CS406: Compilers

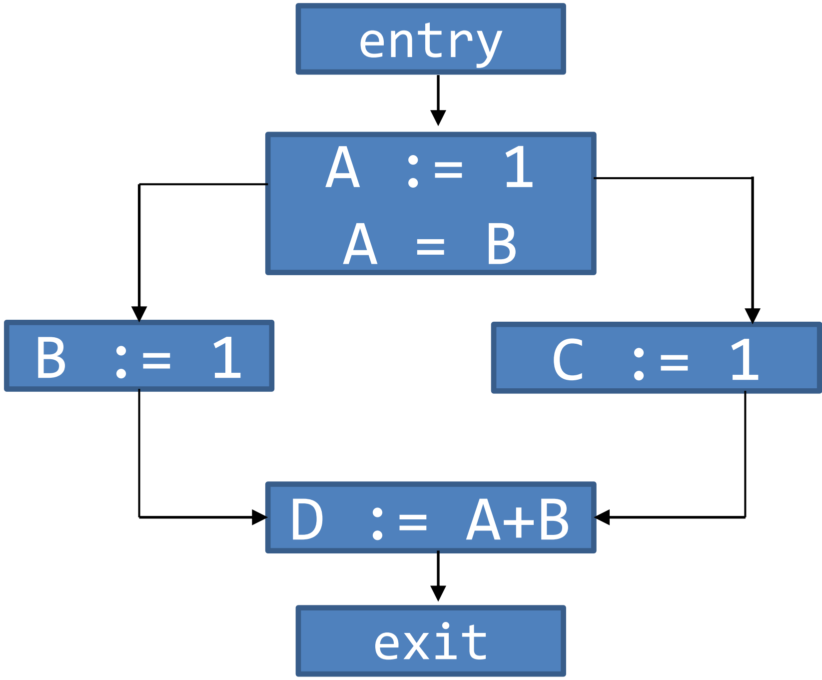
Spring 2022

Week 12: Dataflow Analysis – Constant Propagation, ..

Slides Acknowledgements: Milind Kulkarni

Recap: Liveness

- Variables are live if there exists *some path* leading to its use
- Start from exit block and proceed *backwards* against the control flow to compute



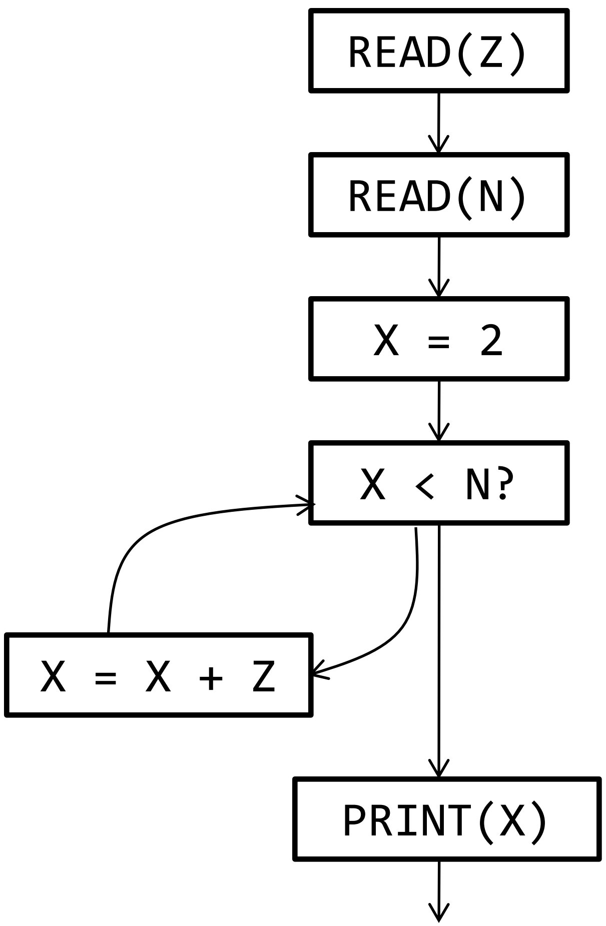
LiveOut(b) $= \cup_{i \in Succ(b)} \mathrm{LiveIn}(i)$

LiveIn(b) = LiveUse(b) U (LiveOut(b) – Def(b))

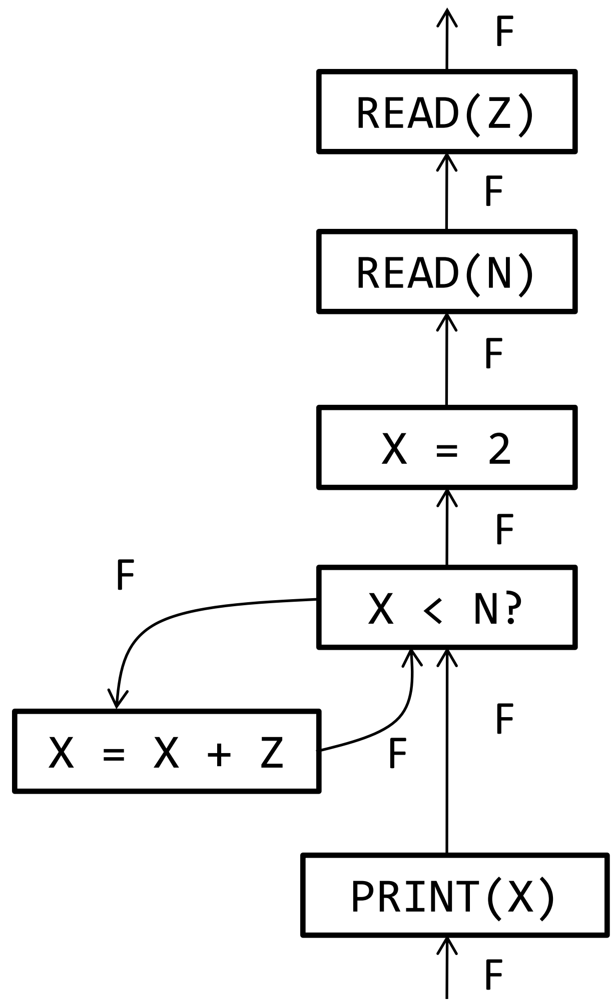
//set that contains all variables used by block b

