CS323: Compilers

Spring 2023

Week 4: Parsers - Top-Down Parsing (table-driven approach contd. and background concepts), Bottom-up parsing (use of goto and action tables)

Parsing using stack-based model

- How do we use the Parse Table constructed?

Top-Down Parsing - Example

`string: xacc$`

Stack	Rem. Input	Action
?	xacc$	?

What do you put on the stack?

Top-Down Parsing - Example

`string: xacc$`

Stack	**Rem. Input**	**Action**
?	xacc$	?

What do you put on the stack? – strings that you derive

Top-Down Parsing - Example

`string: xacc$`

Stack*	**Rem. Input**	**Action**
S	xacc$	?

Top-down parsing. So, start with S.

* Stack top is on the left-side (first symbol) of the column

Top-Down Parsing - Example

string: xacc$

Stack*	**Rem. Input**	**Action**
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Top-down parsing. So, start with S.

What action do you take when stack-top has symbol S and the string to be matched has terminal x in front?

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Top-Down Parsing - Example

string: xacc$

Stack*	**Rem. Input**	**Action**
S	xacc$	Predict(1) S->ABc$

Top-down parsing. So, start with S.

What action do you take when stack-top has symbol S and the string to be matched has terminal x in front? – consult parse table

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B				5	6	

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ABc$		

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Top-Down Parsing - Example

string: xacc$

Stack*	**Rem. Input**	**Action**
S	xacc$	Predict(1) S->ABc$
ABc$	xacc$	

What action do you take when stack-top has symbol A and the string to be matched has terminal x in front? – consult parse table

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B				5	6	

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Top-Down Parsing - Example

string: xacc$

Stack*	**Rem. Input**	**Action**
S	xacc$	Predict(1) S->ABc$
ABc$	xacc$	Predict(2) A->xaA

What action do you take when stack-top has symbol A and the string to be matched has terminal x in front? – consult parse table

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xaABc$	xacc$	match(x)

	x	y	a	b	c	$
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ABc$	cc$	?

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ABc$	cc$	Predict(4) A->c
cBc$	cc$	match(c)
Bc$	c$	Predict(6) B->λ
c$	c$	match(c)
$	$	Done!

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Identifying LL(1) Grammar

- What we saw was an example of LL(1) Grammar
  - Scan input **L**eft-to-right, produce **L**eft-most derivation with 1 symbol look-ahead

Identifying LL(1) Grammar

- What we saw was an example of LL(1) Grammar
  - Scan input **L**eft-to-right, produce **L**eft-most derivation with 1 symbol look-ahead
- Not all Grammars are LL(1)

A Grammar is LL(1) iff for a production A -> α | β, where

α and β are distinct:

1. For no terminal `a` do both α and β derive strings beginning with `a` (i.e. no common prefix)
2. At most one of α and β can derive an empty string
3. If $\beta \stackrel{*}{\Rightarrow} \epsilon$, then α does not derive any string beginning with a terminal in `Follow(A)`. If $\alpha \stackrel{*}{\Rightarrow} \epsilon$, then β does not derive any string beginning with a terminal in `Follow(A)`

Example (Left Factoring)

- Consider

  <stmt> → if <expr> then <stmt list> endif

  <stmt> → if <expr> then <stmt list> else <stmt list> endif

- This is not LL(1) (why?)
- We can turn this in to

  <stmt> → if <expr> then <stmt list> <if suffix>

  <if suffix> → endif

  <if suffix> → else <stmt list> endif

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Left Factoring

$$A \rightarrow \alpha\ \beta \mid \alpha\ \mu$$

$$\Downarrow$$

$$A \rightarrow \alpha\ N$$
$$N \rightarrow \beta$$
$$N \rightarrow \mu$$

Left recursion

- *Left recursion* is a problem for LL(1) parsers
  - LHS is also the first symbol of the RHS
- Consider:

  $E \rightarrow E + T$

- What would happen with the stack-based algorithm?

