

Exercises on Jan 29th

CS355/555 Probability and Statistics

Please do your best to respond to each of the following questions within a 10-minute time-frame:

1. Suppose that x and y are continuous random variables ranging from $[0,1]$, find the probability that $2x - y < 1$.
2. In a software-development class, there are two project teams: *Team A* and *Team B*. Past data show that

$$P(\text{A is successful}) = \frac{4}{5}, \quad P(\text{B is successful}) = \frac{3}{5}, \quad P(\text{at least one team succeeds}) = \frac{14}{15}.$$

Find the probability that *Team B* is successful given that at least one team succeeds. In symbols, compute

$$P(B \text{ is successful} \mid (A \text{ is successful}) \cup (B \text{ is successful})).$$

3. A medical test is used to detect a specific disease. The characteristics of the test are as follows: If the disease is present, the test correctly identifies it with a probability of 95%. If the disease is not present, the test incorrectly indicates the disease with a probability of 5%. The probability of any randomly selected individual having the disease is 1%.

What is $P(\text{no disease} \mid \text{positive test})$? This is the probability that the person does not have the disease given that the test is positive. What is $P(\text{disease present} \mid \text{negative test})$? This is the probability that the disease is present given that the test is negative.

4. We have two fair dice: one die has six faces, all numbered six (a "loaded" die), and the other die is a standard six-sided die with faces numbered from 1 to 6.
 - a. What is the probability of rolling a six if a die is chosen at random and tossed?
 - b. If the roll results in a six, what is the probability that the die used is the standard die?

Good luck!