//set that contains all variables defined by block b

Recap: Liveness

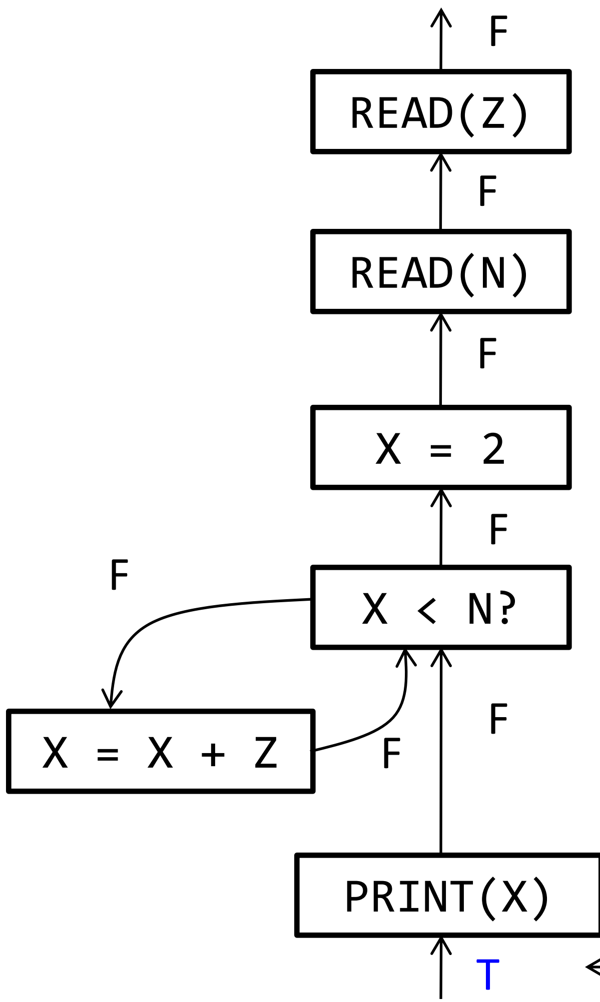


Original CFG



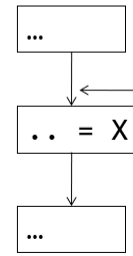
CFG with edges reversed (and initialized) for backwards analysis: is X live? (F=`false`, T=`true`)

Recap: Liveness



X must be live here
(refer week11 slide)

Liveness in a CFG



X *must be* live here (i.e. before the statement)

- Define a set `LiveUse(b)`, where b is a basic block, as the set of all variables that are used within block `b`. `LiveIn(b)` ⊇ `LiveUse(b)`

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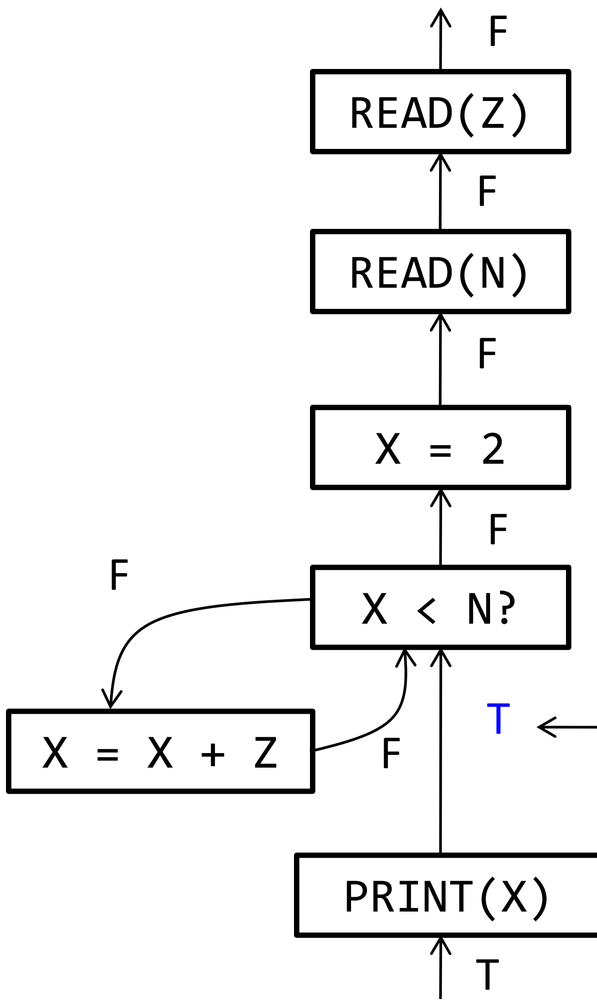
Recap: Liveness



X must be live here
(refer week11 slide)

Liveness in a CFG

- Under what scenarios can a variable be live at the entrance of a basic block?
  - Either the variable is used in the basic block
  - OR the variable is live at exit and not defined within the block

```
LiveIn(b) = LiveUse(b) ∪ (LiveOut(b) – Def(b))
```

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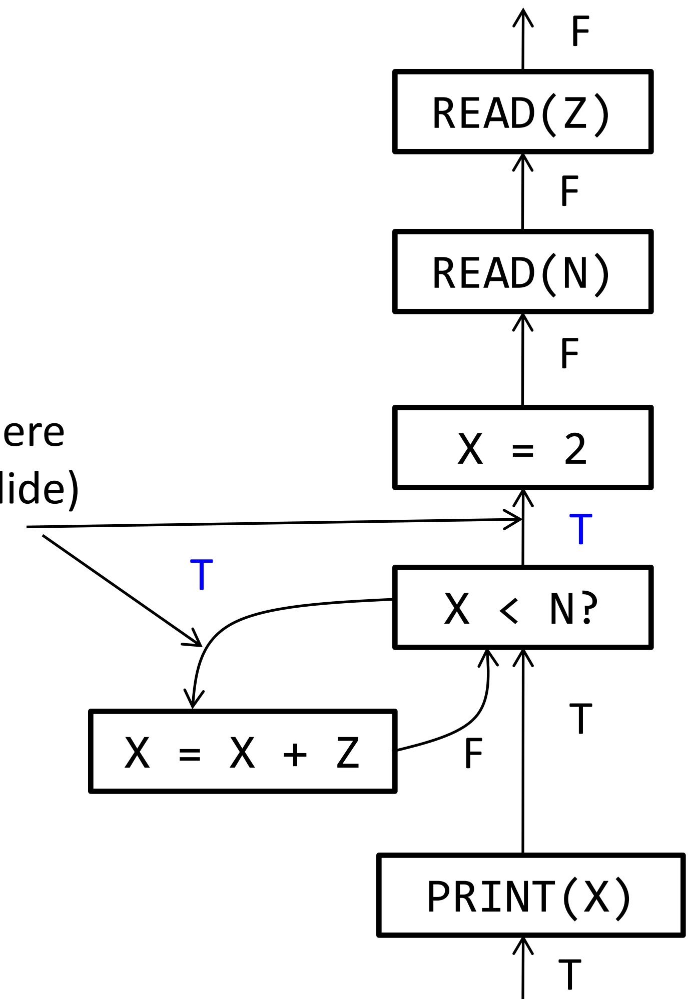
Recap: Liveness

