

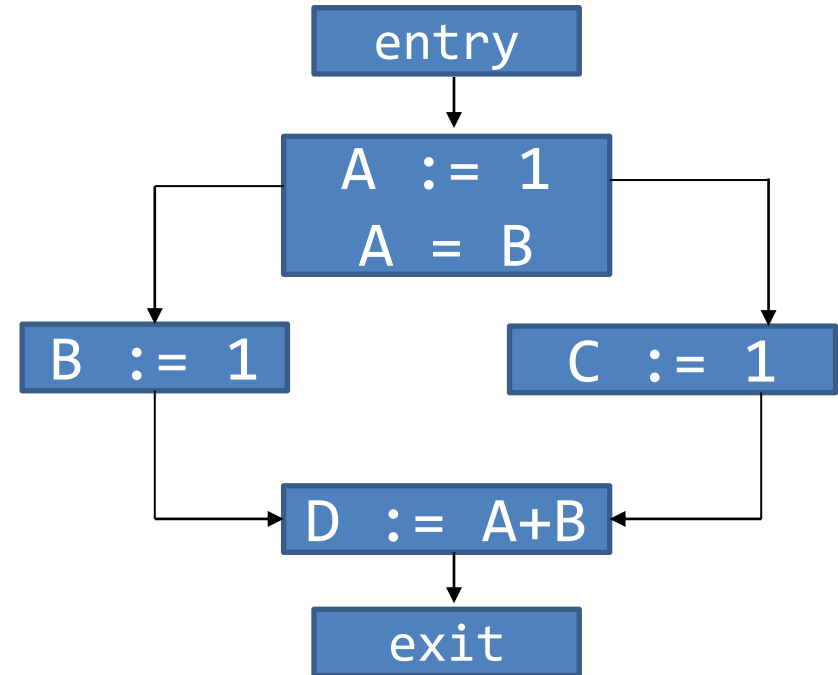
CS406:Compilers

Spring 2021

Week 14: Dataflow Analysis (Recap.) and
Higher-level Loop Optimizations

Recap: Liveness

- Variables are live if *some path* leading to its use exists
- Start from exit block and proceed *backwards* against the control flow



$$\text{LiveOut}(b) = \bigcup_{i \in \text{Succ}(b)} \text{LiveIn}(i)$$

$$\text{LiveIn}(b) = \text{LiveUse}(b) \cup (\text{LiveOut}(b) - \text{Def}(b))$$

\uparrow
 $\text{gen}(b)$

//set that contains all variables
used by block b

\uparrow
 $\text{kill}(b)$

//set that contains all
variables defined by block b

Recap: Reaching Definitions

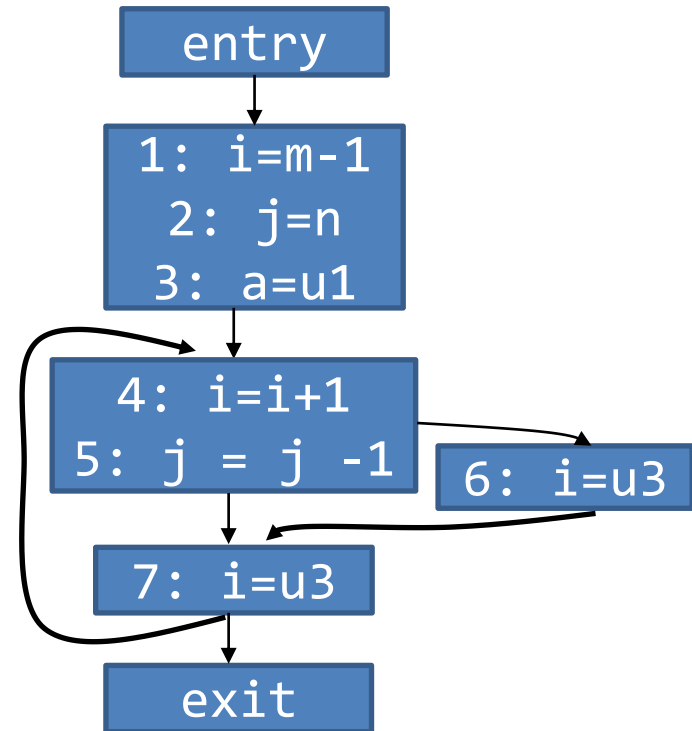
- **Goal:** to know where in a program each variable x may have been defined when control reaches block b
- Definition d reaches block b if there is a path from point immediately following d to b , such that the variable defined in d is not redefined / killed along that path

$$\text{In}(b) = \bigcup_{i \in \text{Pred}(b)} \text{Out}(i)$$

$$\text{Out}(b) = \text{gen}(b) \cup (\text{In}(b) - \text{kill}(b))$$

//set that contains all statements that **may** define some variable x in b
 $\text{gen}(1:a=3; 2:a=4) = \{2\}$

