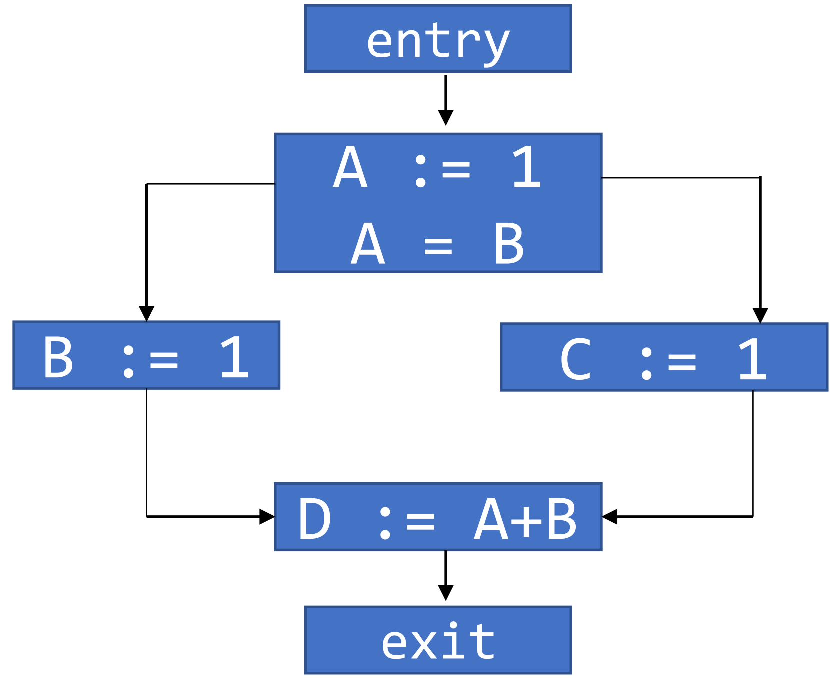
CS323: Compilers
Spring 2023

Week 13: Dataflow Analysis (liveness (recap), Constant Propagation..)

Recap: Liveness

- Variables are live if there exists *some path* leading to its use
- Start from exit block and proceed *backwards* against the control flow to compute



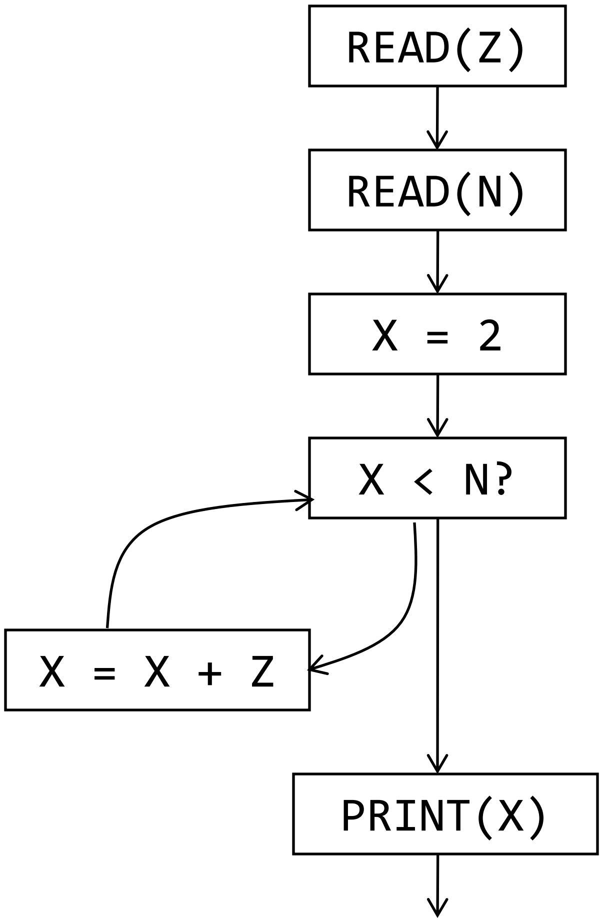
$$\mathtt{LiveOut(b)} = \bigcup_{i \in Succ(b)} LiveIn(i)$$

$$\mathtt{LiveIn(b)} = \mathtt{LiveUse(b)} \cup (\mathtt{LiveOut(b)} - \mathtt{Def(b)})$$

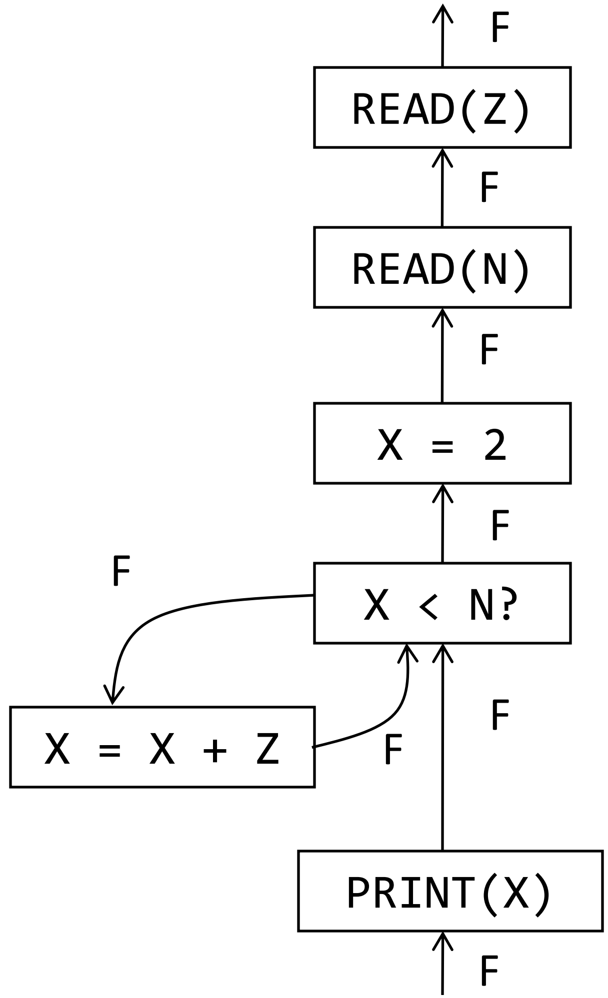
```
//set that contains all variables
used by block b
```

```
//set that contains all
variables defined by block b
```

Recap: Liveness

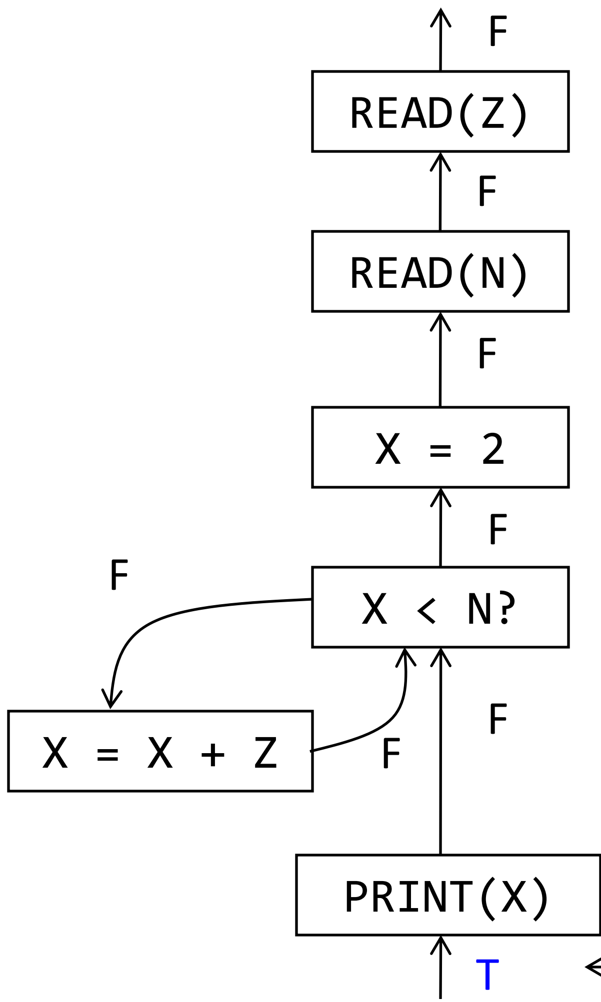


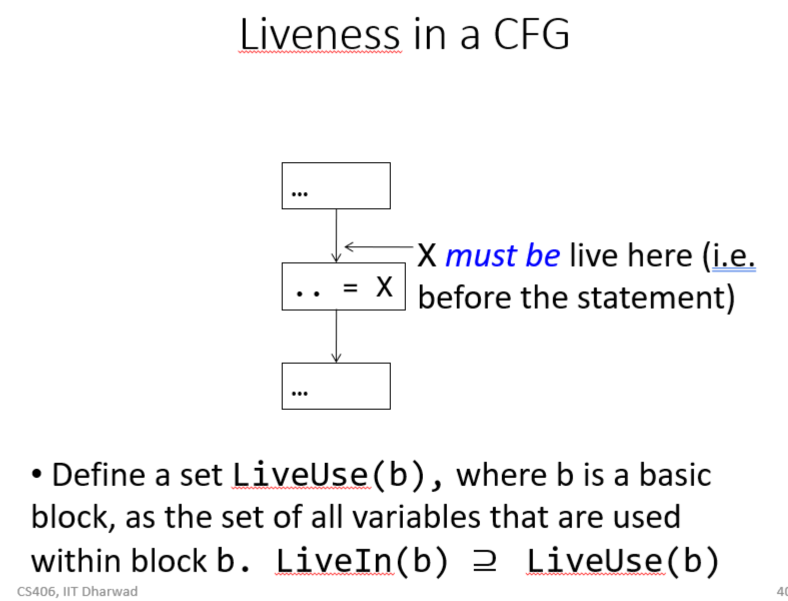
Original CFG



CFG with edges reversed (and initialized) for backwards analysis: is X live? (F=`false`, T=`true`)

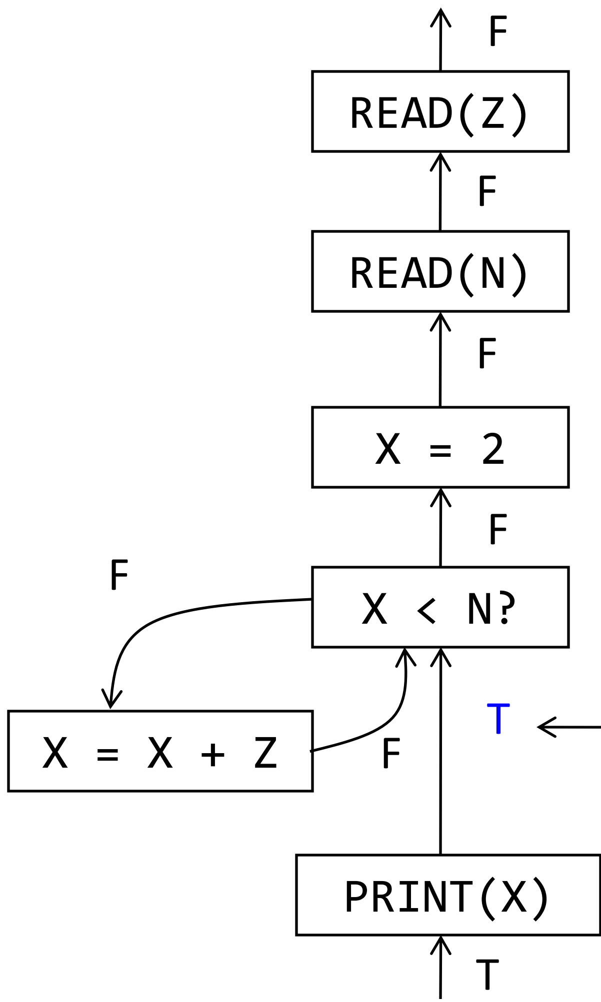
Recap: Liveness

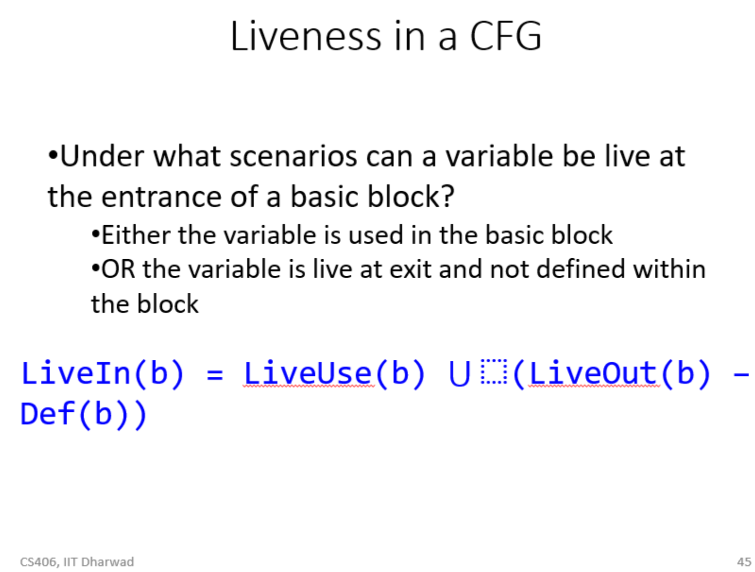




X must be live here
(refer week11 slide)