X must be live here
(refer week11 slide)



Liveness in a CFG

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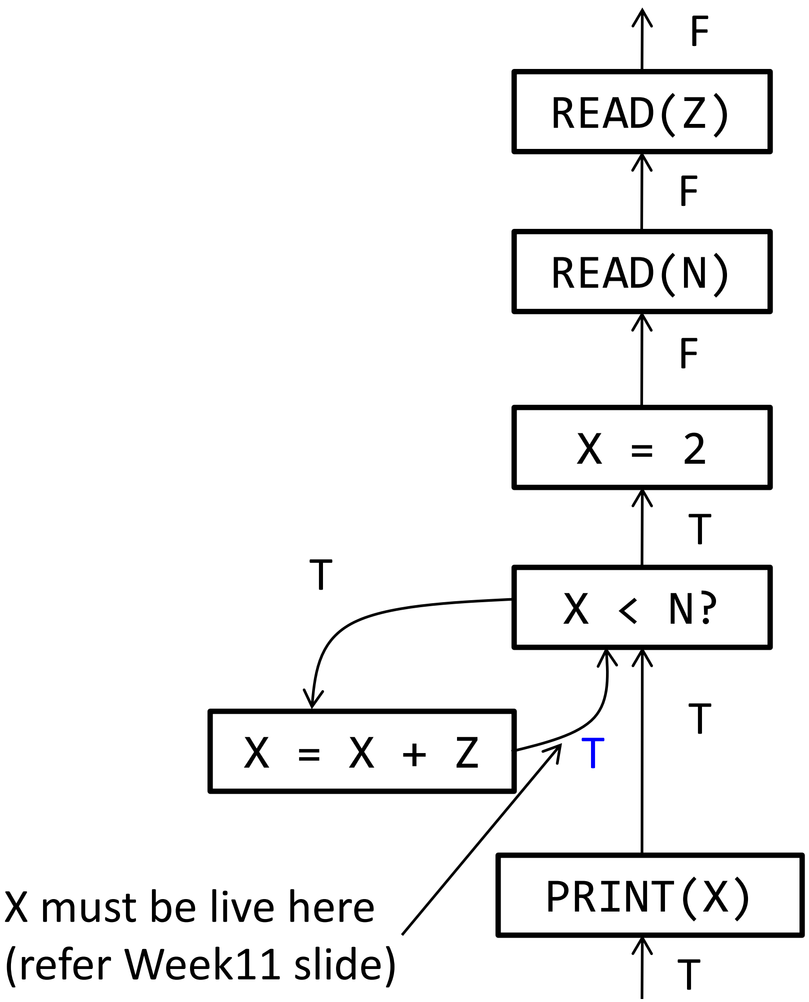
Recap: Liveness