Left recursion

- *Left recursion* is a problem for LL(1) parsers
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- Consider:

  $E \rightarrow E + T$

- What would happen with the stack-based algorithm?

  E
  E + T
  E + T + T
  E + T + T + T

Eliminating Left Recursion

A -> A α | β

A -> NT
N -> β
T -> αT
T -> λ

Eliminating Left Recursion

E -> E + T | T

$\Downarrow$

E -> E1 Etail

E1 -> T

Etail -> + T Etail

Etail -> λ

LL(k) parsers

- Can look ahead more than one symbol at a time
  - *k*-symbol lookahead requires extending first and follow sets
  - 2-symbol lookahead can distinguish between more rules:

    $A \rightarrow ax \mid ay$
- More lookahead leads to more powerful parsers
- What are the downsides?

LL(k)? - Example

1) S - > E
2) E -> (E+E)
3) E -> (E-E)
4) E -> x

string: ((x+x))$

Stack*	Rem. Input	Action
S	((x+x))$	Predict(1) S->E
E		Predict(2) or Predict(3)?

LL(1)

	(	+	-	)	x
S	1				1
E	2,3				4

LL(2)

	((	+(	-(	)$	(x
S	1				1
E	2,3				4

Are all grammars LL(k)?

- No! Consider the following grammar:

$$
\begin{aligned}
S &\rightarrow E \\
E &\rightarrow (E + E) \\
E &\rightarrow (E - E) \\
E &\rightarrow x
\end{aligned}
$$

- When parsing E, how do we know whether to use rule 2 or 3?
  - Potentially unbounded number of characters before the distinguishing '+' or '–' is found
  - No amount of lookahead will help!

In real languages?

- Consider the if-then-else problem
- `if x then y else z`
- Problem: else is optional
- `if a then if b then c else d`
    - Which `if` does the `else` belong to?
- This is analogous to a "bracket language": $[^i\ ]^j$ $(i \geq j)$

$$
\begin{aligned}
S &\rightarrow [\ S\ C \\
S &\rightarrow \lambda \\
C &\rightarrow\ ] \\
C &\rightarrow \lambda
\end{aligned}
$$

[[ ] can be parsed: SSλC or SSCλ
(it's ambiguous!)

Solving the if-then-else problem

- The ambiguity exists at the language level. To fix, we need to define the semantics properly
  - "] matches nearest unmatched ["
  - This is the rule C uses for if-then-else
  - What if we try this?

$$
\begin{aligned}
S &\rightarrow [\ S \\
S &\rightarrow S1 \\
S1 &\rightarrow [\ S1\ ] \\
S1 &\rightarrow \lambda
\end{aligned}
$$

This grammar is still not LL(1) (or LL(k) for any k!)

Two possible fixes

- If there is an ambiguity, prioritize one production over another
    - e.g., if C is on the stack, always match "]" before matching "λ"

$$
\begin{array}{ll}
S & \rightarrow [\ S\ C \\
S & \rightarrow \lambda \\
C & \rightarrow\ ] \\
C & \rightarrow \lambda
\end{array}
$$

- Another option: change the language!
    - e.g., all if-statements need to be closed with an endif

$$
\begin{array}{ll}
S & \rightarrow \text{if } S\ E \\
S & \rightarrow \text{other} \\
E & \rightarrow \text{else } S \text{ endif} \\
E & \rightarrow \text{endif}
\end{array}
$$

Parsing if-then-else

- What if we don't want to change the language?
  - C does not require { } to delimit single-statement blocks
- To parse if-then-else, *we need to be able to look ahead at the entire rhs of a production* before deciding which production to use
  - In other words, we need to determine how many "]" to match before we start matching "["s
- *LR parsers* can do this!