Recap: Liveness




Liveness in a CFG

- Under what scenarios can a variable be live at the entrance of a basic block?
  - Either the variable is used in the basic block
  - OR the variable is live at exit and not defined within the block

```
LiveIn(b) = LiveUse(b) ∪ (LiveOut(b) – Def(b))
```


X must be live here
(refer week11 slide)

Recap: Liveness

X must be live here
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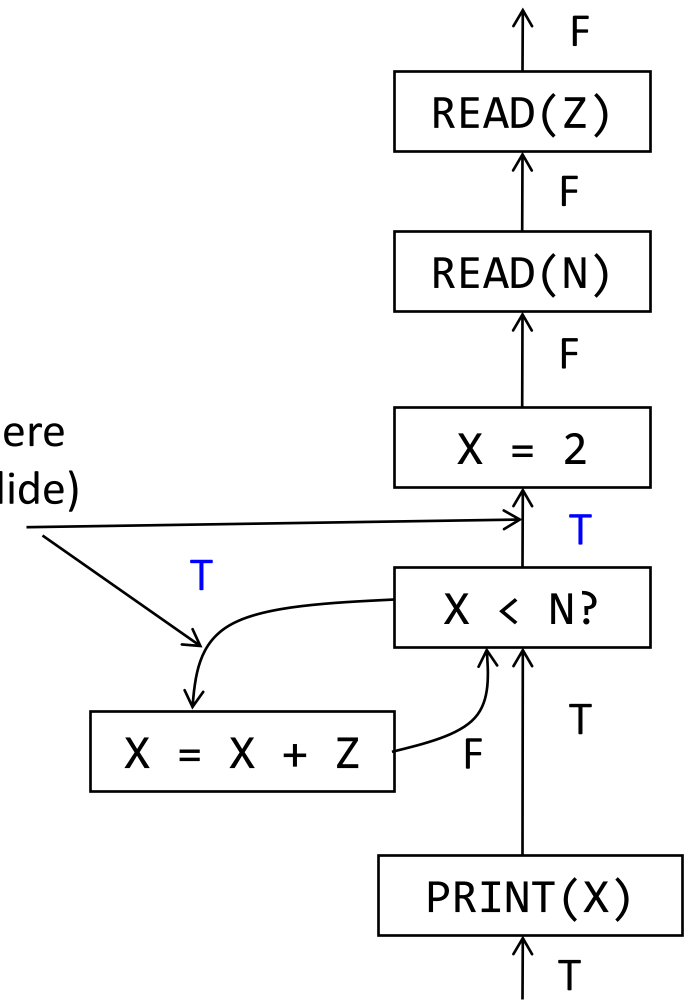

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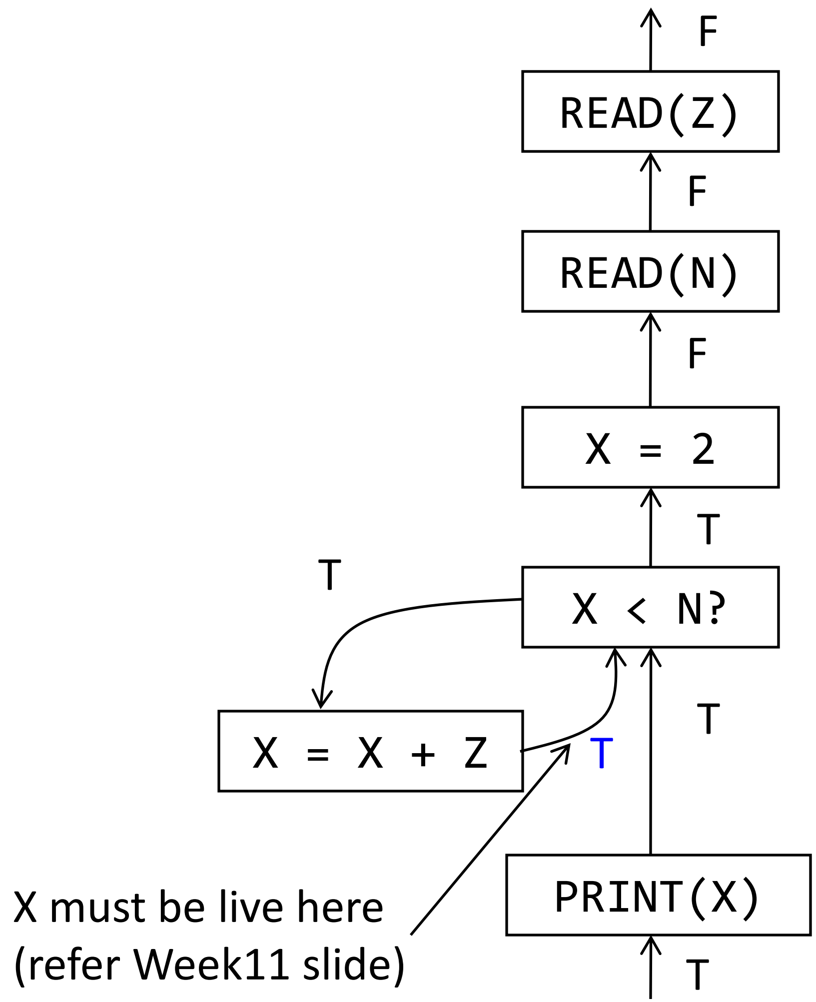
Recap: Liveness



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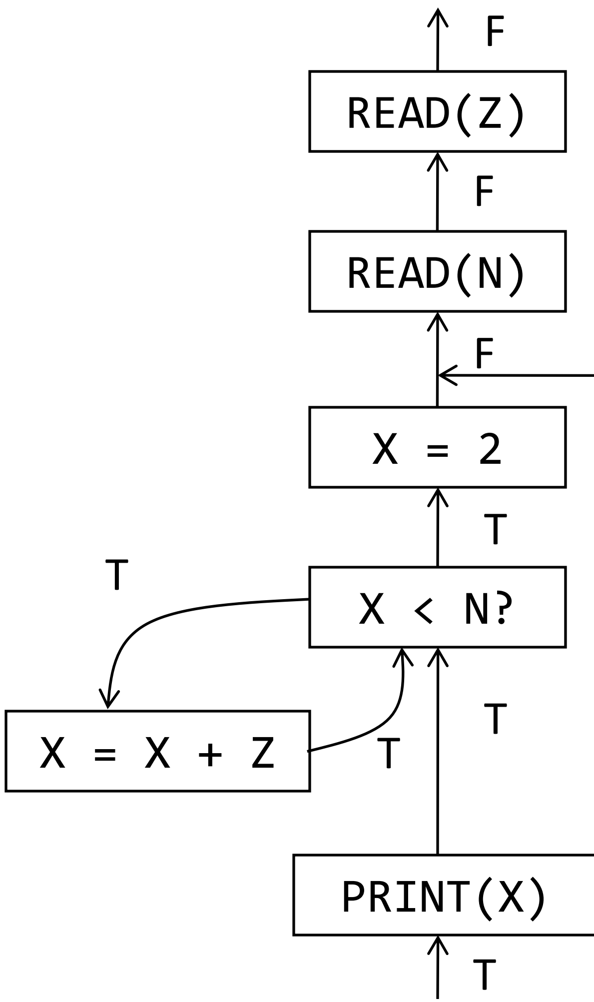
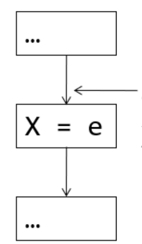


X dead here (refer Week11 slide).
No change in information.

Liveness in a CFG



Given that e does not use X, X is ***definitely dead*** here (i.e. before the statement).

- Define a set `LiveIn(b)`, where b is a basic block, as: the set of all variables live at the entrance of a basic block

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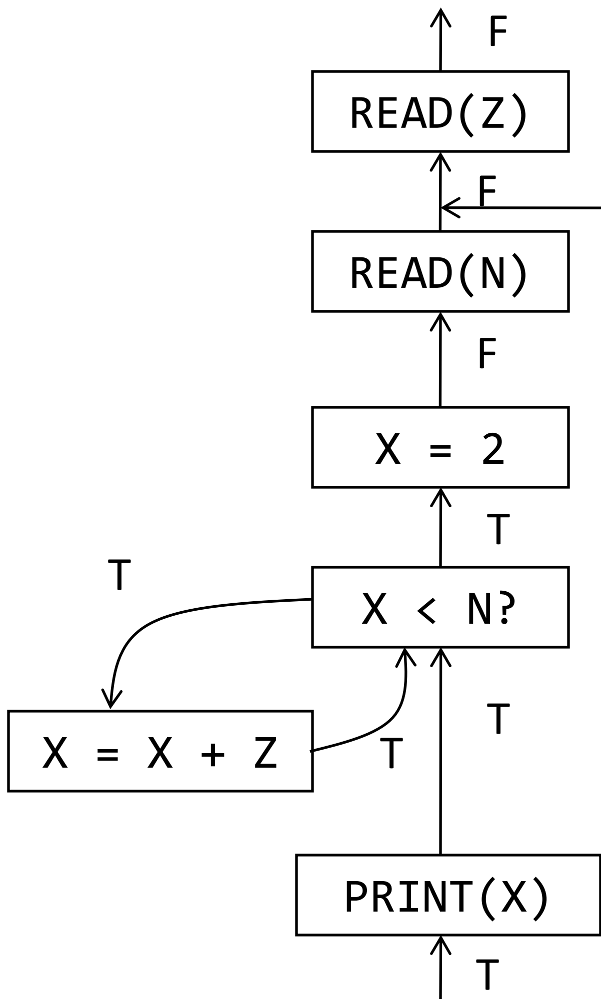
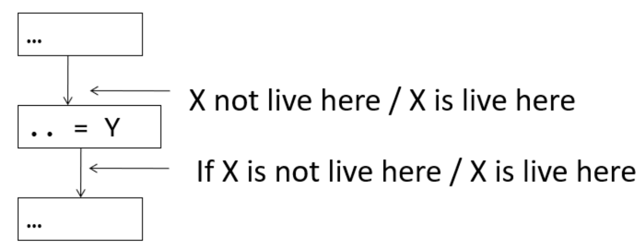
Recap: Liveness



X dead here (refer Week11 slide).
No change in information.

