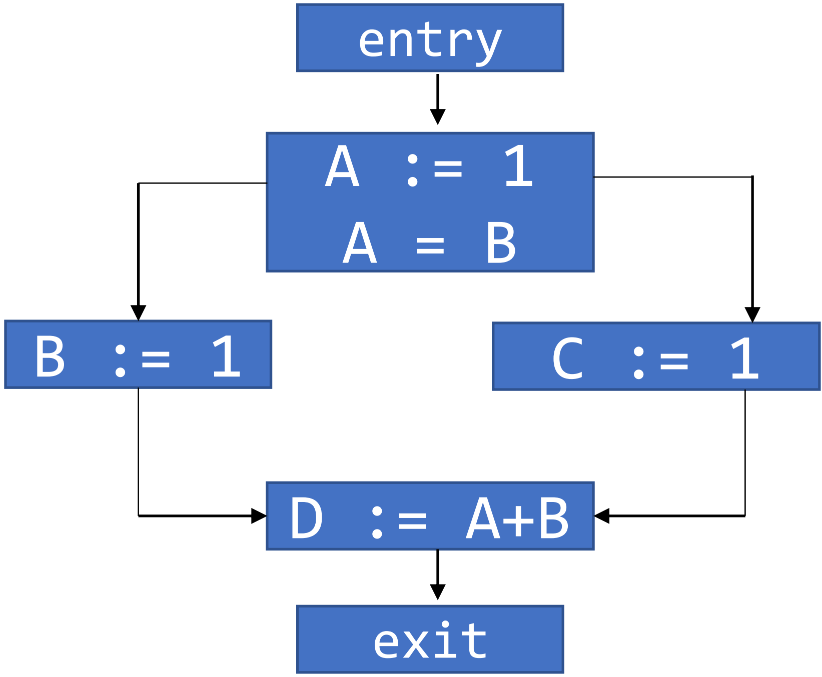
CS323: Compilers

Spring 2023

Week 13: Dataflow Analysis (liveness (recap), Constant Propagation..)

Recap: Liveness

- Variables are live if there exists *some path* leading to its use
- Start from exit block and proceed *backwards* against the control flow to compute



entry

A := 1
A = B

B := 1

C := 1

D := A+B

exit

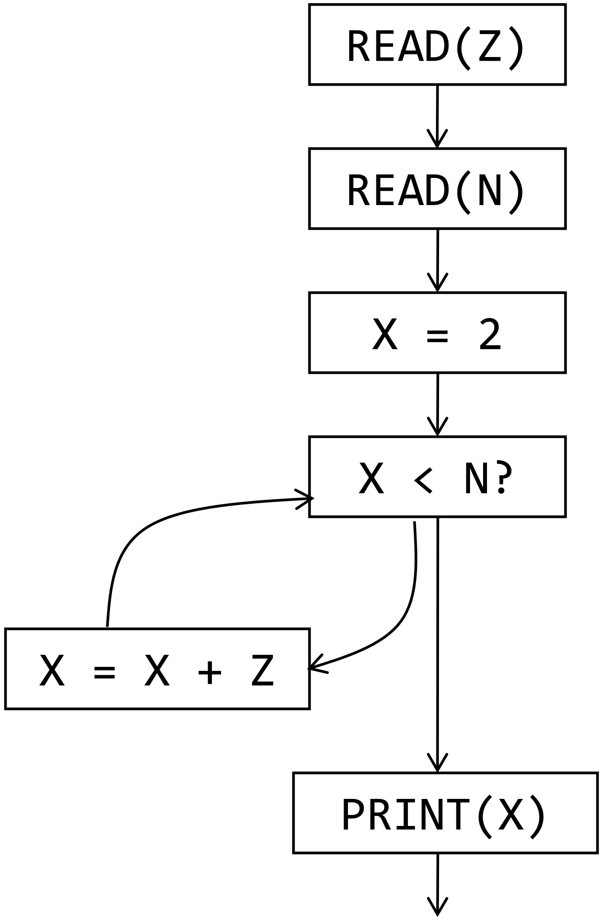
$$\mathtt{LiveOut(b)} = \bigcup_{i \in \mathrm{Succ}(b)} \mathrm{LiveIn}(i)$$

$$\mathtt{LiveIn(b)} = \mathtt{LiveUse(b)} \cup (\mathtt{LiveOut(b)} - \mathtt{Def(b)})$$

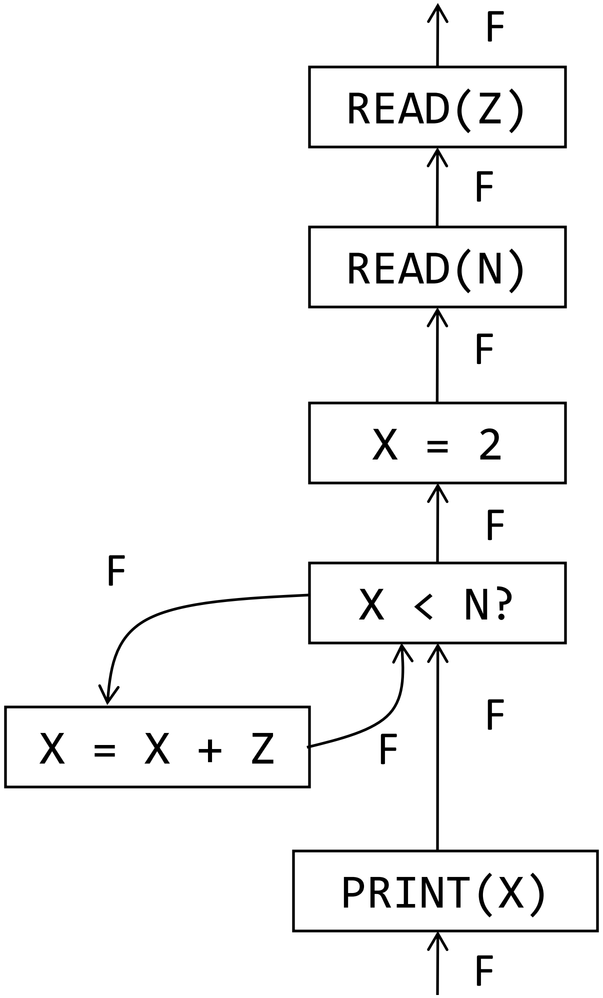
//set that contains all variables used by block b

//set that contains all variables defined by block b

Recap: Liveness

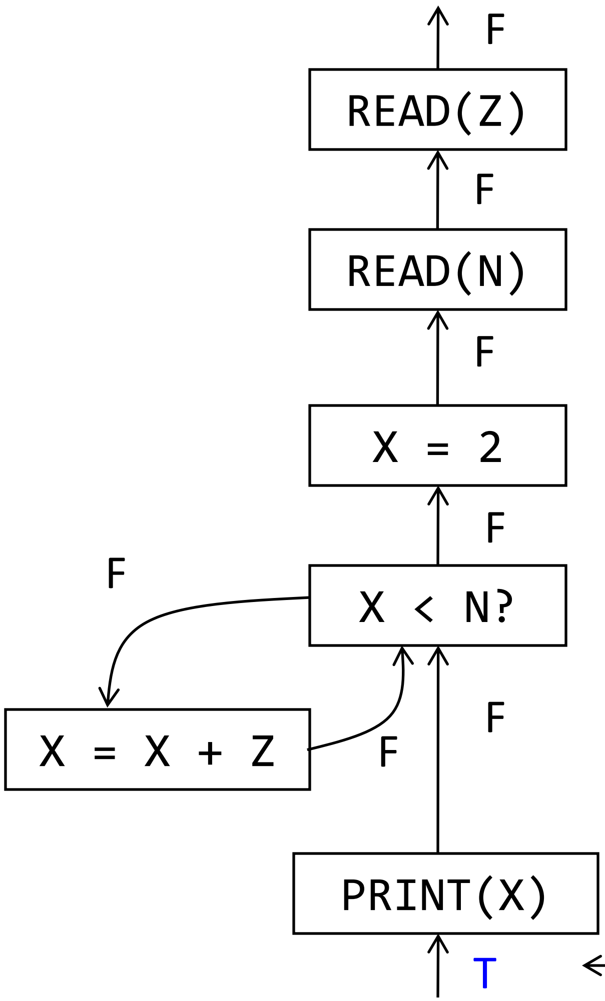


Original CFG



CFG with edges reversed (and initialized) for backwards analysis: is X live? (F=false, T=true)

Recap: Liveness



READ(Z)

READ(N)

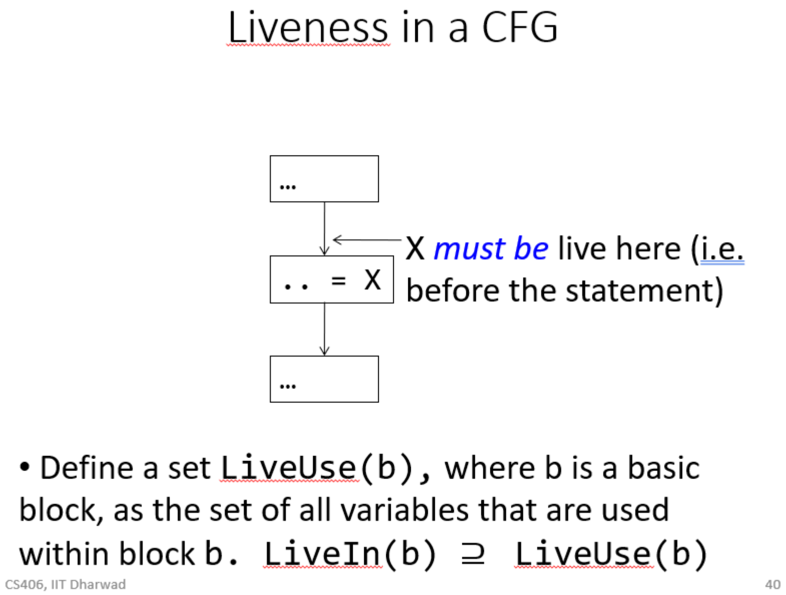
X = 2

X < N?

X = X + Z

PRINT(X)

X must be live here (refer week11 slide)



Liveness in a CFG

X *must be* live here (i.e. before the statement)

- Define a set `LiveUse(b)`, where b is a basic block, as the set of all variables that are used within block `b`. `LiveIn(b)` ⊇ `LiveUse(b)`

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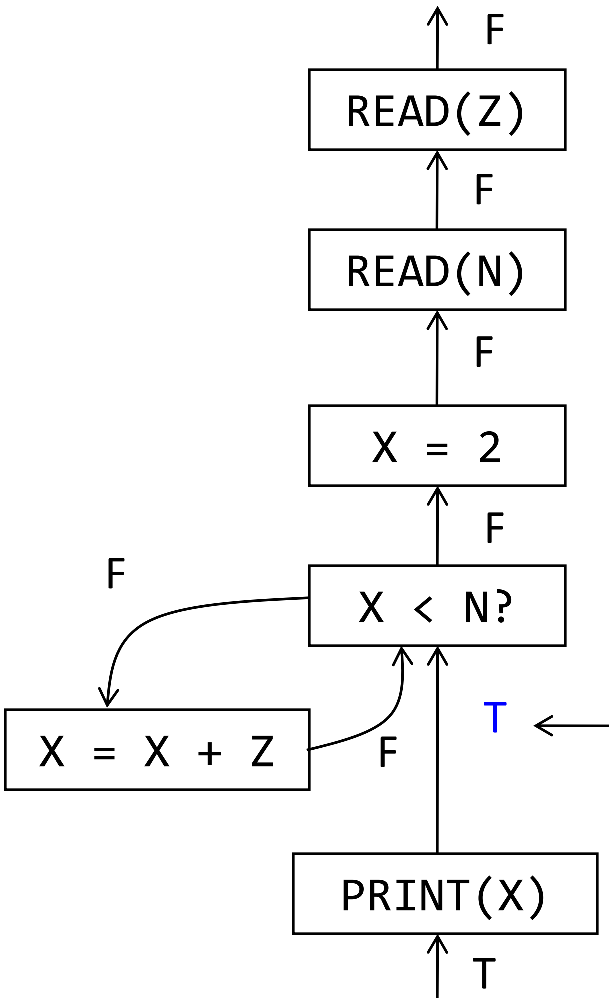
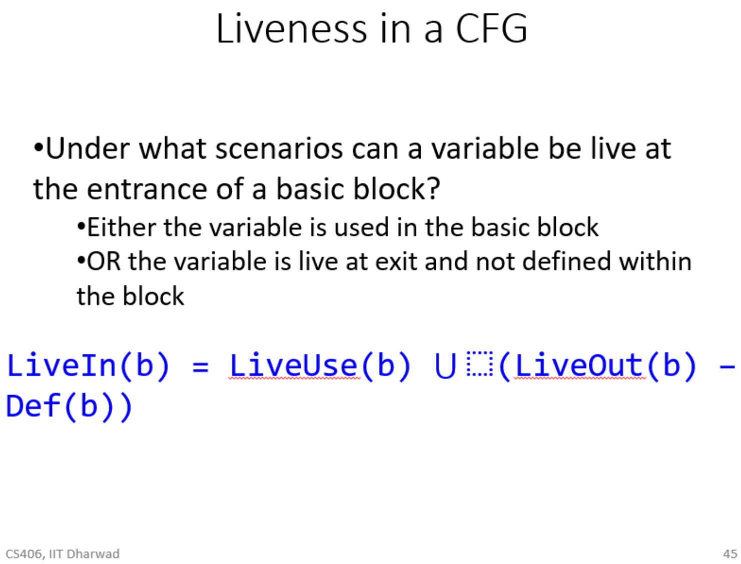
Recap: Liveness



X must be live here
(refer week11 slide)



Liveness in a CFG

- Under what scenarios can a variable be live at the entrance of a basic block?
  - Either the variable is used in the basic block
  - OR the variable is live at exit and not defined within the block

`LiveIn(b) = LiveUse(b) ∪ (LiveOut(b) – Def(b))`

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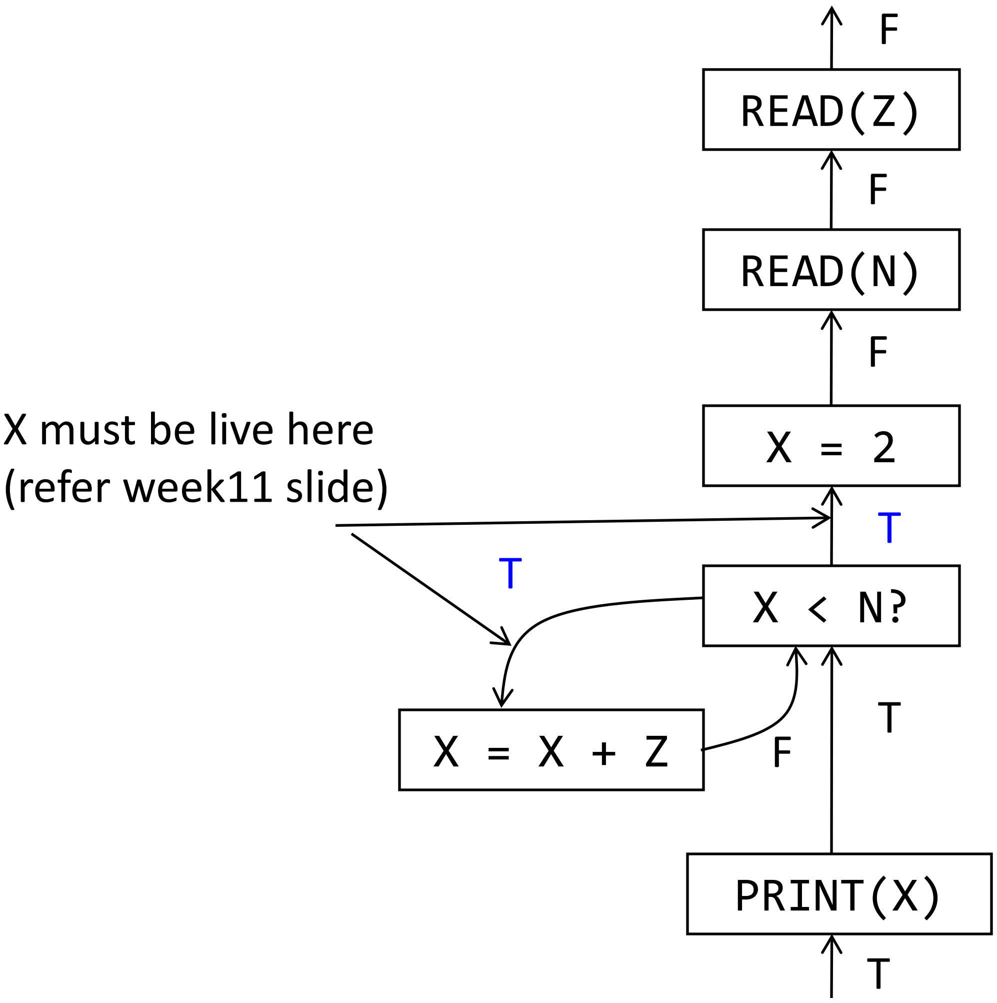
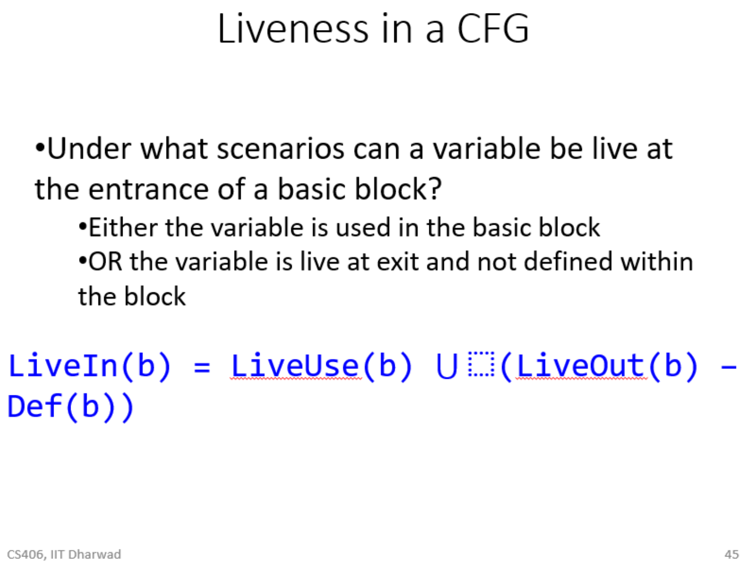
Recap: Liveness



X must be live here
(refer week11 slide)