//set that contains all statements that define a variable x that is also defined in b
 $\text{kill}(1:a=3; 2:a=4) = \{1, 2\}$



Recap: Dataflow Analysis

- Any-path problem
 - The previous two analysis (liveness and reaching definitions) determine if some property holds true along *some path* (no guarantees)
 - a variable is used/live along some path starting from its definition (use-def chain)
 - a definition reaches a block b along some path (without intervening redefinition of the variable involved along that path) (def-use chain)

Recap: Dataflow Analysis

- Forward-flow vs. Backward-flow
 - The previous two analysis (liveness and reaching definitions) determine the properties by computing IN and OUT sets backward and forward to the control flow resp.

Recap: Dataflow Analysis

- Applications of RD (reaching definitions)

By building def-use chains:

- RD helps us to analyze if a variable is defined in the program before it is used (think: “uninitialized variable” warnings)
- RD helps us know what all definitions of ‘x’ reaching a block b. This can enable constant folding if all those definitions assign the same constant to x

Recap: Dataflow Analysis

- Applications of Liveness analysis
 - By building use-def chains:
 - “Undefined variable” warnings
 - Register allocation

Recap: Available Expressions

- Expression “x+y” is available at block b if *every* path from entry node to b computes “x+y” (and x and y are not assigned to/defined after initial computation of “x+y”)

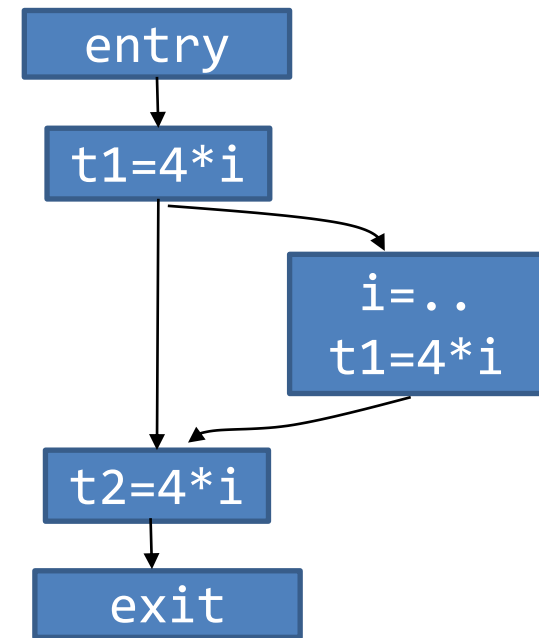
$$\text{In}(b) = \bigcap_{i \in \text{Pred}(b)} \text{Out}(i)$$

$$\text{Out}(b) = \text{gen}(b) \cup (\text{In}(b) - \text{kill}(b))$$

//set that contains expressions
generated by b.

//set that contains expressions
killed by b

//initialization: $\text{out}(\text{entry}) = \{\}$, $\text{out}(\text{all } b \text{ except entry block}) = U$,
the set of all expressions in the program



Recap: Dataflow Analysis

- RD vs. Available Expressions
 - Meet operator: Union vs. Intersection
 - Expression is available at the beginning of a block only if it is available at the end of *all* the predecessor blocks
- vs.
- A definition reaches the beginning of a block whenever it reaches the end of any one or more of its predecessors

Recap: Dataflow Analysis

- All-path, Forward-flow problem
 - The previous analysis (Available expressions) determines if some property holds true along *all paths* (guarantees exist)
 - Whether an expression is computed along all paths

What is an all-path backward-flow problem?

Summary: Dataflow Analysis

	Any-path	All-path
Forward-flow	Reaching Definitions	Available Expressions
Backward-flow	Liveness Analysis	Very-busy expressions*

*also called *anticipated expressions*.