Liveness in a CFG - Observation



- If a node neither uses nor defines X, the liveness property remains the same before and after executing the node

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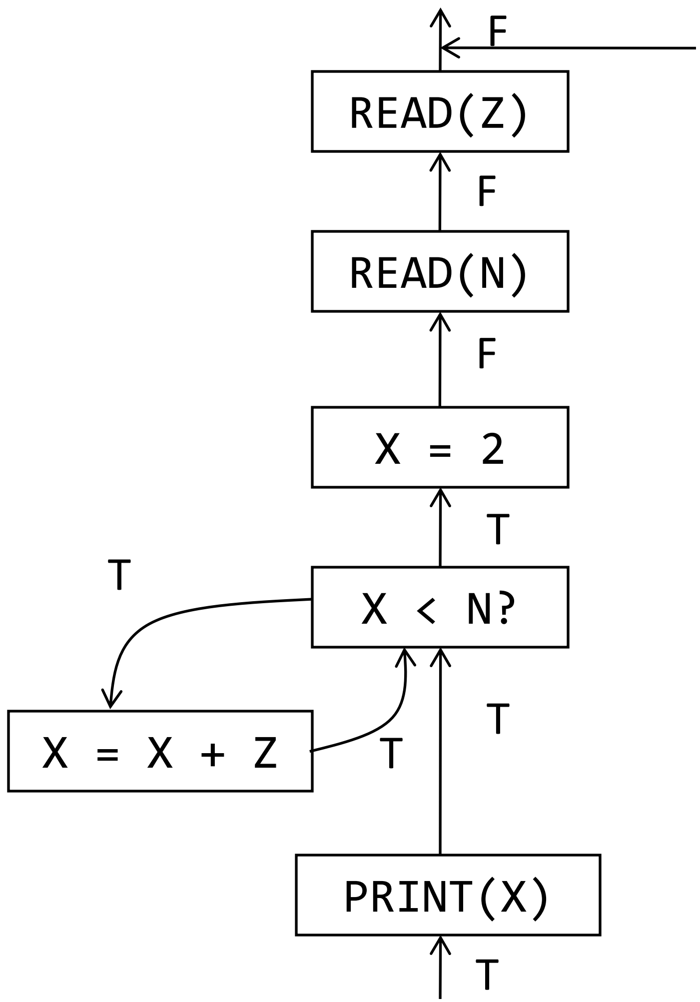
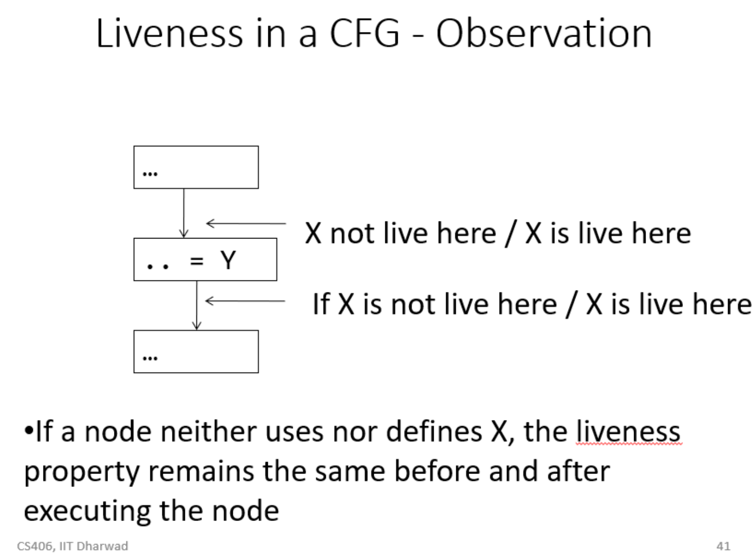
Recap: Liveness



X dead here (refer Week11 slide).
No change in information.



Exercise: Repeat for Z and N

Constant Propagation

- Bigger problem size:
  - Which lines using X could be replaced with a constant value? (apply only constant propagation)
  - How can we automate to find an answer to this question?

```
1. X := 2
2. Label1:
3. Y := X + 1
4. if Z > 8 goto Label2
5. X := 3
6. X := X + 5
7. Y := X + 5
8. X := 2
9. if Z > 10 goto Label1
10.X := 3
11.Label2:
12.Y := X + 2
13.X := 0
14.goto Label3
15.X := 10
16.X := X + X
17.Label3:
18.Y := X + 1
```

Constant Propagation

- Problem statement:
  - Replace use of a variable X by a constant K
- Requirement:
  - **property**: on every path to the use of X, the last assignment to X is: X=K

    Same as: “is X=K at a program point?”

    At any program point where the above property holds, we can apply constant propagation.

How can we find constants?

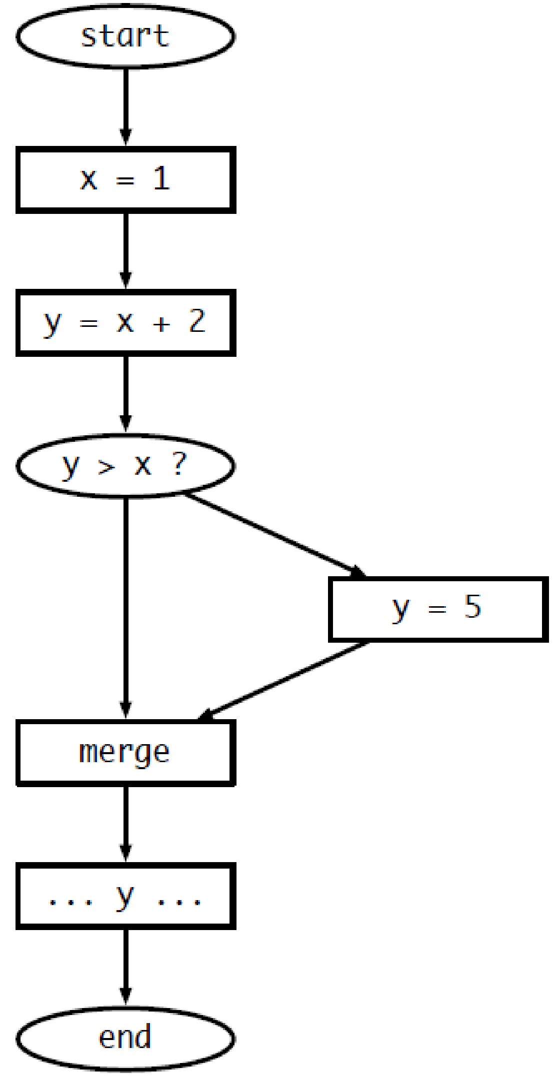
- Ideal: run program and see which variables are constant
  - Problem: variables can be constant with some inputs, not others – need an approach that works for all inputs!
  - Problem: program can run forever (infinite loops?) – need an approach that we know will finish
- Idea: run program *symbolically*
  - Essentially, keep track of whether a variable is constant or not constant (but nothing else)

Overview of algorithm

- Build control flow graph
  - We'll use statement-level CFG (with merge nodes) for this
- Perform symbolic evaluation
  - Keep track of whether variables are constant or not
- Replace constant-valued variable uses with their values, try to simplify expressions and control flow

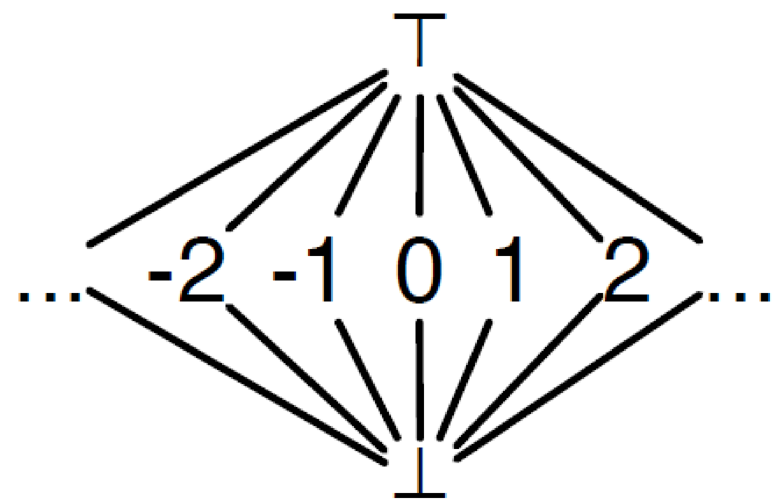
Build CFG

```
x = 1;
y = x + 2;
if (y > x) then y = 5;
... y ...
```



Symbolic evaluation

- Idea: replace each value with a symbol
  - constant (specify which), no information, definitely not constant
- Can organize these possible values in a *lattice*
  - Set of possible values, arranged from least information to most information



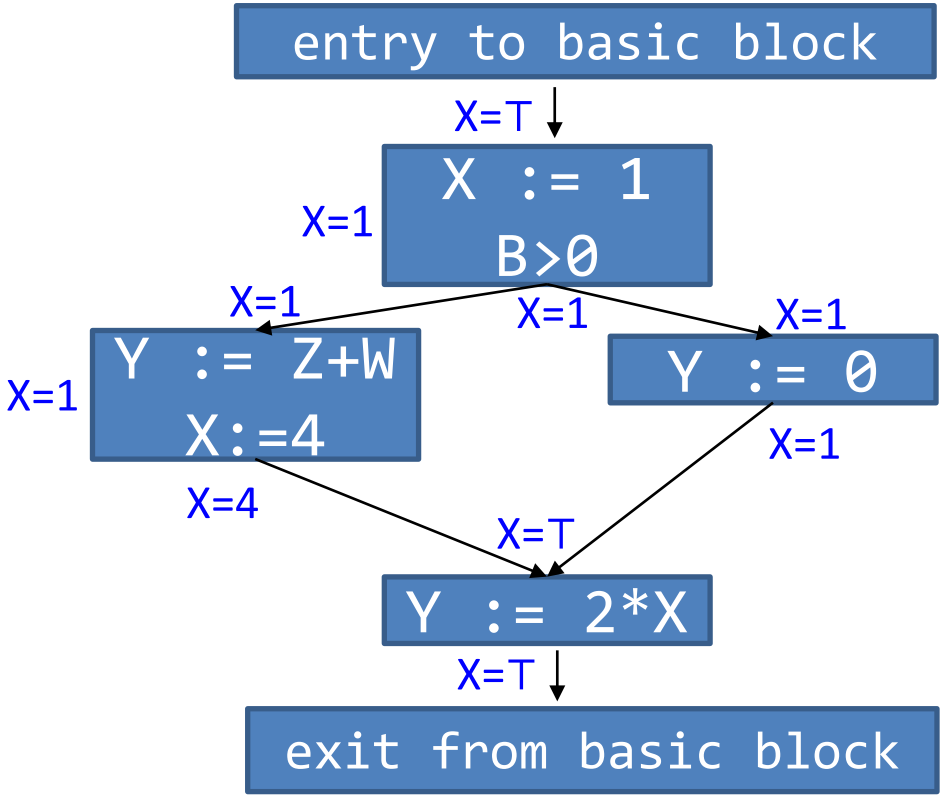
Symbolic Evaluation

- Associate with X one of the following values:

Value	Meaning
⊥ (“bottom”)	This statement never executes
K (“constant”)	X = K
T (“top”)	X is not a constant

- Idea of symbolic execution: at all program points, determine the value of X

Constant Propagation



If X=K at some program point, we can apply constant propagation (replace the use of X with value of K at that program point)

