

1. (9 points) Write a program to simulate the expected waiting time of Example 3.14 (page 168 of the textbook).

Solution: results should be close to 6.25 minutes, see the pdf derived in Pg 168 of the textbook.

2. (8 points) pg 190 problem 18

(a)

$$E[X] = \int_1^3 \frac{x^2}{4} dx = \frac{x^3}{12} \Big|_1^3 = \frac{26}{12} = \frac{13}{6}$$

$$P(A) = \int_2^3 \frac{x}{4} dx = \frac{x^2}{8} \Big|_2^3 = \frac{5}{8}$$

$$f_{X|A} = \begin{cases} \frac{f_X(x)}{P(A)} & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases} = \begin{cases} \frac{2x}{5} & \text{if } 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$E[X|A] = \int_2^3 x \frac{2x}{5} = \frac{2x^3}{15} \Big|_2^3 = \frac{38}{15}$$

(b)

$$E[Y] = [E[X^2]] = \int_1^3 \frac{x^3}{4} dx = \frac{1}{16} x^4 \Big|_1^3 = \frac{80}{16} = 5$$

$$E[y^2] = E[x^4] = \int_1^3 \frac{x^5}{4} dx = \frac{1}{24} x^6 \Big|_1^3 = \frac{1}{24} (729 - 1) = \frac{728}{24} = \frac{91}{3}$$

$$\text{var}(Y) = E[Y^2] - (E[Y])^2 = \frac{91}{3} - 25 = \frac{16}{3}$$

3. (8 points) pg 190 problem 19 a, b

$$1 = \int_1^2 cx^{-2} dx = c \left(-\frac{1}{2} - -\frac{1}{1} \right) = c \frac{1}{2}$$

(a)

$$c = 2$$

(b)

$$P(A) = \int_{1.5}^2 2x^{-2} dx = \frac{1}{3}$$

$$F_{X|A}(x) = \begin{cases} 6x^{-2} & \text{if } 1.5 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$