Liveness in a CFG

- Under what scenarios can a variable be live at the entrance of a basic block?
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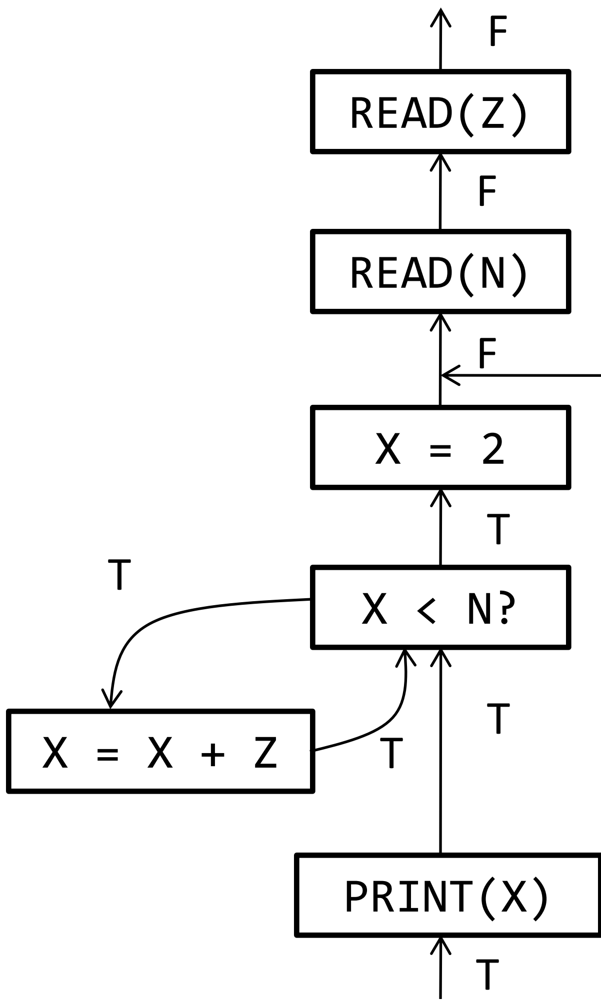
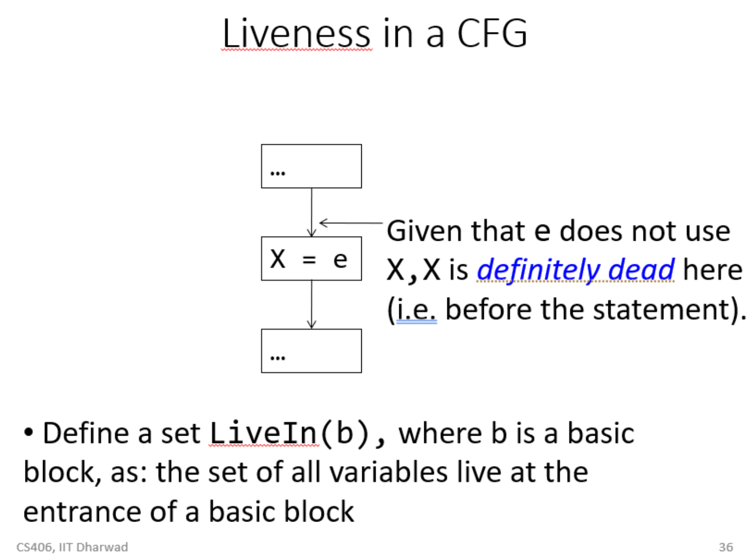
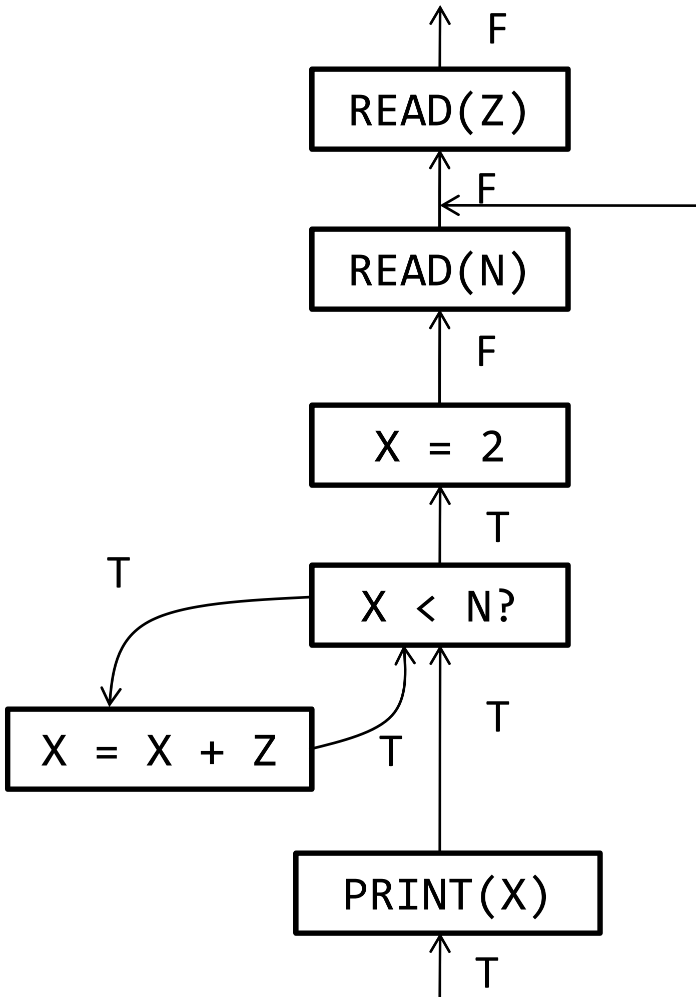
Recap: Liveness



X dead here (refer Week11 slide).
No change in information.

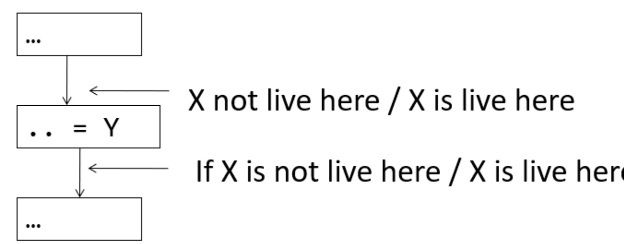


Recap: Liveness



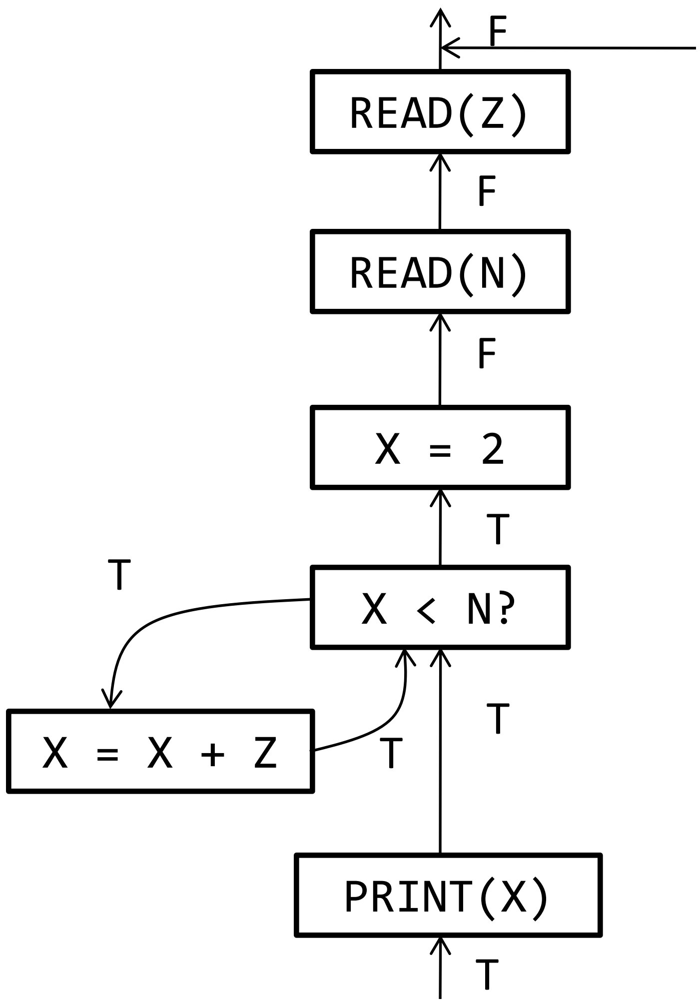
X dead here (refer Week11 slide).
No change in information.

