Problem 1) [9 points] pg 188, #11

**Solution to Problem 3.11.** (a) $X$ is a standard normal, so by using the normal table, we have $\mathbf{P}(X \leq 1.5) = \Phi(1.5) = 0.9332$. Also $\mathbf{P}(X \leq -1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$.

(b) The random variable $(Y-1)/2$ is obtained by subtracting from $Y$ its mean (which is 1) and dividing by the standard deviation (which is 2), so the PDF of $(Y-1)/2$ is the standard normal.

(c) We have, using the normal table,

$$\begin{aligned}
\mathbf{P}(-1 \leq Y \leq 1) &= \mathbf{P}\big(-1 \leq (Y-1)/2 \leq 0\big) \\
&= \mathbf{P}(-1 \leq Z \leq 0) \\
&= \mathbf{P}(0 \leq Z \leq 1) \\
&= \Phi(1) - \Phi(0) \\
&= 0.8413 - 0.5 \\
&= 0.3413,
\end{aligned}$$

where $Z$ is a standard normal random variable.

Note: answers left with decimal points or $\Phi()$ are all correct.

Problem 2) [7 points] pg 189, #13

**Solution to Problem 3.13.** Let $X$ and $Y$ be the temperature in Celsius and Fahrenheit, respectively, which are related by $X = 5(Y-32)/9$. Therefore, 59 degrees Fahrenheit correspond to 15 degrees Celsius. So, if $Z$ is a standard normal random variable, we have using $\mathbf{E}[X] = \sigma_X = 10$,

$$\mathbf{P}(Y \leq 59) = \mathbf{P}(X \leq 15) = \mathbf{P}\left(Z \leq \frac{15 - \mathbf{E}[X]}{\sigma_X}\right) = \mathbf{P}(Z \leq 0.5) = \Phi(0.5).$$

From the normal tables we have $\Phi(0.5) = 0.6915$, so $\mathbf{P}(Y \leq 59) = 0.6915$.

Note: answers left with decimal points or $\Phi()$ are all correct.

Problem 3) [9 points] Two continuous random variables $X$ and $Y$ have a joint PDF given by

$$f_{X,Y}(x,y) = \begin{cases} c(x+y), & \text{if } 0 \leq x \leq 2, 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) [3 points] Find $c$
(b) [3 points] Find $f_X(x)$, $f_Y(y)$
(c) [3 points] Find $E(X)$, $E(Y)$, and $E(2X+3Y)$

Solution: (a) $\int_0^2 \int_0^2 c(x+y)dxdy = 1 \rightarrow \; c = \frac{1}{8}$

(b)

$$f_X(x) = \begin{cases} \int_0^2 f_{X,Y}(x,y)dy = \frac{x+1}{4}, \text{if } 0 \leq x \leq 2 \\ 0, \text{ otherwise} \end{cases}$$

Similarly,

$$f_Y(y) = \begin{cases} \frac{\text{y}+1}{4}, \text{if } 0 \leq \text{y} \leq 2 \\ 0, \text{ otherwise} \end{cases}$$

(c) $E[X] = E[Y] = \frac{7}{6}$, $E[2X + 3Y] = \frac{35}{6}$