Bottom-up Parsing

- More general than top-down parsing
- Used in most parser-generator tools
- Need not have left-factored grammars (i.e. can have left recursion)
- E.g. can work with the bracket language

Bottom-up Parsing

- Reduce a string to start symbol by reverse 'inverting' productions

Bottom-up Parsing

- Reduce a string to start symbol by reverse 'inverting' productions

```
E -> T + E
E -> T
T -> id * T
T -> id
```

```
id * id + id
```

Bottom-up Parsing

- Reduce a string to start symbol by reverse 'inverting' productions

```
id * id + id
id * T + id
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T + id
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E
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T + E
E
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T -> id * T
T -> id
```

```
id * id + id
id * T + id
T + id
T + T
T + E
E
```

Right-most derivation

Bottom-up Parsing

- Scan the input left-to-right and shift tokens – put them on the stack.

```
| id * id + id
id | * id + id
id * | id + id
id * id | + id
```

E -> T + E
E -> T
T -> id * T
T -> id

Bottom-up Parsing

- Replace a set of symbols at the top of the stack that are RHS of a production. Put the LHS of the production on stack – Reduce

```
| id * id + id
id | * id + id
id * | id + id
id * id | + id
```

```
E -> T + E
E -> T
T -> id * T
T -> id
```

Bottom-up Parsing

- Did not discuss when and why a particular production was chosen

```
id * id + id
id * T + id
```

```
E -> T + E
E -> T
T -> id * T
T -> id
```

- *i.e. why replace the id highlighted in input string?*

LR Parsers

- Parser which does a Left-to-right, Right-most derivation
  - Rather than parse top-down, like LL parsers do, parse bottom-up, starting from leaves
- Basic idea: put tokens on a stack until an entire production is found
- Issues:
  - Recognizing the endpoint of a production
  - Finding the length of a production (RHS)
  - Finding the corresponding nonterminal (the LHS of the production)

Data structures

- At each state, given the next token,
  - A *goto table* defines the successor state
  - An *action table* defines whether to
    - *shift* – put the next state and token on the stack
    - *reduce* – an RHS is found; process the production
    - *terminate* – parsing is complete

Simple example

1. P → S
2. S → x ; S
3. S → e

		Symbol					Action
		x	;	e	P	S	
State	0	1		3		5	Shift
	1		2				Shift
	2	1		3		4	Shift
	3						Reduce 3
	4						Reduce 2
	5						Accept

Example

I) P->S
II) S->x;S
III) S->e

Input string

x;x;e

	Symbol					
State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
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Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	?

Start with state 0

Example

I) P->S
II) S->x;S
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Input string

x;x;e

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Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)

Example

I) P->S
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<u>Input string</u>

x;x;e

	Symbol					
State	x	;	e	P	S	Action
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Step	**Parse Stack**	**Rem. Input**	**Parser Action**
1	0	x;x;e	Shift(1)
2	0 1	;x;e	?

Example

I) P->S
II) S->x;S
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<u>Input string</u>

x;x;e

	Symbol					
State	x	;	e	P	S	Action
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I) P->S
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1	0	x;x;e	Shift(1)
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3	0 1 2	x;e	?

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<u>Input string</u>

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1	0	x;x;e	Shift(1)
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3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	?

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1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

x;x;e

State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

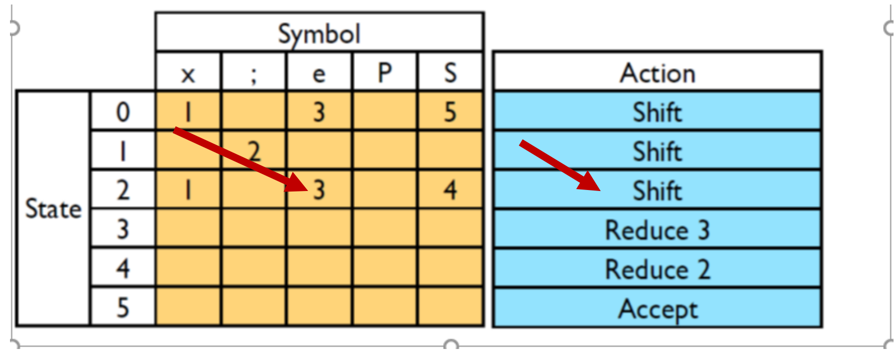
Step	**Parse Stack**	**Rem. Input**	**Parser Action**
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	?