Constant Propagation

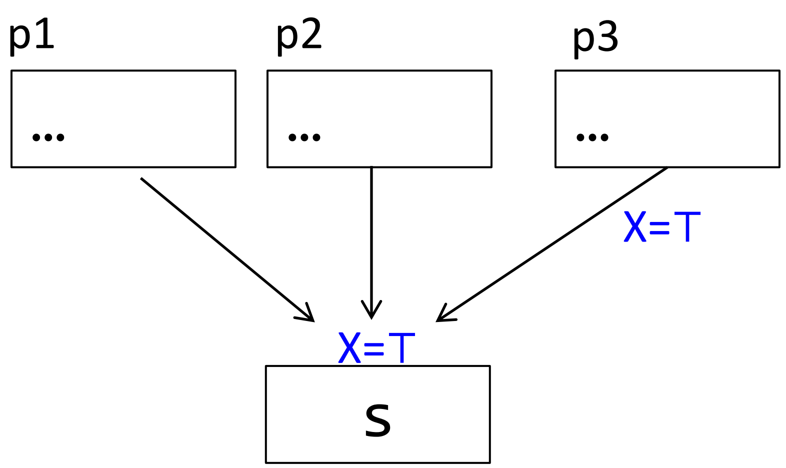
- Determining the value of X at program points:
  - Just like in Liveness Computation in a CFG, the information required for constant propagation flows from one statement to adjacent statement
  - For each statement `s`, compute the information just before and after `s`. C is the function that computes the information:

```
C(X,s,flag)
//if flag=IN, before s what is the value of X
//if flag=OUT, after s what is the value of X
```

    - **Transfer function** (pushes / transfers information from one statement to another)

Constant Propagation

- Determining the value of X at program points (Rule 1):

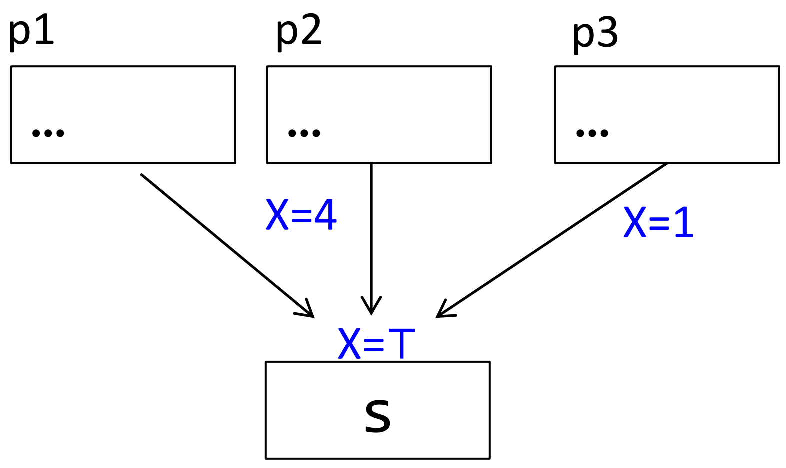


If X=T at exit of *any* of the predecessors, X=T at the entrance of S

`if C(`p_i`,s,OUT)=T`
for any i, then `C(X,s,IN)=T`

Constant Propagation

- Determining the value of X at program points (Rule 2):

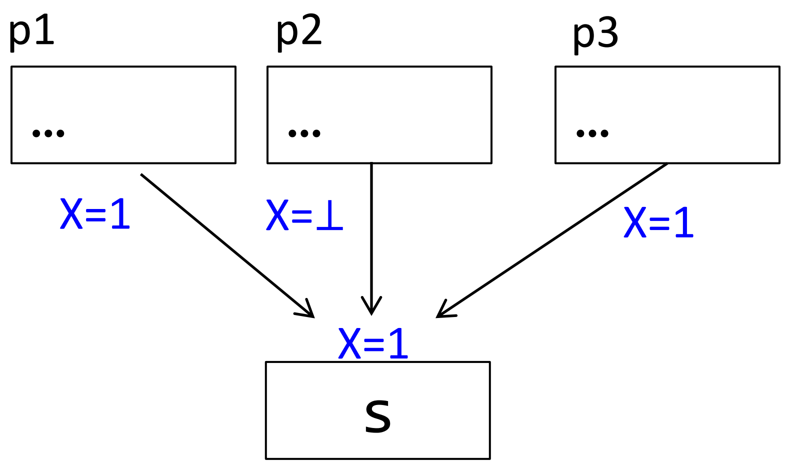


If X=K1 at one predecessor and X=K2 at another predecessor and K1 $\neq$ K2, then X=T at the entrance of S

`if C(`p_i`,s,OUT)=K1 and C(`p_j`,s,OUT)=K2 and K1` $\neq$ `K2 then C(X,s,IN)=T`

Constant Propagation

- Determining the value of X at program points (Rule 3):

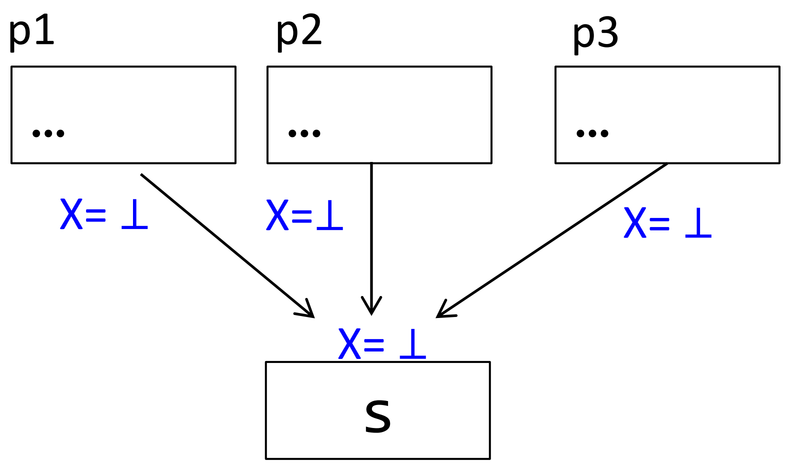


If X=K at some of the predecessors and X= ⊥ at all other predecessors, then X=K at the entrance of S

if $C(p_i,s,OUT)=K$ or ⊥ for all i then $C(X,s,IN)= K$

Constant Propagation

- Determining the value of X at program points (Rule 4):

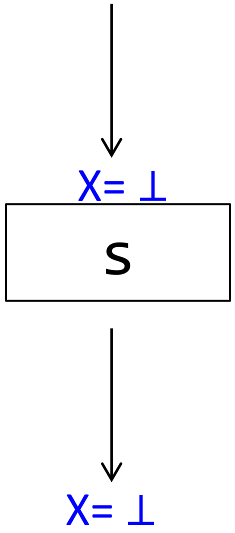


If X= ⊥ at all predecessors, then X= ⊥ at the entrance of S

`if C(`p_i`,s,OUT)=`⊥ for all `i` then `C(X,s,IN)= ⊥`

Constant Propagation

- Determining the value of X at program points (Rule 5):

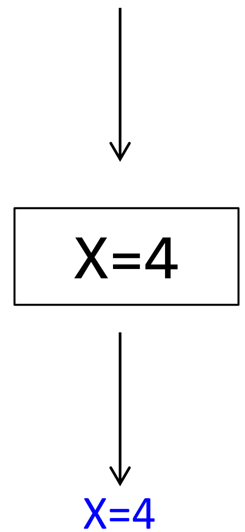


If X= ⊥ at entrance of s, then X= ⊥ at the exit of S

if C(X,s,IN)=⊥ then C(X,s,OUT)= ⊥

Constant Propagation

- Determining the value of X at program points (Rule 6):



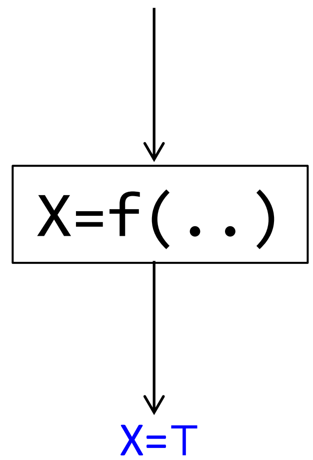
No matter what the value of X is at entrance of `s(X:=K)`, X=K at the exit of s

`C(X,s(X:=K),OUT)=K`

But previous slide said `if C(X,s,IN)=`⊥ then `C(X,s,OUT)=` ⊥. *So, we give priority to this.*

Constant Propagation

- Determining the value of X at program points (Rule 7):



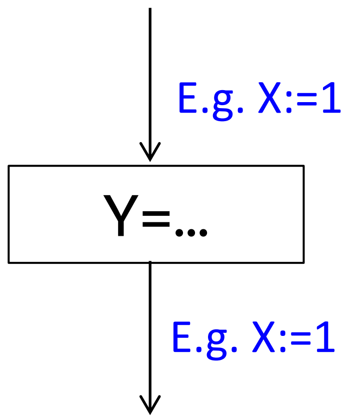
In `s`, assignment to X is any complicated expression (not a constant assignment).

`C(X,s(X:=f()),OUT)=T`

But earlier slide said `if C(X,s,IN)=⊥` then `C(X,s,OUT)= ⊥`. *So, we give priority to this.*

Constant Propagation

- Determining the value of X at program points (Rule 8):



Value of X remains unchanged before and after s(Y:=..) when s doesn't assign to X and X $\neq$ Y

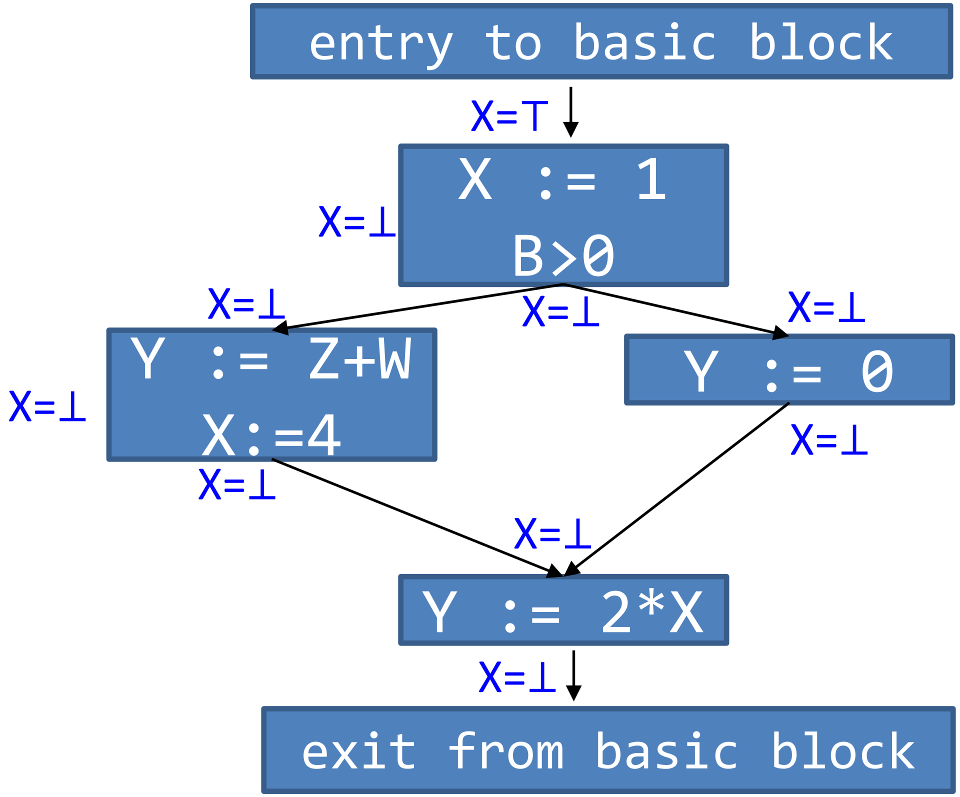
```
C(X,s(Y:=..),OUT)=C(X,s(Y:=..),IN)
```

Constant Propagation

- Putting it all together
  1. For entry s in the program, initialize `C(X,s,IN)=⊤` and initialize `C(X,s,IN)=C(X,s,OUT)=⊥` everywhere else
  2. Repeat until all program points (i.e. any s) satisfy rules 1-8
     1. Pick s in the CFG that doesn't satisfy any one of rules 1-8 and update information.