Liveness in a CFG

- Under what scenarios can a variable be live at the entrance of a basic block?
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```
LiveIn(b) = LiveUse(b) U⬚(LiveOut(b) –
Def(b))
```

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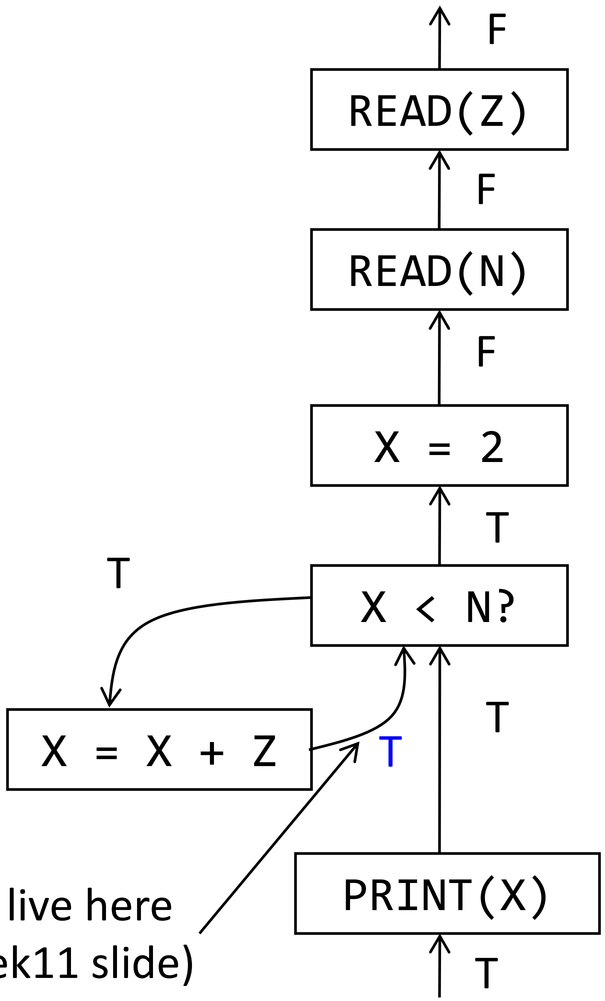
Recap: Liveness



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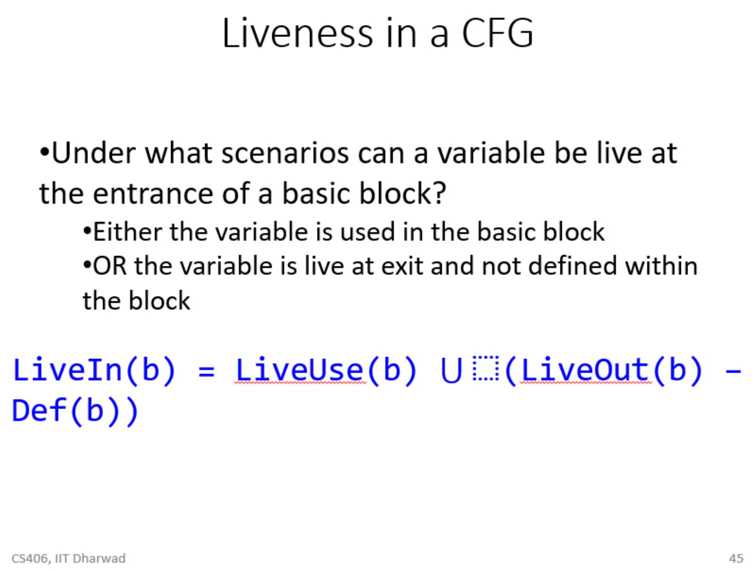

Liveness in a CFG

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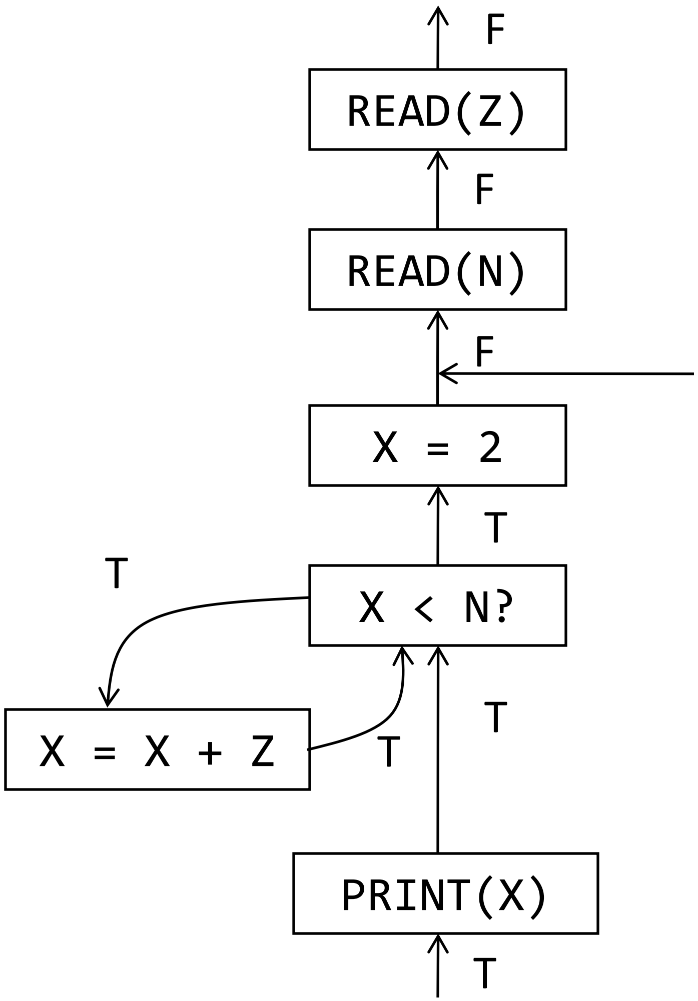
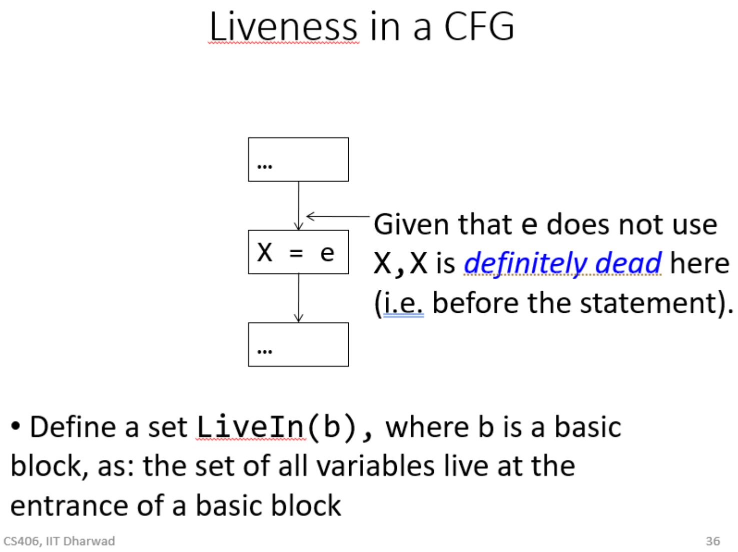
Recap: Liveness



X dead here (refer Week11 slide).
No change in information.



Liveness in a CFG

Given that e does not use X, X is *definitely dead* here (i.e. before the statement).

- Define a set `LiveIn(b)`, where b is a basic block, as: the set of all variables live at the entrance of a basic block

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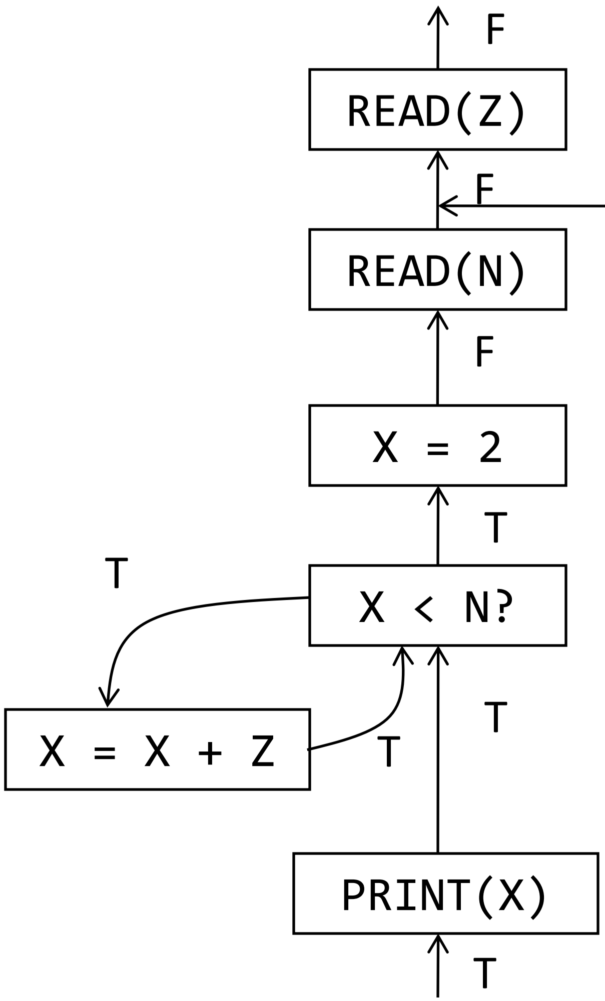
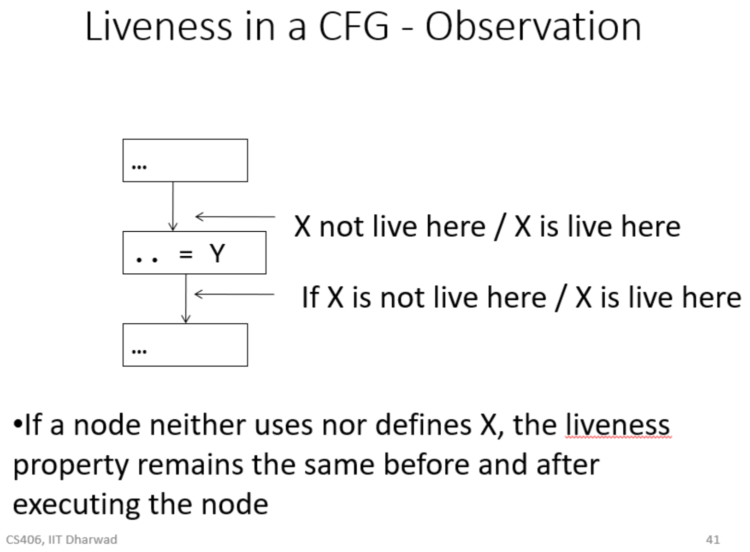
Recap: Liveness



X dead here (refer Week11 slide).
No change in information.


Liveness in a CFG - Observation

…

X not live here / X is live here

.. = Y

If X is not live here / X is live here

…

- If a node neither uses nor defines X, the liveness property remains the same before and after executing the node

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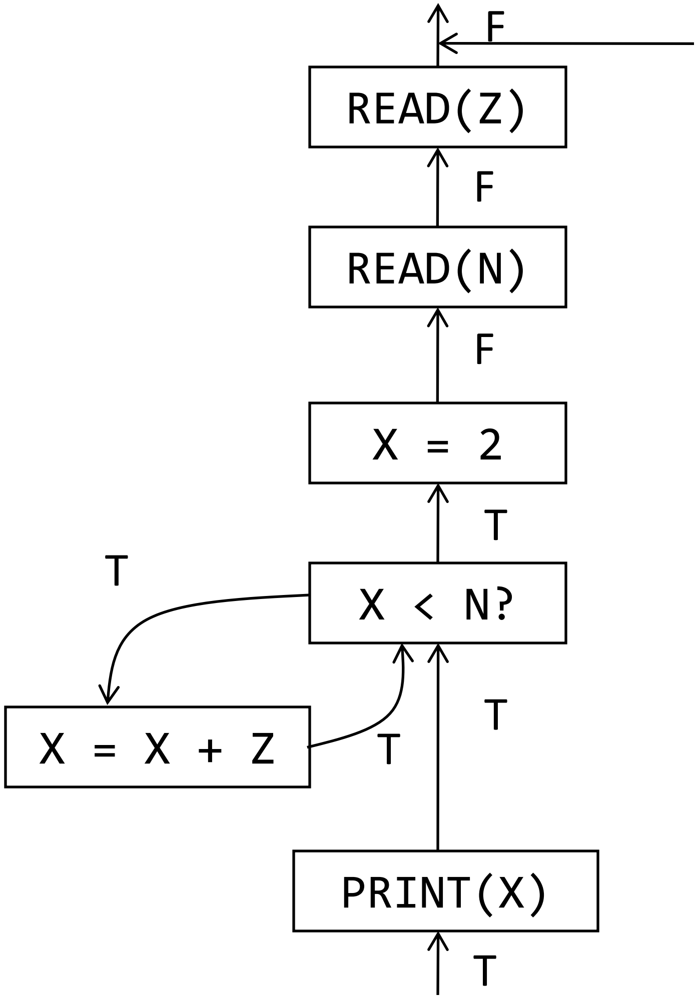
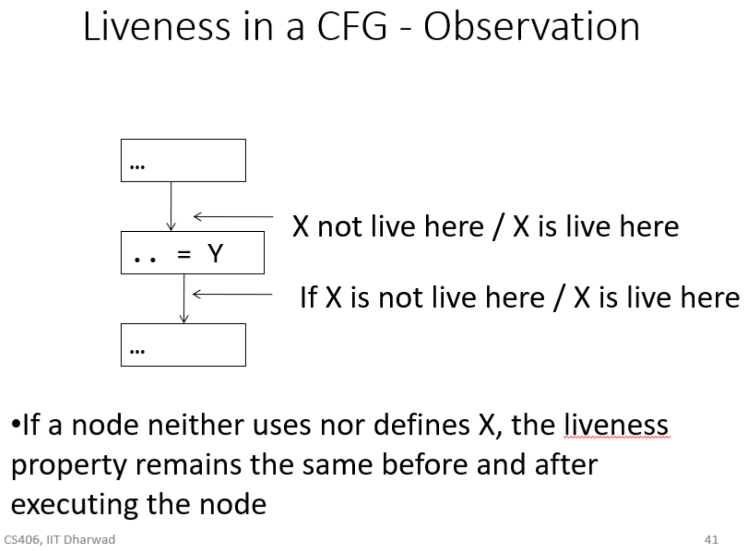
Recap: Liveness



X dead here (refer Week11 slide).
No change in information.



Liveness in a CFG - Observation

…

X not live here / X is live here

.. = Y

If X is not live here / X is live here

…

•If a node neither uses nor defines X, the liveness property remains the same before and after executing the node

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Exercise: Repeat for Z and N

Constant Propagation

- Bigger problem size:
  - Which lines using X could be replaced with a constant value? (apply only constant propagation)
  - How can we automate to find an answer to this question?

```
1. X := 2
2. Label1:
3. Y := X + 1
4. if Z > 8 goto Label2
5. X := 3
6. X := X + 5
7. Y := X + 5
8. X := 2
9. if Z > 10 goto Label1
10.X := 3
11.Label2:
12.Y := X + 2
13.X := 0
14.goto Label3
15.X := 10
16.X := X + X
17.Label3:
18.Y := X + 1
```

Constant Propagation

- Problem statement:
  - Replace use of a variable X by a constant K

- Requirement:
  - **property**: on every path to the use of X, the last assignment to X is: X=K

    Same as: “is X=K at a program point?”

    At any program point where the above property holds, we can apply constant propagation.

How can we find constants?

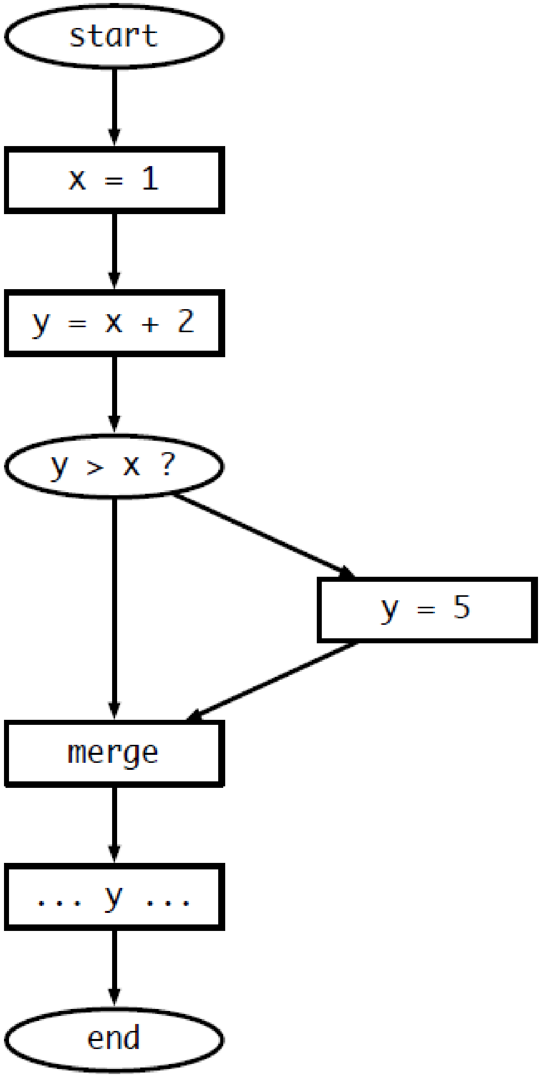
- Ideal: run program and see which variables are constant
  - Problem: variables can be constant with some inputs, not others – need an approach that works for all inputs!
  - Problem: program can run forever (infinite loops?) – need an approach that we know will finish
- Idea: run program *symbolically*
  - Essentially, keep track of whether a variable is constant or not constant (but nothing else)

Overview of algorithm

- Build control flow graph
  - We'll use statement-level CFG (with merge nodes) for this
- Perform symbolic evaluation
  - Keep track of whether variables are constant or not
- Replace constant-valued variable uses with their values, try to simplify expressions and control flow

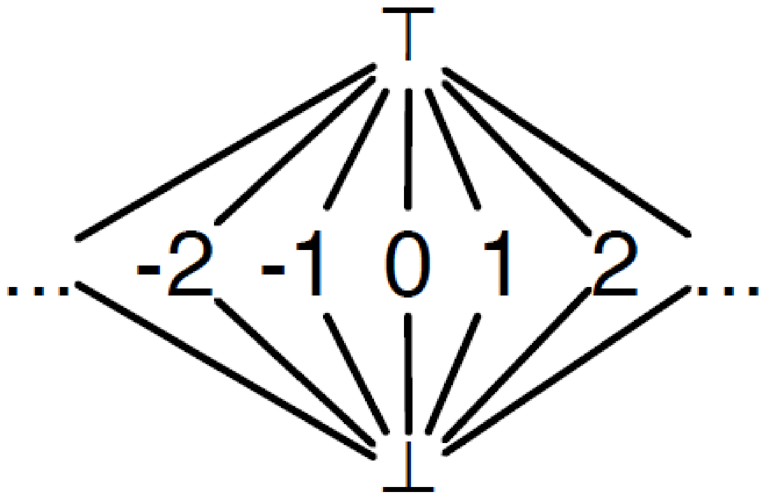
Build CFG

```
x = 1;
y = x + 2;
if (y > x) then y = 5;
... y ...
```



Symbolic evaluation

- Idea: replace each value with a symbol
  - constant (specify which), no information, definitely not constant
- Can organize these possible values in a *lattice*
  - Set of possible values, arranged from least information to most information



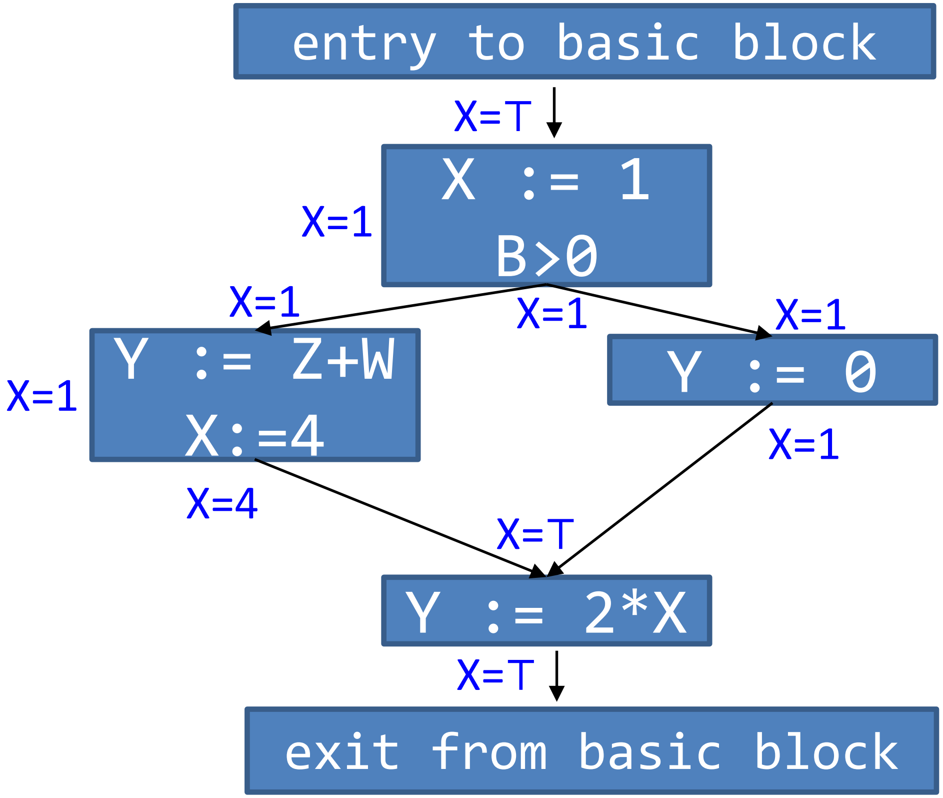
Symbolic Evaluation

- Associate with X one of the following values:

Value	Meaning
⊥ (“bottom”)	This statement never executes
K (“constant”)	X = K
T (“top”)	X is not a constant

- Idea of symbolic execution: at all program points, determine the value of X

Constant Propagation



If X=K at some program point, we can apply constant propagation (replace the use of X with value of K at that program point)

Constant Propagation

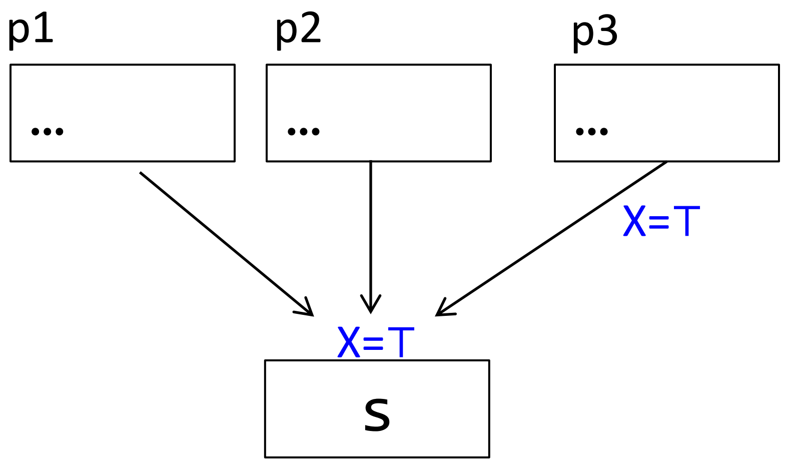
- Determining the value of X at program points:
  - Just like in Liveness Computation in a CFG, the information required for constant propagation flows from one statement to adjacent statement
  - For each statement `s`, compute the information just before and after `s`. C is the function that computes the information:

