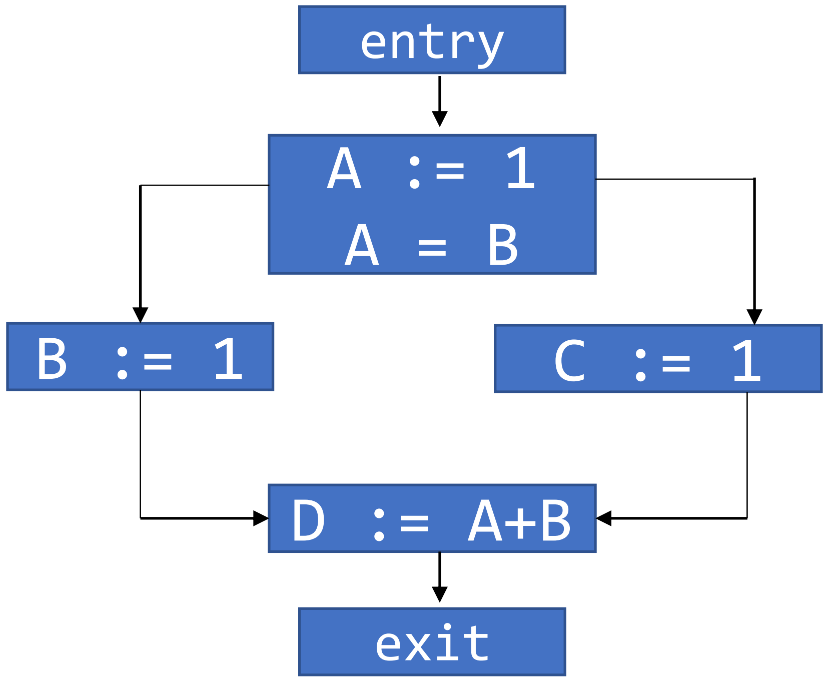
CS323: Compilers

Spring 2023

Week 13: Dataflow Analysis (liveness (recap), Constant Propagation..)

Recap: Liveness

- Variables are live if there exists *some path* leading to its use
- Start from exit block and proceed *backwards* against the control flow to compute



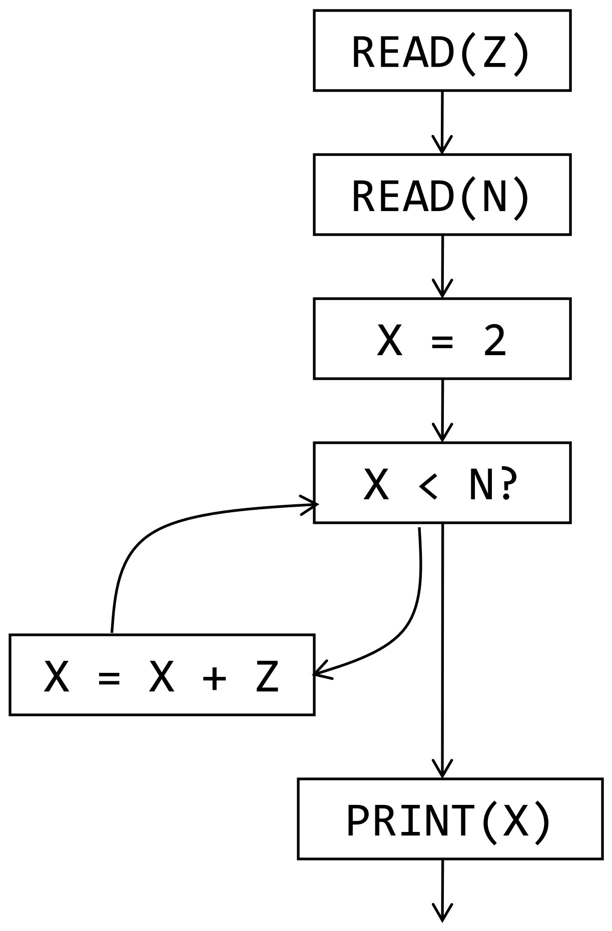
$$\text{LiveOut}(b) = \bigcup_{i \in \text{Succ}(b)} \text{LiveIn}(i)$$

$$\text{LiveIn}(b) = \text{LiveUse}(b) \cup (\text{LiveOut}(b) - \text{Def}(b))$$

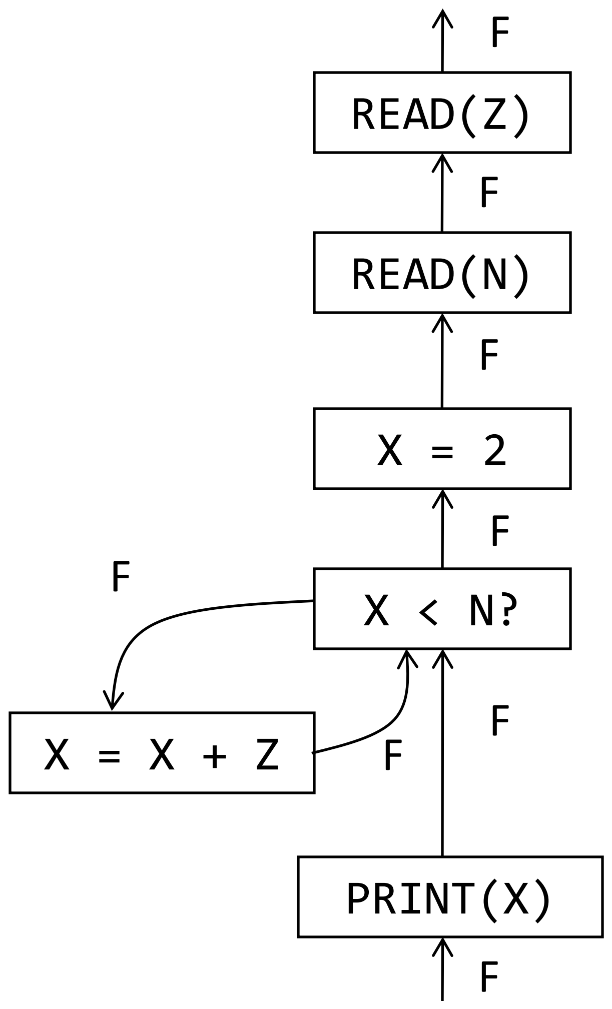
```
//set that contains all variables
used by block b
```

```
//set that contains all
variables defined by block b
```

Recap: Liveness

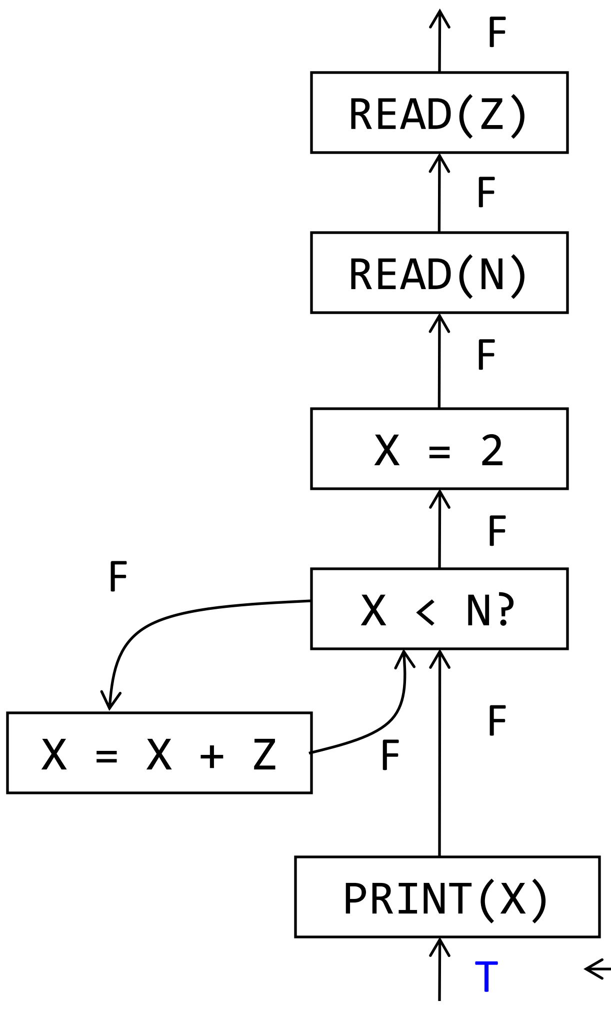


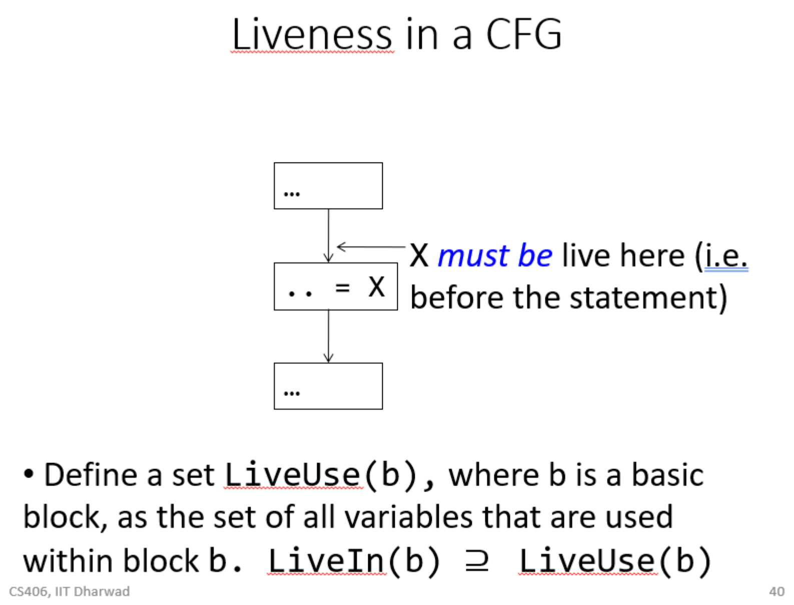
Original CFG



CFG with edges reversed (and initialized) for backwards analysis: is X live? (F=`false`, T=`true`)

Recap: Liveness





Liveness in a CFG

X *must be* live here (i.e. before the statement)