- Avoid recomputing and save space
- Can be used to identify candidates in loop-invariant code motion (see: slide 65, week 12)

Recap: Optimize Loops

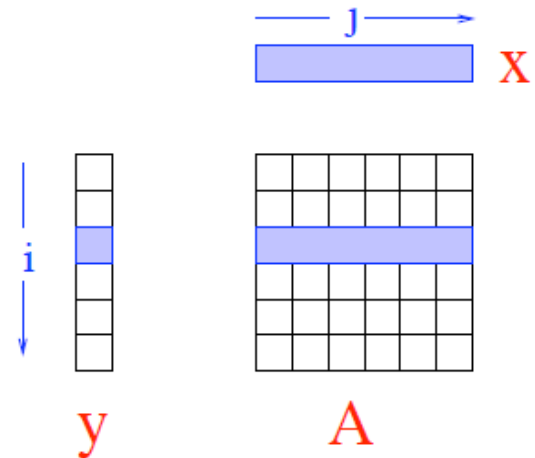
- Low level optimization
 - Moving code around in a single loop
 - Examples: loop invariant code motion, strength reduction, loop unrolling
- High level optimization
 - Restructuring loops, often affects multiple loops
 - Examples: loop fusion, loop interchange, loop tiling

High level loop optimizations

- Many useful compiler optimizations require *restructuring* loops or sets of loops
 - Combining two loops together (*loop fusion*)
 - Switching the order of a nested loop (*loop interchange*)
 - Completely changing the traversal order of a loop (*loop tiling*)
- These sorts of high level loop optimizations usually take place at the AST level (where loop structure is obvious)

Cache behavior

- Most loop transformations target cache performance
 - Attempt to increase *spatial* or *temporal* locality
 - Locality can be exploited when there is reuse of data (for temporal locality) or recent access of nearby data (for spatial locality)
- Loops are a good opportunity for this: many loops iterate through matrices or arrays
- Consider matrix-vector multiply example
 - Multiple traversals of vector: opportunity for spatial and temporal locality
 - Regular access to array: opportunity for spatial locality



$$y = Ax$$

```
for (i = 0; i < N; i++)  
    for (j = 0; j < N; j++)  
        y[i] += A[i][j] * x[j]
```

Loop fusion

```
do i = 1, n
```

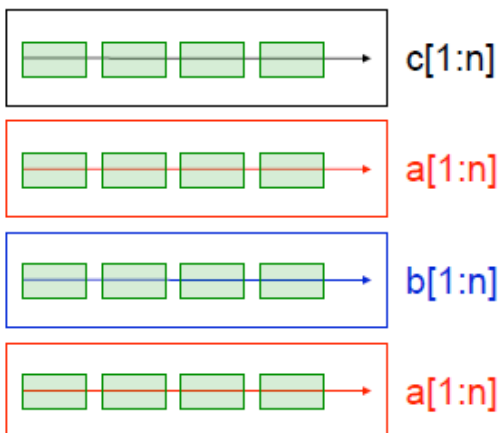
```
  c[i] = a[i]
```

```
end do
```

```
do i = 1, n
```

```
  b[i] = a[i]
```

```
end do
```



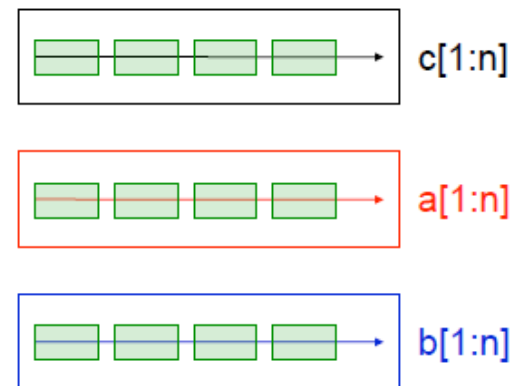
- Combine two loops together into a single loop
- Why is this useful?
- Is this always legal?

```
do i = 1, n
```

```
  c[i] = a[i]
```