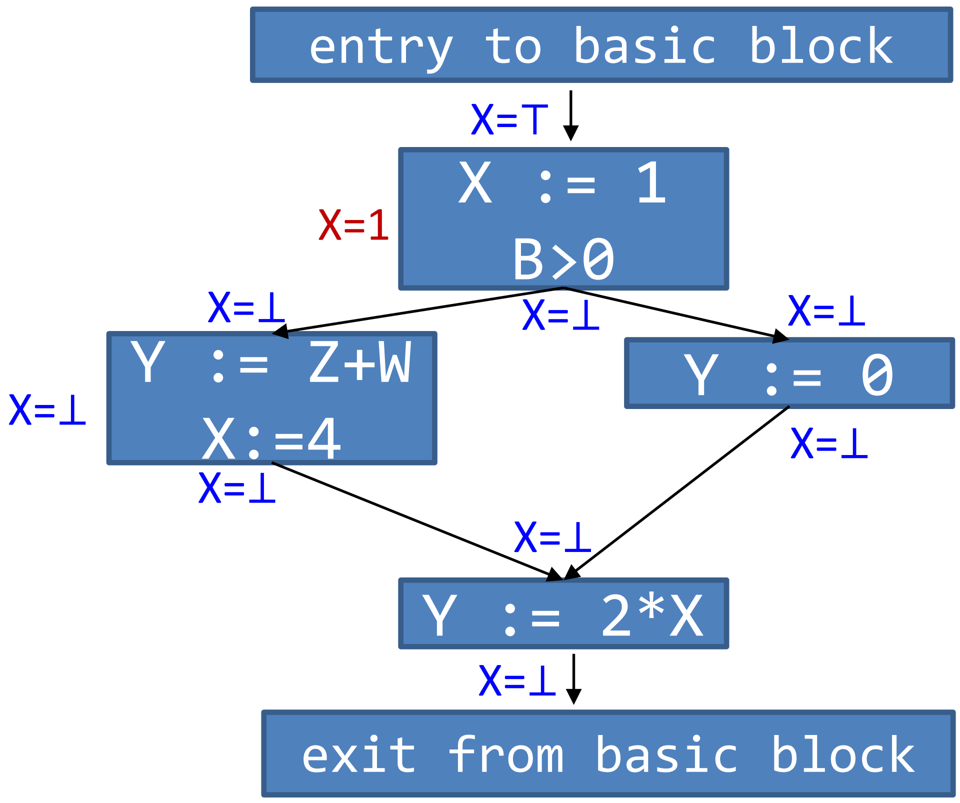
Constant Propagation

- Putting it all together



Constant Propagation

- Putting it all together



Constant Propagation

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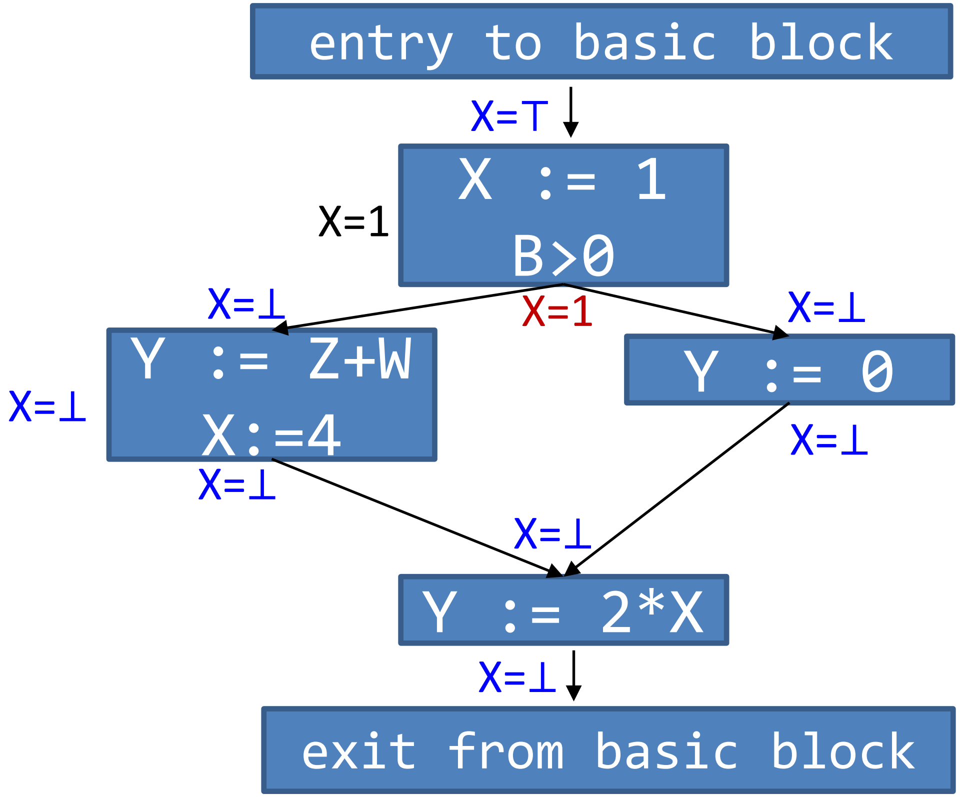

Constant Propagation

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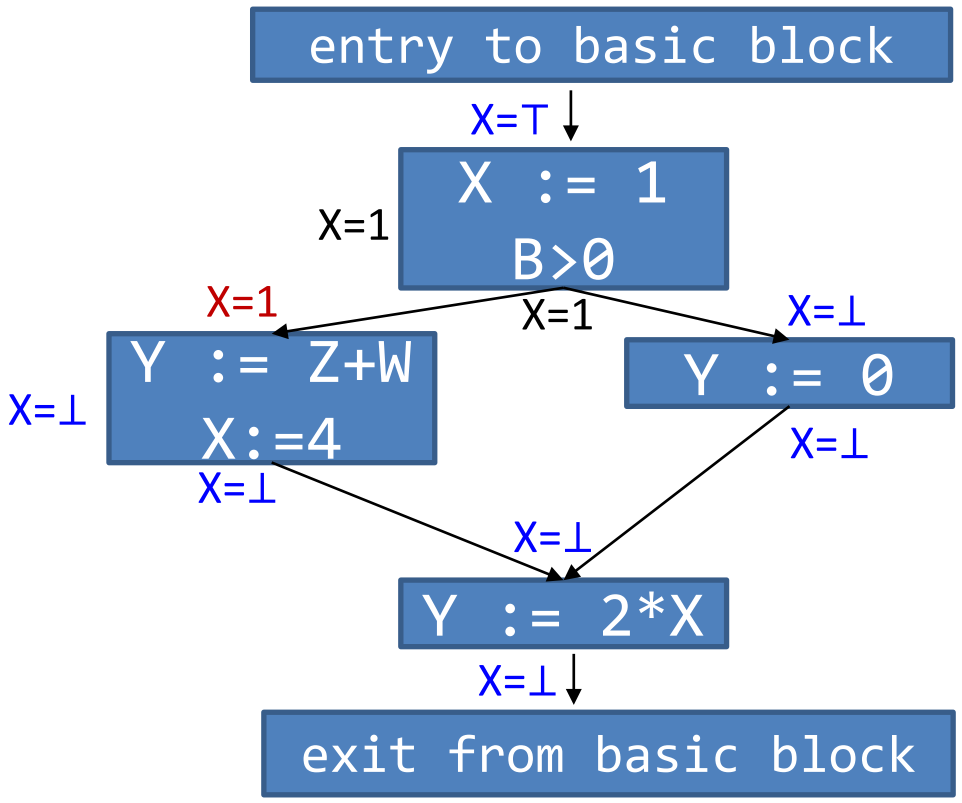

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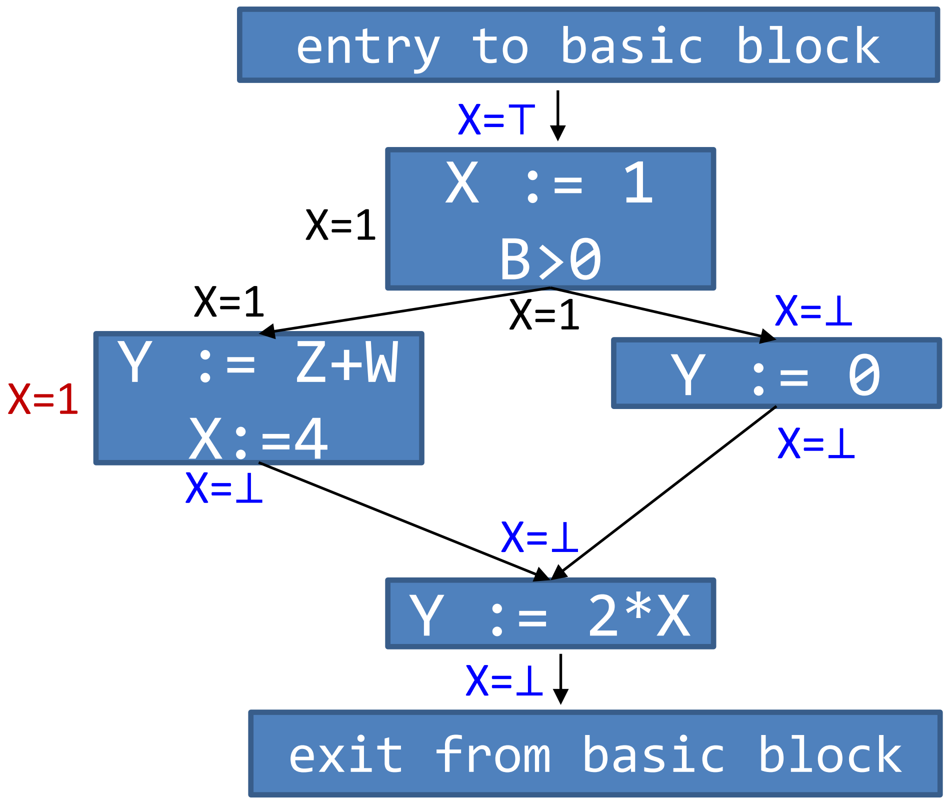