```
                C(X,s,flag)
//if flag=IN, before s what is the value of X
//if flag=OUT, after s what is the value of X
```

    - **Transfer function** (pushes / transfers information from one statement to another)

Constant Propagation

- Determining the value of X at program points (Rule 1):

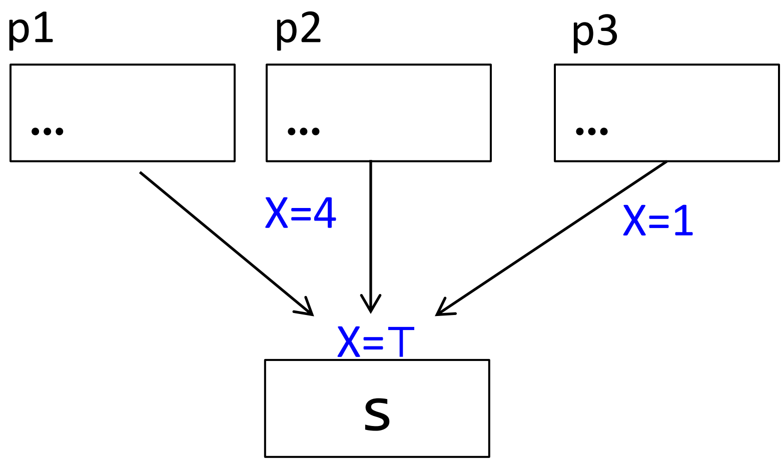


If X=T at exit of *any* of the predecessors, X=T at the entrance of S

if C(p_i,s,OUT)=T
for any i, then C(X,s,IN)=T

Constant Propagation

- Determining the value of X at program points (Rule 2):

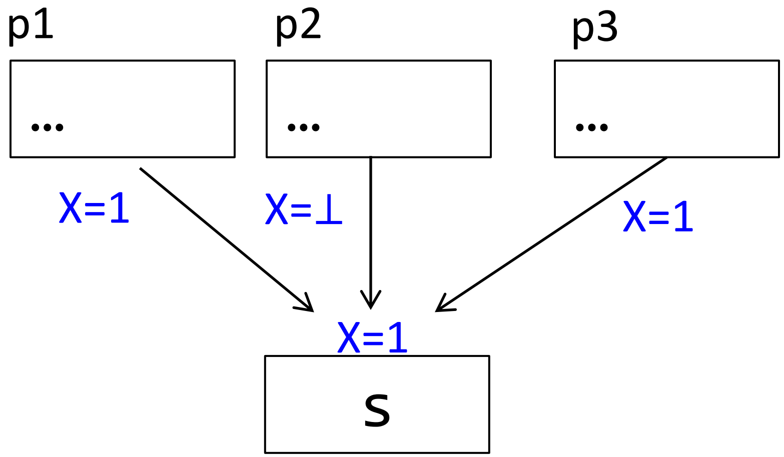


If X=K1 at one predecessor and X=K2 at another predecessor and K1 ≠ K2, then X=T at the entrance of S

`if C(`p_i`,s,OUT)=K1 and C(`p_j`,s,OUT)=K2 and K1 ≠ K2 then C(X,s,IN)=T`

Constant Propagation

- Determining the value of X at program points (Rule 3):

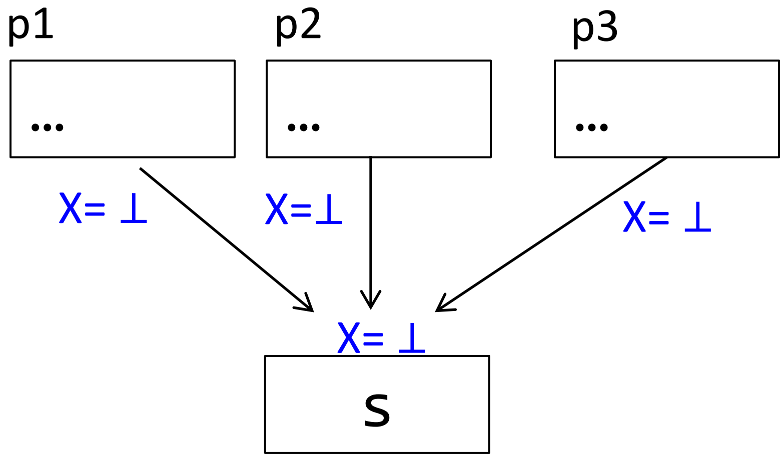


If X=K at some of the predecessors and X= ⊥ at all other predecessors, then X=K at the entrance of S

if C(p_i,s,OUT)=K or ⊥ for all i then C(X,s,IN)= K

Constant Propagation

- Determining the value of X at program points (Rule 4):

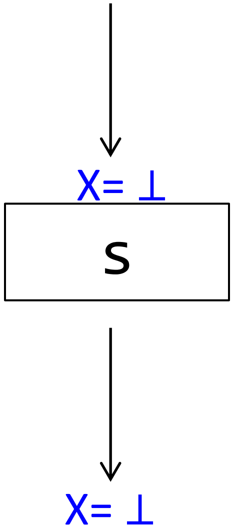


If X= ⊥ at all predecessors, then X= ⊥ at the entrance of S

`if C(`p_i`,s,OUT)=`⊥ for all `i` then `C(X,s,IN)=` ⊥

Constant Propagation

- Determining the value of X at program points (Rule 5):

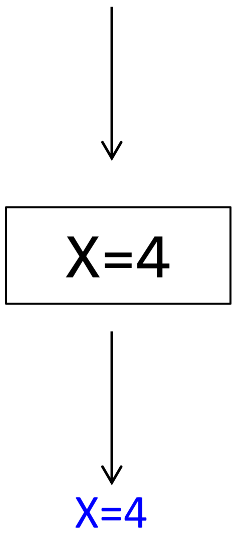


If X= ⊥ at entrance of s, then X= ⊥ at the exit of S

`if C(X,s,IN)=⊥` then `C(X,s,OUT)= ⊥`

Constant Propagation

- Determining the value of X at program points (Rule 6):



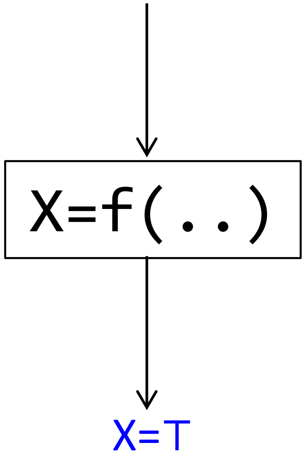
No matter what the value of X is at entrance of `s(X:=K)`, X=K at the exit of s

`C(X,s(X:=K),OUT)=K`

But previous slide said `if C(X,s,IN)=⊥` then `C(X,s,OUT)= ⊥`. *So, we give priority to this.*

Constant Propagation

- Determining the value of X at program points (Rule 7):



X=f(..)

X=⊤

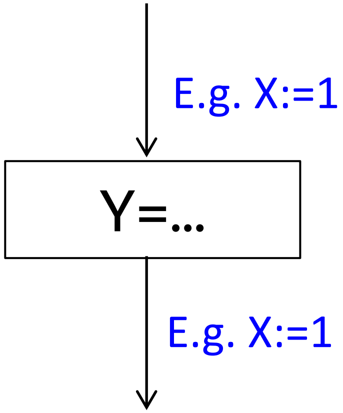
In s, assignment to X is any complicated expression (not a constant assignment).

`C(X,s(X:=f()),OUT)=⊤`

But earlier slide said `if C(X,s,IN)=⊥` then `C(X,s,OUT)= ⊥`. *So, we give priority to this.*

Constant Propagation

- Determining the value of X at program points (Rule 8):



Value of X remains unchanged before and after s(Y:=..) when s doesn't assign to X and X ≠ Y

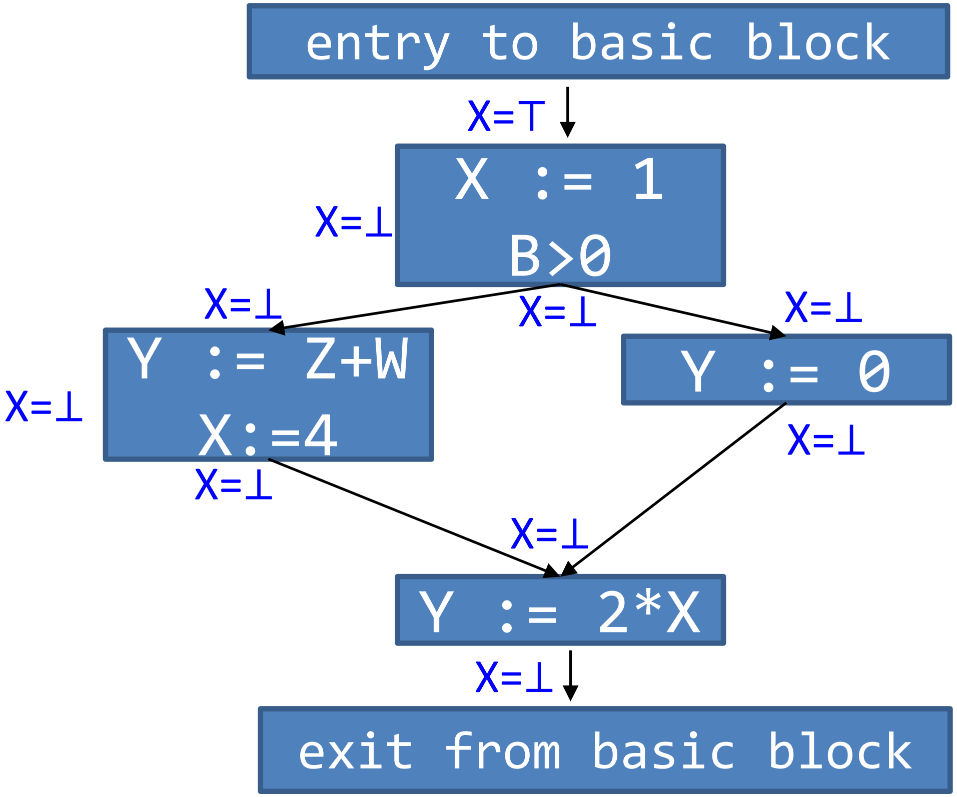
```
C(X,s(Y:=..),OUT)=C(X,s(Y:=..),IN)
```

Constant Propagation

- Putting it all together
  1. For entry s in the program, initialize `C(X,s,IN)=⊤` and initialize `C(X,s,IN)=C(X,s,OUT)=⊥` everywhere else
  2. Repeat until all program points (i.e. any s) satisfy rules 1-8
     1. Pick s in the CFG that doesn't satisfy any one of rules 1-8 and update information.

Constant Propagation

- Putting it all together



entry to basic block

X=⊤

X=⊥

X := 1

B>0

X=⊥

X=⊥

X=⊥

Y := Z+W

X:=4

X=⊥

Y := 0

X=⊥

X=⊥

X=⊥

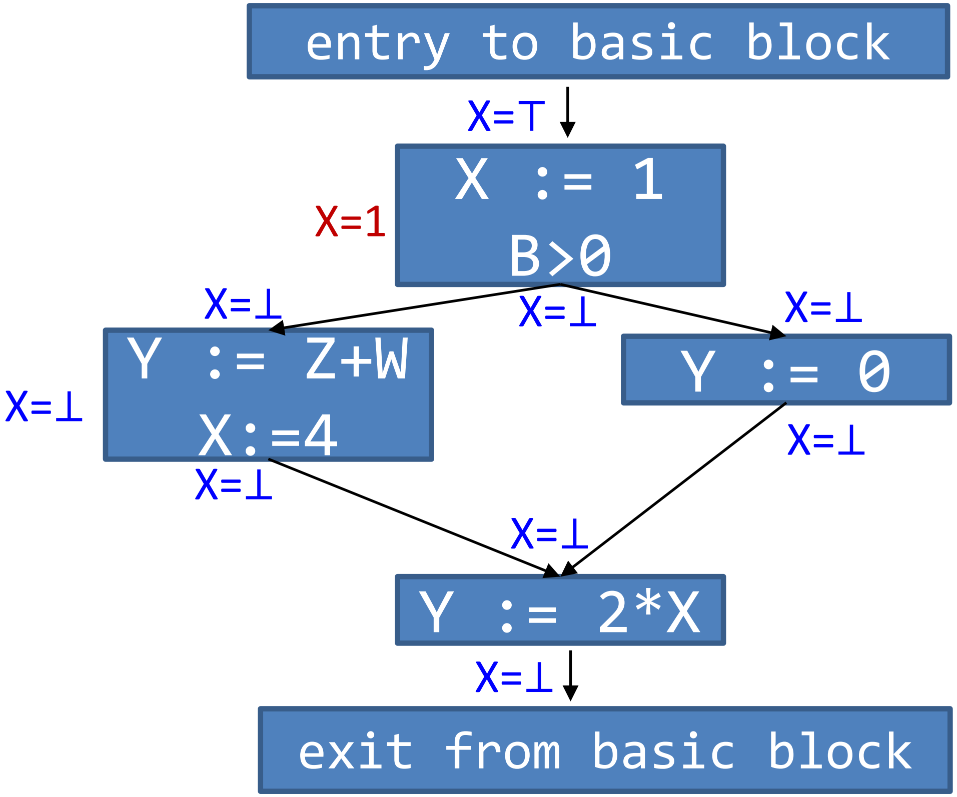
Y := 2*X

X=⊥

exit from basic block

Constant Propagation

- Putting it all together



Constant Propagation

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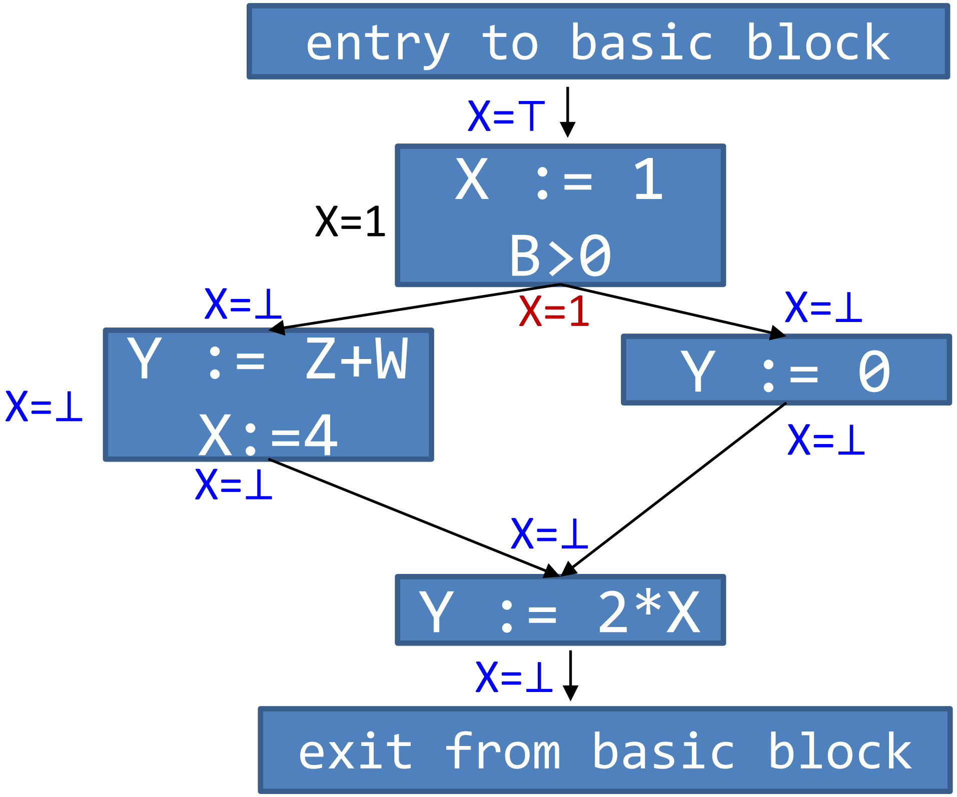

Constant Propagation

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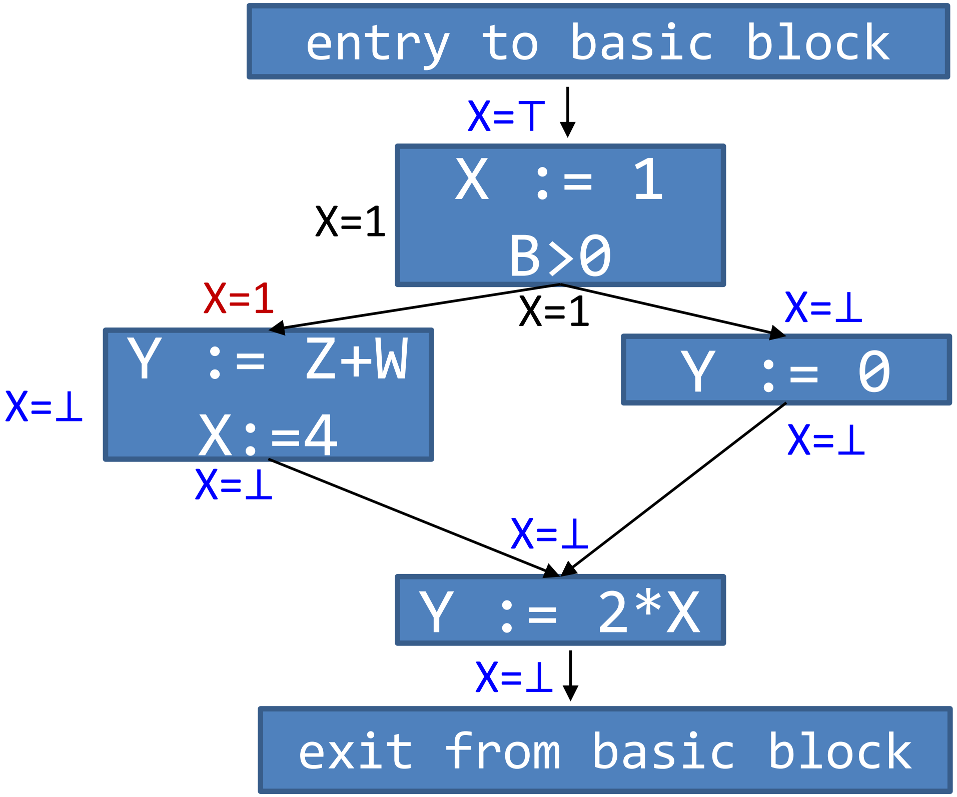

Constant Propagation

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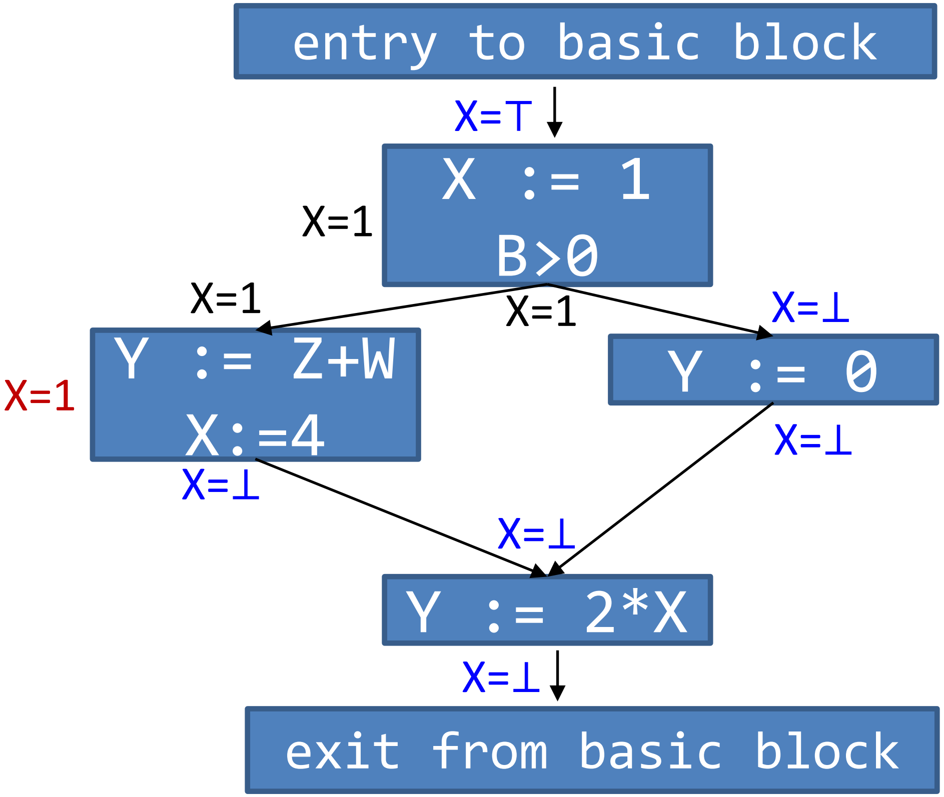
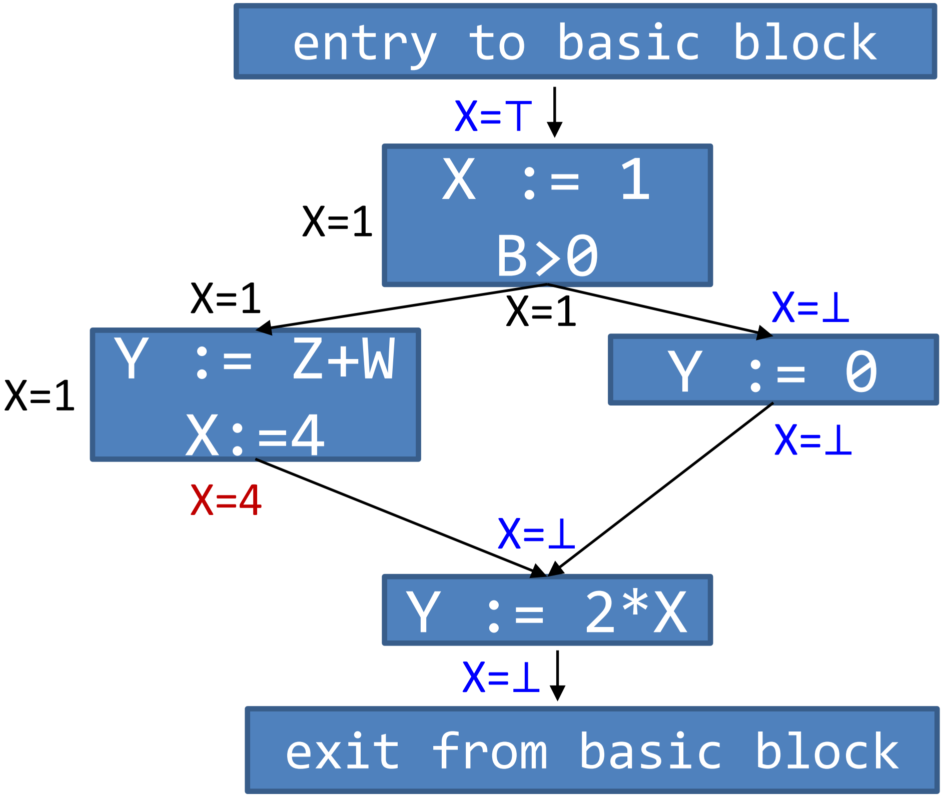


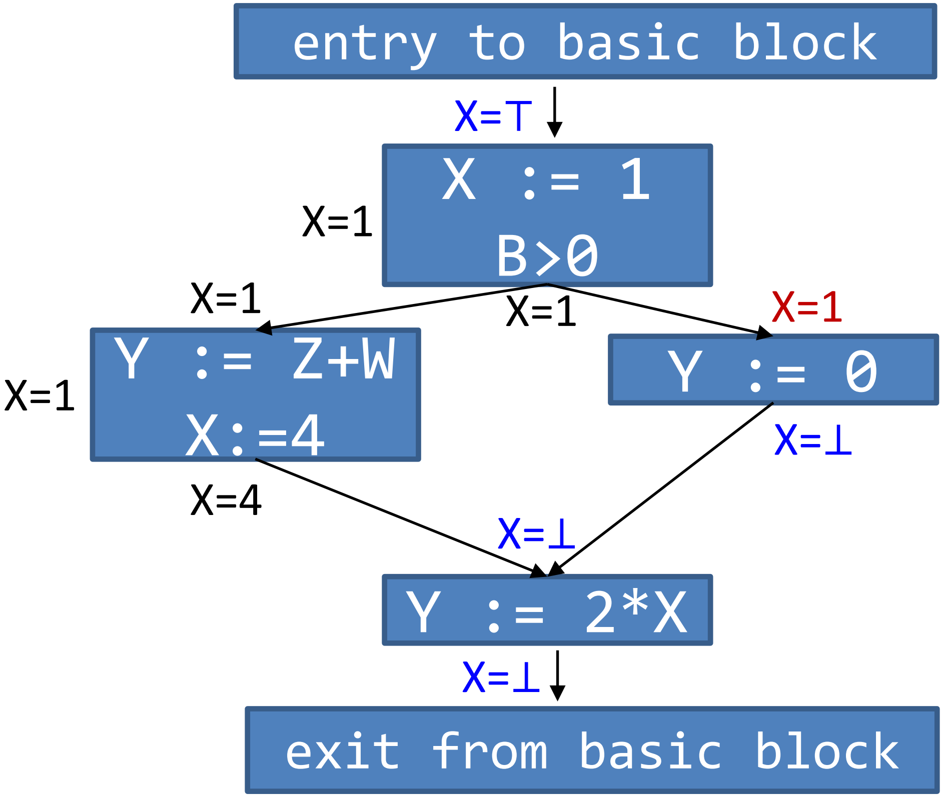
Constant Propagation

- Putting it all together



Constant Propagation

- Putting it all together



entry to basic block

X=⊤

X := 1

B>0

X=1

X=1

X=1

X=1

Y := Z+W

X:=4

X=1

Y := 0

X=⊥

X=4

X=⊥

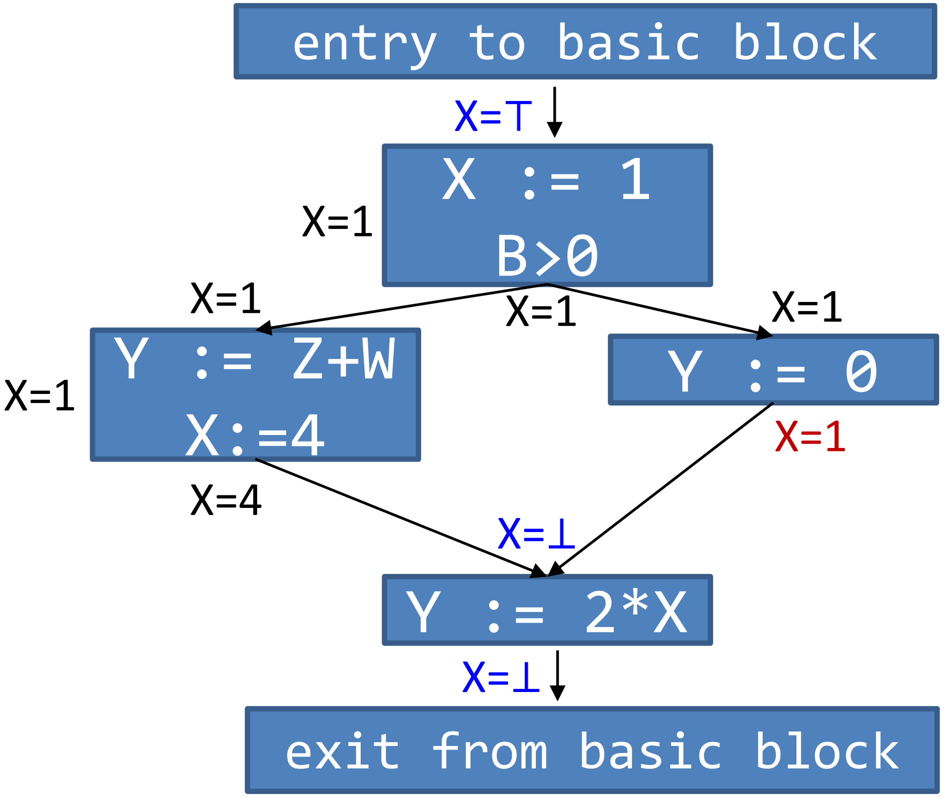
Y := 2*X

X=⊥

exit from basic block

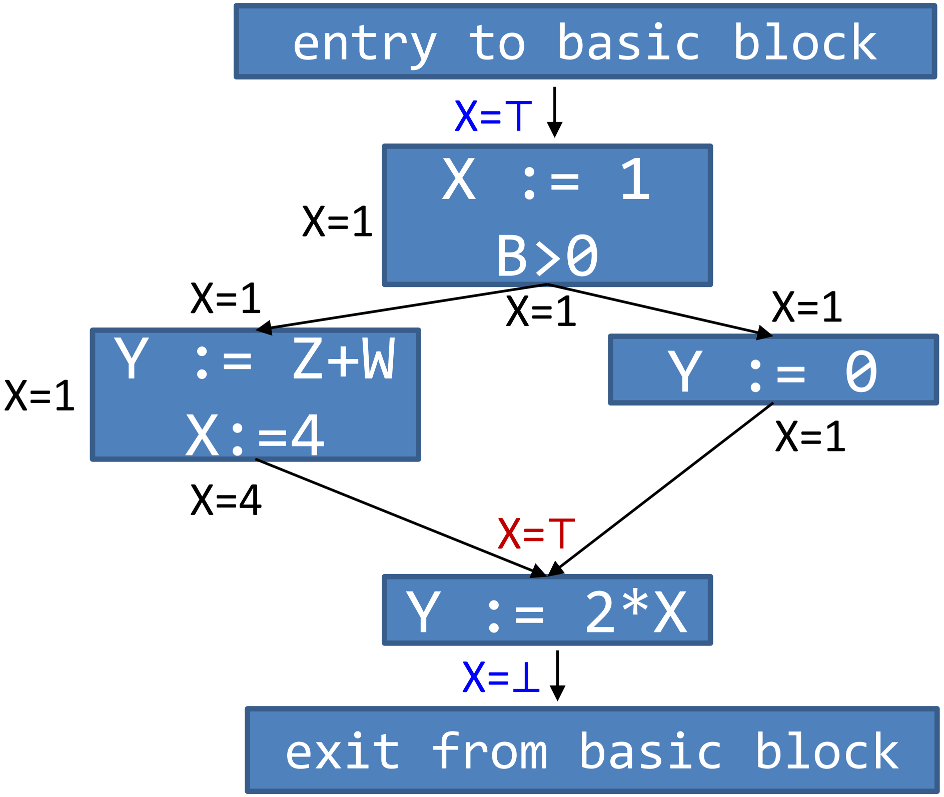
Constant Propagation

- Putting it all together



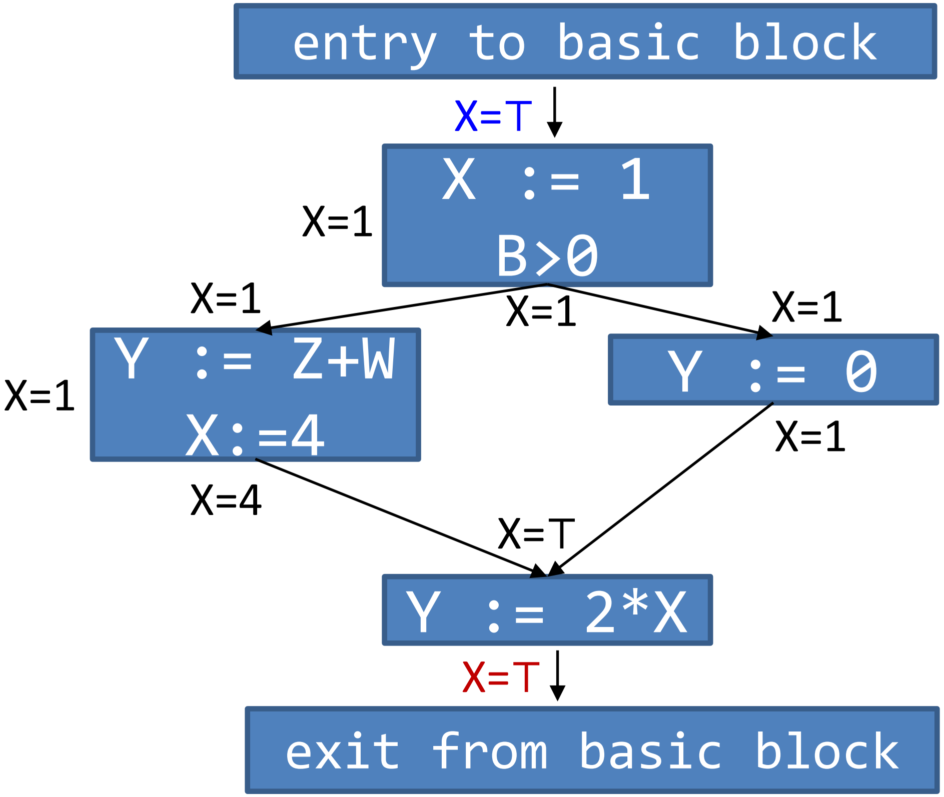
Constant Propagation

- Putting it all together



Constant Propagation

- Putting it all together



entry to basic block

X=T

X := 1
B>0

X=1

X=1

X=1

X=1

Y := Z+W
X:=4

X=1

Y := 0

X=1

X=4

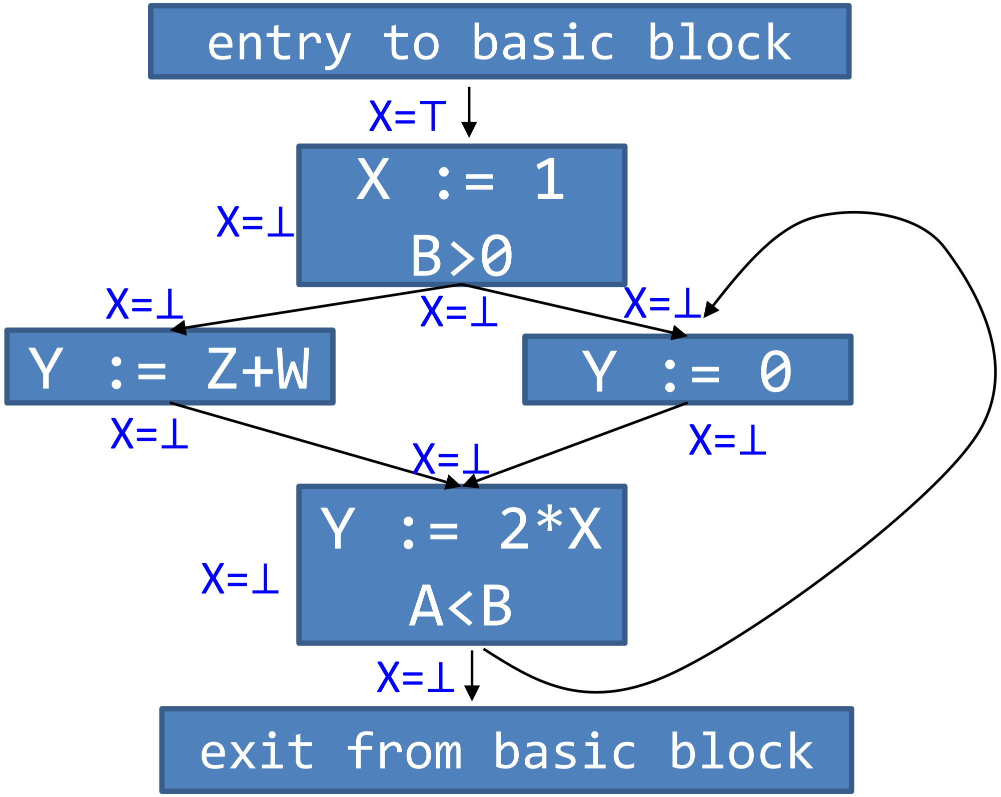
X=T

Y := 2*X

X=T

exit from basic block

Constant Propagation - Loops



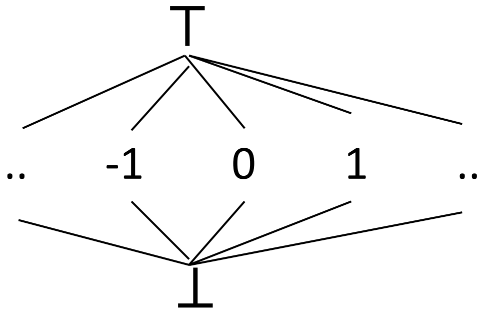
Ordering of information: Generalizing

- We have been executing with symbols ⊥, ⊤ , and K. These are called *abstract values*
- Order these values as:

$$\bot < K < \top$$

Can also be thought of as an ordering from least information to most information

Pictorially:



Ordering of information: Generalizing

- Least Upper Bound (`lub`) : smallest element (abstract value) that is greater than or equal to values in the input
  - E.g. $\text{lub}(\bot,\bot) =\bot$, $\text{lub}(\top,\bot) = \top$,$\text{lub}(-1,1) = \top$, $\text{lub}(1\ \ \bot) =?$
  - Rewriting rules 1-4: `C(X,s,IN)=lub{C(`p_i`,s,OUT) for all predecessors i)}`
  - Also called as join operator. Written as: A ⊔ B

Ordering of information: Generalizing

- Recall that in determining information at all program points:

  "2. Repeat until all program points (i.e. any s) satisfy rules 1-8

  - Pick s in the CFG that doesn't satisfy any one of rules 1-8 and update information. "

  - How do we know that this terminates?
    - lub ensures that the information changes from lower value to higher value
    - In the constant propagation algorithm:
      - ⊥ can change to constant and then to T
      - ⊥ can change to T
      - `C(X, s, flag)` can change at most twice

Constant Propagation

- Exercise: what is the complexity of our constant propagation algorithm?

= NumS* 4 (NumS = number of statements in the program).

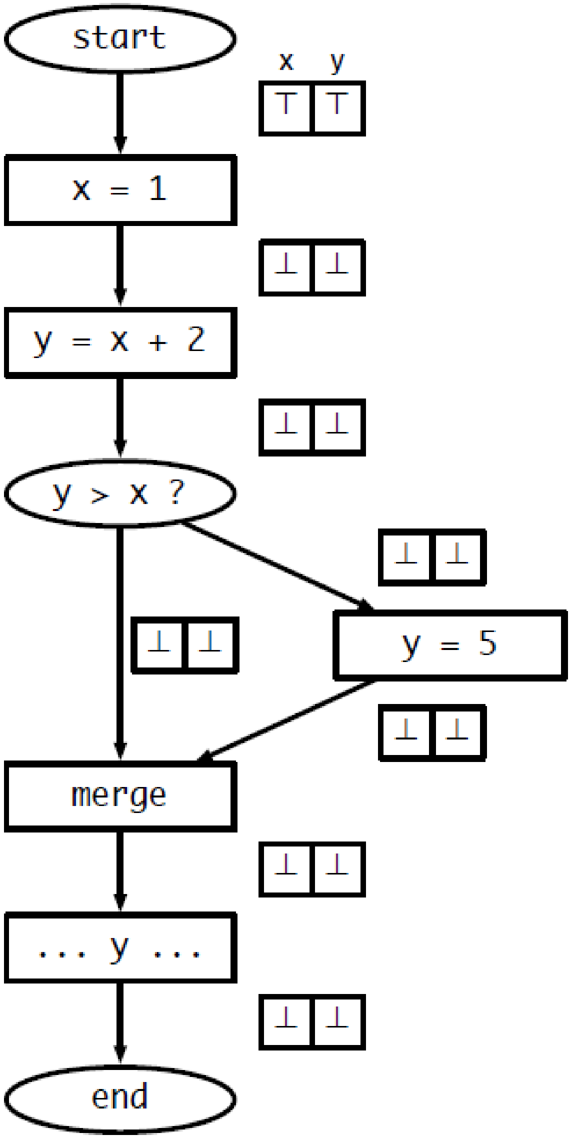
- Per program point, we evaluate the C function.
- The C function changes value at most two times (initialized to ⊥ first and then could change to K and then to ⊤).
- There are two program points (entry/IN and exit/OUT) for every statement.

This is the complexity of the analysis per variable

How do we do the analysis considering all variables that exist in the program?

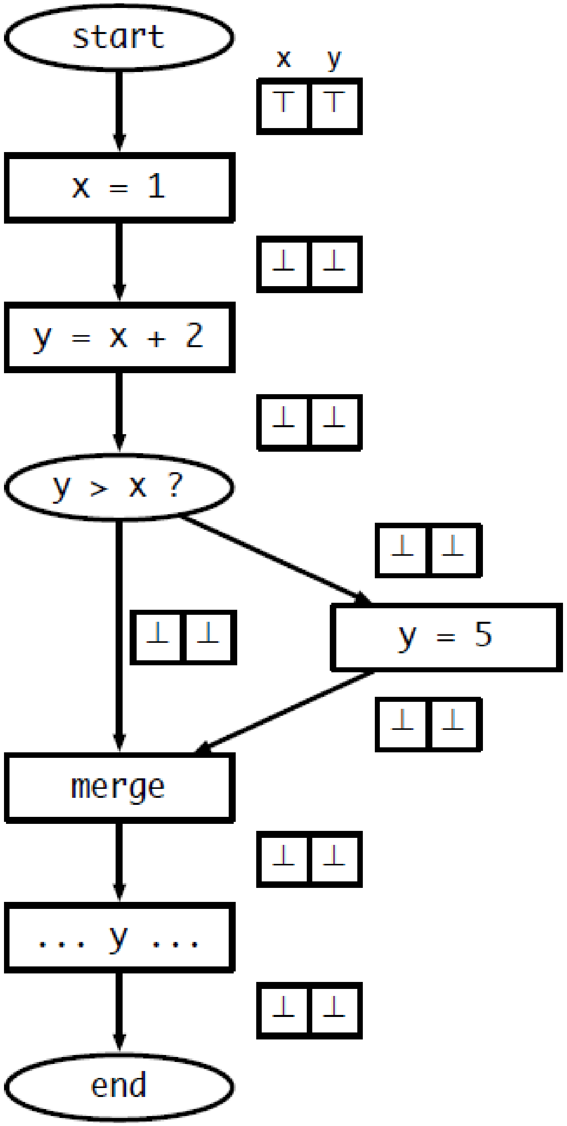
Constant Propagation (Multiple Variables)

- Keep track of the symbolic value of a variable at every program point (on every CFG edge)
  - State vector V
- What should our initial value be?
  - Starting state vector is all ⊤
    - Can't make any assumptions about inputs – must assume not constant
  - Everything else starts as ⊥, since we have no information about the variable at that point



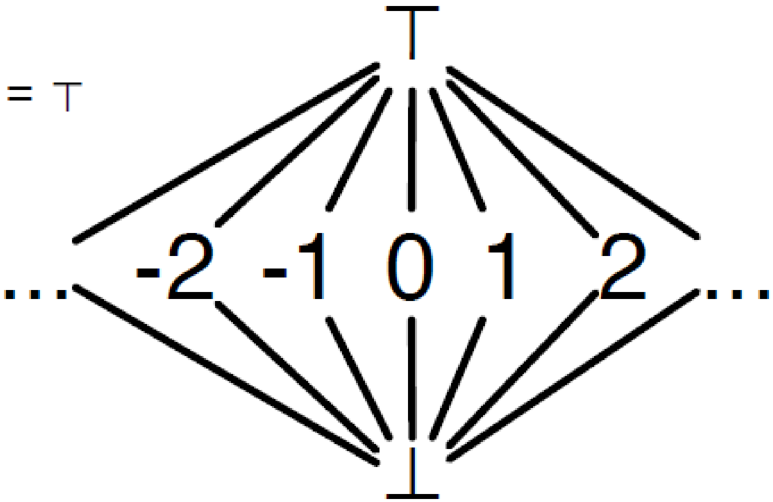
Constant Propagation (Multiple Variables)

- For each statement `t = e` evaluate `e` using V_{in}, update value for `t` and propagate state vector to next statement
- What about switches?
  - If `e` is true or false, propagate V_{in} to both appropriate branch
  - What if we can't tell?
    - Propagate V_{in} to both branches, and symbolically execute both sides
- What do we do at merges?



Handling merges

- Have two different V_{in}s coming from two different paths
- Goal: want new value for V_{in} to be *safe* (shouldn't generate wrong information), and we don't know which path we actually took
- Consider a single variable. Several situations:
  - $V_1 = \bot, V_2 = * \rightarrow V_{out} = *$
  - V_1 = constant x, V_2 = x → V_{out} = x