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

x;x;e



State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

x;x;e

State	Symbol: x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		**?**

Example

I) P->S
II) S->x;S
III) S->e

Input string

x;x;e

State		Symbol					Action
		x	;	e	P	S	
	0	1		3		5	Shift
	1		2				Shift
	2	1		3		4	Shift
	3						Reduce 3
	4						Reduce 2
	5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

x;x;e

State	Symbol					Action
	x	;	e	P	S	
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	**Parse Stack**	**Rem. Input**	**Parser Action**
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3
7	0 1 2 1 2		

- *Look at rule III and pop 1 symbol of the stack because RHS of rule III has just 1 symbol*

Example

```
I) P->S
II) S->x;S
III) S->e
```

<u>Input string</u>

x;x;e

State	Symbol					Action
	x	;	e	P	S	
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3
7	0 1 2 1 2		

- *Now stack top has symbol 2 and LHS of rule III has S (imagine you saw S at input). Consult goto and action table.*

Example

I) P->S
II) S->x;S
III) S->e

Input string

x;x;e

	Symbol					
State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		

- *Now stack top has symbol 2 and LHS of rule III has S (imagine you saw S at input). Consult goto and action table. Shift(4)*

Example

I) P->S
II) S->x;S
III) S->e

Input string

x;x;e

		Symbol					
		x	;	e	P	S	Action
State	0	1		3		5	Shift
	1		2				Shift
	2	1		3		4	Shift
	3						Reduce 3
	4						Reduce 2
	5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		?

- *Now stack top has symbol 2 and LHS of rule III has S (imagine you saw S at input). Consult goto and action table. Shift(4)*

Example

I) P->S
II) S->x;S
III) S->e

Input string

x;x;e

State	Symbol					Action
	x	;	e	P	S	
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

x;x;e

		Symbol					Action
		x	;	e	P	S	
State	0	1		3		5	Shift
	1		2				Shift
	2	1		3		4	Shift
	3						Reduce 3
	4						Reduce 2
	5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2
8	0 1 2		

- *Look at rule II and pop 3 symbols of the stack because RHS of rule II has 3 symbols*

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

x;x;e

	Symbol					
State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2
8	0 1 2		

- *Now stack top has symbol 2 and LHS of rule II has S (imagine you saw S at input). Consult goto and action table.*

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

x;x;e

		Symbol					
		x	;	e	P	S	Action
State	0	1		3		5	Shift
	1		2				Shift
	2	1		3		4	Shift
	3						Reduce 3
	4						Reduce 2
	5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2 (shift(4))
8	0 1 2 4		

- *Now stack top has symbol 2 and LHS of rule II has S (imagine you saw S at input). Consult goto and action table. Shift(4)*

Example

```
I) P->S
II) S->x;S
III) S->e
```

Input string

`x;x;e`

State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	`x;x;e`	Shift(1)
2	0 1	`;x;e`	Shift(2)
3	0 1 2	`x;e`	Shift(1)
4	0 1 2 1	`;e`	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2 (shift(4))
8	0 1 2 4		**?**

Example

```
I) P->S
II) S->x;S
III) S->e
```

Input string

x;x;e

State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2 (shift(4))
8	0 1 2 4		Reduce 2

Example

```
I) P->S
II) S->x;S
III) S->e
```

<u>Input string</u>

x;x;e

	Symbol					
State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	`x;x;e`	Shift(1)
2	0 1	`;x;e`	Shift(2)
3	0 1 2	`x;e`	Shift(1)
4	0 1 2 1	`;e`	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2 (shift(4))
8	0 1 2 4		Reduce 2
9	0		

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

`x;x;e`

	Symbol					
State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2 (shift(4))
8	0 1 2 4		Reduce 2 (shift(5))
9	0 5		

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

x;x;e

State	Symbol					Action
	x	;	e	P	S	
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2 (shift(4))
8	0 1 2 4		Reduce 2 (shift(5))
9	0 5		?