- Define a set `LiveUse(b)`, where b is a basic block, as the set of all variables that are used within block `b`. `LiveIn(b)` ⊇ `LiveUse(b)`

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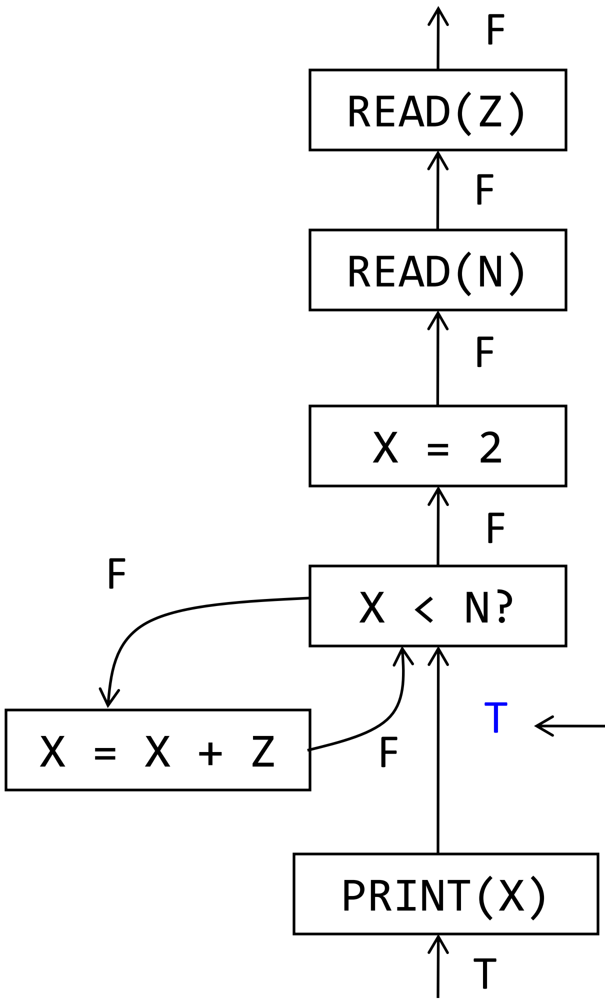
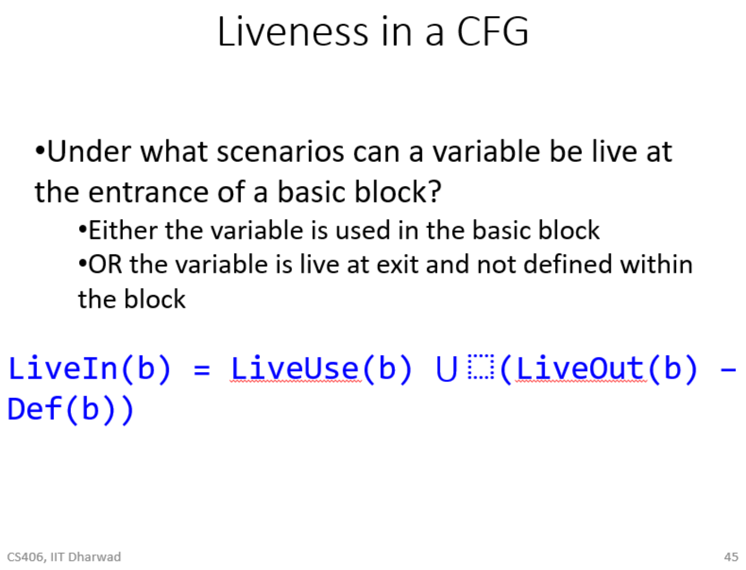
X must be live here (refer week11 slide)

Recap: Liveness



X must be live here
(refer week11 slide)



Liveness in a CFG

- Under what scenarios can a variable be live at the entrance of a basic block?
  - Either the variable is used in the basic block
  - OR the variable is live at exit and not defined within the block

```
LiveIn(b) = LiveUse(b) ∪ (LiveOut(b) –
Def(b))
```

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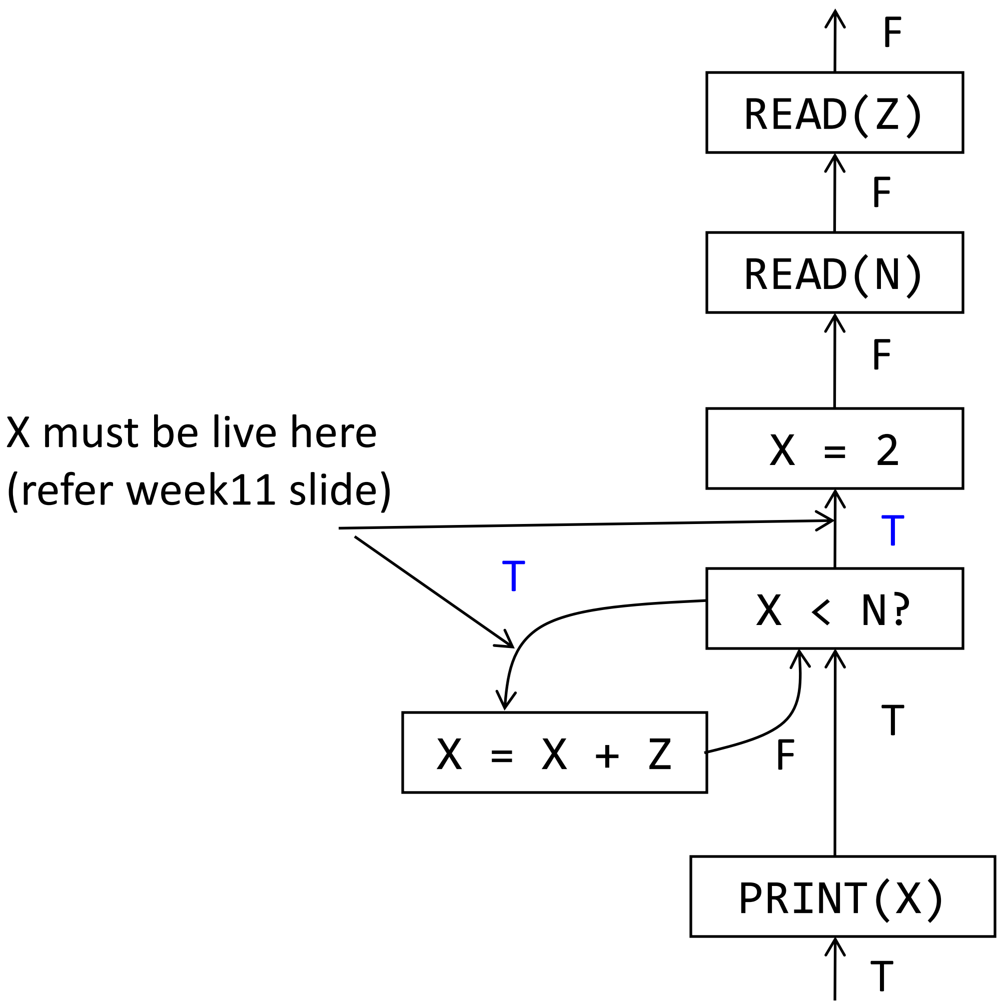
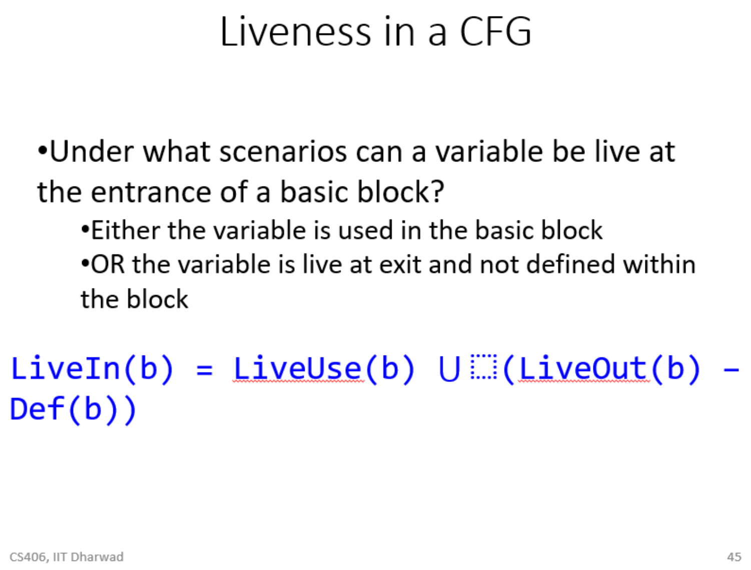
Recap: Liveness

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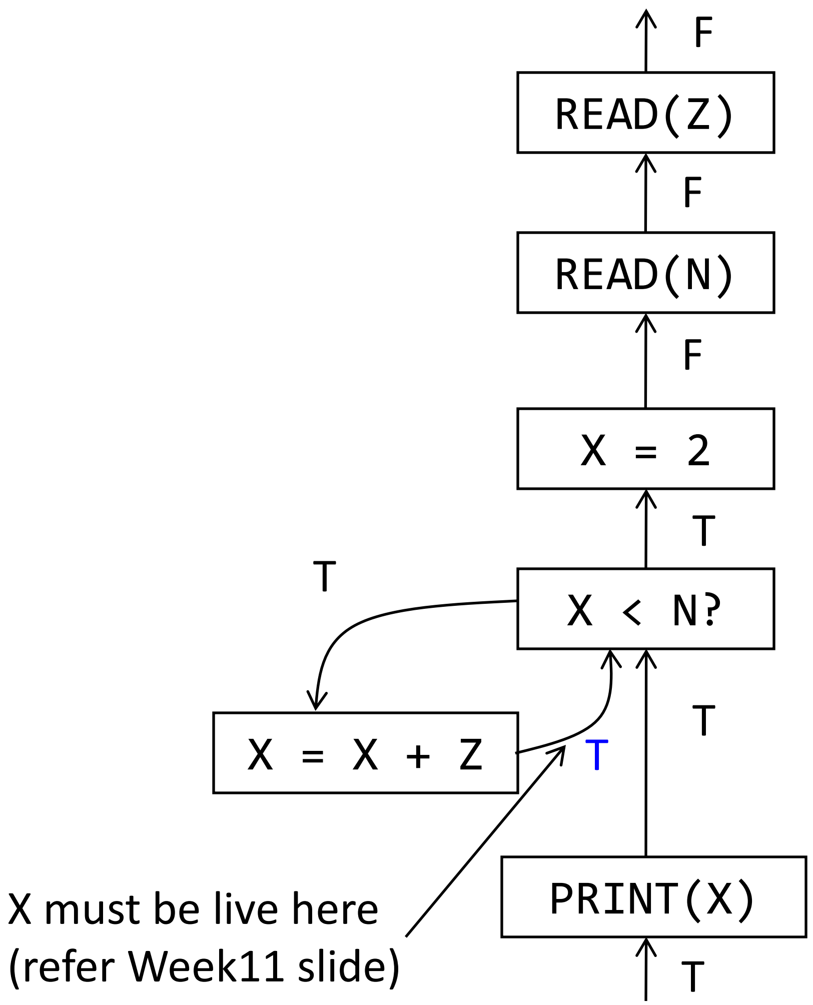