Liveness in a CFG - Observation

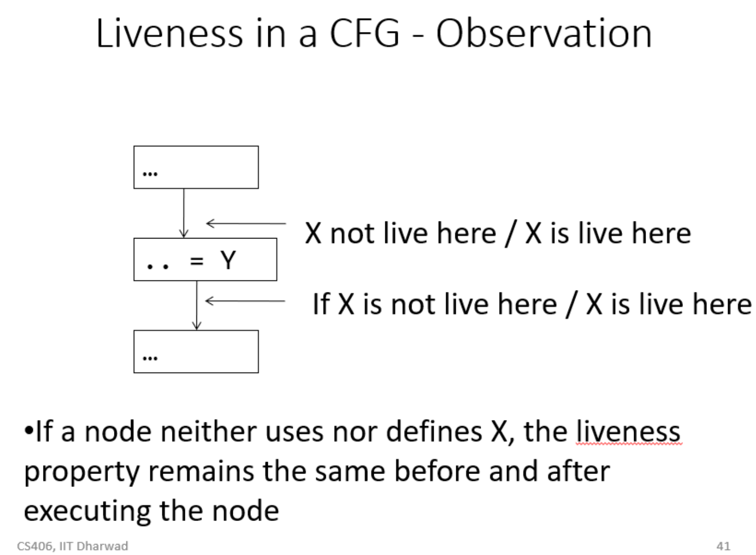


- If a node neither uses nor defines X, the liveness property remains the same before and after executing the node

Recap: Liveness



X dead here (refer Week11 slide).
No change in information.



Exercise: Repeat for Z and N

Constant Propagation

```
X = 1
Y = X + 2
if( Y> X)
   Y = 5
..
```

⇨

```
X = 1
Y = 1 + 2
if( Y> X)
   Y = 5
..
```

Constant Propagation

⇨

```
X = 1
Y = 3
if( Y> X)
   Y = 5
..
```

Using Constant Propagation, we can optimize further: do constant folding

Constant Propagation

```
X = 1
Y = X + 2
if( Y> X)
   Y = 5
..
```

```
X = 1
Y = 3
if( Y> X)
   Y = 5
..
```

Constant Propagation

~~X = 1~~

~~Y = 3~~ //dead code

`if(true)`

`   Y = 5`

Using Liveness information leads to further optimizations: Dead Code Elimination

Constant Propagation

- Bigger problem size:
  - Which lines using X could be replaced with a constant value? (apply only constant propagation)
  - How can we automate to find an answer to this question?

```
1. X := 2
2. Label1:
3. Y := X + 1
4. if Z > 8 goto Label2
5. X := 3
6. X := X + 5
7. Y := X + 5
8. X := 2
9. if Z > 10 goto Label1
10.X := 3
11.Label2:
12.Y := X + 2
13.X := 0
14.goto Label3
15.X := 10
16.X := X + X
17.Label3:
18.Y := X + 1
```

Constant Propagation

- Problem statement:
  - Replace use of a variable X by a constant K

- Requirement:
  - **property**: on every path to the use of X, the last assignment to X is: X=K

    Same as: “is X=K at a program point?”

    At any program point where the above property holds, we can apply constant propagation.

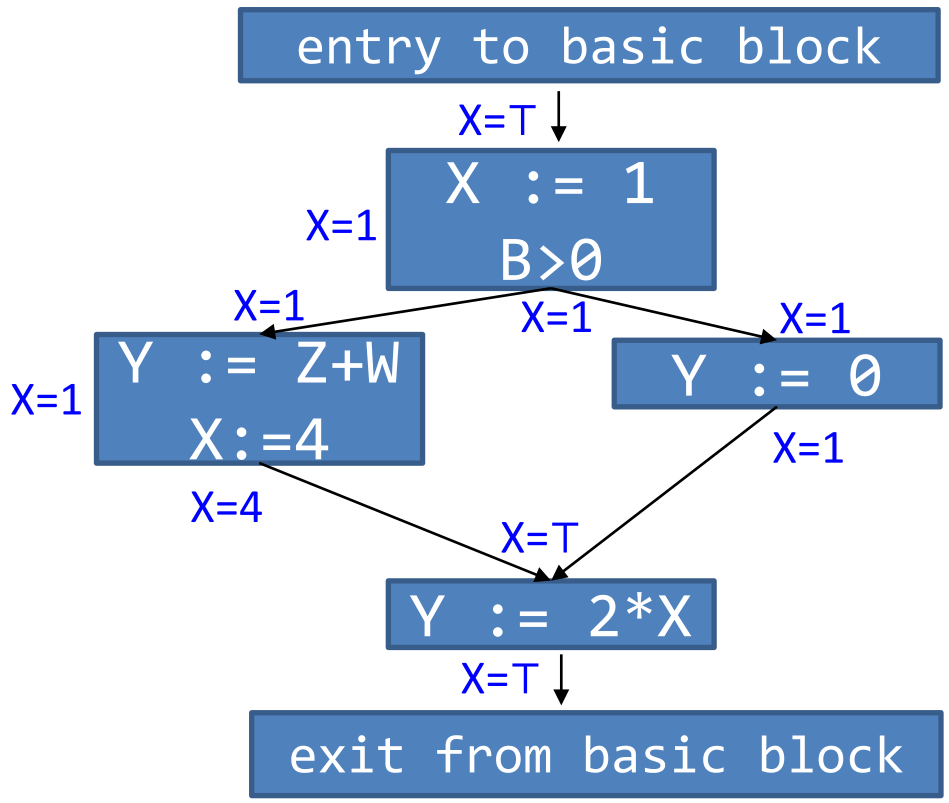
Constant Propagation

- Associate with X one of the following values:

Value	Meaning
⊥ (“bottom”)	This statement never executes
K (“constant”)	X = K
T (“top”)	X is not a constant

- Idea of symbolic execution: at all program points, determine the value of X

Constant Propagation



If X=K at some program point, we can apply constant propagation (replace the use of X with value of K at that program point)