  - V_1 = constant x, V_2 = constant y → $V_{out} = \top$
  - $V_1 = \top, V_2 = * \rightarrow V_{out} = \top$
- Generalization:
  - $V_{out} = V_1 \sqcup V_2$

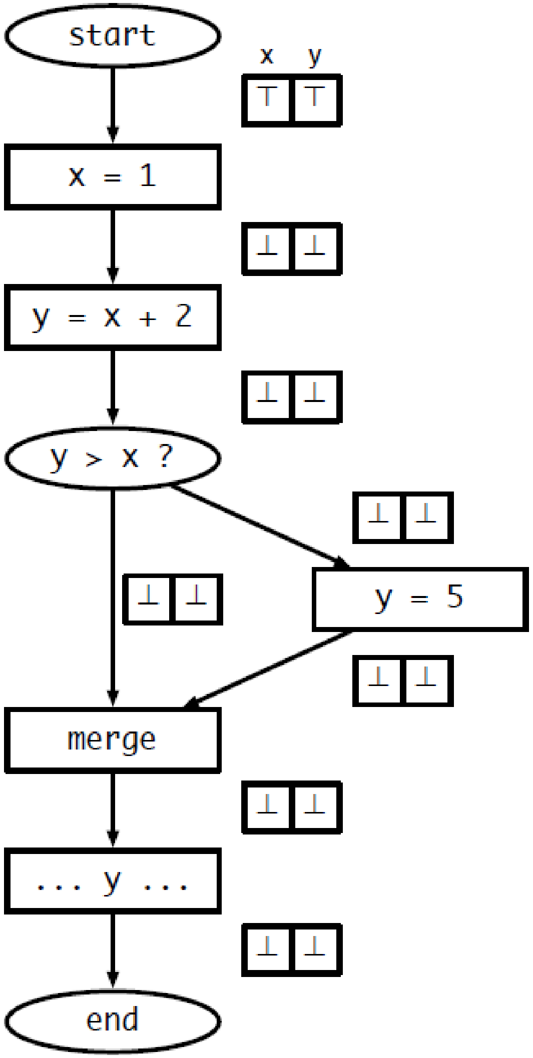


Result: worklist algorithm

- Associate state vector with each edge of CFG, initialize all values to ⊥, worklist has just start edge
  - While worklist not empty, do:

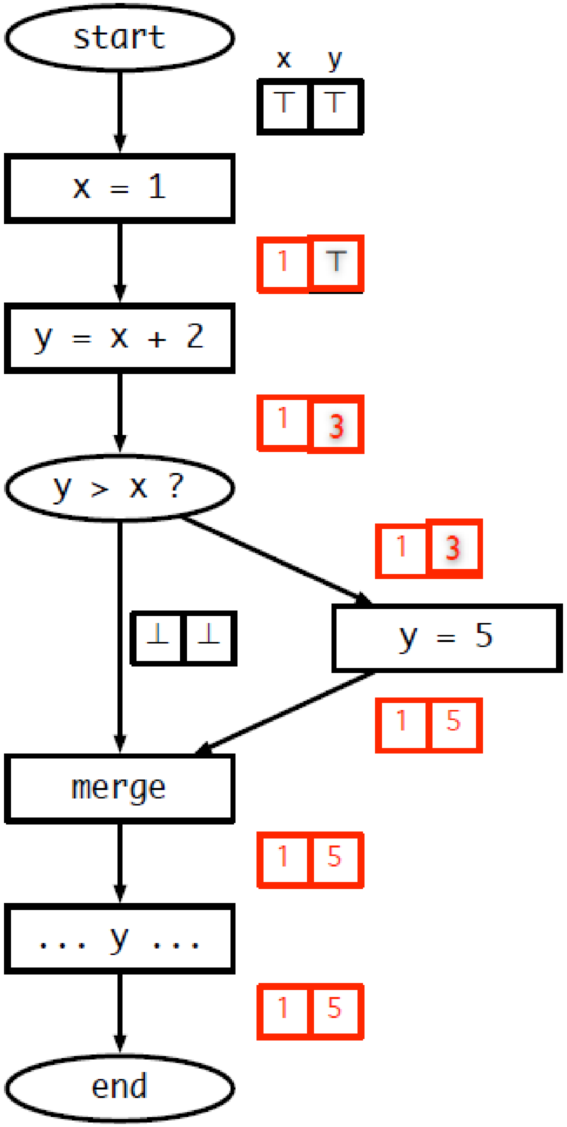
```
Process the next edge from worklist
Symbolically evaluate target node of edge using input state vector
If target node is assignment (x = e), propagate $V_{in}$[eval(e)/x] to output edge
If target node is branch (e?)
 If eval(e) is true or false, propagate $V_{in}$ to appropriate    output edge
 Else, propagate $V_{in}$ along both output edges
If target node is merge, propagate join(all $V_{in}$) to output edge
If any output edge state vector has changed, add it to worklist
```

Running example



Worklist

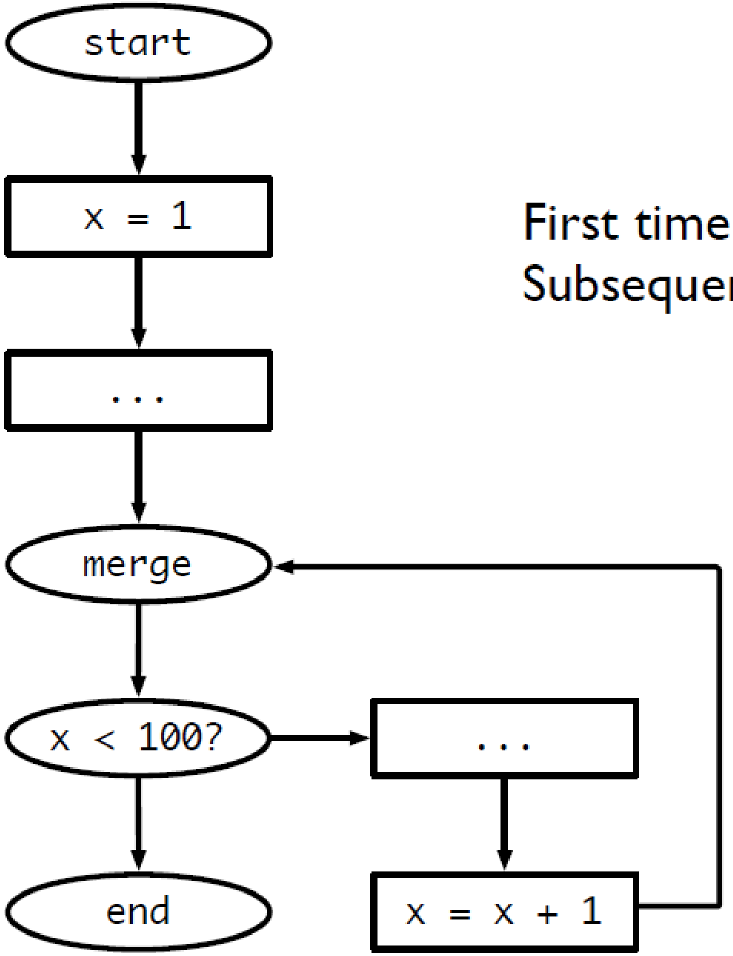
Running example



What do we do about loops?

- Unless a loop never executes, symbolic execution looks like it will keep going around to the same nodes over and over again
- Insight: if the input state vector(s) for a node don't change, then its output doesn't change
  - If input stops changing, then we are done!
- Claim: input will eventually stop changing. Why?

Loop example



First time through loop, x = 1
Subsequent times, x = ⊤

Complexity of algorithm

- V = # of variables, E = # of edges
- Height of lattice = 2 → each state vector can be updated at most 2 * V times.
- So each edge is processed at most 2 * V times, so we process at most 2 * E * V elements in the worklist.
- Cost to process a node: O(V)
- Overall, algorithm takes $O(EV^2)$ time

Question

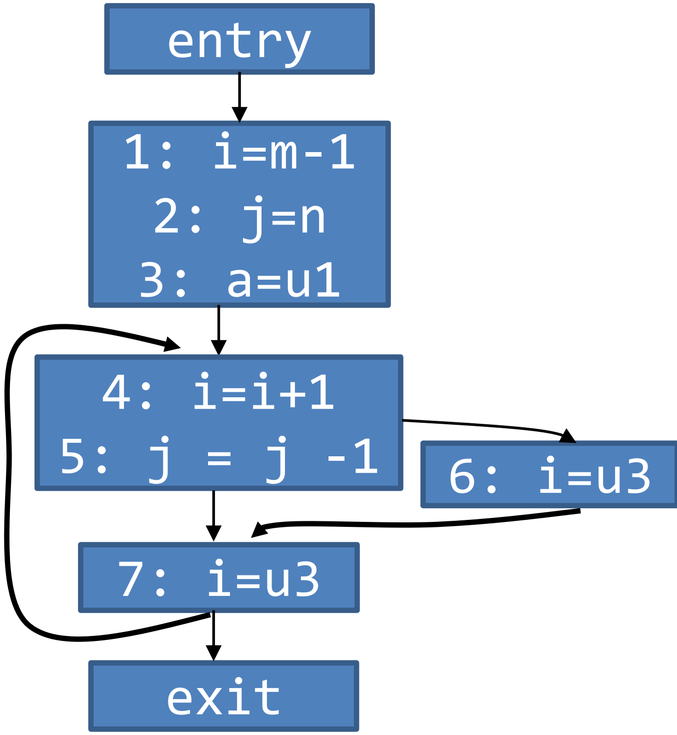
- Can we generalize this algorithm and use it for more analyses?

Constant propagation

- Step 1: choose lattice (which values are you going to track during symbolic execution)?
  - Use constant lattice
- Step 2: choose direction of dataflow (if executing symbolically, can run program backwards!)
  - Run forward through program
- Step 3: create *transfer functions*
  - How does executing a statement change the symbolic state?
- Step 4: choose *confluence operator*
  - What do do at merges? For constant propagation, use join

Reaching Definitions - Example

- **Goal:** to know where in a program each variable x may have been defined when control reaches block b
- Definition d reaches block b if there is a path from point immediately following d to b, such that the variable defined in d is not redefined / killed along that path



entry

1: i=m-1
2: j=n
3: a=u1

4: i=i+1
5: j = j -1

6: i=u3

7: i=u3

exit

$$\mathrm{In}(b) = \bigcup_{i \in \mathrm{Pred}(b)} \mathrm{Out}(i)$$

$$\mathrm{Out}(b) = \mathrm{gen}(b) \cup (\mathrm{In}(b) - \mathrm{kill}(b))$$

//set that contains all statements that may define some variable x in b. E.g. gen(1:a=3;2:a=4)={2}

//set that contains all statements that define a variable x that is also defined in b. E.g. kill(1:a=3; 2:a=4)={1,2}

Reaching definitions

- What definitions of a variable *reach* a particular program point
    - A definition of variable x from statement s reaches a statement t if there is a path from s to t where x is not redefined
- Especially important if x is used in t
    - Used to build *def-use* chains and *use-def* chains, which are key building blocks of other analyses
        - Used to determine dependences: if x is defined in s and that definition reaches t then there is a flow dependence from s to t
    - We used this to determine if statements were loop invaraint
        - All definitions that reach an expression must originate from outside the loop, or themselves be invariant

Creating a reaching-def analysis

- Can we use a powerset lattice?
- At each program point, we want to know which definitions have reached a particular point
  - Can use powerset of set of definitions in the program
    - *V* is set of variables, *S* is set of program statements
    - Definition: $d \in V \times S$
      - Use a tuple, <v, s>
  - How big is this set?
    - At most $|V \times S|$ definitions

Forward or backward?

- What do you think?

Choose confluence operator

- Remember: we want to know if a definition *may* reach a program point
- What happens if we are at a merge point and a definition reaches from one branch but not the other?
  - We don't know which branch is taken!
  - We should union the two sets – any of those definitions can reach
- We want to avoid getting too many reaching definitions → should start sets at $\perp$

Transfer functions for RD

- Forward analysis, so need a slightly different formulation
  - Merged data flowing into a statement

$$\begin{aligned} IN(s) &= \bigcup_{t \in pred(s)} OUT(t) \\ OUT(s) &= \mathbf{gen}(s) \cup (IN(s) - \mathbf{kill}(s)) \end{aligned}$$

- What are gen and kill?
  - gen(s): the set of definitions that *may* occur at s
    - e.g., gen(s_1: x = e) is <x, s_1>
  - kill(s): all previous definitions of variables that are *definitely* redefined by s
    - e.g., kill(s_1: x = e) is <x, *>

Generalization (Recap)

- **Direction of the analysis:**
  - How does information flow w.r.t. control flow?
- **Join operator:**
  - What happens at merge points? E.g. what operator to use Union or Intersection?
- **Transfer function:**
  - Define sets gen(b), kill(b), IN(b), OUT(b)
- **Initializations?**

Available Expressions

- **Goal:** determine a set of expressions that have already been computed.
  - E.g. to perform global CSE
- **Direction of the analysis:**
  - How does information flow w.r.t. control flow?
- **Join operator:**
  - What happens at merge points? E.g. what operator to use Union or Intersection?
- **Transfer function:**
  - Define sets AvailIn(b), AvailOut(b), Compute(b), Kill(b)
- **Initializations?**

Transfer functions for meet

- What do the transfer functions look like if we are doing a meet?

$$
\begin{aligned}
IN(S) &= \cap_{t \in pred(s)} OUT(t) \\
OUT(S) &= \mathbf{gen}(s) \cup (IN(S) - \mathbf{kill}(s))
\end{aligned}
$$

- gen(s): expressions that *must be* computed in this statement
- kill(s): expressions that use variables that *may* be defined in this statement
  - Note difference between these sets and the sets for reaching definitions or liveness
- Insight: gen and kill must never lead to incorrect results
  - Must not decide an expression is available when it isn't, but OK to be safe and say it isn't
  - Must not decide a definition *doesn't* reach, but OK to overestimate and say it does

Analysis initialization

- How do we initialize the sets?
  - If we start with everything initialized to $\bot$, we compute the smallest sets
  - If we start with everything initialized to $\top$, we compute the largest
- Which do we want? It depends!
  - Reaching definitions: a definition that *may* reach this point
    - We want to have as few reaching definitions as possible → $\bot$
  - Available expressions: an expression that *was definitely* computed earlier
    - We want to have as many available expressions as possible → $\top$
  - Rule of thumb: if confluence operator is $\sqcup$, start with $\bot$, otherwise start with $\top$