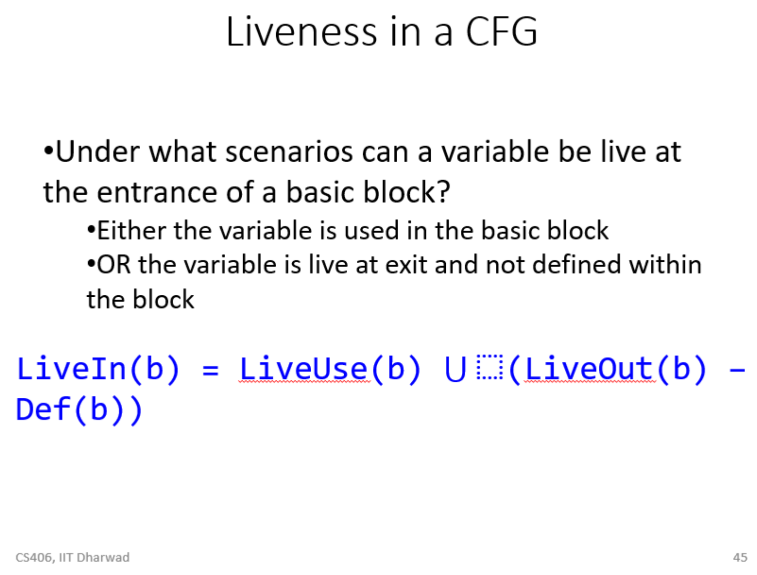

Liveness in a CFG

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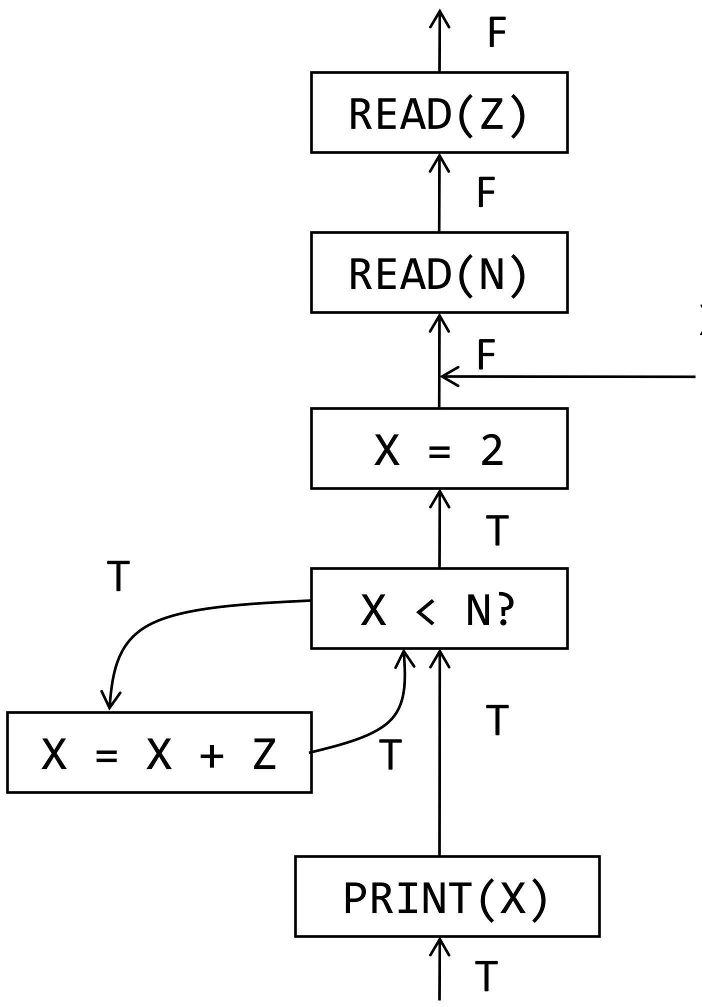
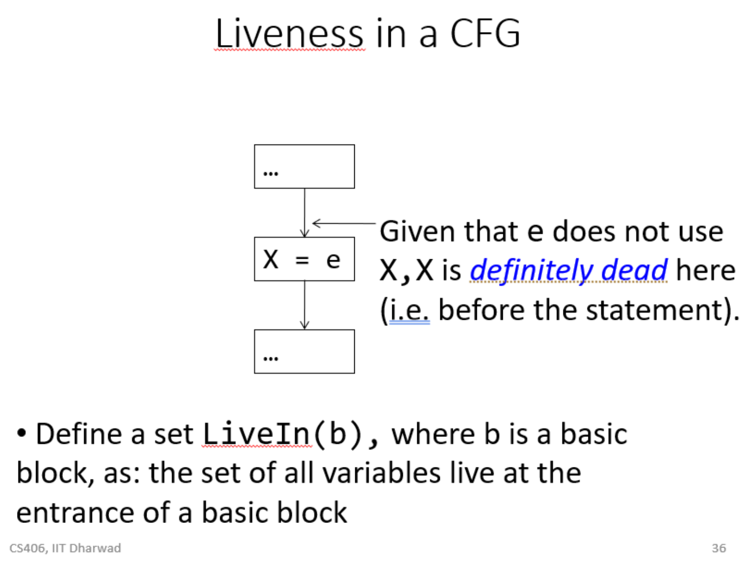
Recap: Liveness



X dead here (refer Week11 slide).
No change in information.



Liveness in a CFG

Given that e does not use X, X is *definitely dead* here (i.e. before the statement).

- Define a set `LiveIn(b)`, where b is a basic block, as: the set of all variables live at the entrance of a basic block

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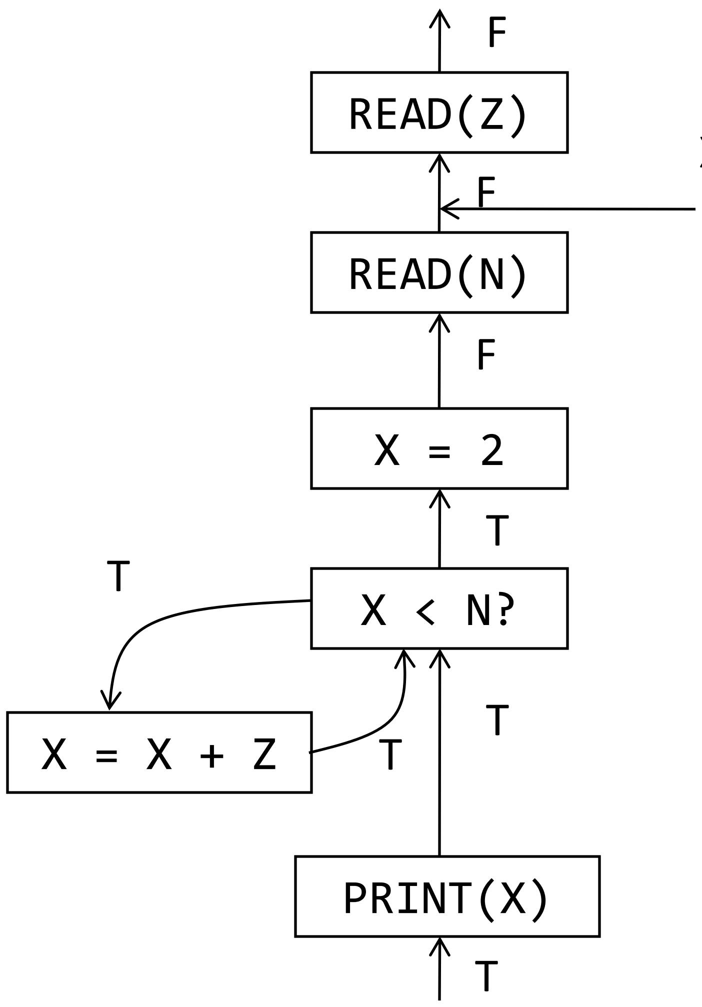
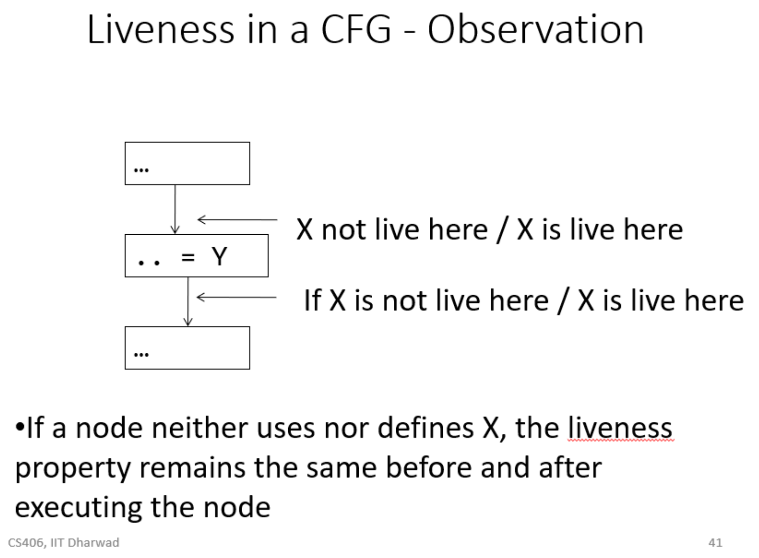
Recap: Liveness



X dead here (refer Week11 slide).
No change in information.