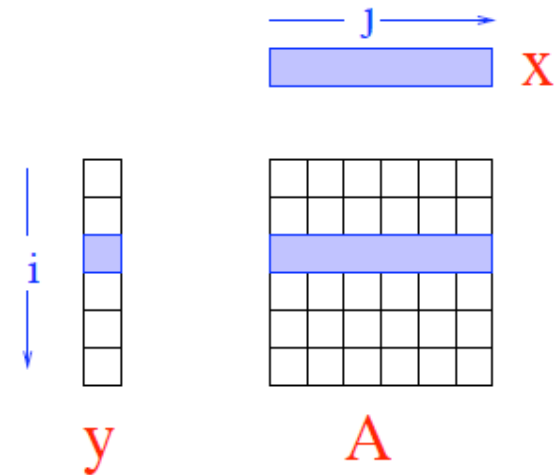
```
  b[i] = a[i]
```

```
end do
```



Loop interchange

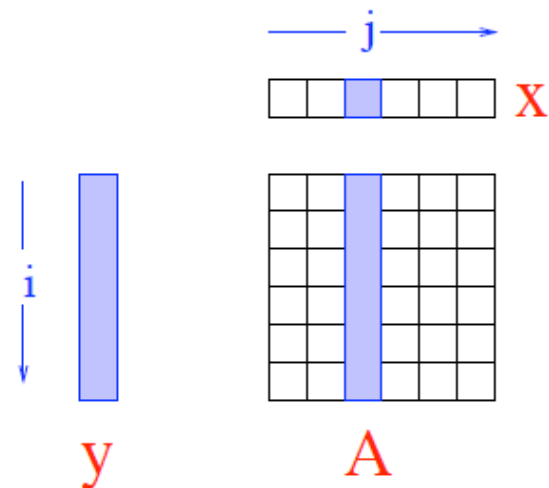
- Change the order of a nested loop
- This is not always legal – it changes the order that elements are accessed!
- Why is this useful?
- Consider matrix-matrix multiply when A is stored in column-major order (i.e., each column is stored in contiguous memory)



```
for (i = 0; i < N; i++)  
  for (j = 0; j < N; j++)  
    y[i] += A[i][j] * x[j]
```


Loop interchange

- Change the order of a nested loop
- This is not always legal – it changes the order that elements are accessed!
- Why is this useful?
- Consider matrix-matrix multiply when A is stored in column-major order (i.e., each column is stored in contiguous memory)



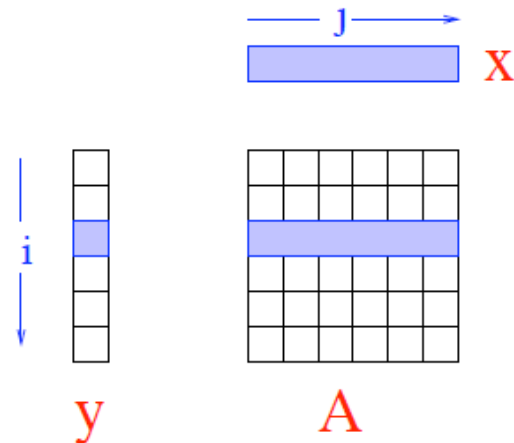
```
for (j = 0; j < N; j++)  
  for (i = 0; i < N; i++)  
    y[i] += A[i][j] * x[j]
```

Loop tiling

- Also called “loop blocking”
- One of the more complex loop transformations
- Goal: break loop up into smaller pieces to get spatial and temporal locality
- Create new inner loops so that data accessed in inner loops fit in cache
- Also changes iteration order, so may not be legal

```
for (i = 0; i < N; i++)  
  for (j = 0; j < N; j++)  
    y[i] += A[i][j] * x[j]
```

```
for (ii = 0; ii < N; ii += B)  
  for (jj = 0; jj < N; jj += B)  
    for (i = ii; i < ii+B; i++)  
      for (j = jj; j < jj+B; j++)  
        y[i] += A[i][j] * x[j]
```