```
void            (int m, int n)

{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /* swap a[i], a[j] */
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* swap a[i], a[n] */
    /* fragment ends here */
               (m,j);              (i+1,n);
}
```

What is this piece of code doing?

[1] R. Sedgewick, "Implementing Quicksort Programs," *Comm. ACM*, **21**, 1978, pp. 847–857.

Intermediate code (assuming int is 4 bytes):

(Ignore the temporary counter value for now)

```
void quicksort(int m, int n)
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /* swap a[i], a[j] */
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* swap a[i], a[n] */
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

available expression		Can be rewritten:
{}		
	t6 = 4*i	
{“4*i”}		
	x = a[t6]	
S_1 ={“4*i”, “a+t6”}		
	t7 = 4*i	t7 = t6
set S_1		
	t8 = 4*j	
S_2 =S_1 U {“4*j”}		
	t9 = a[t8]	
S_3 =S_2 U {“a+t8”}		
	a[t7] = t9	a[t6] = t9
set S_3		
	t10 = 4*j	t10 = t8
set S_3		
	a[t10] = x	a[t8] = x
set S_3		

copy propagation

Intermediate code (assuming int is 4 bytes):

(Ignore the temporary counter value for now)

```
void quicksort(int m, int n)

{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /* swap a[i], a[j] */
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* swap a[i], a[n] */
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

available expression

apply dead-code elim.

{}		
	t6 = 4*i	
{“4*i”}		
	x = a[t6]	
S_1 ={“4*i”, “a+t6”}		
	t7 = 4*i	~~t7 = t6~~
set S_1		
	t8 = 4*j	
S_2 =S_1 U {“4*j”}		
	t9 = a[t8]	
S_3 =S_2 U {“a+t8”}		
	a[t7] = t9	a[t6] = t9
set S_3		
	t10 = 4*j	~~t10 = t8~~
set S_3		
	a[t10] = x	a[t8] = x
set S_3		

```
void quicksort(int m, int n)

{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /* swap a[i], a[j] */
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* swap a[i], a[n] */
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

Intermediate code
(after local CSE+copy prop.+dead-code elim.)

```
t6 = 4*i
x = a[t6]
t7 = 4*i
t8 = 4*j
t9 = a[t8]
a[t7] = t9
t10 = 4*j
a[t10] = x
```

```
t6 = 4*i
x = a[t6]
t8 = 4*j
t9 = a[t8]
a[t6] = t9
a[t8] = x
```

```
void quicksort(int m, int n)

{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /* swap a[i], a[j] */
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* swap a[i], a[n] */
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

Intermediate code (assuming int is 4 bytes):

(assume next temporary counter value=11)

```
t11 = 4*i
x = a[t11]
t12 = 4*i        t12=t11
t13 = 4*n
t14 = a[t13]
a[t12] = t14     a[t11]=x
t15 = 4*n        t15=t13
a[t15] = x       a[t13]=x
```

Intermediate code (assuming int is 4 bytes):

```
void quicksort(int m, int n)

{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /* swap a[i], a[j] */
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* swap a[i], a[n] */
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

`t11 = 4*i`	after dead-code
`x = a[t11]`	elim.
`t12 = 4*i`	~~t12=t11~~
`t13 = 4*n`	
`t14 = a[t13]`	
`a[t12] = t14`	a[t11]=x
`t15 = 4*n`	~~t15=t13~~
`a[t15] = x`	a[t13]=x

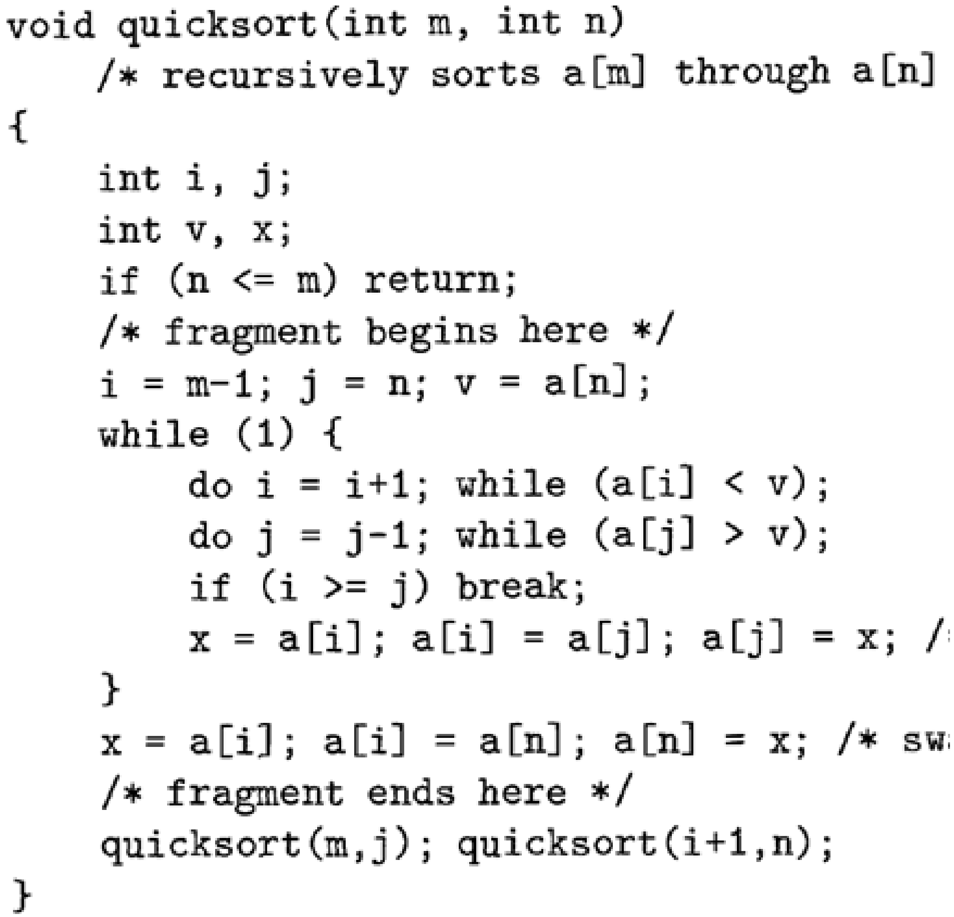
Intermediate code (after local CSE+copy prop.+dead-code elim.)

```
void quicksort(int m, int n)

{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /* swap a[i], a[j] */
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* swap a[i], a[n] */
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

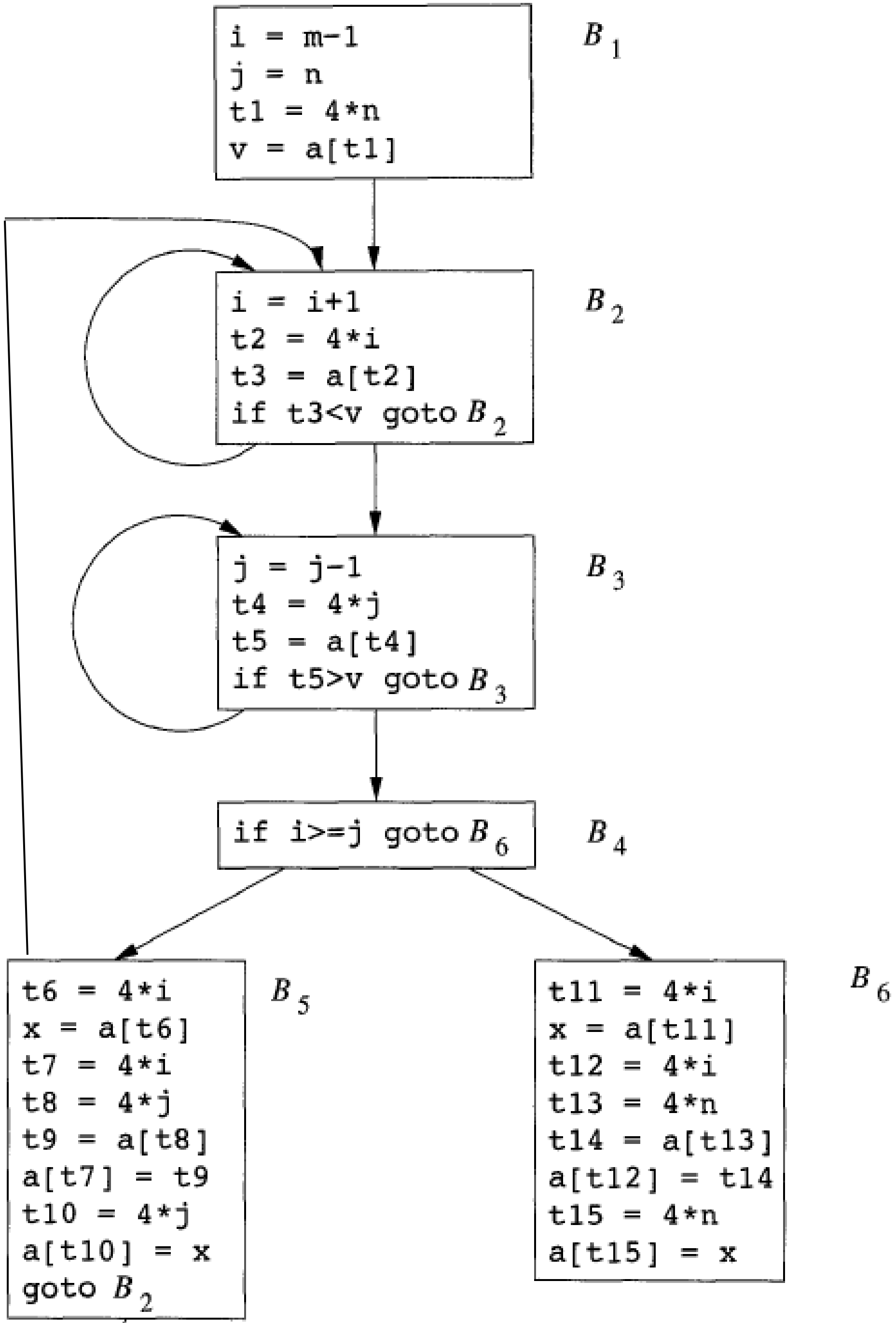
```
t11 = 4*i
x = a[t11]
t12 = 4*i
t13 = 4*n
t14 = a[t13]
a[t12] = t14
t15 = 4*n
a[t15] = x
```

t11=4*I
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=x
a[t13]=x