Liveness in a CFG - Observation

X not live here / X is live here

If X is not live here / X is live here

•If a node neither uses nor defines X, the liveness property remains the same before and after executing the node

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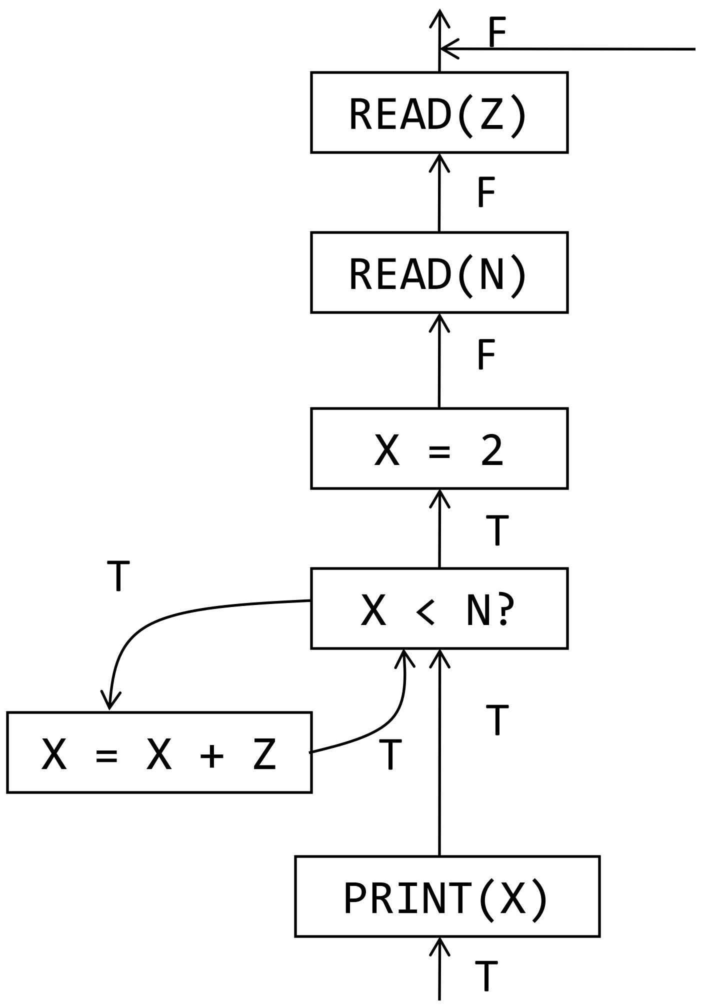
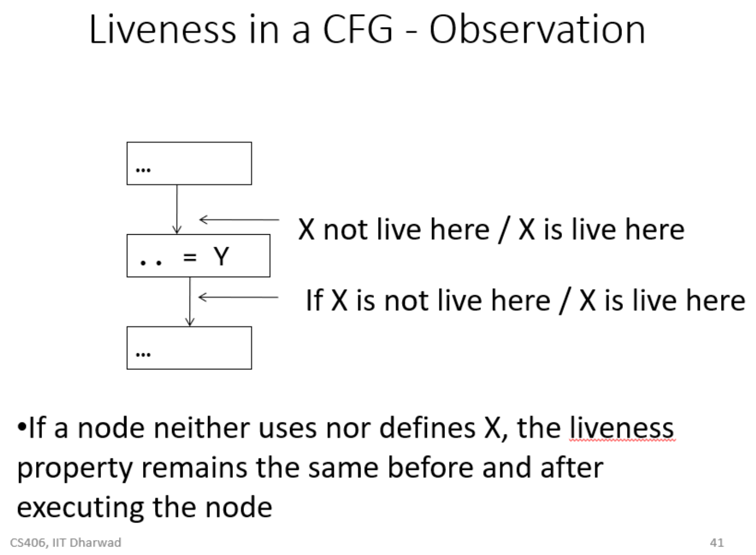
Recap: Liveness



X dead here (refer Week11 slide). No change in information.


Liveness in a CFG - Observation

…
.. = Y
…
X not live here / X is live here
If X is not live here / X is live here

•If a node neither uses nor defines X, the liveness property remains the same before and after executing the node

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Exercise: Repeat for Z and N

Constant Propagation

- Bigger problem size:
  - Which lines using X could be replaced with a constant value? (apply only constant propagation)
  - How can we automate to find an answer to this question?

```
1. X := 2
2. Label1:
3. Y := X + 1
4. if Z > 8 goto Label2
5. X := 3
6. X := X + 5
7. Y := X + 5
8. X := 2
9. if Z > 10 goto Label1
10.X := 3
11.Label2:
12.Y := X + 2
13.X := 0
14.goto Label3
15.X := 10
16.X := X + X
17.Label3:
18.Y := X + 1
```

Constant Propagation

- Problem statement:
  - Replace use of a variable X by a constant K

- Requirement:
  - **property**: on every path to the use of X, the last assignment to X is: X=K

    Same as: “is X=K at a program point?”

    At any program point where the above property holds, we can apply constant propagation.

How can we find constants?

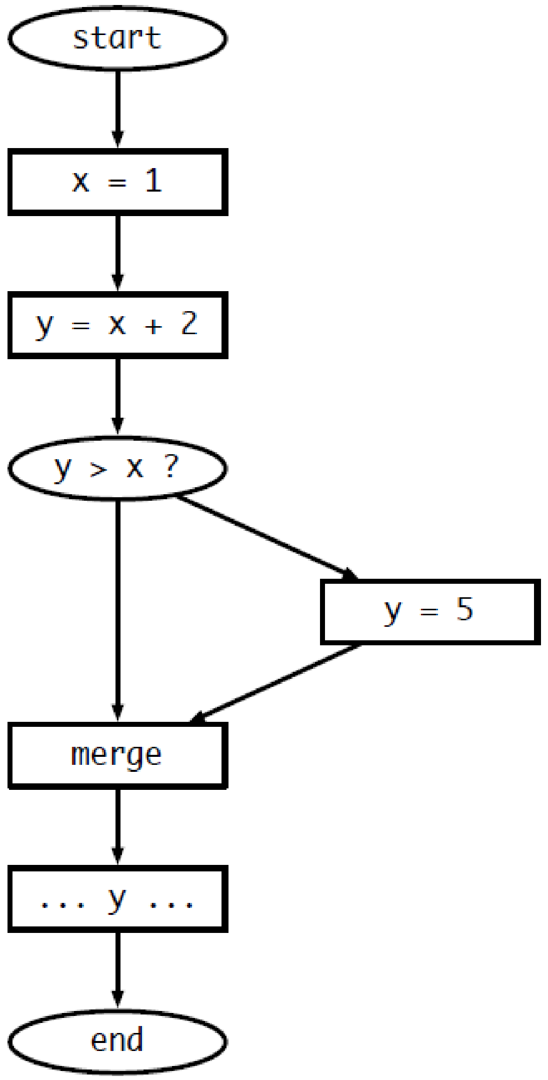
- Ideal: run program and see which variables are constant
  - Problem: variables can be constant with some inputs, not others – need an approach that works for all inputs!
  - Problem: program can run forever (infinite loops?) – need an approach that we know will finish
- Idea: run program *symbolically*
  - Essentially, keep track of whether a variable is constant or not constant (but nothing else)

Overview of algorithm

- Build control flow graph
  - We'll use statement-level CFG (with merge nodes) for this
- Perform symbolic evaluation
  - Keep track of whether variables are constant or not
- Replace constant-valued variable uses with their values, try to simplify expressions and control flow

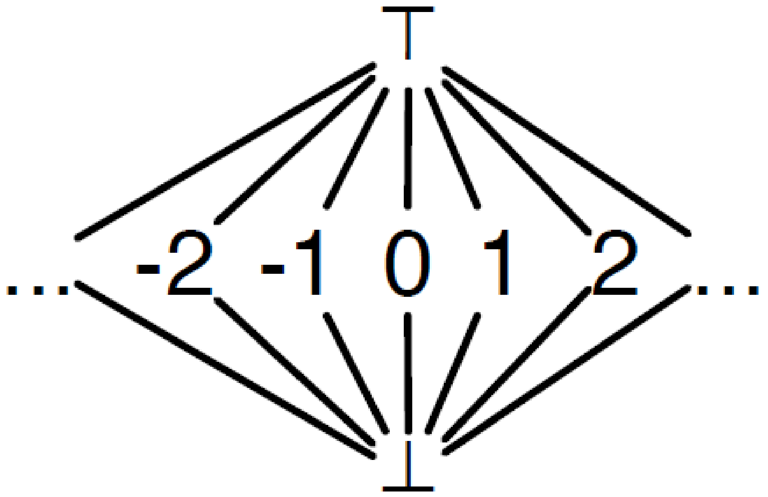
Build CFG

```
x = 1;
y = x + 2;
if (y > x) then y = 5;
... y ...
```



Symbolic evaluation

- Idea: replace each value with a symbol
  - constant (specify which), no information, definitely not constant
- Can organize these possible values in a *lattice*
  - Set of possible values, arranged from least information to most information