Constant Propagation

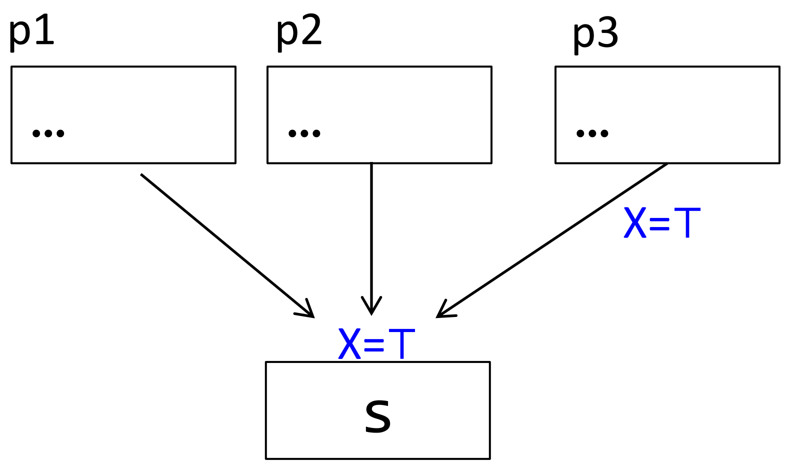
- Determining the value of X at program points:
  - Just like in Liveness Computation in a CFG, the information required for constant propagation flows from one statement to adjacent statement
  - For each statement `s`, compute the information just before and after `s`. C is the function that computes the information:

```
C(X,s,flag)
//if flag=IN, before s what is the value of X
//if flag=OUT, after s what is the value of X
```

    - **Transfer function** (pushes / transfers information from one statement to another)

Constant Propagation

- Determining the value of X at program points (Rule 1):

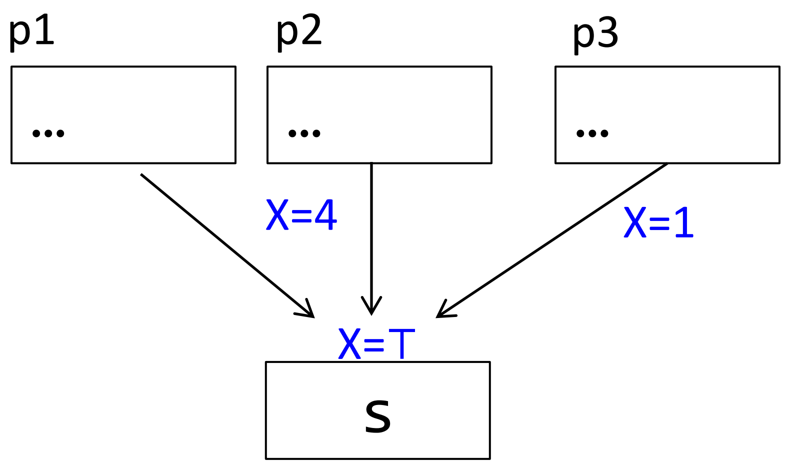


If X=T at exit of *any* of the predecessors, X=T at the entrance of S

`if C(`p_i`,s,OUT)=T`
for any i, then `C(X,s,IN)=T`

Constant Propagation

- Determining the value of X at program points (Rule 2):

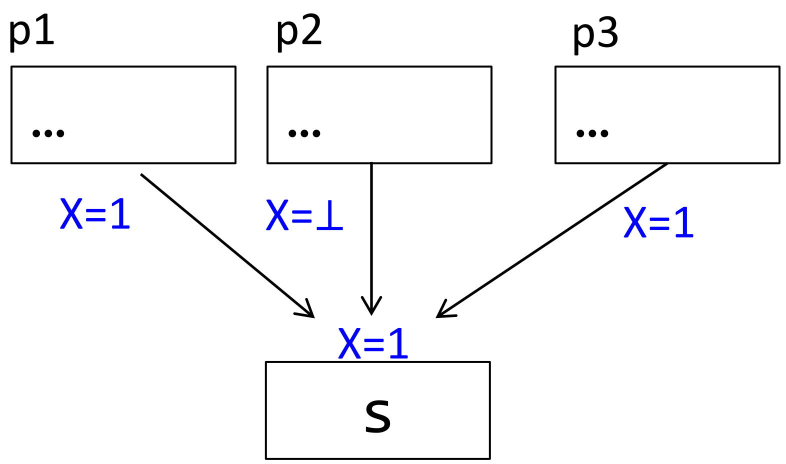


If X=K1 at one predecessor and X=K2 at another predecessor and K1 $\neq$ K2, then X=T at the entrance of S

`if C(`p_i`,s,OUT)=K1 and C(`p_j`,s,OUT)=K2 and K1` $\neq$ `K2 then C(X,s,IN)=T`

Constant Propagation

- Determining the value of X at program points (Rule 3):

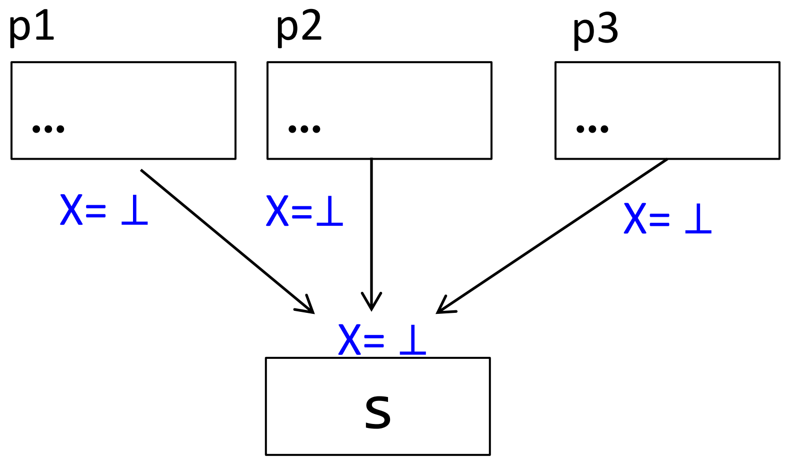


If X=K at some of the predecessors and X= ⊥ at all other predecessors, then X=K at the entrance of S

$$\texttt{if } C(p_i, s, OUT) = K \text{ or } \perp \text{ for all } i \text{ then } C(X, s, IN) = K$$

Constant Propagation

- Determining the value of X at program points (Rule 4):

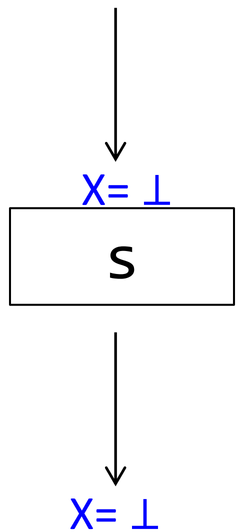


If X= ⊥ at all predecessors, then X= ⊥ at the entrance of S

`if C(`p_i`,s,OUT)=`⊥ for all `i` then `C(X,s,IN)= ⊥`

Constant Propagation

- Determining the value of X at program points (Rule 5):