Example

I) P->S
II) S->x;S
III) S->e

Input string

x;x;e

State	x	;	e	P	S	Action
0	1		3		5	Shift
1		2				Shift
2	1		3		4	Shift
3						Reduce 3
4						Reduce 2
5						Accept

Step	Parse Stack	Rem. Input	Parser Action
1	0	x;x;e	Shift(1)
2	0 1	;x;e	Shift(2)
3	0 1 2	x;e	Shift(1)
4	0 1 2 1	;e	Shift(2)
5	0 1 2 1 2	e	Shift(3)
6	0 1 2 1 2 3		Reduce 3 (shift(4))
7	0 1 2 1 2 4		Reduce 2 (shift(4))
8	0 1 2 4		Reduce 2 (shift(5))
9	0 5		Accept

means replace whatever is there in the stack with the start symbol

Example

I) P->S
II) S->x;S
III) S->e

Input string

|x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept

Example

I) P->S
II) S->x;S
III) S->e

Input string

|x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept

x |; x ; e

Example

I) P->S
II) S->x;S
III) S->e

Input string

|x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept

x ;|x ; e

Example

I) P->S
II) S->x;S
III) S->e

Input string

|x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept

x ; x|; e

Example

I) P->S
II) S->x;S
III) S->e

Input string

x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept

x ; x ; e

Example

I) P->S
II) S->x;S
III) S->e

<u>Input string</u>

|x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept

x ; x ; e|

Example

I) P->S
II) S->x;S
III) S->e

Input string

x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept

x ; x ; e

S

x ; x ; e

Example

I) P->S
II) S->x;S
III) S->e

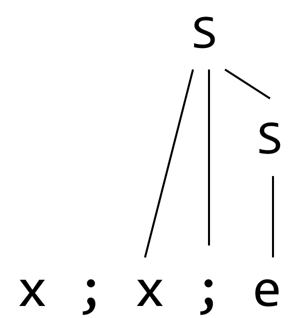
Input string

|x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept

x ; x ; S|



Example

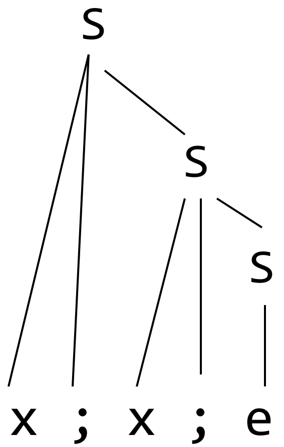
I) P->S
II) S->x;S
III) S->e

Input string

|x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept



x ; S |

Example

I) P->S
II) S->x;S
III) S->e

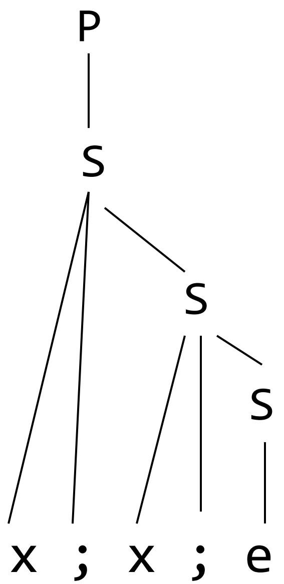
Input string

x;x;e

Initial scan pointer

Step	Parser Action
1	Shift(1)
2	Shift(2)
3	Shift(1)
4	Shift(2)
5	Shift(3)
6	Reduce 3 (shift(4))
7	Reduce 2 (shift(4))
8	Reduce 2 (shift(5))
9	Accept

S



Parsing using an LR(0) parser

- Basic idea: parser keeps track, simultaneously, of all possible productions that *could be matched* given what it's seen so far. When it sees a full production, match it.
- Maintain a *parse stack* that tells you what state you're in
  - Start in state 0
- In each state, look up in action table whether to:
  - *shift*: consume a token off the input; look for next state in goto table; push next state onto stack
  - *reduce*: match a production; pop off as many symbols from state stack as seen in production; look up where to go according to non-terminal we just matched; push next state onto stack
  - *accept*: terminate parse