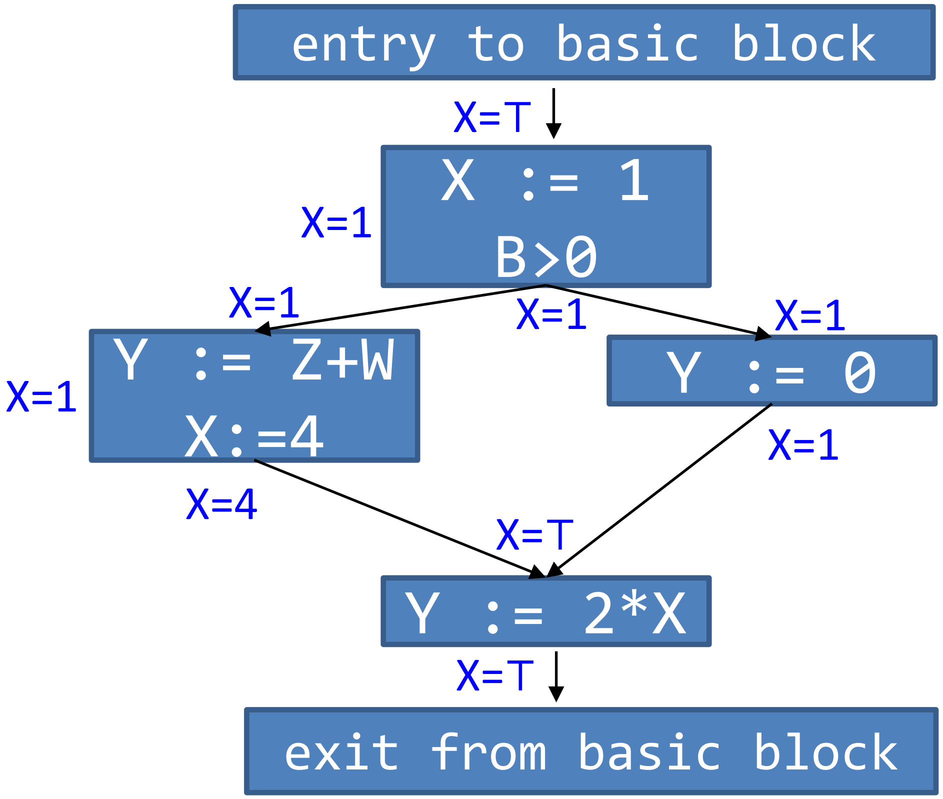
Symbolic Evaluation

- Associate with X one of the following values:

Value	Meaning
⊥ (“bottom”)	This statement never executes
K (“constant”)	X = K
T (“top”)	X is not a constant

- Idea of symbolic execution: at all program points, determine the value of X

Constant Propagation



If X=K at some program point, we can apply constant propagation (replace the use of X with value of K at that program point)

Constant Propagation

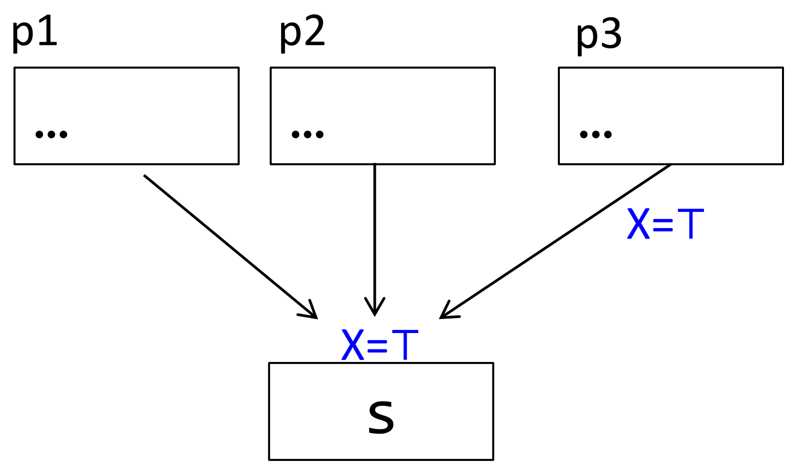
- Determining the value of X at program points:
  - Just like in Liveness Computation in a CFG, the information required for constant propagation flows from one statement to adjacent statement
  - For each statement `s`, compute the information just before and after `s`. C is the function that computes the information:

```
C(X,s,flag)
//if flag=IN, before s what is the value of X
//if flag=OUT, after s what is the value of X
```

    - **Transfer function** (pushes / transfers information from one statement to another)

Constant Propagation

- Determining the value of X at program points (Rule 1):

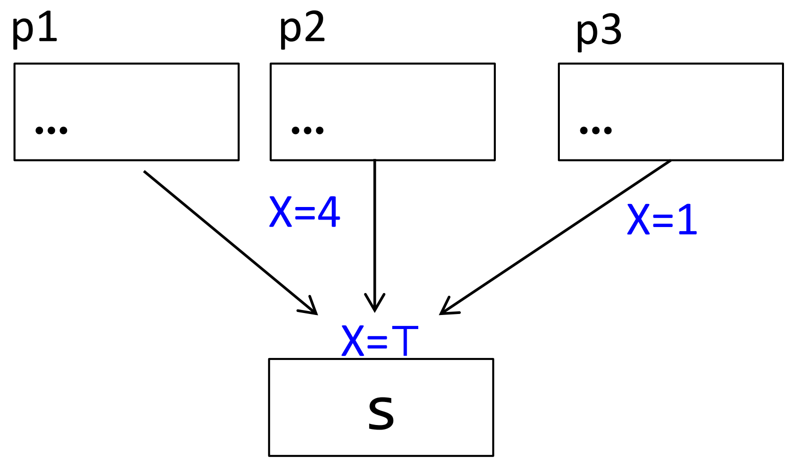


If X=T at exit of *any* of the predecessors, X=T at the entrance of S

`if C(`p_i`,s,OUT)=T`
for any i, then `C(X,s,IN)=T`

Constant Propagation

- Determining the value of X at program points (Rule 2):

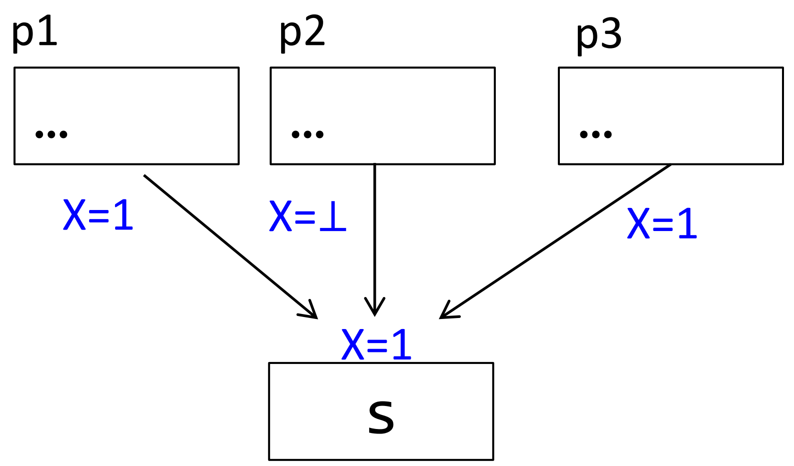


If X=K1 at one predecessor and X=K2 at another predecessor and K1 $\neq$ K2, then X=T at the entrance of S

`if C(`p_i`,s,OUT)=K1 and C(`p_j`,s,OUT)=K2 and K1` $\neq$ `K2 then C(X,s,IN)=T`

Constant Propagation

- Determining the value of X at program points (Rule 3):

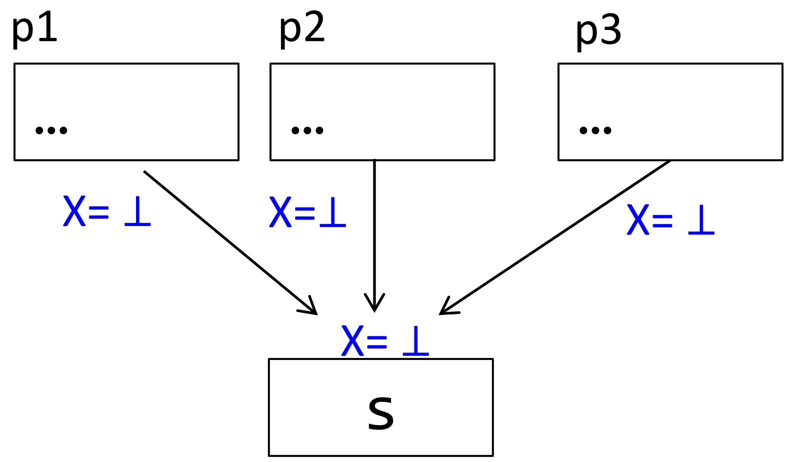


If X=K at some of the predecessors and X= ⊥ at all other predecessors, then X=K at the entrance of S

`if C(`p_i`,s,OUT)=K` or ⊥ for all `i` then `C(X,s,IN)= K`

Constant Propagation

- Determining the value of X at program points (Rule 4):

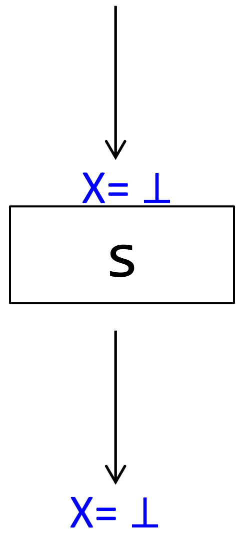


If X= ⊥ at all predecessors, then X= ⊥ at the entrance of S

`if C(`p_i`,s,OUT)=⊥` for all `i` then `C(X,s,IN)= ⊥`

Constant Propagation

- Determining the value of X at program points (Rule 5):

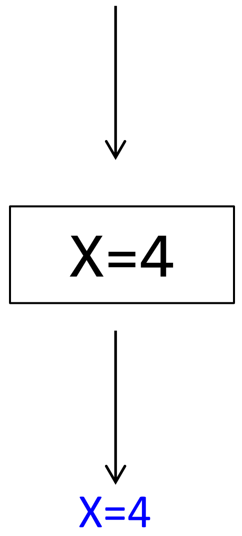


If X= ⊥ at entrance of s, then X= ⊥ at the exit of S

`if C(X,s,IN)=⊥` then `C(X,s,OUT)= ⊥`

Constant Propagation

- Determining the value of X at program points (Rule 6):



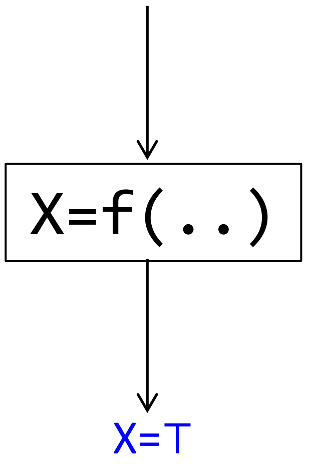
No matter what the value of X is at entrance of `s(X:=K)`, X=K at the exit of s

`C(X,s(X:=K),OUT)=K`

But previous slide said `if C(X,s,IN)=⊥` then `C(X,s,OUT)= ⊥`. *So, we give priority to this.*

Constant Propagation

- Determining the value of X at program points (Rule 7):



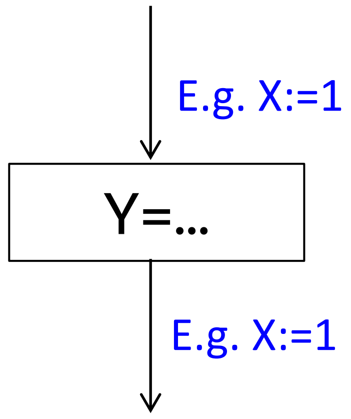
In `s`, assignment to X is any complicated expression (not a constant assignment).