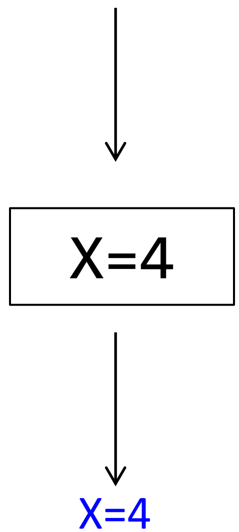


If X= ⊥ at entrance of s, then X= ⊥ at the exit of S

```
if C(X,s,IN)=⊥ then C(X,s,OUT)= ⊥
```

Constant Propagation

- Determining the value of X at program points (Rule 6):



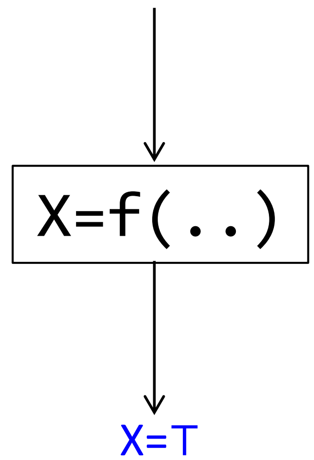
No matter what the value of X is at entrance of `s(X:=K)`, X=K at the exit of s

`C(X,s(X:=K),OUT)=K`

But previous slide said `if C(X,s,IN)=`$\perp$ then `C(X,s,OUT)=` $\perp$. *So, we give priority to this.*

Constant Propagation

- Determining the value of X at program points (Rule 7):



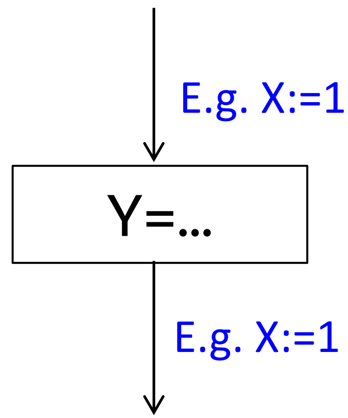
In `s`, assignment to X is any complicated expression (not a constant assignment).

`C(X,s(X:=f()),OUT)=⊤`

But earlier slide said `if C(X,s,IN)=⊥` then `C(X,s,OUT)= ⊥`. *So, we give priority to this.*

Constant Propagation

- Determining the value of X at program points (Rule 8):



Value of X remains unchanged before and after s(Y:=..) when s doesn't assign to X and $X \neq Y$

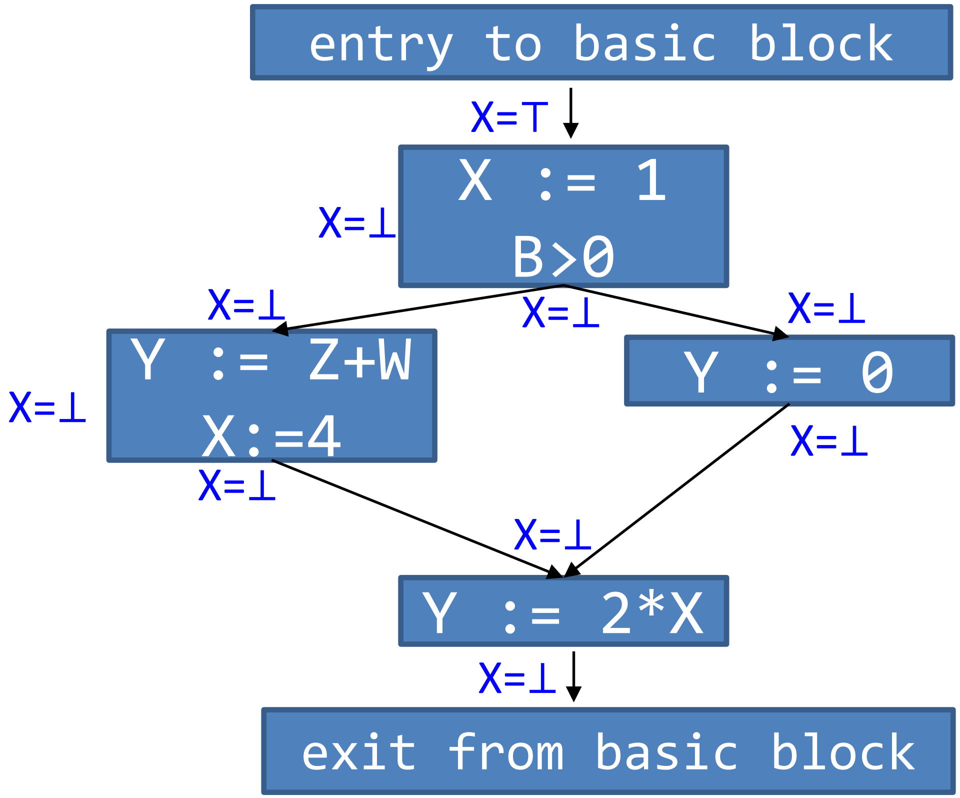
```
C(X,s(Y:=..),OUT)=C(X,s(Y:=..),IN)
```

Constant Propagation

- Putting it all together
  1. For entry s in the program, initialize $C(X,s,IN)=\top$ and initialize $C(X,s,IN)=C(X,s,IN)=\bot$ everywhere else
  2. Repeat until all program points (i.e. any s) satisfy rules 1-8
     1. Pick s in the CFG that doesn't satisfy any one of rules 1-8 and update information.