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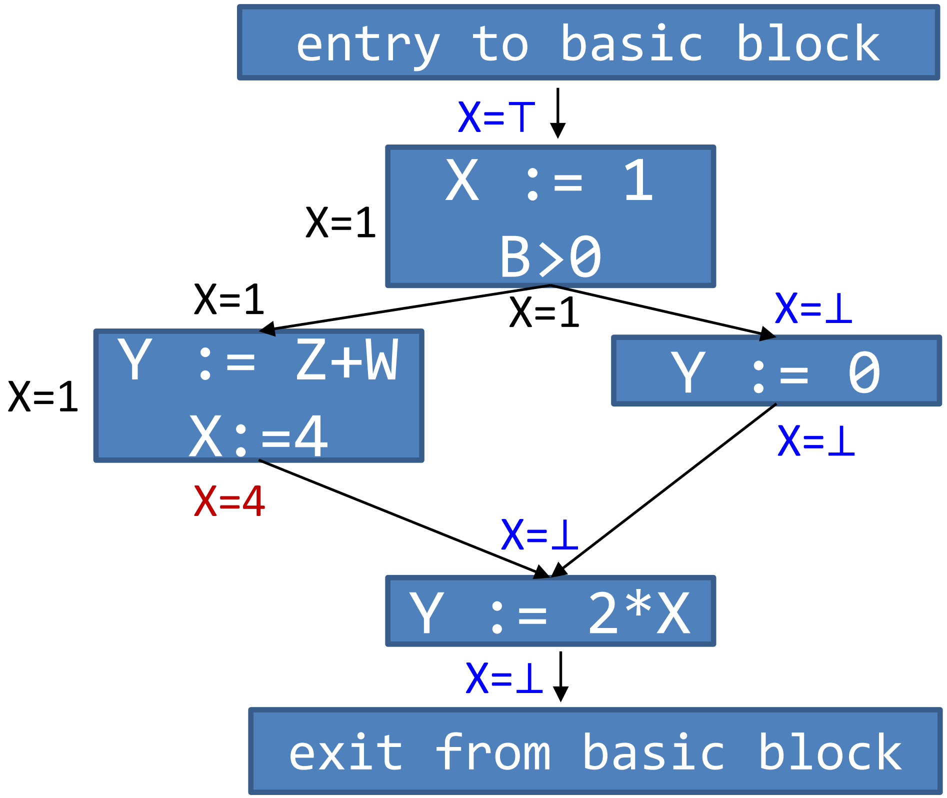

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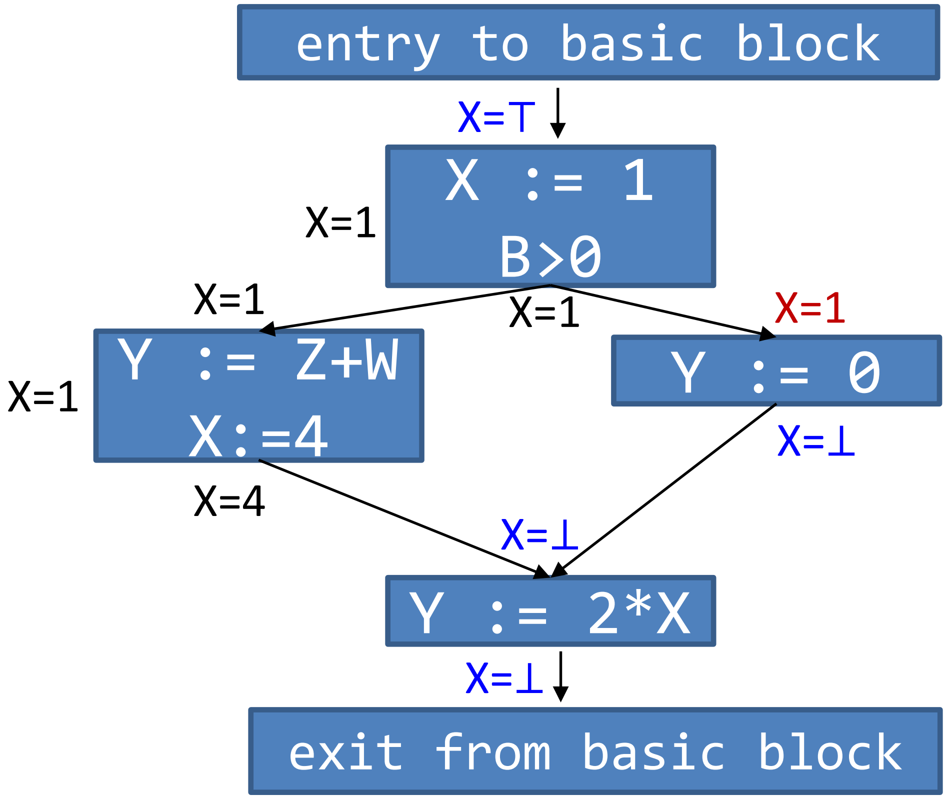

Constant Propagation

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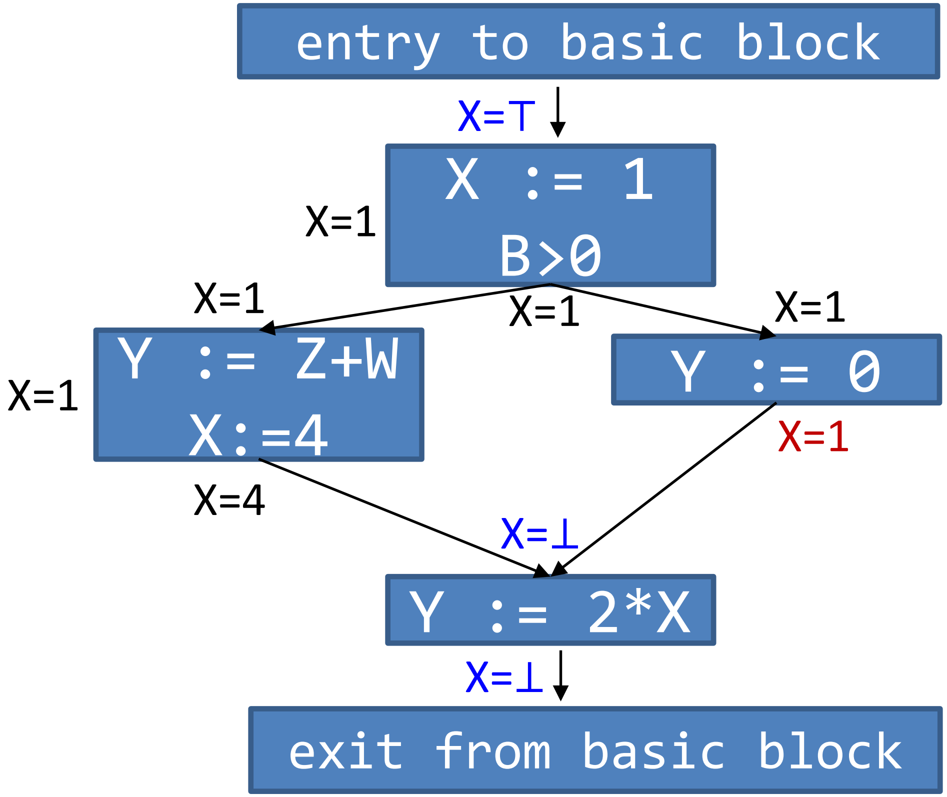

Constant Propagation

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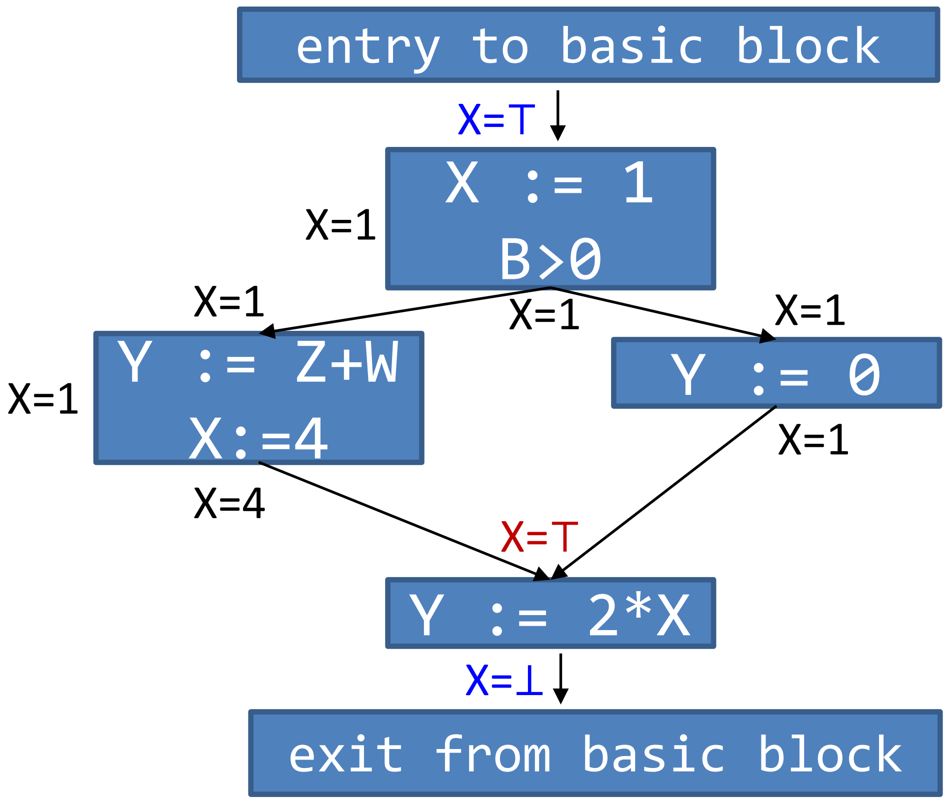
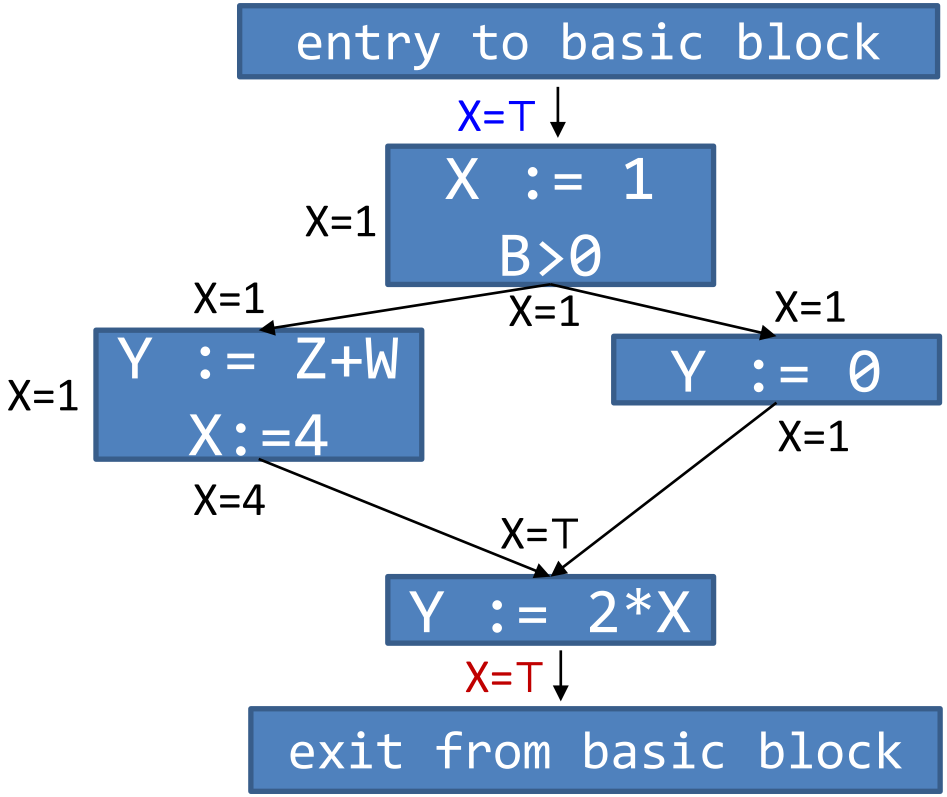
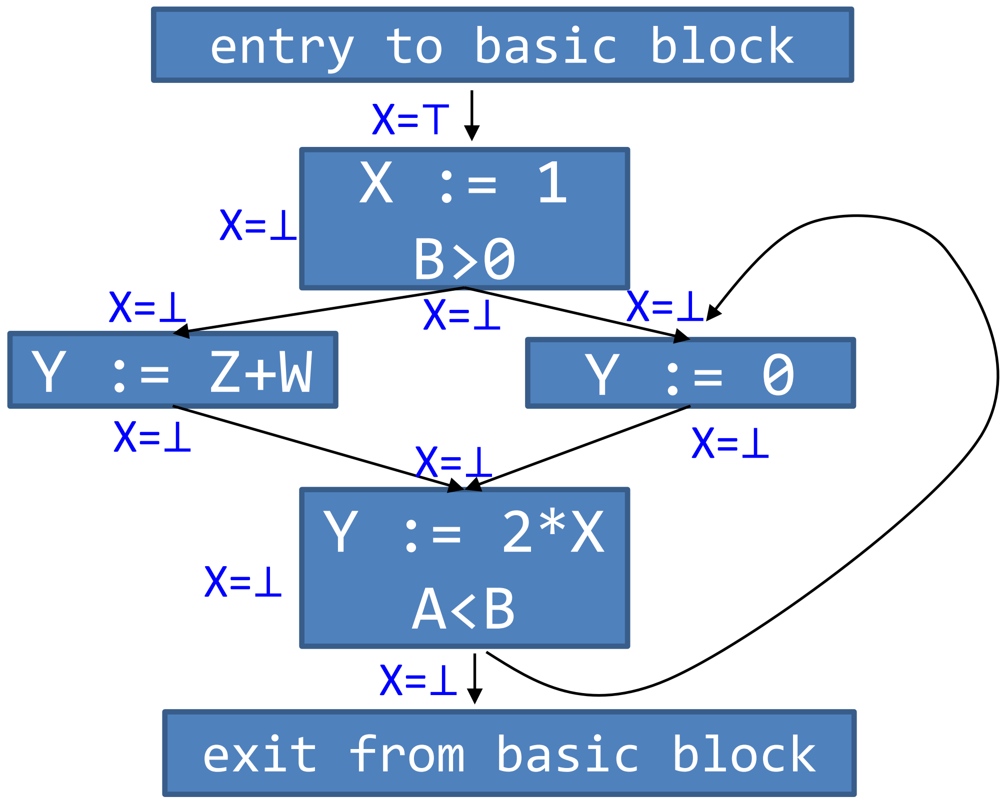


