CS 335 HW 1

1) Power set of $\{a, b, c, d\}$

Power set : $2^A = \{B : B \subset A\}$

& No. of Elements $= A = 4$

$\therefore 2^4 = 16$

$2^4 = \{ \{\}, \{a\}, \{b\}, \{c\}, \{d\},$
$\{a,b\}, \{b,c\}, \{c,d\},$
$\{a,d\}, \{a,c\}, \{b,d\},$
$\{a,b,c\}, \{b,c,d\}, \{a,b,d\},$
$\{a,c,d\}, \{a,b,c,d\} \}$

2) Suppose $A \subset B$ (show: $2^A \subset 2^B$)

let $x \in 2^A$

$\therefore$ Defination of power set :

$2^A = \{B : B \subset A\}$.

$\therefore$ Since that, as $x \in 2^A$

$\Rightarrow x \subset A$

Therefore, $x \subset B$ ($\because A \subset B$)

$\Rightarrow$ ~~Asx~~ As $x \subseteq B$

Implies,

$x \subseteq 2^B$

Hence proved,

If $A \subseteq B$, $2^A \subseteq 2^B$

3) $A = \{2, 4, 6, 8, 10, 12\}$

$B = \{1, 3, 6, 9, 12\}$

$\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

- ~~Q.~~ $\Omega \setminus (A \cup B) = \Omega - (A \cup B)$

$\Rightarrow A \cup B = \{1, 2, 3, 4, 6, 8, 9, 10, 12\}$

$\Rightarrow \underline{\Omega - A \cup B = \{5, 7, 11\}} = \Omega \setminus (A \cup B)$

- $A \cap B = \{6, 12\}$

$\{2, 4, \underline{6}, 8, 10, \underline{12}\} \cap \{1, 3, \underline{6}, 9, \underline{12}\}$

$= \{6, 12\}$

- $A \cup B = \{2, 4, 6, 8, 10, 12\} \cup \{1, 3, 6, 9, 12\}$

$= \{1, 2, 3, 4, 6, 8, 9, 10, 12\}$

4 = undergrad :

$(A \cup B)^c = A^c \cap B^c$

$A^c = \Omega / A \quad ; \quad B^c = \Omega / B$

$\Rightarrow (A \cup B)^c = \Omega / (A \cup B)$

$A^c \cap B^c = (\Omega / A) \cap (\Omega / B)$

$= \Omega / A \cup B$

$= (A \cup B)^c$

$\Rightarrow (A \cup B)^c = A^n \cap B^c$

Proved.

Grad : show

$$A^c = (A^c \cap B) \cup (A^c \cap B^c)$$

$$= A^c \cap (B \cup B^c).$$

$$= A^c \cap \underbrace{(B \cup B^c)}_{\Omega}$$

$$= A^c \cap (\Omega)$$

$$A^c = A^c.$$

Proved.