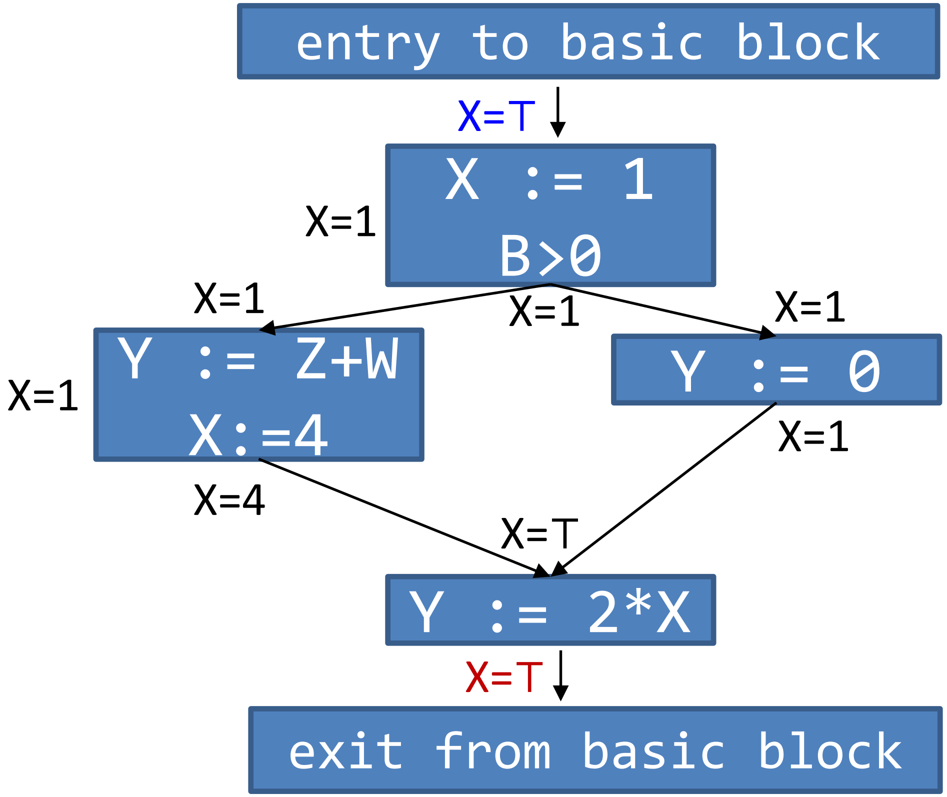
Constant Propagation

- Putting it all together (initialize)

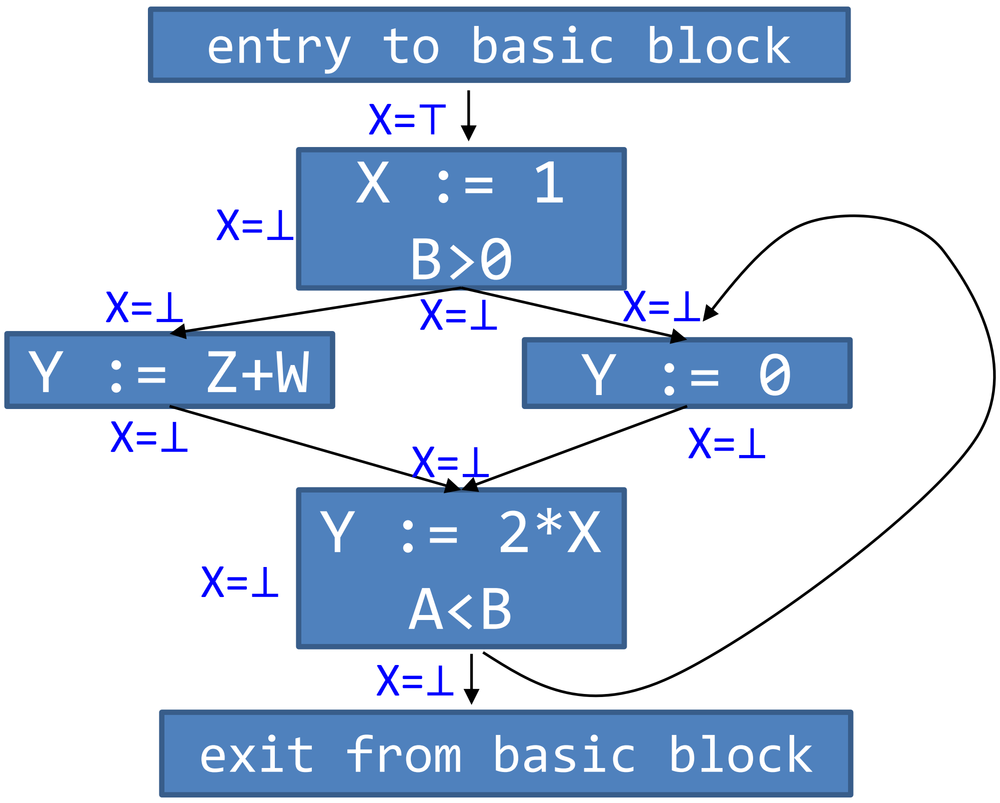


Constant Propagation

- Putting it all together (stable state – no change in information)



Constant Propagation - Loops



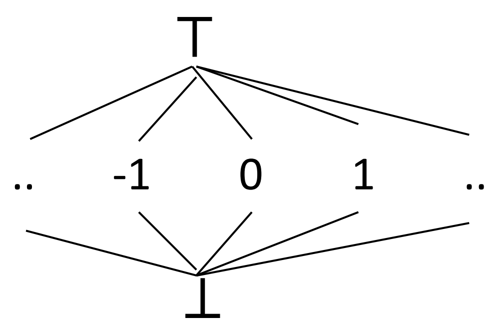
Ordering of information: Generalizing

- We have been executing with symbols $\perp$, $\top$, and K. These are called abstract values
- Order these values as:

$$\perp \; < \; K \; < \; \top$$

Can also be thought of as an ordering from least information to most information

Pictorially:



Ordering of information: Generalizing

- Least Upper Bound (`lub`) : smallest element (abstract value) that is greater than or equal to values in the input
  - E.g. $\mathrm{lub}(\perp, \perp) = \perp$, $\mathrm{lub}(\top, \perp) = \top$, $\mathrm{lub}(-1, 1) = \top$, $\mathrm{lub}(1\ \perp) = ?$
  - Rewriting rules 1-4: `C(X,s,IN)=lub{C(`$\texttt{p}_\texttt{i}$`,s,OUT) for all predecessors i)}`
  - Also called as join operator. Written as: A $\sqcup$ B

Ordering of information: Generalizing

- Recall that in determining information at all program points:

  "2. Repeat until all program points (i.e. any s) satisfy rules 1-8

  - Pick s in the CFG that doesn't satisfy any one of rules 1-8 and update information. "

  – How do we know that this terminates?

- lub ensures that the information changes from lower value to higher value
  – In the constant propagation algorithm:
    - $\perp$ can change to constant and then to $\top$
    - $\perp$ can change to $\top$
    - `C(X, s, flag)` can change at most twice

Constant Propagation

- Exercise: what is the complexity of our constant propagation algorithm?