Constant Propagation

- Putting it all together



Constant Propagation - Loops



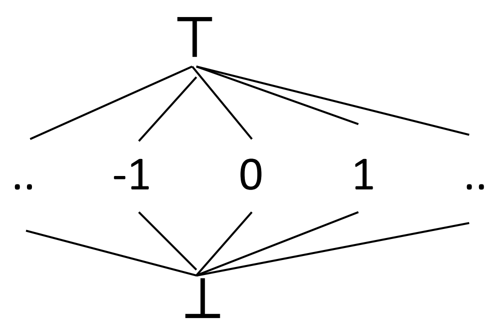
Ordering of information: Generalizing

- We have been executing with symbols $\perp$, $\top$, and K. These are called *abstract values*
- Order these values as:

$$\perp < K < \top$$

Can also be thought of as an ordering from least information to most information

Pictorially:



Ordering of information: Generalizing

- Least Upper Bound (`lub`) : smallest element (abstract value) that is greater than or equal to values in the input
  - E.g. $\mathrm{lub}(\bot, \bot) = \bot$, $\mathrm{lub}(\top, \bot) = \top$, $\mathrm{lub}(-1, 1) = \top$, $\mathrm{lub}(1\ \ \bot) = ?$
  - Rewriting rules 1-4: `C(X,s,IN)=lub{C(`p_i`,s,OUT) for all predecessors i)}`
  - Also called as join operator. Written as: A $\sqcup$ B

Ordering of information: Generalizing

- Recall that in determining information at all program points:

  “2. Repeat until all program points (i.e. any s) satisfy rules 1-8

  - Pick s in the CFG that doesn’t satisfy any one of rules 1-8 and update information. “

  – How do we know that this terminates?
    - lub ensures that the information changes from lower value to higher value
    - In the constant propagation algorithm:
      – $\perp$ can change to constant and then to $\top$
      – $\perp$ can change to $\top$
      – `C(X, s, flag)` can change at most twice

Constant Propagation

- Exercise: what is the complexity of our constant propagation algorithm?

= NumS* 4 (NumS = number of statements in the program).

- Per program point, we evaluate the C function.

- The C function changes value at most two times (initialized to ⊥ first and then could change to K and then to ⊤).

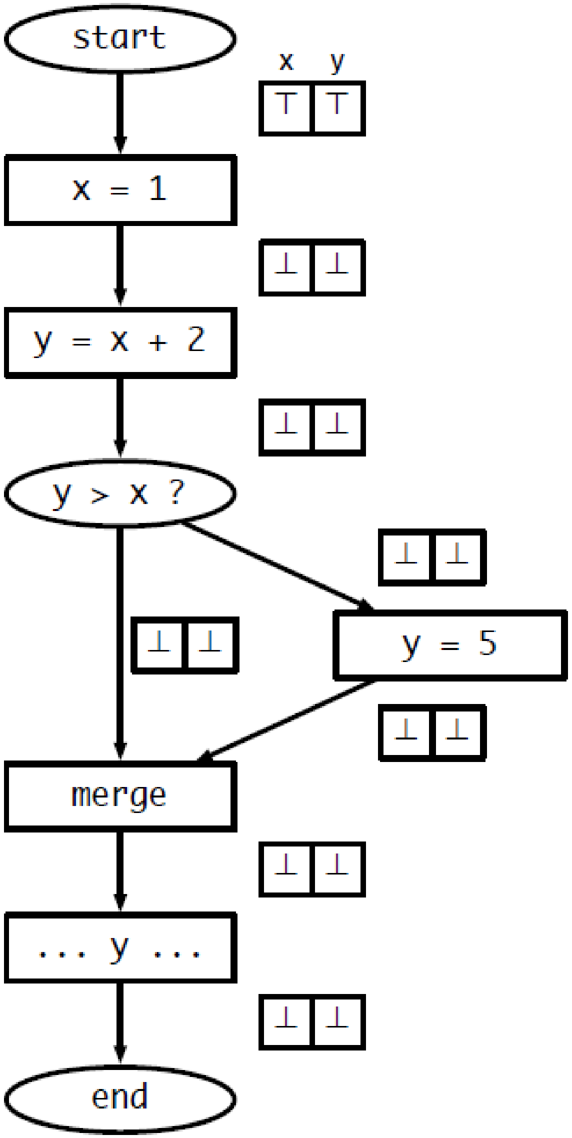
- There are two program points (entry/IN and exit/OUT) for every statement.

This is the complexity of the analysis per variable

How do we do the analysis considering all variables that exist in the program?

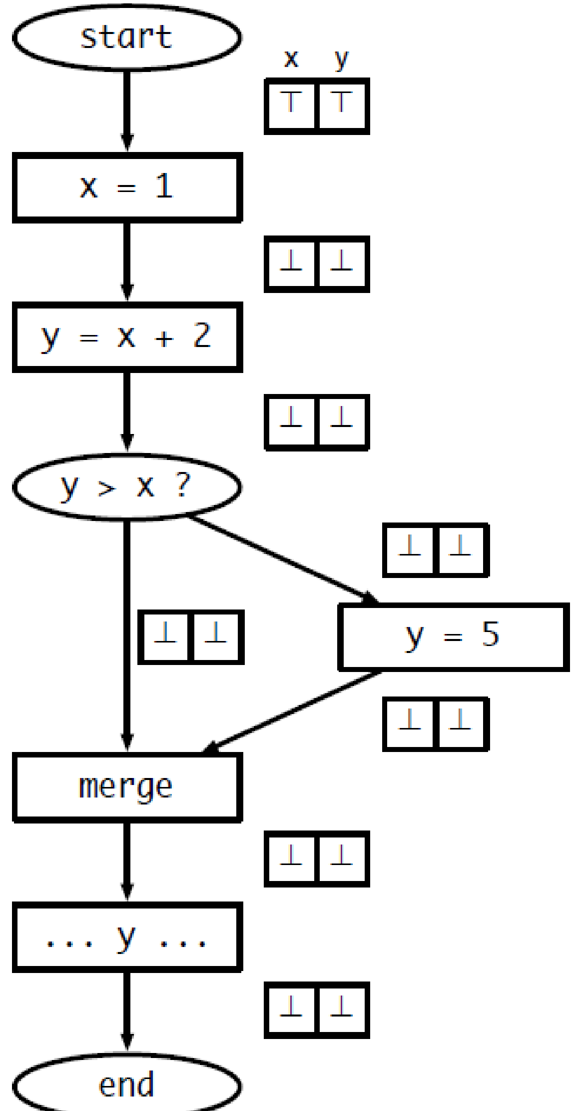
Constant Propagation (Multiple Variables)

- Keep track of the symbolic value of a variable at every program point (on every CFG edge)
  - State vector V
- What should our initial value be?
  - Starting state vector is all $\top$
    - Can't make any assumptions about inputs – must assume not constant
  - Everything else starts as $\bot$, since we have no information about the variable at that point



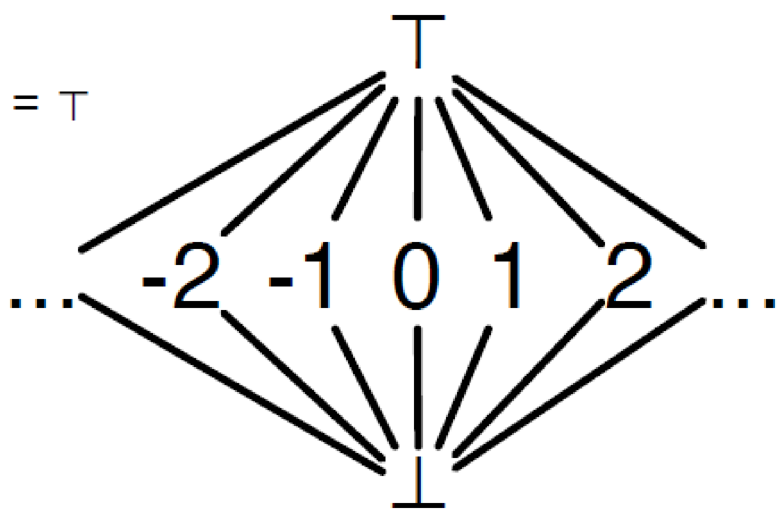
Constant Propagation (Multiple Variables)

- For each statement $\texttt{t} = \texttt{e}$ evaluate $\texttt{e}$ using V_{in}, update value for $\texttt{t}$ and propagate state vector to next statement
- What about switches?
  - If $\texttt{e}$ is true or false, propagate V_{in} to appropriate branch
  - What if we can't tell?
    - Propagate V_{in} to both branches, and symbolically execute both sides
- What do we do at merges?