Shift-Reduce Parsing

The LR parsing seen previously is an example of shift-reduce parsing

- When do we *shift* and when do we *reduce*?
  - *How do we construct goto and action tables?*

Concept: configuration / item

- Configuration or item has a form:

  $$A \to X_1 \ldots X_i \bullet X_{i+1} \ldots X_j$$

- Dot • can appear anywhere
- Represents a production part of which has been matched (what is to the left of Dot)
- LR parsers keep track of multiple (all) productions that can be potentially matched
  - We need a *configuration set*

Concept: configuration / item

- E.g. configuration set

```
stmt -> ID • := expr
stmt -> ID • : stmt
stmt -> ID •
```

Corresponding to productions:

```
stmt -> ID := expr
stmt -> ID : stmt
stmt -> ID
```

- Dot at the extreme left of RHS of a production denotes that production is predicted
- Dot at the extreme right of RHS of a production denotes that production is recognized
- if <u>Dot precedes a Non-Terminal</u> in a configuration set, more configurations need to be added to the set

Concept: closure

➢For each configuration in the configuration set,

$A \rightarrow \alpha \bullet B\gamma$, where B is a non-terminal,

1 add configurations of the form:

$B \rightarrow \bullet\ \delta$

2 if the addition introduces a configuration with Dot behind a new non-Terminal N, add all configurations having the form $N \rightarrow \bullet\ \varepsilon$

Repeat 2 when another new non-terminal is introduced and so on..

Concept: closure

- E.g. closure {S -> •E$}

  ⇩ Non-terminal

  S -> •E$

Grammar

```
S -> E$
E -> E+T | T
T -> ID | (E)
```

Concept: closure

Grammar

```
S -> E$
E -> E+T | T
T -> ID | (E)
```

- E.g. closure {S -> • E$}

Non-terminal

```
S -> •E$
E -> •E+T
```

Concept: closure

- E.g. closure {S ->•E$}

```
S ->•E$
E ->•E+T
E ->•T
```

Non-terminal

Grammar

```
S -> E$
E -> E+T | T
T -> ID | (E)
```

Concept: closure

Grammar

```
S  -> E$
E  -> E+T | T
T  -> ID | (E)
```

- E.g. closure {S -> • E$}

⇩

```
S -> •E$
E -> •E+T
E -> •T
```

New Non-terminal

Concept: closure

- E.g. closure {S -> • E$}

⇩

```
S -> •E$
E -> •E+T
E -> •T
T -> •ID
```

New Non-terminal

Grammar

```
S -> E$
E -> E+T | T
T -> ID | (E)
```

Concept: closure

Grammar

```
S -> E$
E -> E+T | T
T -> ID | (E)
```

- E.g. closure {S -> • E$}

⇩

```
S -> •E$
E -> •E+T
E -> •T
T -> •ID
T -> •(E)
```

New Non-terminal

Concept: closure

- E.g. closure {S -> • E$}

$$\left\{\begin{array}{l} \texttt{S -> •E\$} \\ \texttt{E -> •E+T} \\ \texttt{E -> •T} \\ \texttt{T -> •ID} \\ \texttt{T -> •(E)} \end{array}\right\}$$

Grammar

```
S -> E$
E -> E+T | T
T -> ID | (E)
```

Concept: successor

Grammar

```
S -> E$
E -> E+T | T
T -> ID | (E)
```

- E.g. successor ({S ->•E$}, E)

```
S ->•E$
E ->•E+T                S -> E•$
E ->•T        --E-->    E -> E•+T
T ->•ID
T ->•(E)
```

- Consider all symbols that are to the <u>immediate right of Dot</u> and compute respective successors
  - You must compute closure of successor before finalizing items in successor

Concept: CFSM

- Each configuration set becomes a state
- The symbol used as input for computing the successor becomes the transition
- Configuration-set finite state machine (CFSM)
  - The state diagram obtained after computing the chain of all successors (for all symbols) starting from the configuration involving the first production