```
void quicksort(int m, int n)
    /* recursively sorts a[m] through a[n]
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* sw
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

- CFG for quicksort



B_1

```
i = m-1
j = n
t1 = 4*n
v = a[t1]
```

B_2

```
i = i+1
t2 = 4*i
t3 = a[t2]
if t3<v goto B2
```

B_3

```
j = j-1
t4 = 4*j
t5 = a[t4]
if t5>v goto B3
```

B_4

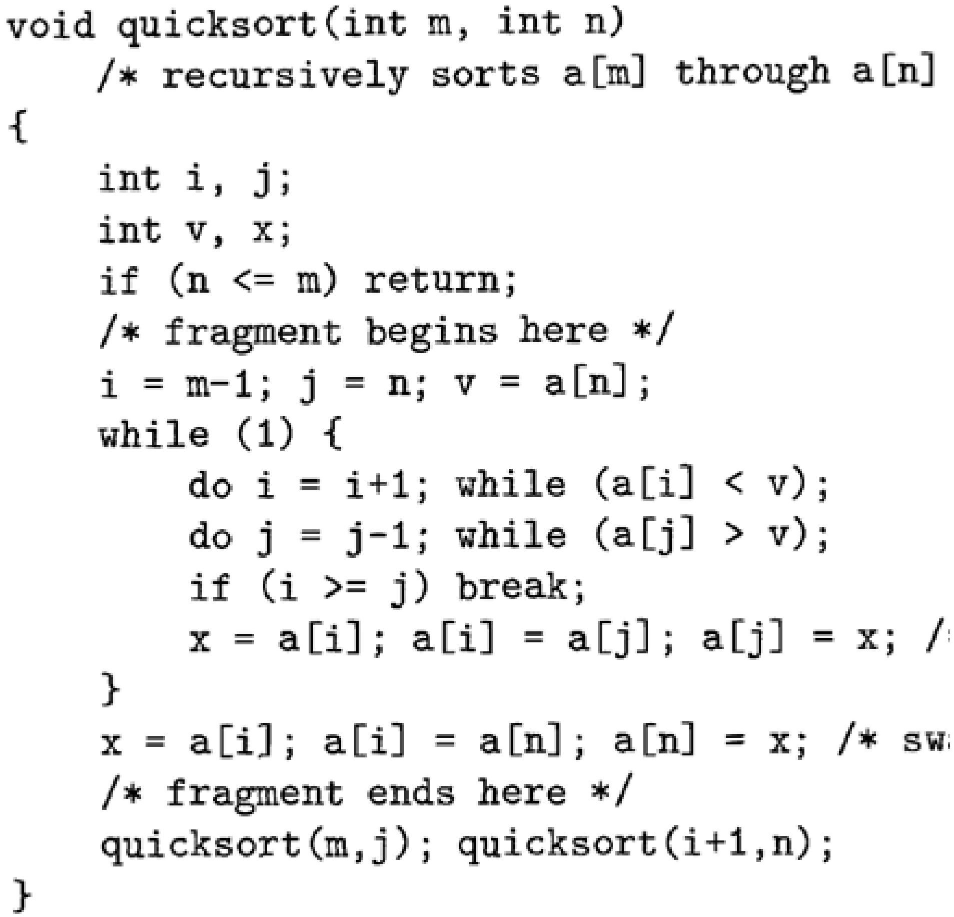
```
if i>=j goto B6
```

B_5

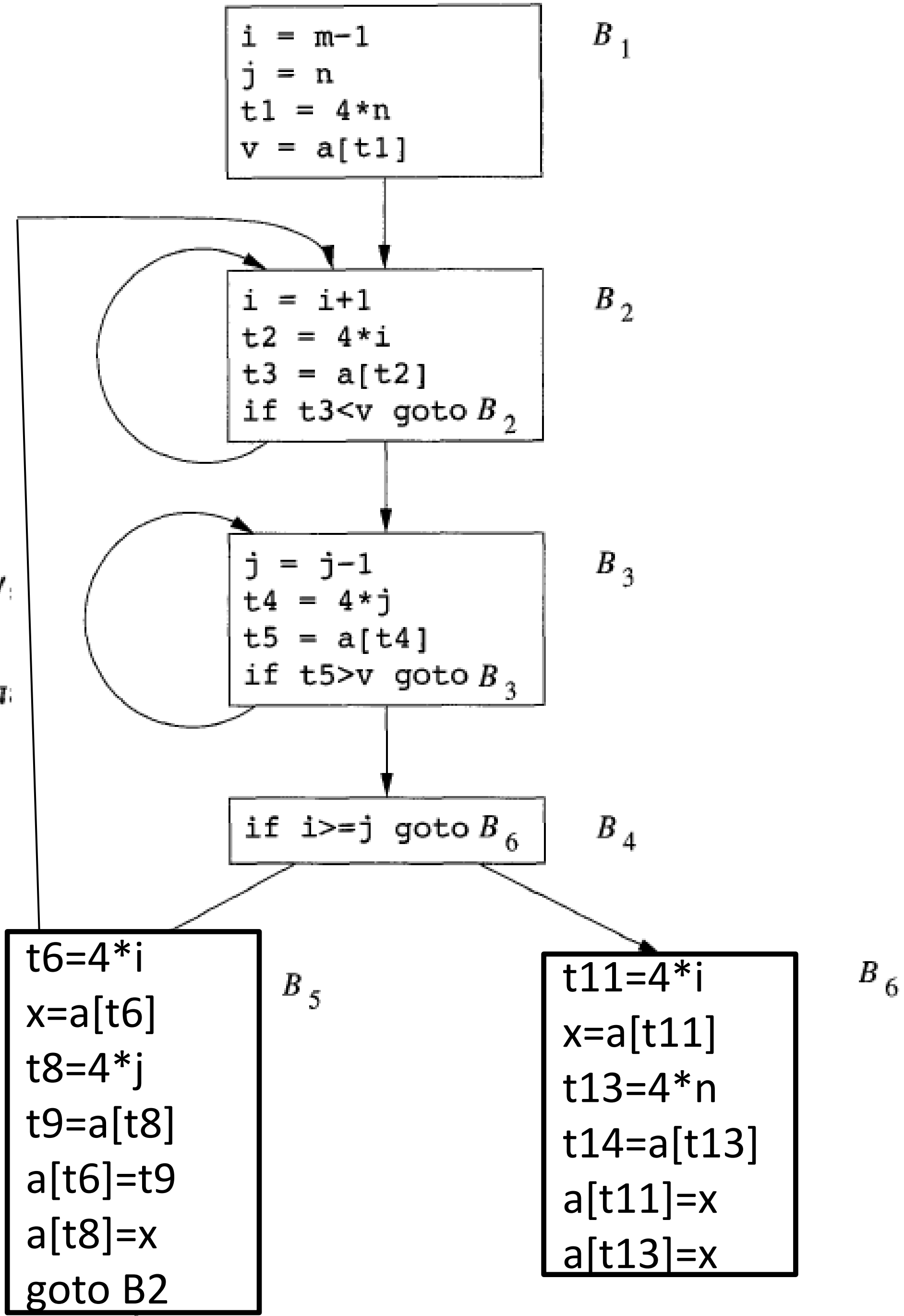
```
t6 = 4*i
x = a[t6]
t7 = 4*i
t8 = 4*j
t9 = a[t8]
a[t7] = t9
t10 = 4*j
a[t10] = x
goto B2
```

B_6

```
t11 = 4*i
x = a[t11]
t12 = 4*i
t13 = 4*n
t14 = a[t13]
a[t12] = t14
t15 = 4*n
a[t15] = x
```



```
void quicksort(int m, int n)
    /* recursively sorts a[m] through a[n]
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* sw
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```



B_1

```
i = m-1
j = n
t1 = 4*n
v = a[t1]
```

B_2

```
i = i+1
t2 = 4*i
t3 = a[t2]
if t3<v goto B2
```

B_3

```
j = j-1
t4 = 4*j
t5 = a[t4]
if t5>v goto B3
```

B_4

```
if i>=j goto B6
```

B_5

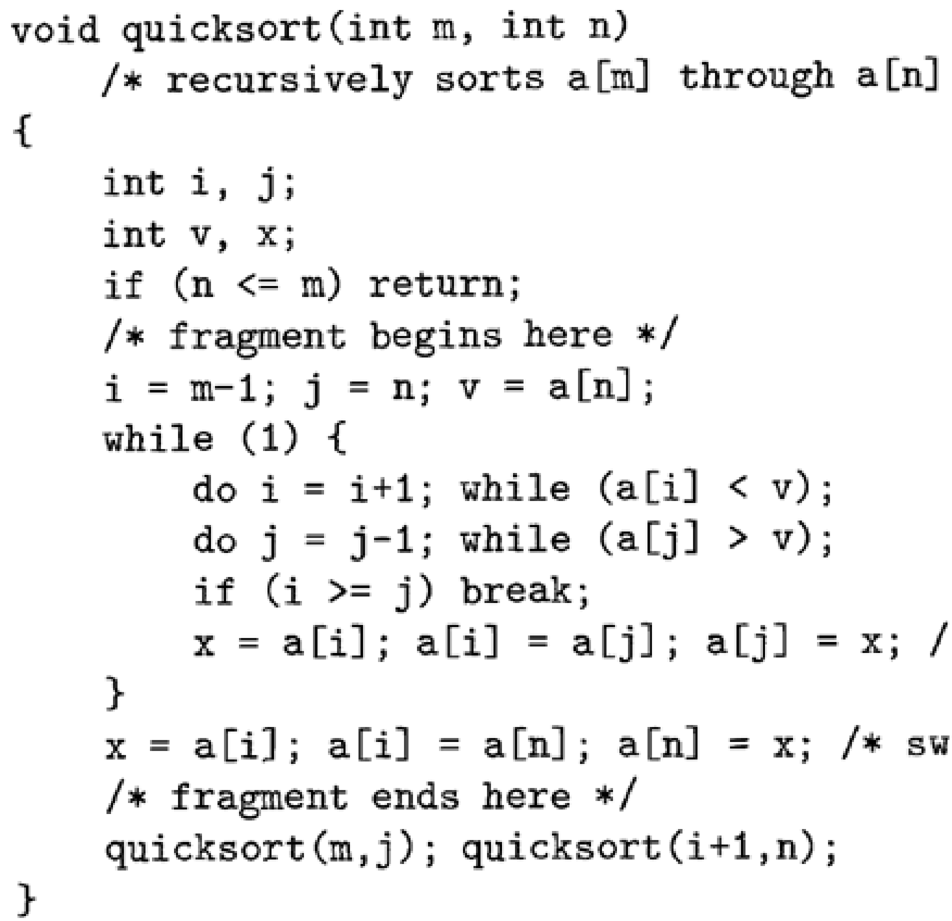
```
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto B2
```

B_6

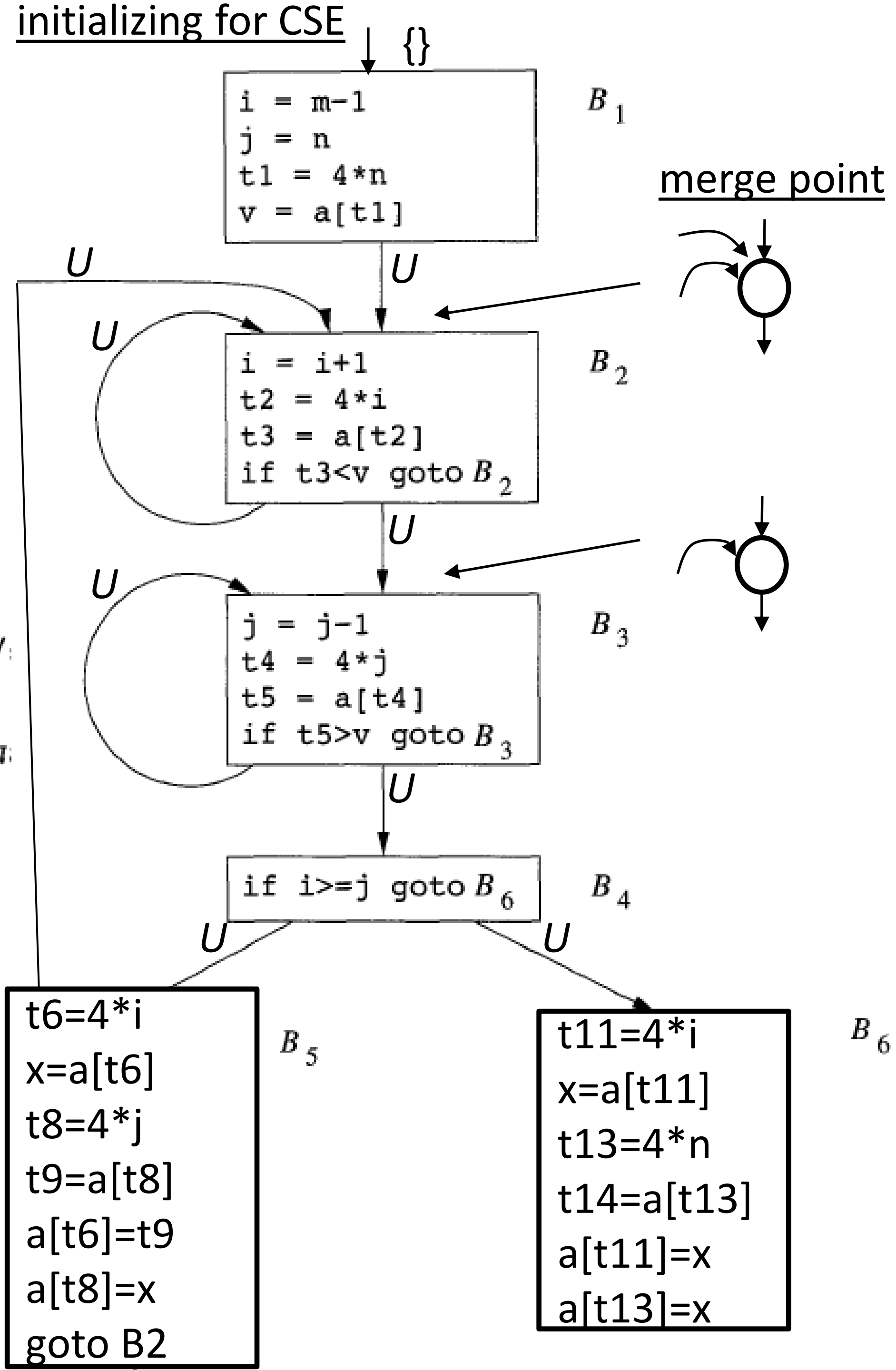
```
t11=4*i
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=x
a[t13]=x
```

- CFG for quicksort

(after optimizing B5 and B6)



```
void quicksort(int m, int n)
    /* recursively sorts a[m] through a[n]
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* sw
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```



initializing for CSE

merge point

```
B1:
i = m-1
j = n
t1 = 4*n
v = a[t1]

B2:
i = i+1
t2 = 4*i
t3 = a[t2]
if t3<v goto B2

B3:
j = j-1
t4 = 4*j
t5 = a[t4]
if t5>v goto B3

B4:
if i>=j goto B6

B5:
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto B2

B6:
t11=4*i
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=x
a[t13]=x
```

