Problem 1) [5 points] pg 184, #1

$$E[Y] = 1 \cdot \frac{1}{3} + 2\frac{2}{3} = \frac{5}{3}$$

Expected Value Rule:

$$E[Y] = \int_0^1 g(x)F_X(x)dx = \int_0^{\frac{1}{3}} 1dx + \int_{\frac{1}{3}}^1 2dx = 1 \cdot \frac{1}{3} + 2\frac{2}{3} = \frac{5}{3}$$

Problem 2) [5 points] The number of hardware failures, X, and the number of software failures, Y , on any day in a small computer lab have the joint distribution P (x, y), where P (0, 0) = 0.6, P (0, 1) = 0.1, P (1, 0) = 0.1, P (1, 1) = .2. Based on the information,

(a) Are X and Y independent? Give a reason.

	0	1	
0	.6	.1	.7
1	.1	.2	.3
	.7	.3	

Not independent $.7 \times .7 \neq .6$

(b) Compute E(X + Y), the total number of failures expected in a day.

$$E[X + Y] = 0 \times .6 + .1 \times 1 + .1 \times 1 + .2 \times 2 = .6$$

Problem 3) [5 points] pg 123, #20

The number C of candy bars you need to eat is a geometric random variable with parameter p. Thus the mean is $E[C] = \frac{1}{p}$, and the variance is $var(C) = \frac{1-p}{p^2}$.

Problem 4) [5 points] Let X be a continuous random variable with PDF given by

$$f_X = \begin{cases} cx^2, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the constant c, [1 point]
(b) Find $E[X]$ and $Var(X)$, [2 points]
(c) Find $P(X \geq \frac{1}{2})$

Solution: (a)

$$1 = \int_{-1}^1 cx^2 dx = \frac{2}{3}c$$

Therefore, $c = \frac{3}{2}$.

(b)

$$E[X] = \int_{-\infty}^{\infty} x f_X(X) dx = \frac{3}{2}\int_{-1}^{1} x^3 dx = 0$$

$$\begin{aligned} Var(X) &= E[X^2] - (E[X])^2 = E[X^2] \\ &= \int_{-1}^{1} x^2 f_X(x) dx \\ &= \frac{3}{2}\int_{-1}^{1} x^4 dx = \frac{3}{5} \end{aligned}$$

(c)

$$P(X \geq 1/2) = \int_{\frac{1}{2}}^{1} \frac{3}{2} x^2 dx = \frac{7}{16}$$

Problem 5) [5 points] Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} 4x^3, & 0 < x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional probability $P(X \leq \frac{2}{3} | X > \frac{1}{3})$

$$\begin{aligned} P(X \leq \frac{2}{3} | X > \frac{1}{3}) &= \frac{P(\frac{1}{3} < X \leq \frac{2}{3})}{P(X > \frac{1}{3})} \\ &= \frac{\int_{\frac{1}{3}}^{\frac{2}{3}} 4x^3 dx}{\int_{\frac{1}{3}}^{1} 4x^3 dx} \\ &= \frac{3}{16} \end{aligned}$$

Problem 6) [Bonus 5 points] pg 128, #31

The marginal PMF p_Y is given by the binomial formula

$$p_Y(y) = \binom{4}{y} \left(\frac{1}{6}\right)^y \left(\frac{5}{6}\right)^{4-y} \quad y = 0, 1, ..., 4$$

To compute the conditional PMF $p_{X|Y}$, note that given that $Y = y$, X is the number of 1s in the remaining $4 - y$ rolls, each of which can take the 5 values 1, 3, 4, 5, 6 with equal probability $\frac{1}{5}$. Thus, the conditional PMF $p_{X|Y}$ is binomial with parameters 4 - y and $p = \frac{1}{5}$

$$p_{X|Y}(x|y) = \binom{4-y}{x} \left(\frac{1}{5}\right)^y \left(\frac{4}{5}\right)^{4-y-x}$$

for all nonnegative integers x and y such that $0 \leq x + y \leq 4$. The joint PMF is now given by

$$p_{X,Y}(x,y) = p_Y(y)p_{X|Y}(x|y) = \binom{4}{x} \left(\frac{1}{6}\right)^y \left(\frac{5}{6}\right)^{4-y-x} \binom{4-y}{x} \left(\frac{1}{5}\right)^y \left(\frac{4}{5}\right)^{4-y-x}$$

for all nonnegative integers x and y such that $0 \leq x + y \leq 4$. For other values of x and y, we have $p_{X,Y}(x,y) = 0$.