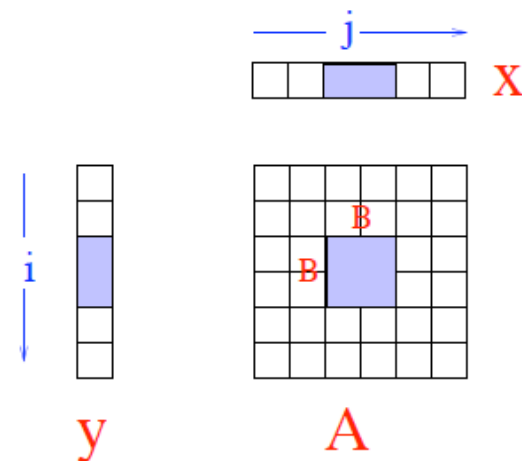


Loop tiling

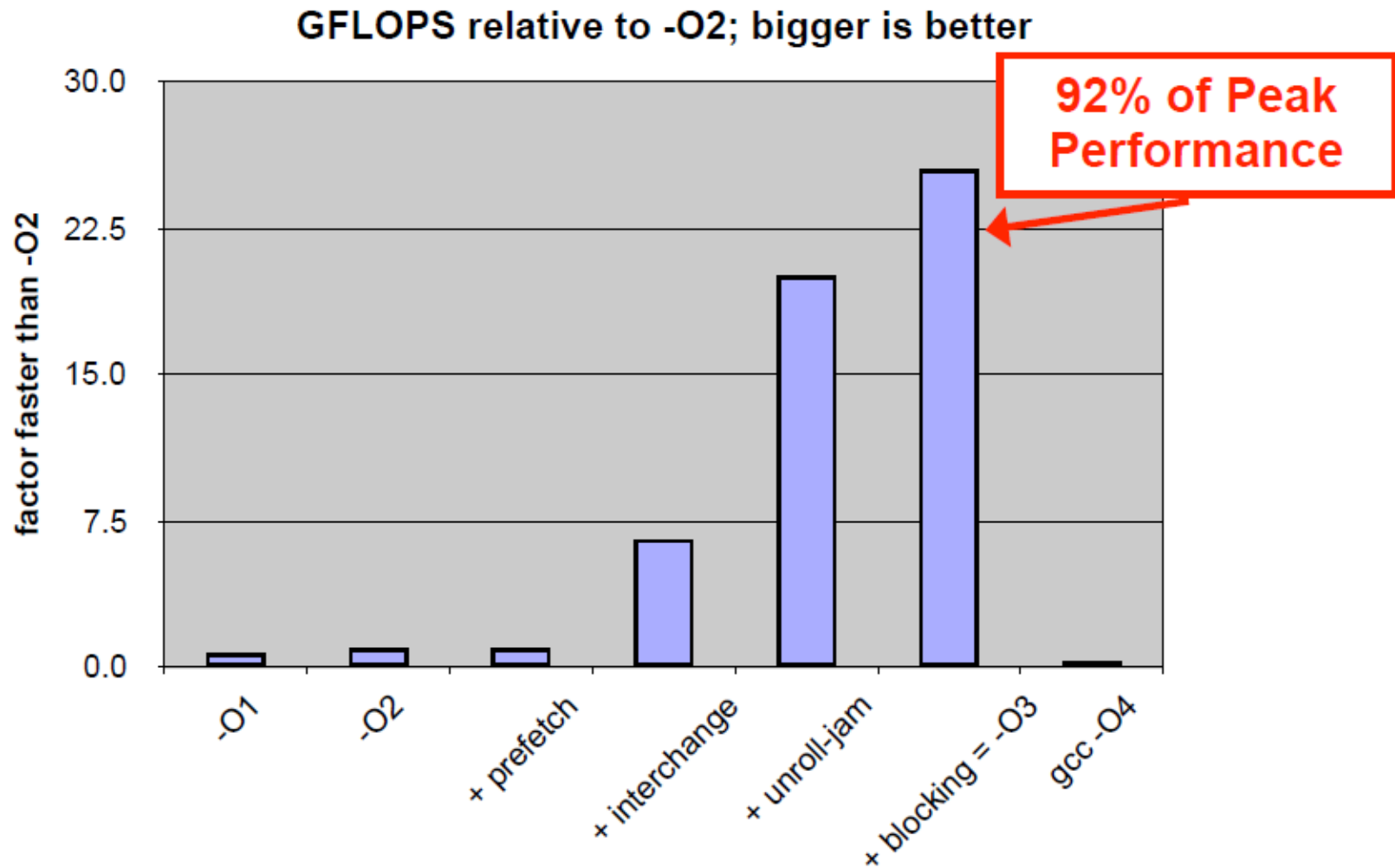
- Also called “loop blocking”
- One of the more complex loop transformations
- Goal: break loop up into smaller pieces to get spatial and temporal locality
- Create new inner loops so that data accessed in inner loops fit in cache
- Also changes iteration order, so may not be legal

```
for (i = 0; i < N; i++)  
  for (j = 0; j < N; j++)  
    y[i] += A[i][j] * x[j]
```

```
for (ii = 0; ii < N; ii += B)  
  for (jj = 0; jj < N; jj += B)  
    for (i = ii; i < ii+B; i++)  
      for (j = jj; j < jj+B; j++)  
        y[i] += A[i][j] * x[j]
```



In a real (Itanium) compiler



Loop transformations

- Loop transformations can have dramatic effects on performance
- Doing this legally and automatically is very difficult!
- Researchers have developed techniques to determine legality of loop transformations and automatically transform the loop
- Techniques like *unimodular transform framework* and *polyhedral framework*
- These approaches will get covered in more detail in advanced compilers course

Dependence Analysis

(separate set of slides posted)

Exercise

1. Draw iteration graph for:

```
for(i=0;i<5;i++)  
    for(j=0;j<5;j++)  
        a[i][j]=a[i-1][j+2]+a[i][j-2]
```

2. What are the distance and direction vectors?
3. Can the loops be interchanged?
4. Repeat 1,2,and 3 for the following loop.

```
for(i=0;i<5;i++)  
    for(j=0;j<5;j++)  
        a[i][j]=a[i-1][j-2]
```