- CFG for quicksort

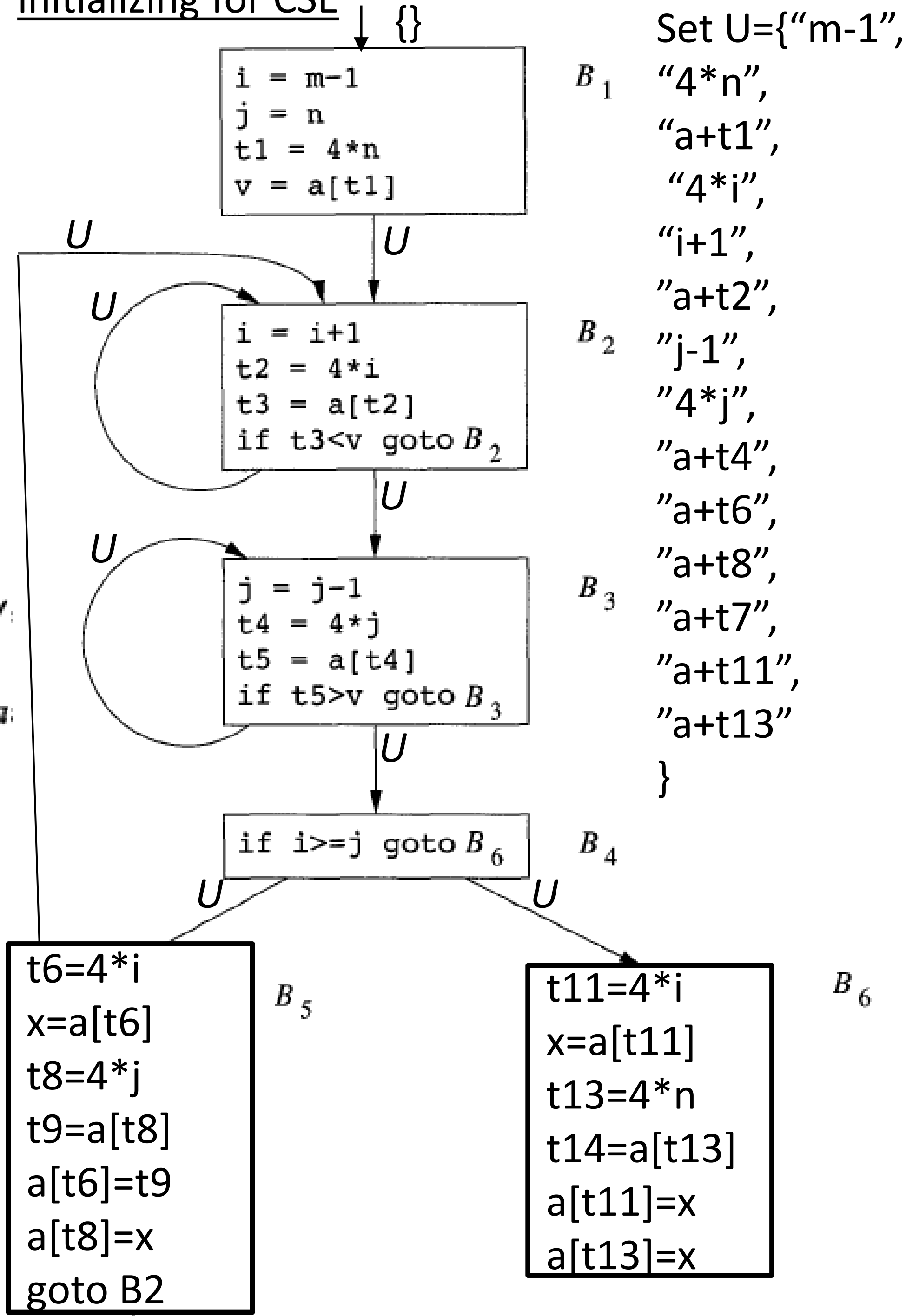
(after optimizing B5 and B6)

```
void quicksort(int m, int n)
    /* recursively sorts a[m] through a[n]
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* sw
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

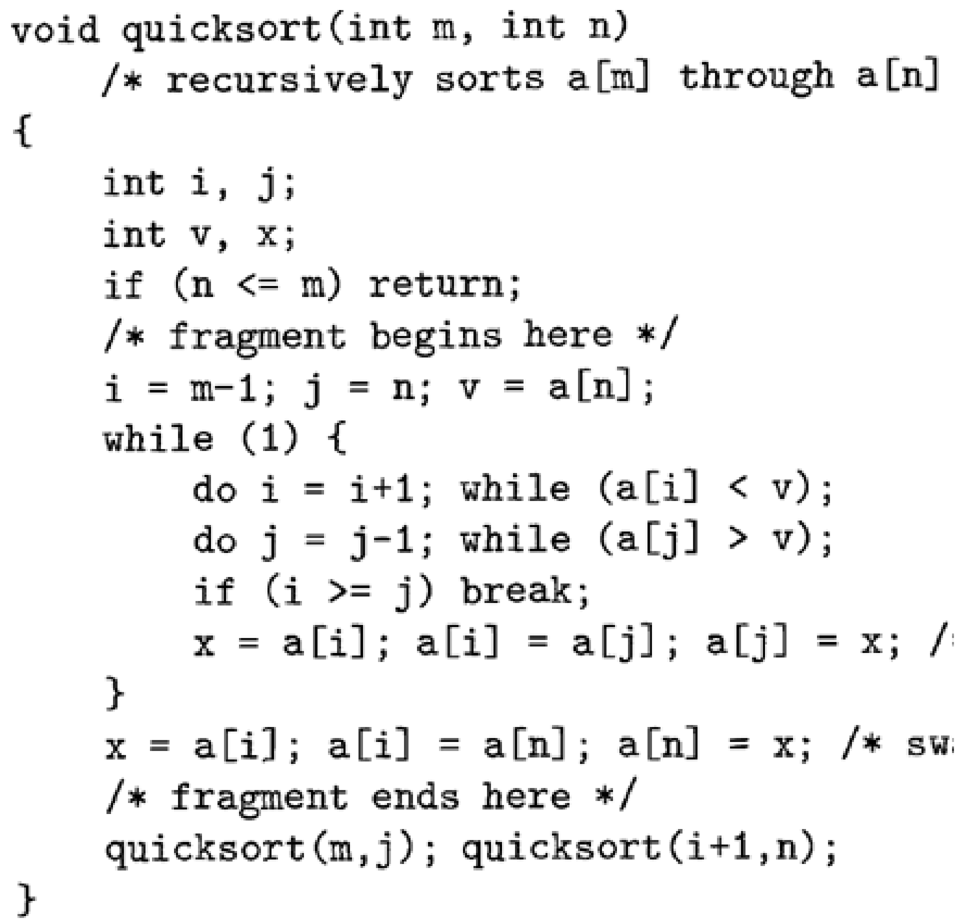
- CFG for quicksort

(after optimizing B5 and B6)

initializing for CSE



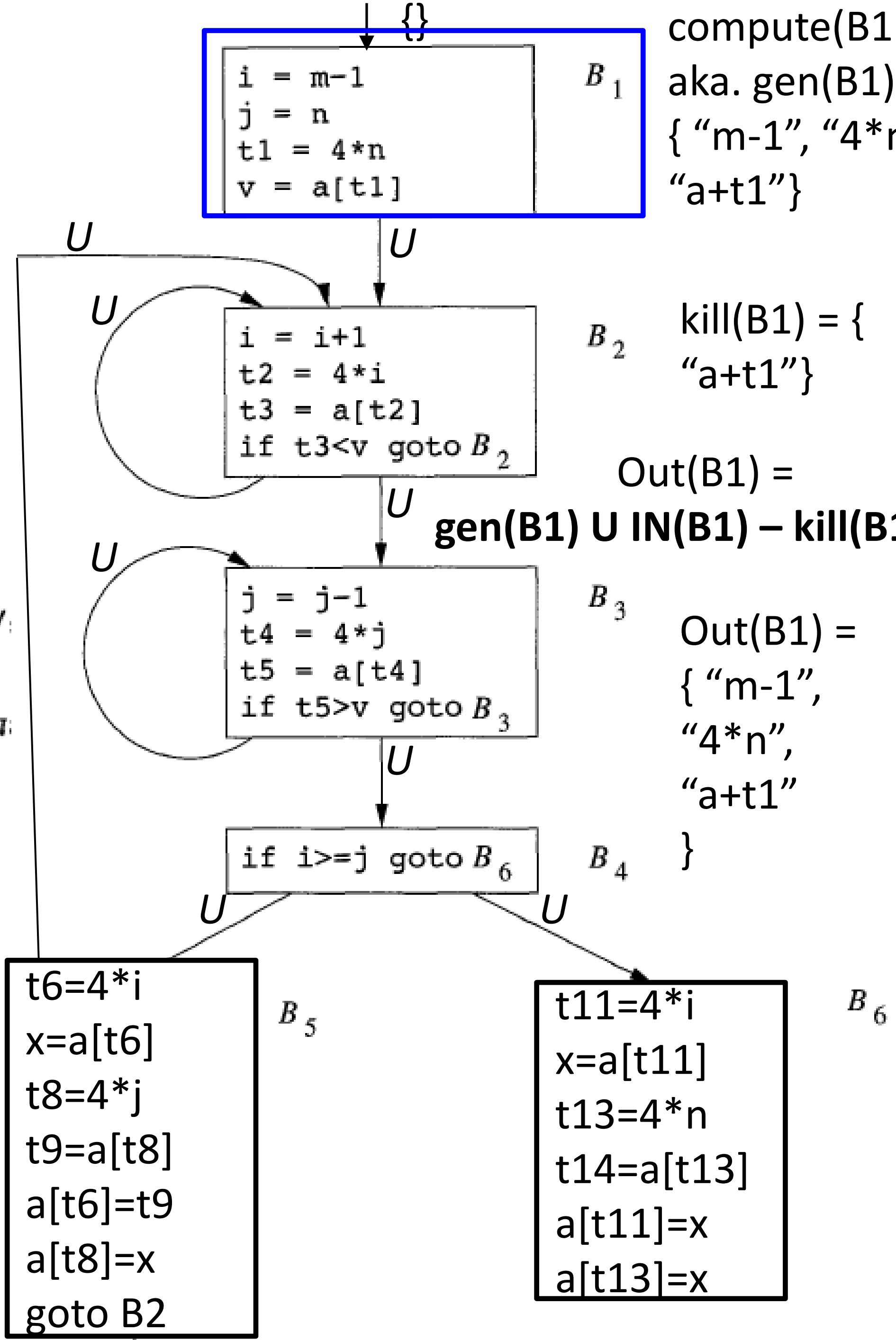
Set U={“m-1”,
“4*n”,
“a+t1”,
“4*i”,
“i+1”,
”a+t2”,
”j-1”,
”4*j”,
”a+t4”,
”a+t6”,
”a+t8”,
”a+t7”,
”a+t11”,
”a+t13”
}



```
void quicksort(int m, int n)
    /* recursively sorts a[m] through a[n]
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* sw
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

- CFG for quicksort

(after optimizing B5 and B6)



{}

B_1

```
i = m-1
j = n
t1 = 4*n
v = a[t1]
```

B_2

```
i = i+1
t2 = 4*i
t3 = a[t2]
if t3<v goto B2
```

B_3

```
j = j-1
t4 = 4*j
t5 = a[t4]
if t5>v goto B3
```

B_4

```
if i>=j goto B6
```

B_5

```
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto B2
```

B_6

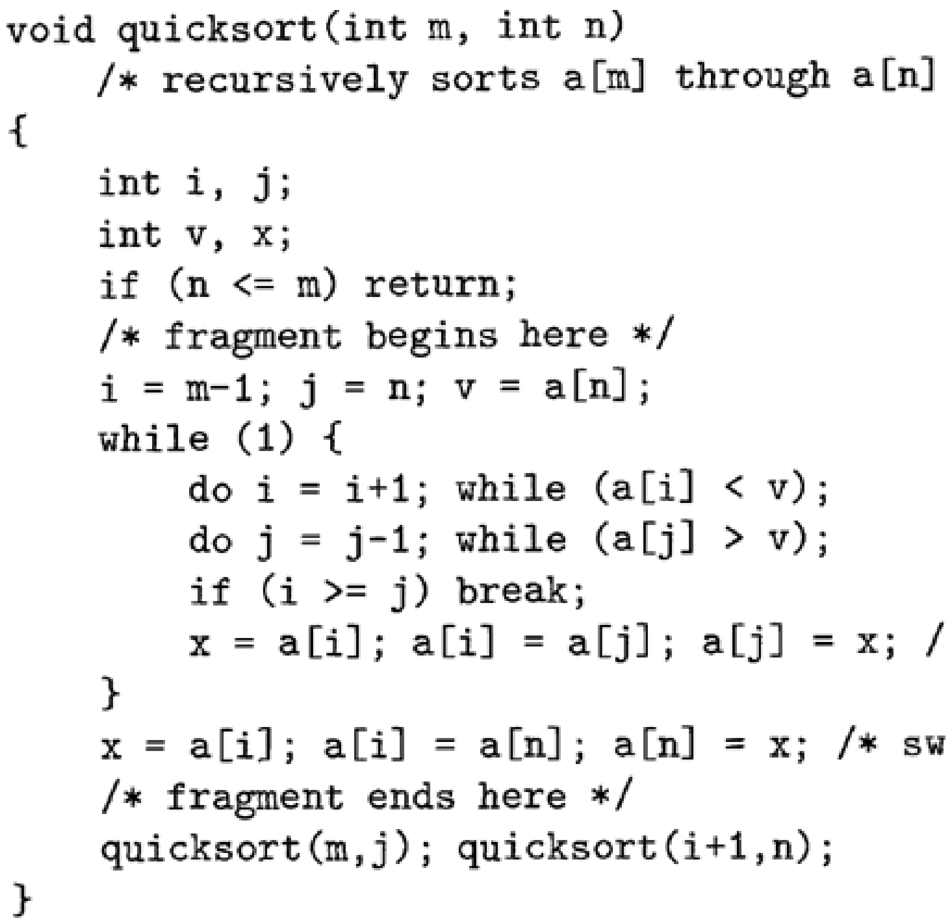
```
t11=4*i
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=x
a[t13]=x
```

compute(B1)
aka. gen(B1) =
{ “m-1”, “4*n”,
“a+t1”}

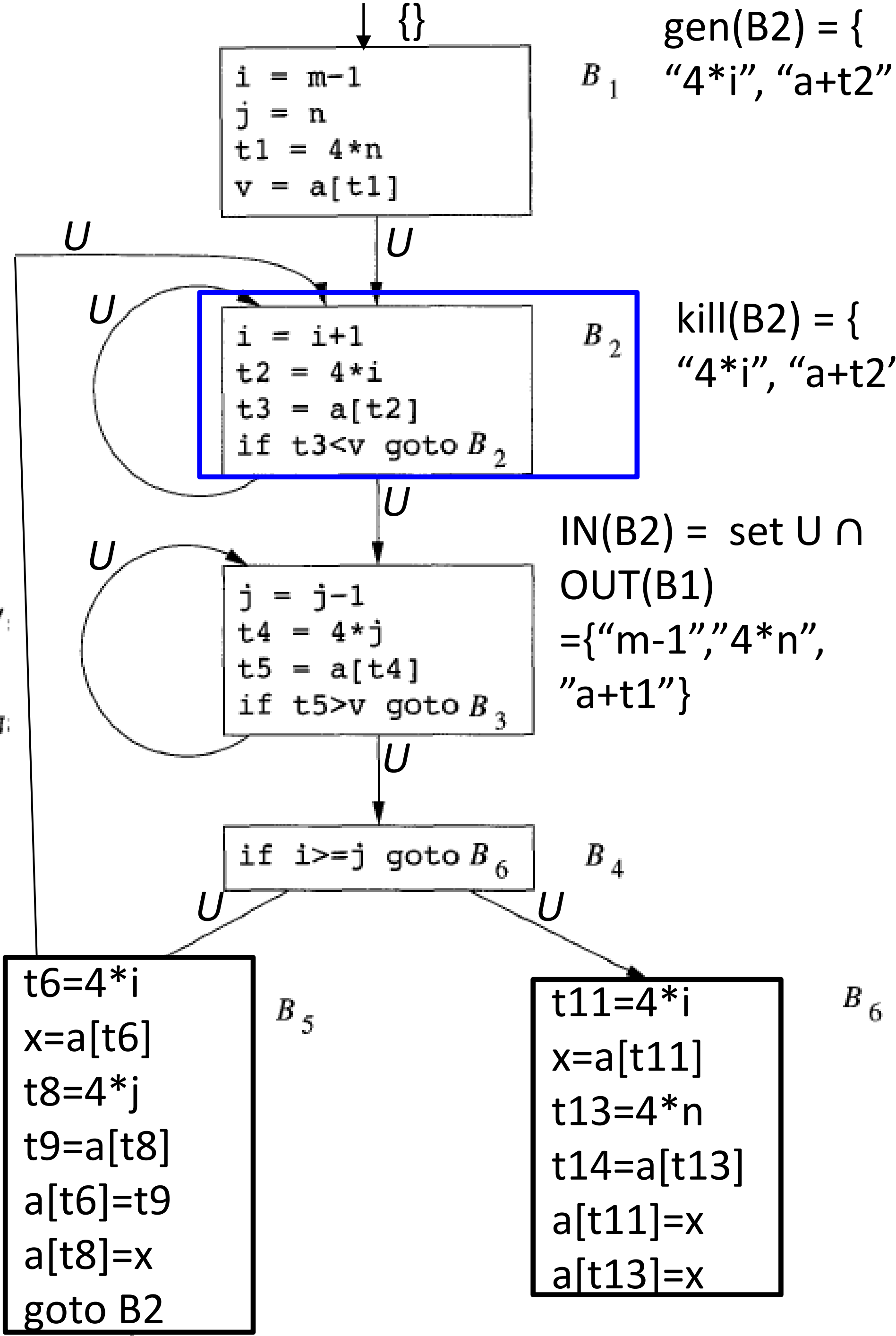
kill(B1) = {
“a+t1”}

Out(B1) =
gen(B1) U IN(B1) – kill(B1)

Out(B1) =
{ “m-1”,
“4*n”,
“a+t1”
}



```
void quicksort(int m, int n)
    /* recursively sorts a[m] through a[n]
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* sw
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```



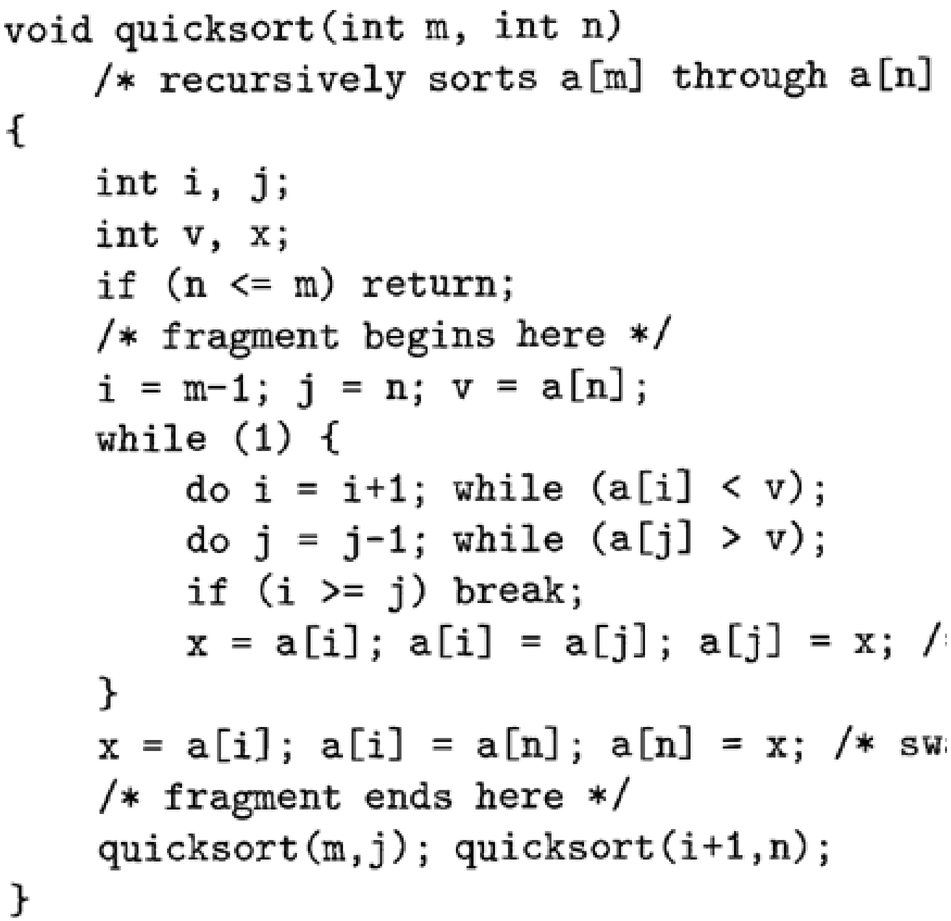
gen(B2) = { “4*i”, “a+t2”}

kill(B2) = { “4*i”, “a+t2”}

IN(B2) = set U ∩ OUT(B1) ={“m-1”,”4*n”, ”a+t1”}

- CFG for quicksort

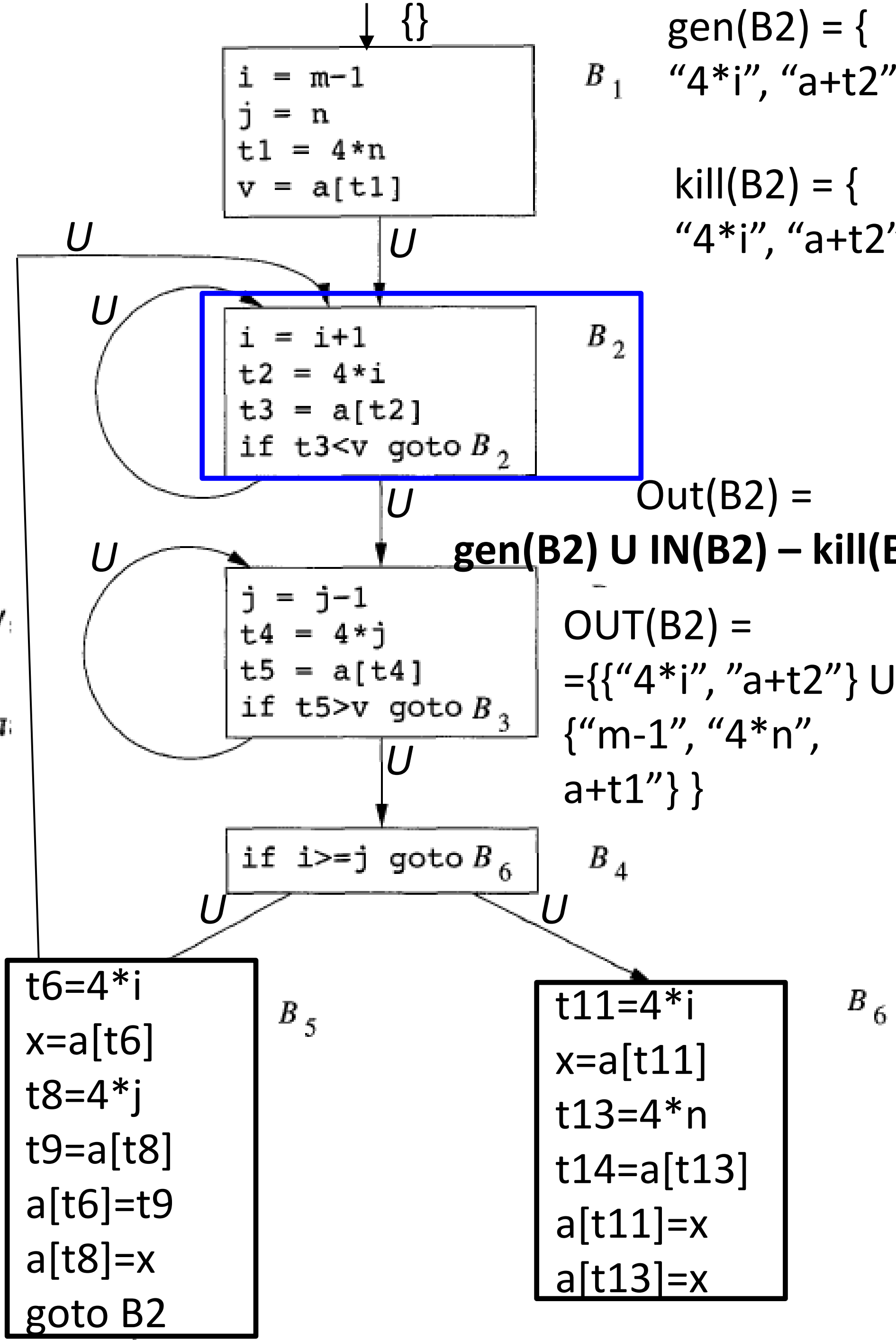
(after optimizing B5 and B6)



```
void quicksort(int m, int n)
    /* recursively sorts a[m] through a[n]
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* sw
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```

- CFG for quicksort

(after optimizing B5 and B6)



```
i = m-1
j = n
t1 = 4*n
v = a[t1]
```

```
i = i+1
t2 = 4*i
t3 = a[t2]
if t3<v goto B2
```

```
j = j-1
t4 = 4*j
t5 = a[t4]
if t5>v goto B3
```

```
if i>=j goto B6
```

```
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto B2
```

```
t11=4*i
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=x
a[t13]=x
```