`C(X,s(X:=f()),OUT)=T`

But earlier slide said `if C(X,s,IN)=⊥` then `C(X,s,OUT)= ⊥`. *So, we give priority to this.*

Constant Propagation

- Determining the value of X at program points (Rule 8):



Value of X remains unchanged before and after s(Y:=..) when s doesn't assign to X and $X \neq Y$

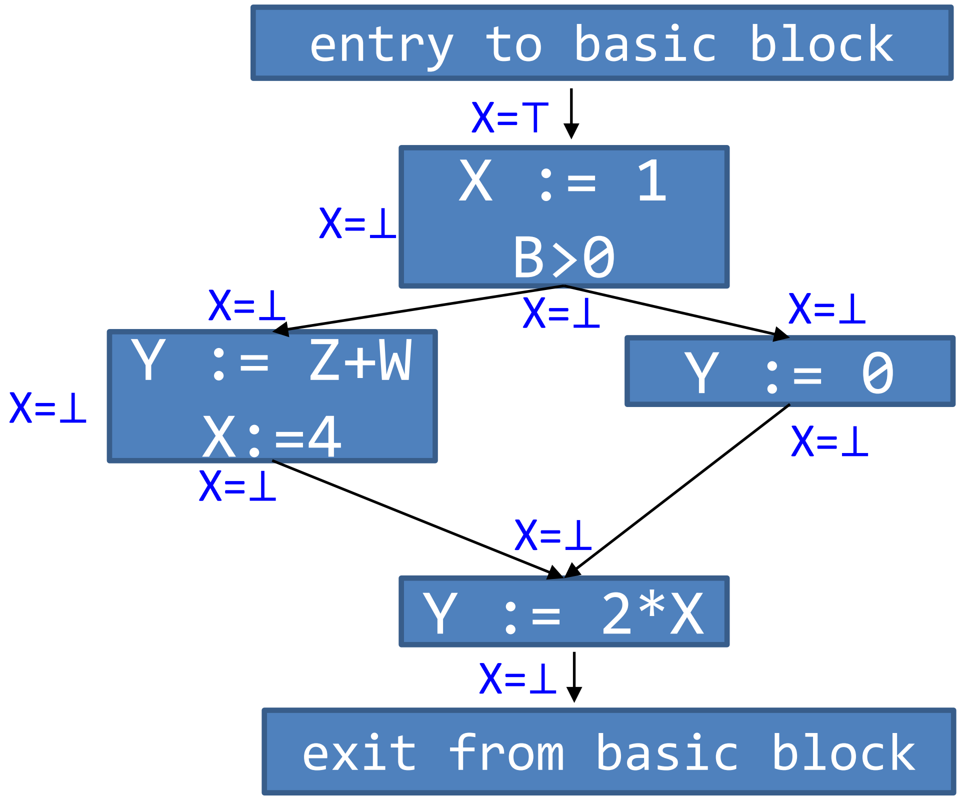
```
C(X,s(Y:=..),OUT)=C(X,s(Y:=..),IN)
```

Constant Propagation

- Putting it all together
  1. For entry s in the program, initialize `C(X,s,IN)=⊤` and initialize `C(X,s,IN)=C(X,s,OUT)=⊥` everywhere else
  2. Repeat until all program points (i.e. any s) satisfy rules 1-8
     1. Pick s in the CFG that doesn't satisfy any one of rules 1-8 and update information.

Constant Propagation

- Putting it all together



Constant Propagation

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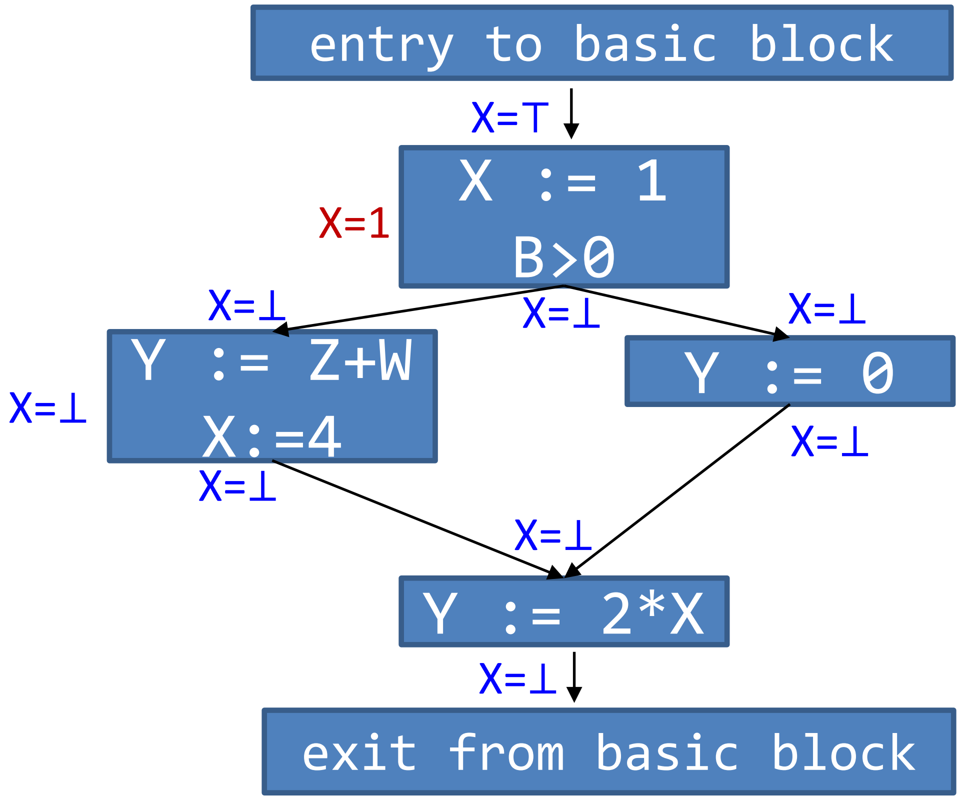

Constant Propagation

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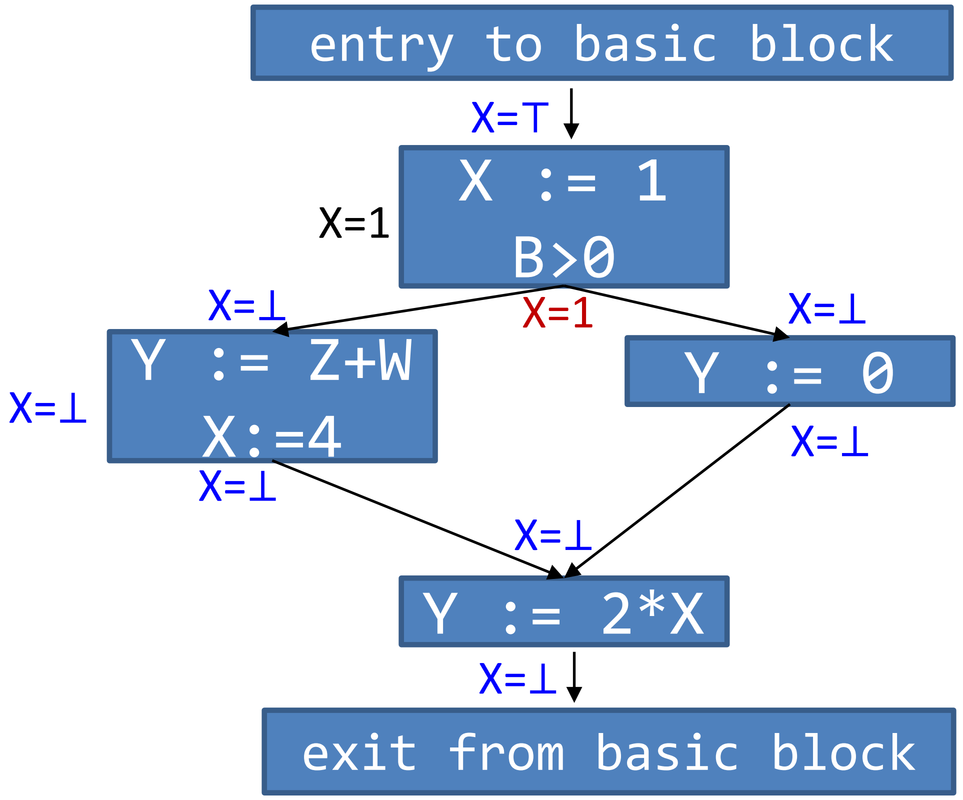

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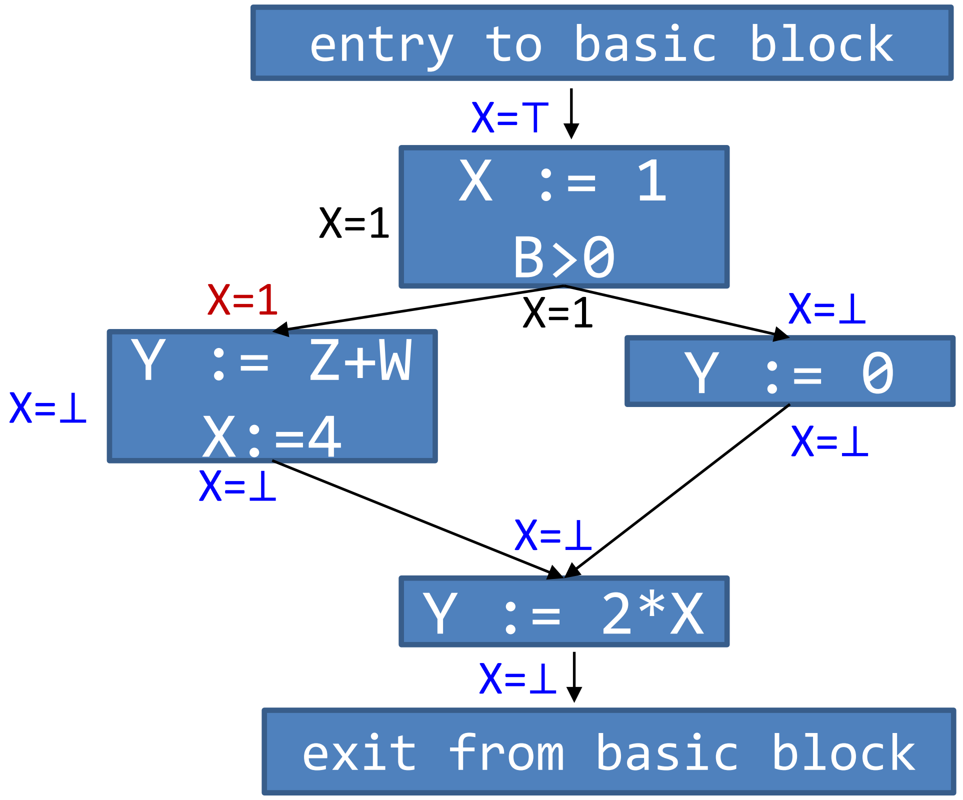

