Problem 1)

Joint PDF :

$$\begin{cases} c(x+y) & ; \quad 0 \le y \le 2 \\ & \quad 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

$$\int_{0}^{2}\int_{0}^{2} c(x+y) \, dy \, dx = 1$$

$$\int_{0}^{2} c\left[2x + \left(\int_{0}^{2} y \, dy \right) \right] dx$$

2) $$\int_{0}^{2} (x+y) \, dy = \int_{0}^{2} x \, dy + \int_{0}^{2} y \, dy$$

$$= 2x + \frac{1}{2} (y)^2$$

$$= 2x + \frac{4}{2} = \boxed{2x + 2}$$

$$\int_{0}^{2} c(2x+2) \, dx = c \int_{0}^{2} 2x+2 \, dx$$

$$= [x^2 + 2x]_{0}^{2}$$

$$= c(4+4) = \boxed{8c}$$

$\therefore$ $8c = 1$

$$\boxed{c = 1/8}$$

2) Marginal of x:

$$\int_0^2 f(x,y)\,dy = \int_0^2 \frac{1}{8}(x+y)\,dy$$

$$= \frac{1}{8}\left[2x + \frac{y^2}{2} \right]_0^2$$

$$= \frac{1}{8}(2x+2) = \boxed{\frac{x+1}{4}} \quad 0 \le x \le 2$$

Marginal of y:

$$\int_0^2 \frac{1}{8}(x+y)\,dx = \frac{1}{8}(2y+2)$$

$$= \boxed{\frac{y+1}{4}} \quad 0 \le y \le 2$$

$E[2x + 4y]$

$$= 2E[x] + 4E[y]$$

$$E[x] = \int_0^2 x f(x)\,dx = \int_0^2 x\left(\frac{x+1}{4}\right)$$

$$= \frac{1}{4}\int_0^2 x(x+1)\,dx$$

$$= \frac{1}{4} \left[\frac{x^3}{3} + \frac{x^2}{2} \right]_0^2 = \frac{1}{4} \left(\frac{8}{3} + 2 \right)^2$$

$$= \boxed{\frac{7}{6}}$$

$$E[y] = E[x] = 7/6$$

$$\downarrow \qquad\qquad \downarrow$$

$$\frac{y+1}{4} \qquad\qquad \frac{x+1}{4}$$

$$\therefore \quad 2E[x] + 4E[y] = 2\left(\frac{7}{6}\right) + 4\left(\frac{7}{6}\right)$$

$$= \frac{14 + 28}{6}$$

$$= \frac{42}{6} = 7$$

(b) $\text{Cov}(x,y)$
$$= E[xy] - E[x] \, E[y]$$

$$E[xy] = \int_0^2 \int_0^2 xy \, \frac{1}{8} (x+y) \, dy \, dx.$$

$$= \frac{1}{8} \int_0^2 x \left[\int_0^2 y (x+y) \, dy \right] dx$$

$$= \frac{1}{8} \int_0^2 x \left[\frac{x y^2}{2} + \frac{y^3}{3} \right]_0^2 dx$$

$$= \frac{1}{8} \int_0^2 x \cdot x \left(2 \right) + \left(\frac{8}{3} \right) dx$$

$$= \frac{1}{8} \int_0^2 2x^2 + \frac{8x}{3} \, dx$$

$$= \frac{1}{8}\left(\frac{2n^3}{3} + \frac{4n^2}{3}\right)_0^2$$

$$= \frac{2}{8 \cdot 3}\left(8 + 8\right)$$

$$= \frac{(2)(16)}{(8)(3)} = \boxed{\frac{4}{3}}$$

$$E[xy] - E[x]E[y]$$

$$\Rightarrow \left(\frac{4}{3}\right) - \left(\frac{7}{6}\right)\left(\frac{7}{6}\right)$$

$$= \frac{4}{3} - \frac{49}{36} = \frac{48 - 49}{36}$$

$$= \boxed{\frac{-1}{36}}$$

Problem 2 Pg 290

10 $\mu = 5, \quad \sigma^2 = 9 \Rightarrow \sigma = 3$

variance.

For production in 100 days

$$\Rightarrow 5 \cdot 100 = 500 = E[s]$$

$$var[s] = 100 \cdot 9 = 900 \quad 30$$

$$\sigma = \frac{9}{10}$$

$\therefore \left(R < \dfrac{440 - 500}{30} \right) = P\left(R < \dfrac{-60}{30} \right)$

$\qquad\qquad\qquad\qquad = P(R < -2)$.

$\qquad = \boxed{-\phi(2) + 1} \qquad \phi(-2)$

$\qquad = \boxed{1 - \phi(2)}$

$\therefore$ For $\quad P(y \geqslant 200 + 5n) \leq 0.05$

$\qquad\qquad P(S_n \geqslant 200 + 5n) \leq 0.05$

$S_n = x_1 + x_2 + \cdots \cdots + x_n.$

$\therefore \quad 1 - 0.05 = 0.95$

$\Rightarrow \quad P(S_n \leq 200 + 5n) \geqslant 0.95.$

$\Rightarrow \qquad R = \dfrac{(200 + 5n) - 5n}{3\sqrt{n}}.$

$\qquad\qquad = \dfrac{200 + 5n - 5n}{3\sqrt{n}} \qquad = \dfrac{200}{3\sqrt{n}}$

$\Rightarrow P\left(B_n < \dfrac{200}{3\sqrt{n}} \right) \geqslant 0.95.$

$\Rightarrow \qquad \sqrt{n} = \dfrac{200}{(3)(0.95)} \qquad = \dfrac{(200)^2}{(9)(0.95)}$

So $n \leqslant \left(\dfrac{200}{(3)(0.95)} \right)^2$

(c)   Exceeds 1000: $P(N \geqslant 220)$

$n = 219$ (for)

$P(S_n \leqslant 1000)$

$\mu = 5$ $\sigma = 3$ $\Rightarrow$ $E[S_n] = 5 \cdot 219$
$= 1095$

$\therefore$ $\sigma = (3)(\sqrt{219})$.

So $Z = \dfrac{1060 - 1095}{3\sqrt{219}} = \dfrac{-95}{(3)(\sqrt{219})}$

$\therefore$ $P(N \geqslant 220) = P(S_{219} \leqslant 1000)$

$= \phi\left(\dfrac{-95}{3\sqrt{219}} \right)$

$= 1 - \phi\left(\dfrac{95}{3\sqrt{219}} \right)$