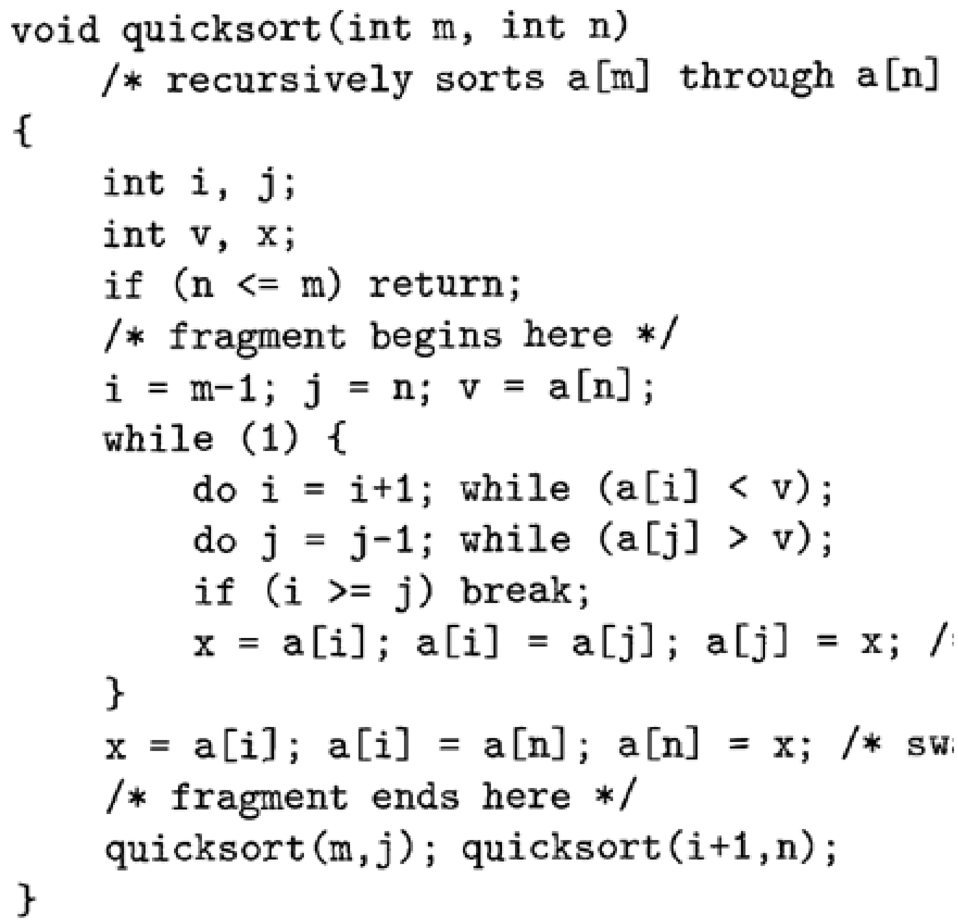
gen(B2) = {
“4*i”, “a+t2”}

kill(B2) = {
“4*i”, “a+t2”}

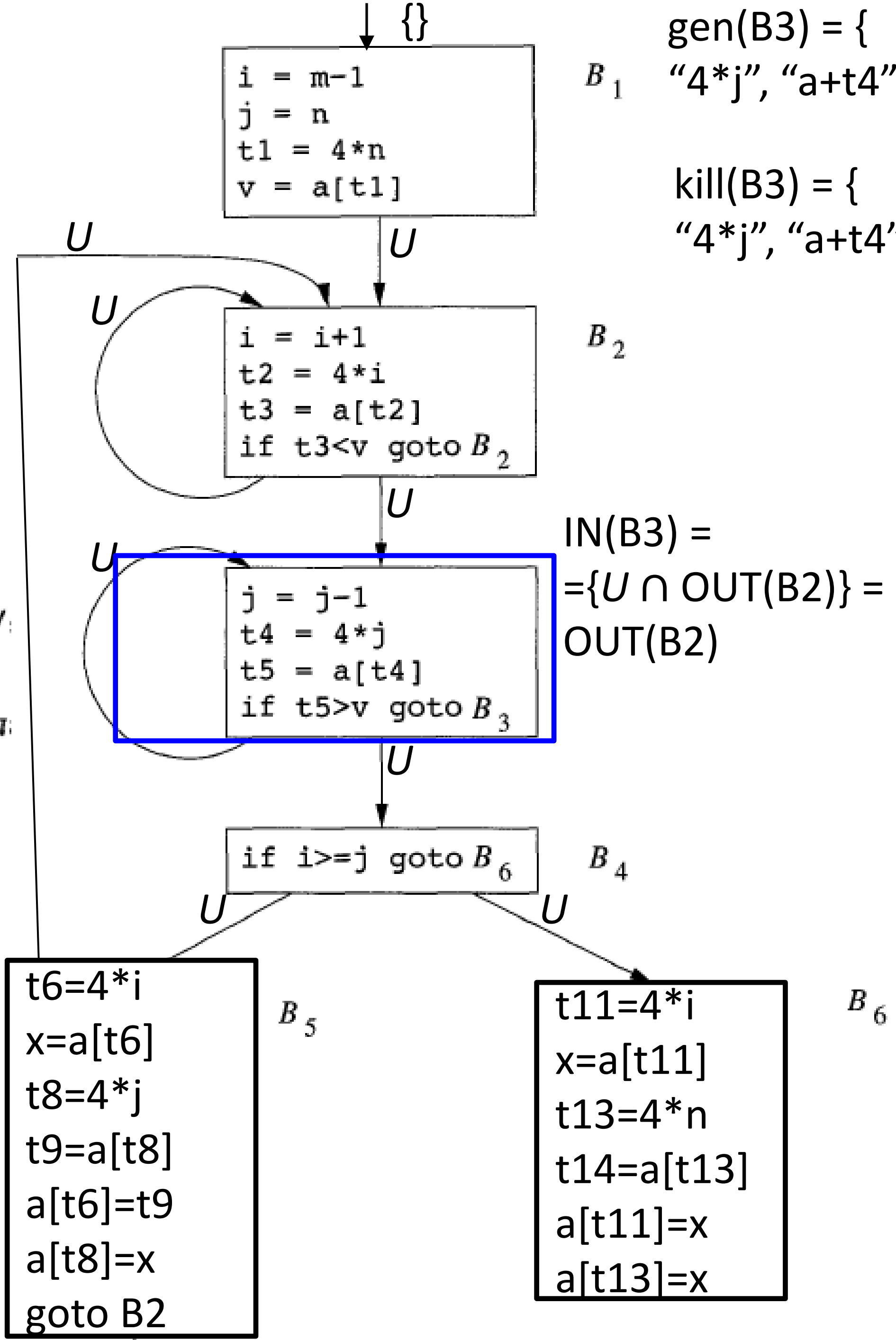
Out(B2) =

gen(B2) U IN(B2) – kill(B2

OUT(B2) =
={{“4*i”, ”a+t2”} U
{“m-1”, “4*n”,
a+t1”} }



```
void quicksort(int m, int n)
    /* recursively sorts a[m] through a[n]
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* sw
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```



{}

B_1

```
i = m-1
j = n
t1 = 4*n
v = a[t1]
```

B_2

```
i = i+1
t2 = 4*i
t3 = a[t2]
if t3<v goto B2
```

B_3

```
j = j-1
t4 = 4*j
t5 = a[t4]
if t5>v goto B3
```

B_4

```
if i>=j goto B6
```

B_5

```
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto B2
```

B_6

```
t11=4*i
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=x
a[t13]=x
```

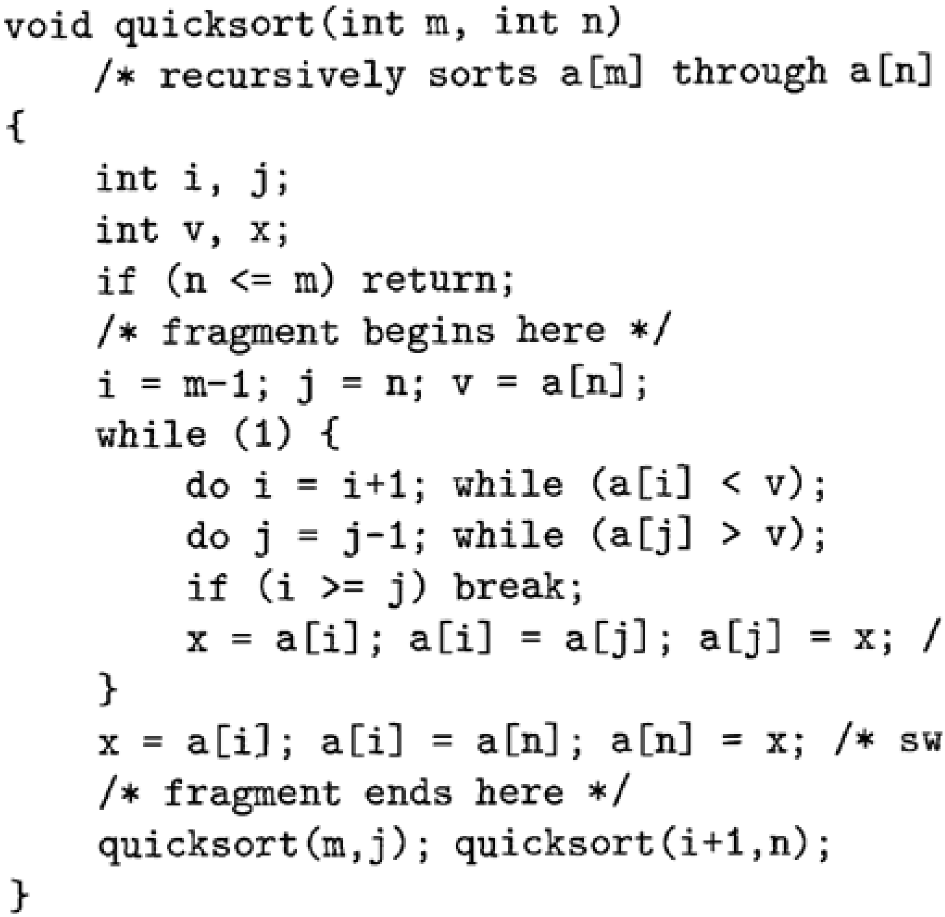
gen(B3) = { “4*j”, “a+t4”}

kill(B3) = { “4*j”, “a+t4”}

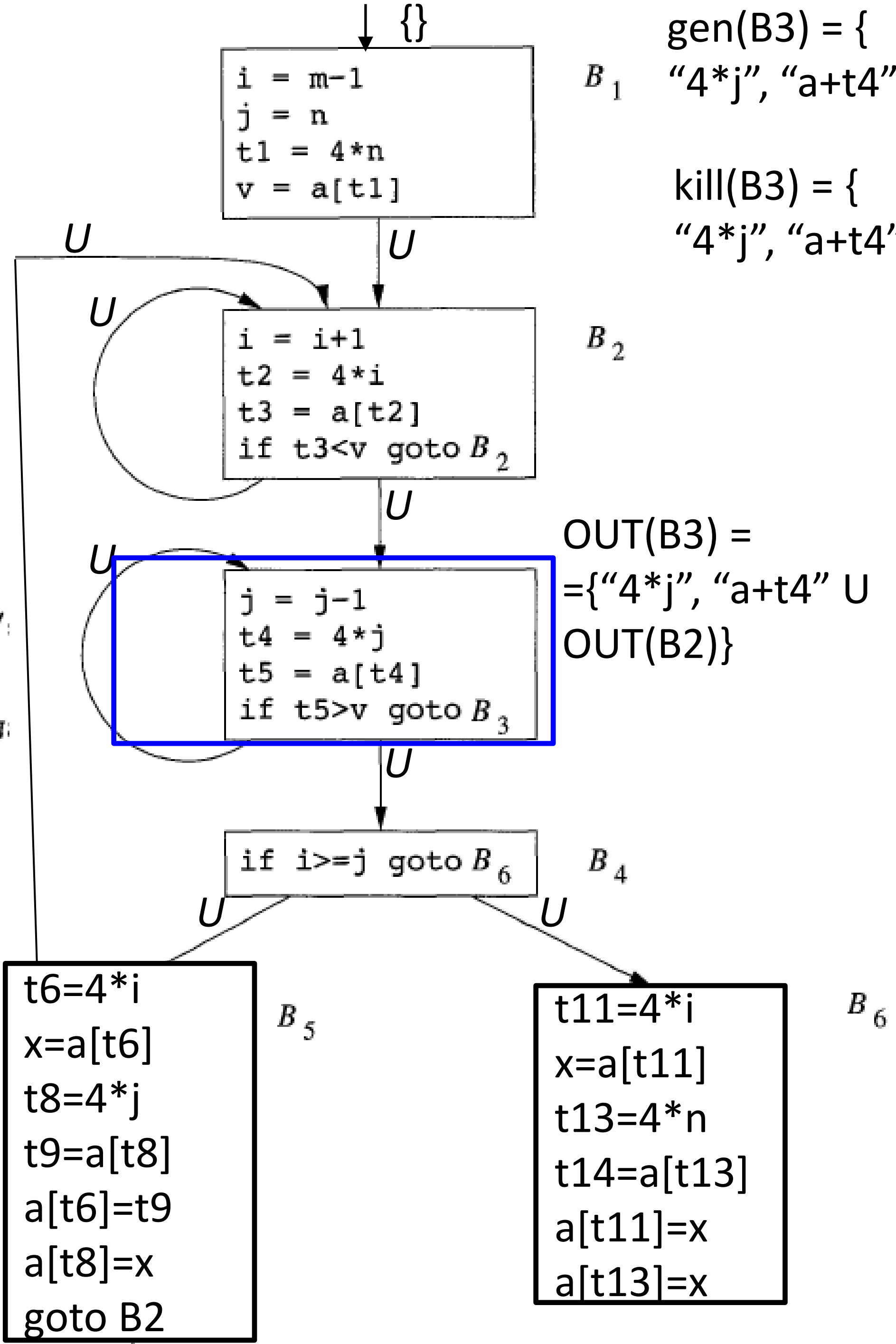
IN(B3) = ={*U* ∩ OUT(B2)} = OUT(B2)

- CFG for quicksort

(after optimizing B5 and B6)



```
void quicksort(int m, int n)
    /* recursively sorts a[m] through a[n]
{
    int i, j;
    int v, x;
    if (n <= m) return;
    /* fragment begins here */
    i = m-1; j = n; v = a[n];
    while (1) {
        do i = i+1; while (a[i] < v);
        do j = j-1; while (a[j] > v);
        if (i >= j) break;
        x = a[i]; a[i] = a[j]; a[j] = x; /
    }
    x = a[i]; a[i] = a[n]; a[n] = x; /* sw
    /* fragment ends here */
    quicksort(m,j); quicksort(i+1,n);
}
```



```
i = m-1
j = n
t1 = 4*n
v = a[t1]
```

```
i = i+1
t2 = 4*i
t3 = a[t2]
if t3<v goto B2
```

```
j = j-1
t4 = 4*j
t5 = a[t4]
if t5>v goto B3
```

```
if i>=j goto B6
```

```
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto B2
```

```
t11=4*i
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=x
a[t13]=x
```

gen(B3) = {
“4*j”, “a+t4”}

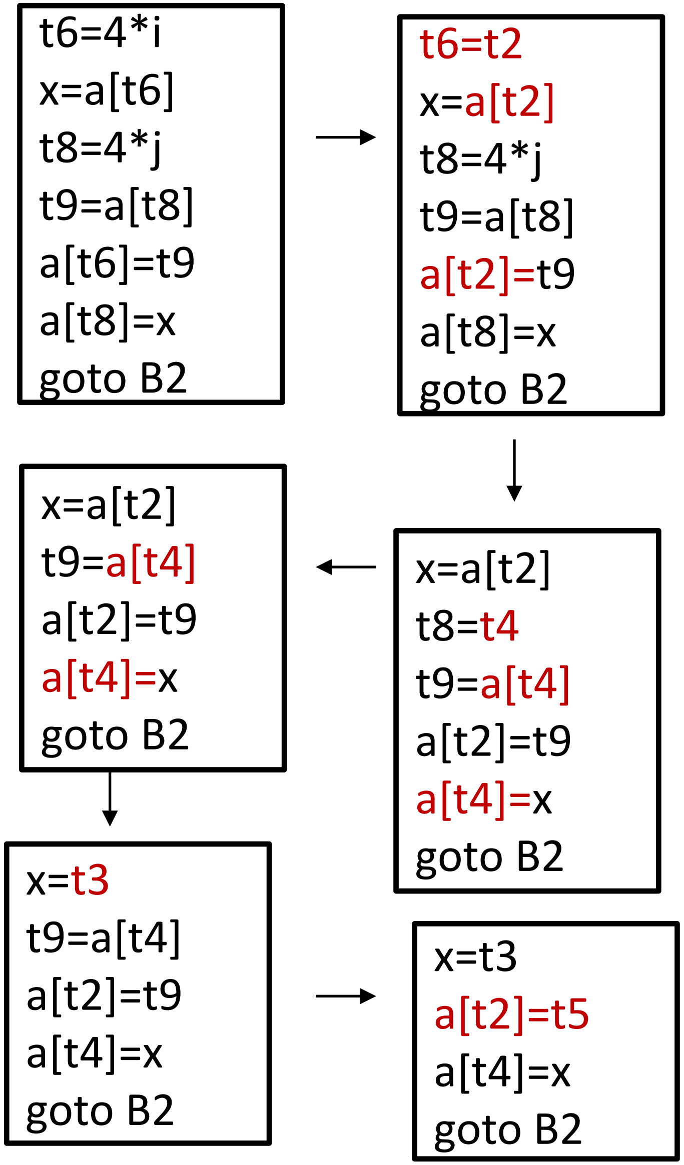
kill(B3) = {
“4*j”, “a+t4”}

OUT(B3) =
={“4*j”, “a+t4” U
OUT(B2)}

- CFG for quicksort

(after optimizing B5 and B6)

IN(B5)=“4*j”, “a+t4”, “4*i”, “a+t2”, “m-1”, “4*n”, “a+t1”



```
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto B2
```

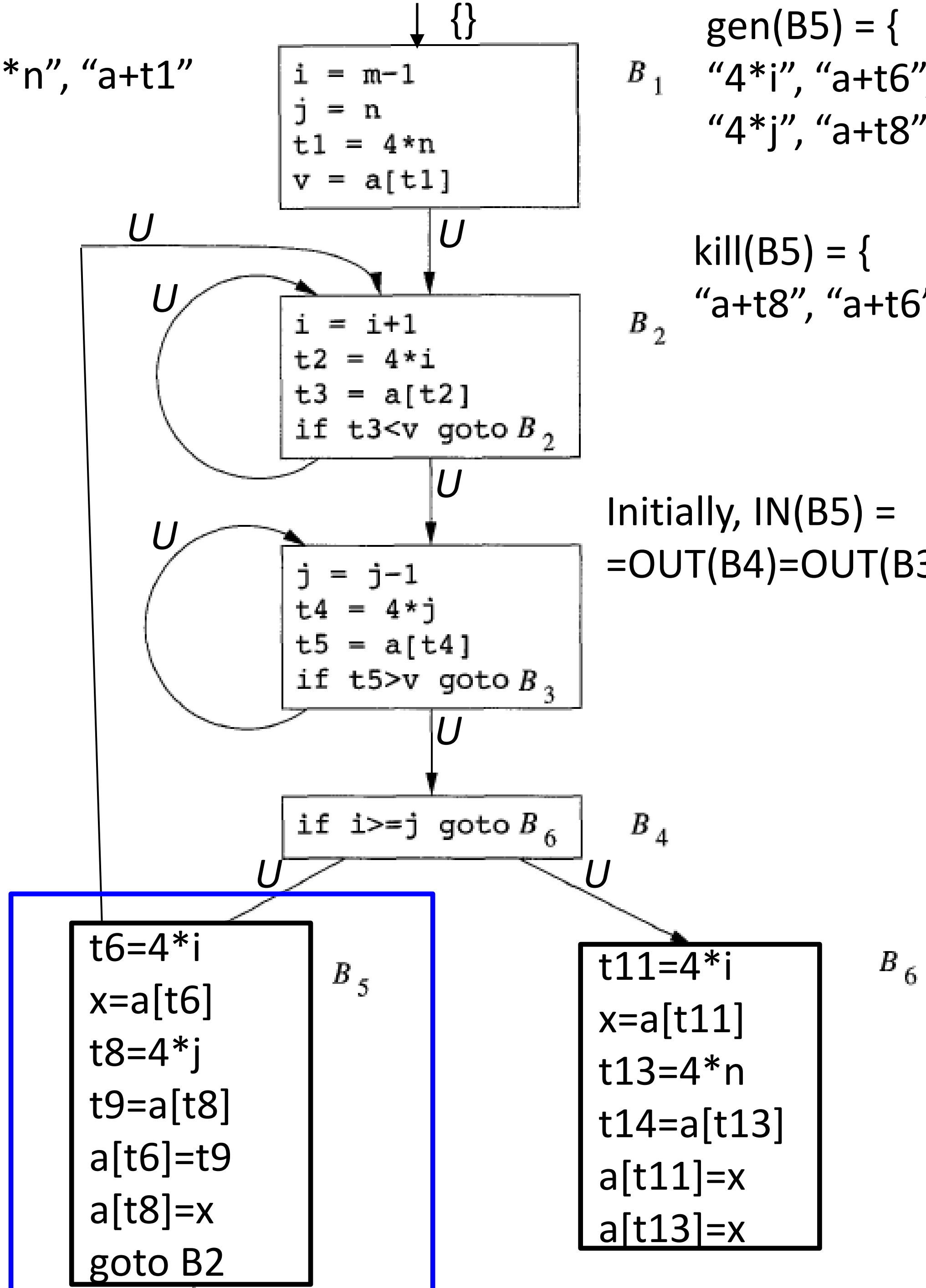
```
t6=t2
x=a[t2]
t8=4*j
t9=a[t8]
a[t2]=t9
a[t8]=x
goto B2
```

```
x=a[t2]
t8=t4
t9=a[t4]
a[t2]=t9
a[t4]=x
goto B2
```

```
x=a[t2]
t9=a[t4]
a[t2]=t9
a[t4]=x
goto B2
```

```
x=t3
t9=a[t4]
a[t2]=t9
a[t4]=x
goto B2
```

```
x=t3
a[t2]=t5
a[t4]=x
goto B2
```



```
i = m-1
j = n
t1 = 4*n
v = a[t1]
```

```
i = i+1
t2 = 4*i
t3 = a[t2]
if t3<v goto B2
```

```
j = j-1
t4 = 4*j
t5 = a[t4]
if t5>v goto B3
```

```
if i>=j goto B6
```

```
t6=4*i
x=a[t6]
t8=4*j
t9=a[t8]
a[t6]=t9
a[t8]=x
goto B2
```

```
t11=4*i
x=a[t11]
t13=4*n
t14=a[t13]
a[t11]=x
a[t13]=x
```

gen(B5) = {
“4*i”, “a+t6”,
“4*j”, “a+t8”}

kill(B5) = {
“a+t8”, “a+t6”}

Initially, IN(B5) =
=OUT(B4)=OUT(B3)

Dataflow Analysis – Problem Categorization

- All path problem:
  - we want the property to hold at all the paths reaching a program point.
- Any path problem:
  - we want the property to hold at some path reaching a program point.

Orthogonal to the above categorization we can have:

- Forward flow problem:
  - Transfer of information done along the direction of the control flow
- Backward flow problem:
  - Transfer of information done opposite to the direction of the control flow