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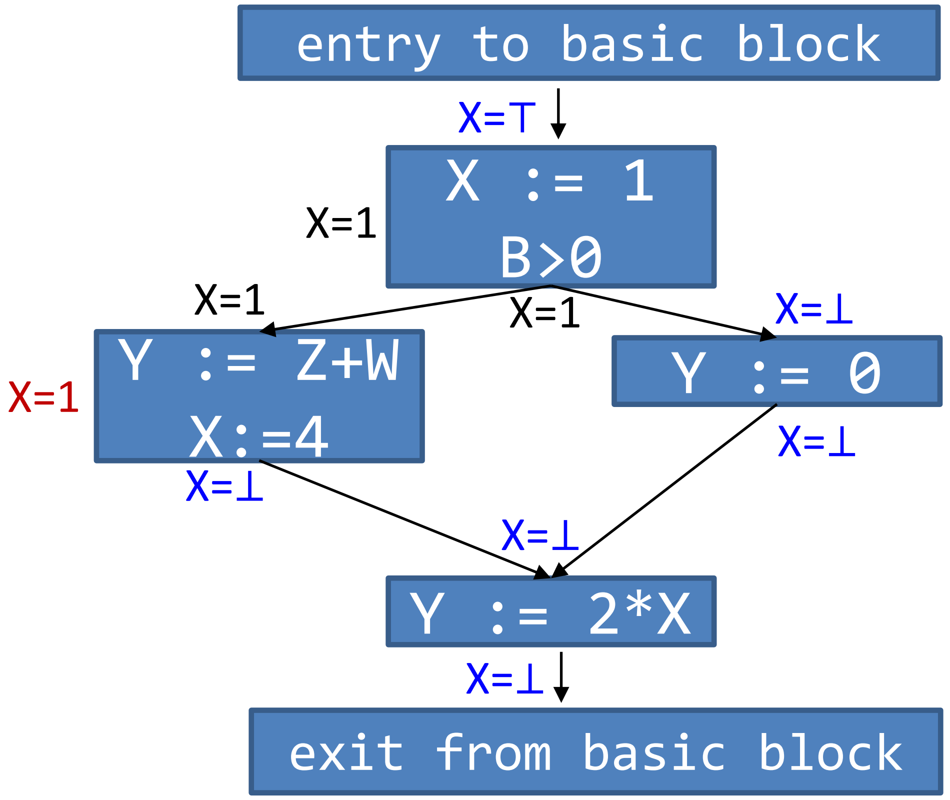

Constant Propagation

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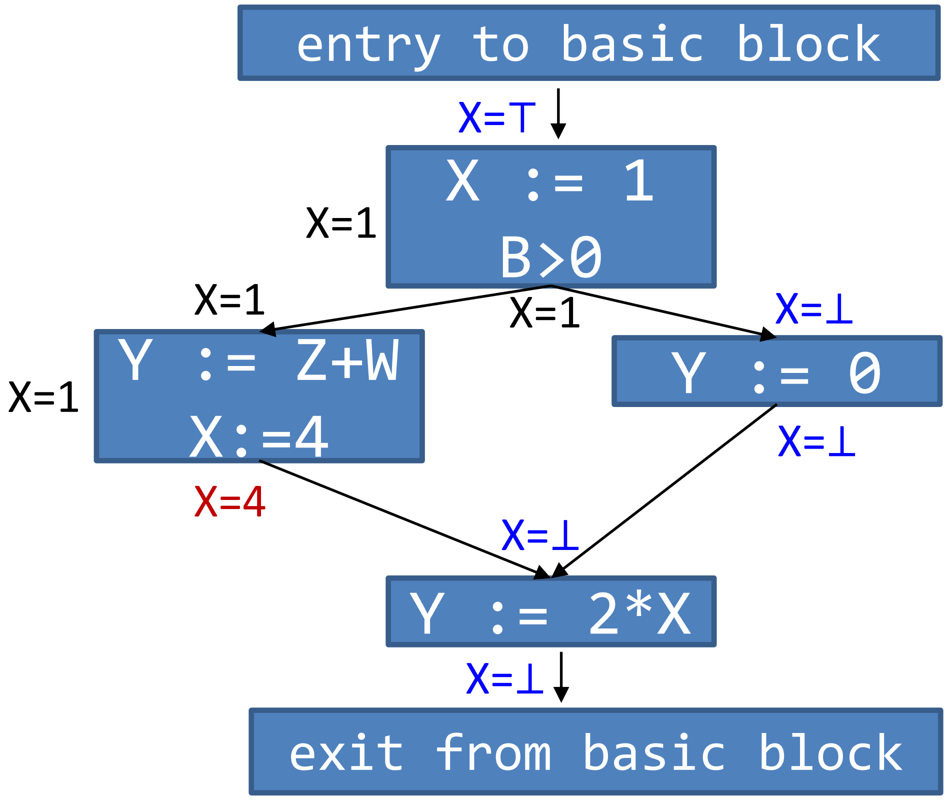

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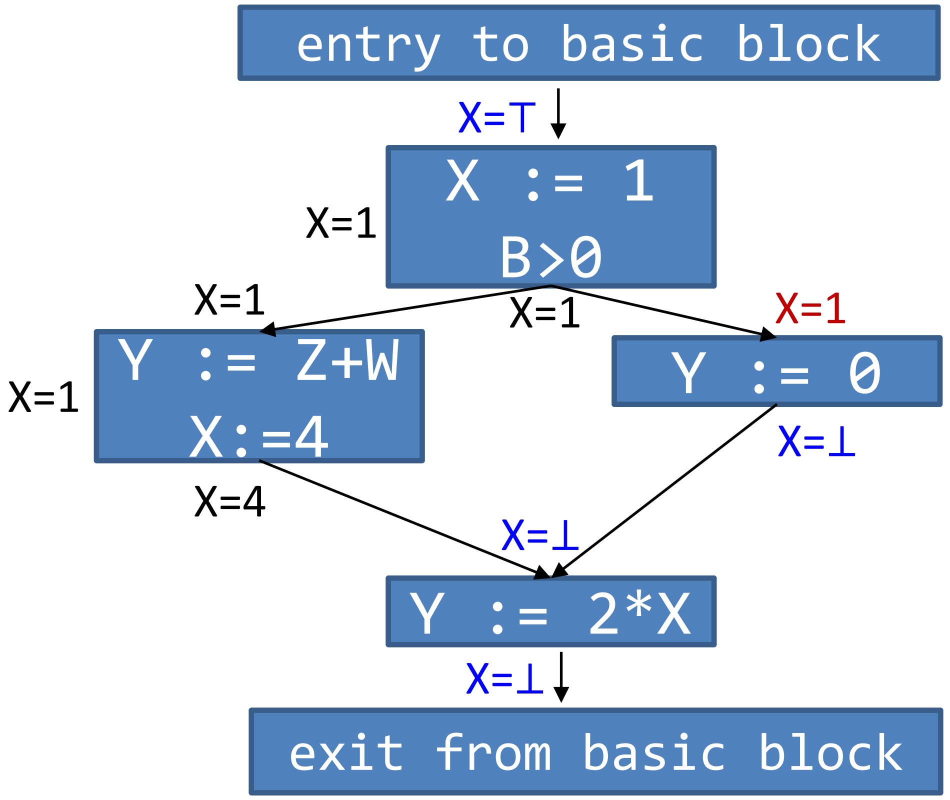

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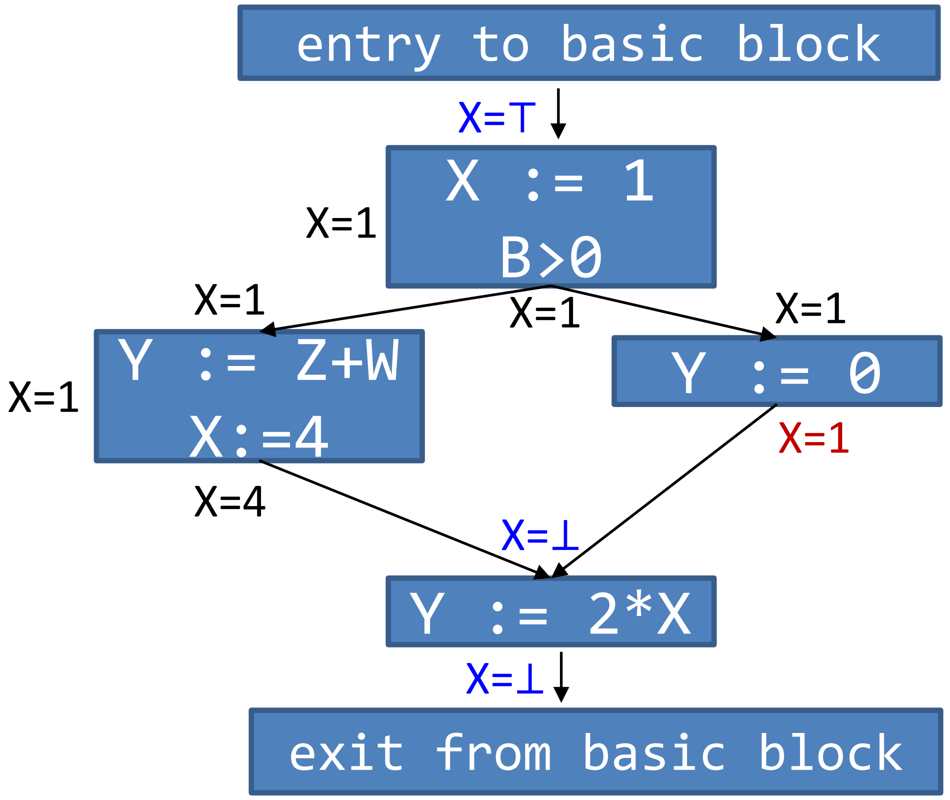