Handling merges

- Have two different V_{in}s coming from two different paths
- Goal: want new value for V_{in} to be *safe* (shouldn't generate wrong information), and we don't know which path we actually took
- Consider a single variable. Several situations:
  - $V_1 = \bot, V_2 = * \rightarrow V_{out} = *$
  - $V_1 =$ constant $x, V_2 = x \rightarrow V_{out} = x$
  - $V_1 =$ constant $x, V_2 =$ constant $y \rightarrow V_{out} = \top$
  - $V_1 = \top, V_2 = * \rightarrow V_{out} = \top$
- Generalization:
  - $V_{out} = V_1 \sqcup V_2$

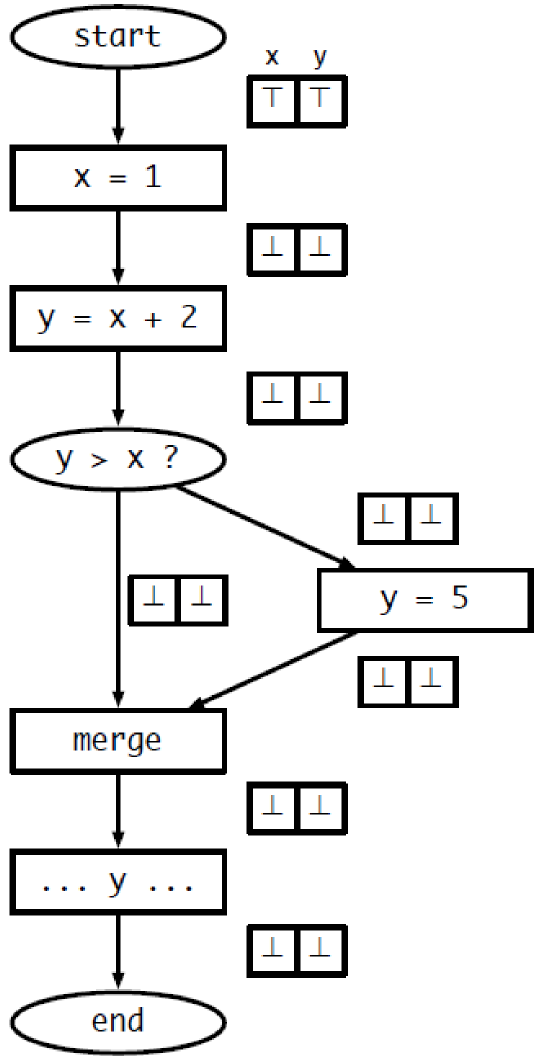


Result: worklist algorithm

- Associate state vector with each edge of CFG, initialize all values to $\perp$, worklist has just start edge
  - While worklist not empty, do:

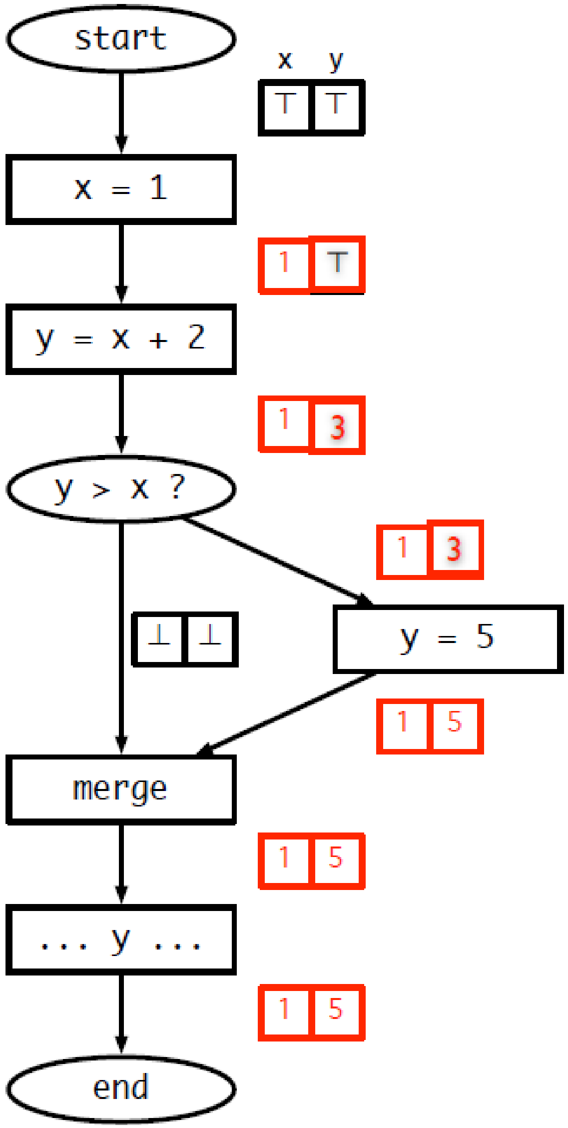
```
Process the next edge from worklist

Symbolically evaluate target node of edge using input state vector

If target node is assignment (x = e), propagate Vin[eval(e)/x] to
output edge

If target node is branch (e?)

 If eval(e) is true or false, propagate Vin to appropriate    output
edge

 Else, propagate Vin along both output edges

If target node is merge, propagate join(all Vin) to output edge

If any output edge state vector has changed, add it to worklist
```

Running example



Worklist

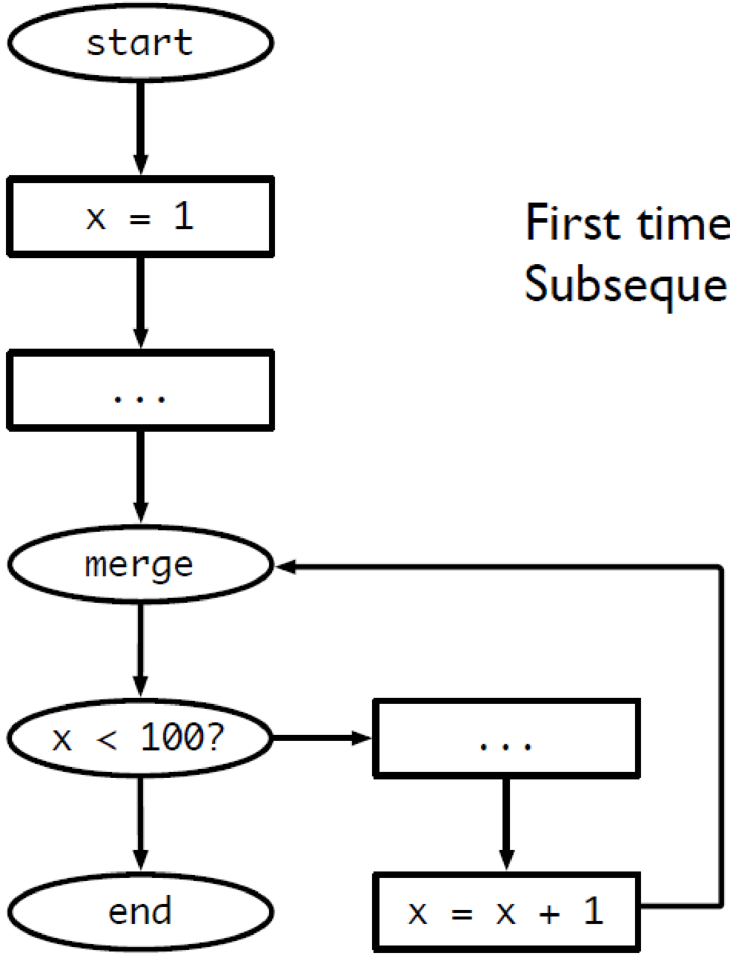
Running example



What do we do about loops?

- Unless a loop never executes, symbolic execution looks like it will keep going around to the same nodes over and over again
- Insight: if the input state vector(s) for a node don't change, then its output doesn't change
    - If input stops changing, then we are done!
- Claim: input will eventually stop changing. Why?

Loop example



First time through loop, x = 1
Subsequent times, x = $\top$

Complexity of algorithm

- V = # of variables, E = # of edges
- Height of lattice = 2 → each state vector can be updated at most 2 * V times.
- So each edge is processed at most 2 * V times, so we process at most 2 * E * V elements in the worklist.
- Cost to process a node: O(V)
- Overall, algorithm takes $O(EV^2)$ time

Question

- Can we generalize this algorithm and use it for more analyses?

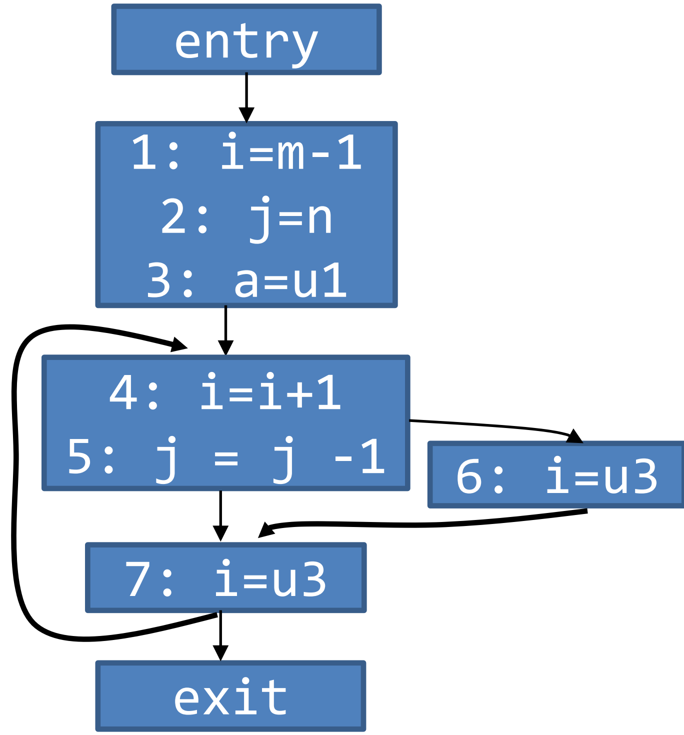
Constant propagation

- Step 1: choose lattice (which values are you going to track during symbolic execution)?
  - Use constant lattice
- Step 2: choose direction of dataflow (if executing symbolically, can run program backwards!)
  - Run forward through program
- Step 3: create *transfer functions*
  - How does executing a statement change the symbolic state?
- Step 4: choose *confluence operator*
  - What do do at merges? For constant propagation, use join

Reaching Definitions - Example

- **Goal:** to know where in a program each variable x may have been defined when control reaches block b
- Definition d reaches block b if there is a path from point immediately following d to b, such that the variable defined in d is not redefined / killed along that path

$$In(b) = \bigcup_{i \in Pred(b)} Out(i)$$



$$Out(b) = gen(b) \cup (In(b) - kill(b))$$

`//set that contains all statements that may define some variable x in b. E.g. gen(1:a=3;2:a=4)={2}`

`//set that contains all statements that define a variable x that is also defined in b. E.g. kill(1:a=3; 2:a=4)={1,2}`

Reaching definitions

- What definitions of a variable *reach* a particular program point
  - A definition of variable x from statement s reaches a statement t if there is a path from s to t where x is not redefined
- Especially important if x is used in t
  - Used to build *def-use* chains and *use-def* chains, which are key building blocks of other analyses
    - Used to determine dependences: if x is defined in s and that definition reaches t then there is a flow dependence from s to t
  - We used this to determine if statements were loop invaraint
    - All definitions that reach an expression must originate from outside the loop, or themselves be invariant

Creating a reaching-def analysis

- Can we use a powerset lattice?
- At each program point, we want to know which definitions have reached a particular point
  - Can use powerset of set of definitions in the program
    - V is set of variables, S is set of program statements
    - Definition: $d \in V \times S$
      - Use a tuple, $\langle v, s \rangle$
  - How big is this set?
    - At most $|V \times S|$ definitions

Forward or backward?

- What do you think?

Choose confluence operator

- Remember: we want to know if a definition *may* reach a program point
- What happens if we are at a merge point and a definition reaches from one branch but not the other?
  - We don't know which branch is taken!
  - We should union the two sets – any of those definitions can reach
- We want to avoid getting too many reaching definitions → should start sets at ⊥

Transfer functions for RD

- Forward analysis, so need a slightly different formulation
  - Merged data flowing into a statement

$$
\begin{aligned}
IN(s) &= \bigcup_{t \in pred(s)} OUT(t) \\
OUT(s) &= \mathbf{gen}(s) \cup (IN(s) - \mathbf{kill}(s))
\end{aligned}
$$

- What are gen and kill?
  - gen(s): the set of definitions that *may* occur at s
    - e.g., gen(s_1: x = e) is <x, s_1>
  - kill(s): all previous definitions of variables that are *definitely* redefined by s
    - e.g., kill(s_1: x = e) is <x, *>