\frac{1}{3}\right)^4$$

$$= \frac{\cancel{15}^{5}}{\cancel{81}_{27}} = \boxed{\frac{5}{27}} \quad \text{— (1)}$$

$$P(x > 1/3) = \int_{1/3}^{1} 4x^3\,dx = \frac{4}{4}\left(x^4\right)_{1/3}^{1}$$

$$= 1 - \left(\frac{1}{3}\right)^4$$

$$= 1 - \frac{1}{81}$$

$$= \boxed{\frac{80}{81}} \quad \text{— (2)}$$

$\therefore$ $(1)/(2) = \dfrac{5/27}{80/81}$

$= \dfrac{5 \times \cancel{81}^{3}}{\cancel{80}_{16} \times \cancel{27}} = \boxed{\dfrac{3}{16}}$

$\frac{\cancel{27}}{\cancel{81}}$

Problem $\boxed{6}$ Pg 128, # 31

4 independent rolls (6 sided die)

$x \to$ Getting 1's $\to$ Probability $= 1/6$
$y \to$ Getting 2's $\to$ Prob. $= 1/6$

prob for other no. $= 1 - \frac{2}{6} = \boxed{\frac{4}{6}}$

$\therefore$ $P(X = x, Y = y) = \binom{4}{x,\, y,\, 4-x-y} \left(\frac{1}{6}\right)^{x} \left(\frac{1}{6}\right)^{y} \left(\frac{4}{6}\right)^{4-x-y}$

$\therefore$ $x + y \leq 4$