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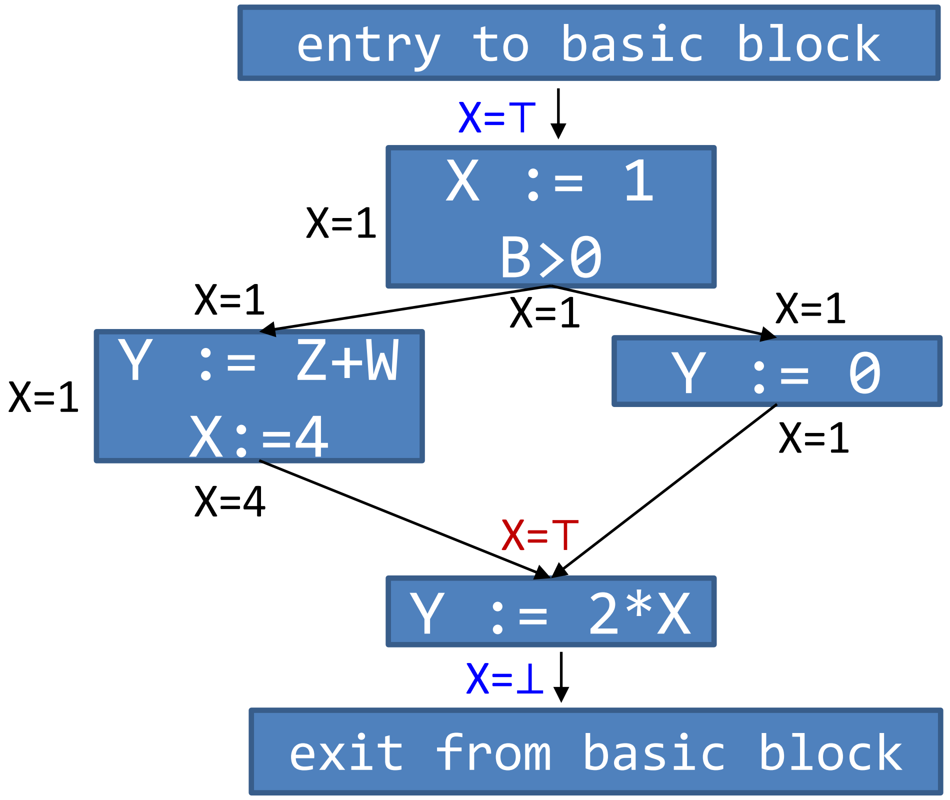

Constant Propagation

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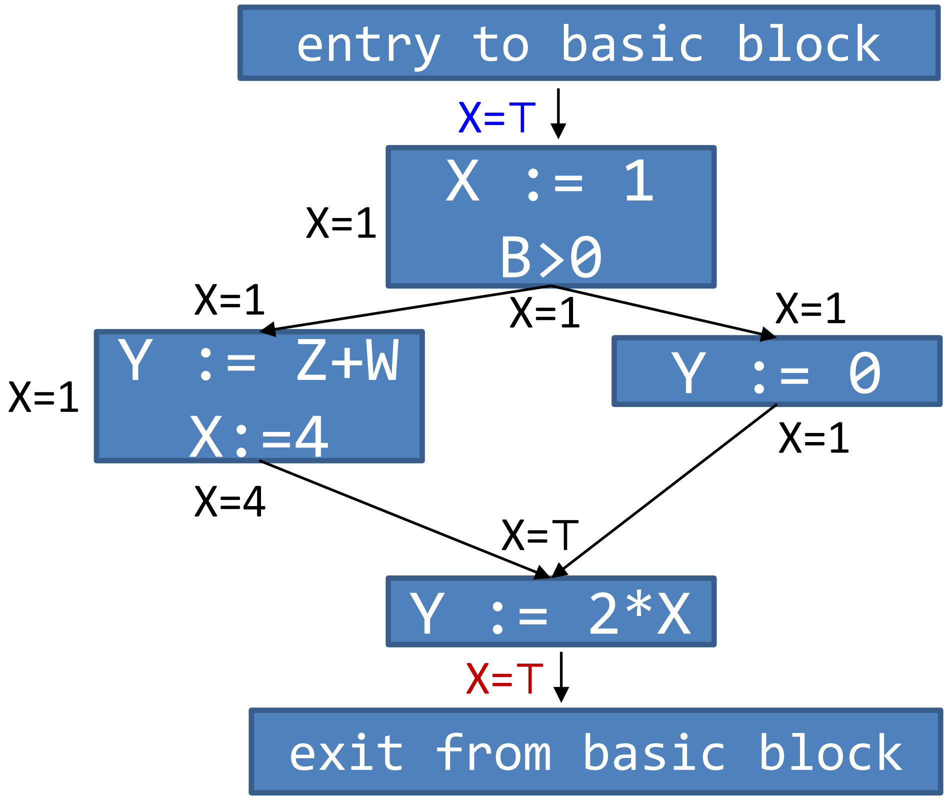
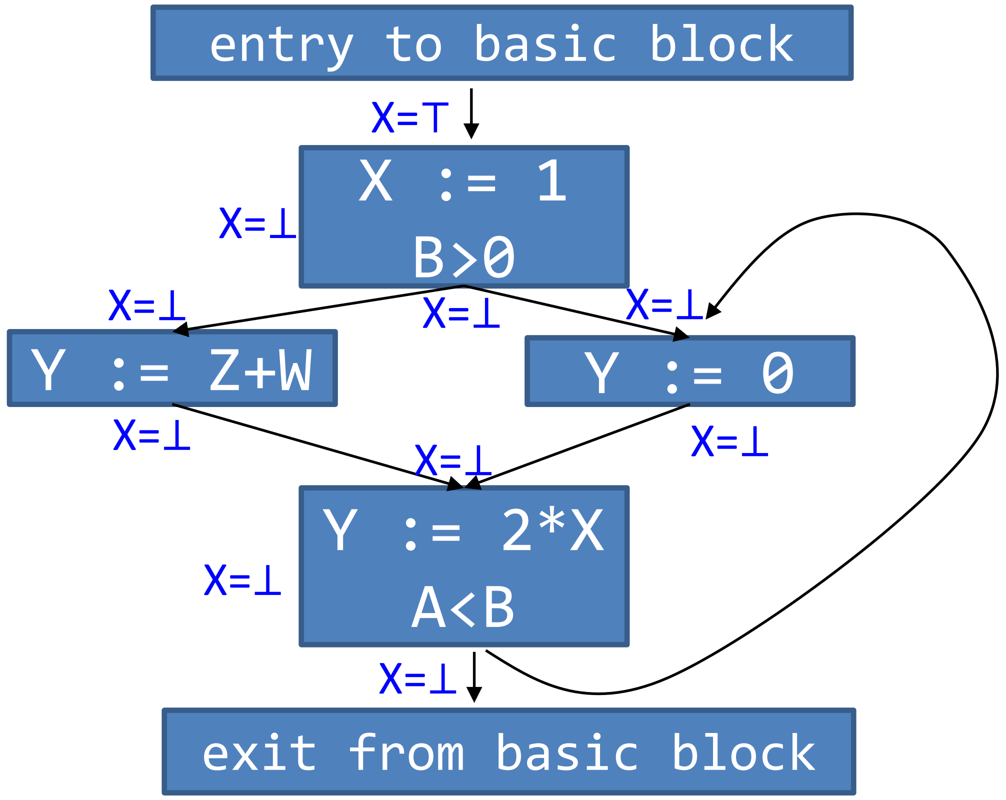


Constant Propagation - Loops



Ordering of information: Generalizing

- We have been executing with symbols ⊥, ⊤ , and K. These are called *abstract values*
- Order these values as:

$$\bot < K < \top$$

Can also be thought of as an ordering from least information to most information

Pictorially:

$$\begin{tikzcd}
 & & \top \arrow[dll, dash] \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] \arrow[drr, dash] & & \\
.. \arrow[drr, dash] & -1 \arrow[dr, dash] & 0 \arrow[d, dash] & 1 \arrow[dl, dash] & .. \arrow[dll, dash] \\
 & & \bot & &
\end{tikzcd}$$

Ordering of information: Generalizing

- Least Upper Bound (`lub`) : smallest element (abstract value) that is greater than or equal to values in the input
  - E.g. $\text{lub}(\perp, \perp) = \perp$, $\text{lub}(\top, \perp) = \top$, $\text{lub}(-1, 1) = \top$, $\text{lub}(1\ \ \perp) = ?$
  - Rewriting rules 1-4: `C(X,s,IN)=lub{C(`p_i`,s,OUT) for all predecessors i)}`
  - Also called as join operator. Written as: $A \sqcup B$

Ordering of information: Generalizing

- Recall that in determining information at all program points:

  "2. Repeat until all program points (i.e. any s) satisfy rules 1-8

  - Pick s in the CFG that doesn't satisfy any one of rules 1-8 and update information. "

  – How do we know that this terminates?

    - lub ensures that the information changes from lower value to higher value
    - In the constant propagation algorithm:
      - $\bot$ can change to constant and then to $\top$
      - $\bot$ can change to $\top$
      - `C(X, s, flag)` can change at most twice

Constant Propagation

- Exercise: what is the complexity of our constant propagation algorithm?

= NumS* 4 (NumS = number of statements in the program).

- Per program point, we evaluate the C function.
- The C function changes value at most two times (initialized to ⊥ first and then could change to K and then to ⊤).
- There are two program points (entry/IN and exit/OUT) for every statement.

This is the complexity of the analysis per variable

How do we do the analysis considering all variables that exist in the program?