

# CS323: Compilers

Spring 2023

Week 4: Parsers - Top-Down Parsing (table-driven approach contd. and background concepts), Bottom-up parsing (use of goto and action tables)

# Parsing using stack-based model

- How do we use the Parse Table constructed?

# Top-Down Parsing - Example

string: xacc\$

**Stack**

?

**Rem. Input**

xacc\$

**Action**

?

*What do you put on the stack?*

# Top-Down Parsing - Example

string: xacc\$

**Stack**

?

**Rem. Input**

xacc\$

**Action**

?

*What do you put on the stack? – strings that you derive*

# Top-Down Parsing - Example

string: xacc\$

**Stack\***

S

**Rem. Input**

xacc\$

**Action**

?

Top-down parsing. So, start with S.

# Top-Down Parsing - Example

string: xacc\$

Stack*	Rem. Input	Action
S	xacc\$	?

Top-down parsing. So, start with S.

*What action do you take when stack-top has symbol S and the string to be matched has terminal x in front?*

# Top-Down Parsing - Example

string: xacc\$

**Stack\***

S

**Rem. Input**

xacc\$

**Action**

Predict(1) S → ABc\$

Top-down parsing. So, start with S.

What action do you take when stack-top has **symbol S** and the string to be matched has **terminal x** in front? – consult parse table

	x	y	a	b	c	\$
S	1	1			1	
A	2	3			4	
B				5	6	

# Top-Down Parsing - Example

string: xacc\$

**Stack\***

S  
ABc\$

**Rem. Input**

xacc\$

**Action**

Predict(1) S → ABc\$

	x	y	a	b	c	\$
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# Top-Down Parsing - Example

string: xacc\$

**Stack\***

S  
ABc\$

**Rem. Input**

xacc\$  
xacc\$

**Action**

Predict(1) S → ABc\$

*What action do you take when stack-top has **symbol A** and the string to be matched has **terminal x** in front? – consult parse table*

	x	y	a	b	c	\$
S	1	1			1	
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# Top-Down Parsing - Example

string: xacc\$

Stack*	Rem. Input	Action
S	xacc\$	Predict(1) S → ABc\$
ABc\$	xacc\$	Predict(2) A → xaA

What action do you take when stack-top has **symbol A** and the string to be matched has **terminal x** in front? – consult parse table

	x	y	a	b	c	\$
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# Top-Down Parsing - Example

string: xacc\$

**Stack\***

S  
ABc\$  
xaABc\$

**Rem. Input**

xacc\$  
xacc\$

**Action**

Predict(1) S → ABc\$  
Predict(2) A → xaA

	x	y	a	b	c	\$
S	1	1			1	
A	2	3			4	
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# Top-Down Parsing - Example

string: xacc\$

**Stack\***

S

ABc\$

**x**aABc\$

**Rem. Input**

xacc\$

xacc\$

**x**acc\$

**Action**

Predict(1) S → ABc\$

Predict(2) A → xaA

?

	x	y	a	b	c	\$
S	1	1			1	
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# Top-Down Parsing - Example

string: xacc\$

Stack*	Rem. Input	Action
S	xacc\$	Predict(1) S → ABc\$
ABc\$	xacc\$	Predict(2) A → xaA
<b>x</b> aABc\$	<b>x</b> acc\$	<b>match(x)</b>

	x	y	a	b	c	\$
S	1	1			1	
A	2	3			4	
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# Top-Down Parsing - Example

string: xacc\$

**Stack\***

S  
ABc\$  
xaABc\$  
aABc\$

**Rem. Input**

xacc\$  
xacc\$  
xacc\$  
acc\$

**Action**

Predict(1) S → ABc\$  
Predict(2) A → xaA  
match(x)  
match(a)

	x	y	a	b	c	\$
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# Top-Down Parsing - Example

string: xacc\$

**Stack\***

S  
ABc\$  
xaABc\$  
aABc\$  
ABc\$

**Rem. Input**

xacc\$  
xacc\$  
xacc\$  
acc\$  
cc\$

**Action**

Predict(1) S → ABc\$  
Predict(2) A → xaA  
match(x)  
match(a)  
?

	x	y	a	b	c	\$
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# Top-Down Parsing - Example

string: xacc\$

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**Stack\***

**Rem. Input**

**Action**

S	xacc\$	Predict(1) S->ABc\$
ABc\$	xacc\$	Predict(2) A->xaA
xaABc\$	xacc\$	match(x)
aABc\$	acc\$	match(a)
ABc\$	cc\$	Predict(4) A->c



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cBc\$		

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<b>c</b> Bc\$	<b>c</b> c\$	?

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<b>c</b> Bc\$	<b>cc</b> \$	<b>match(c)</b>

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cBc\$	cc\$	match(c)
Bc\$	c\$	?

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cBc\$	cc\$	match(c)
Bc\$	c\$	Predict(6) B → λ

# Top-Down Parsing - Example

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cBc\$	cc\$	match(c)
<b>B</b> c\$	c\$	Predict(6) B → λ
c\$		

# Top-Down Parsing - Example

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cBc\$	cc\$	match(c)
Bc\$	c\$	Predict(6) B → λ
c\$	c\$	?

# Top-Down Parsing - Example

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	x	y	a	b	c	\$
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c\$	c\$	match(c)



# Top-Down Parsing - Example

string: xacc\$

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cBc\$	cc\$	match(c)
Bc\$	c\$	Predict(6) B → λ
c\$	c\$	match(c)
\$	\$	Done!

# Identifying LL(1) Grammar

- What we saw was an example of LL(1) Grammar
  - Scan input **L**eft-to-right, produce **L**eft-most derivation with **1** symbol look-ahead

# Identifying LL(1) Grammar

- What we saw was an example of LL(1) Grammar
  - Scan input **L**eft-to-right, produce **L**eft-most derivation with 1 symbol look-ahead
- Not all Grammars are LL(1)

A Grammar is LL(1) iff for a production  $A \rightarrow \alpha \mid \beta$ , where  $\alpha$  and  $\beta$  are distinct:

  1. For no terminal  $a$  do both  $\alpha$  and  $\beta$  derive strings beginning with  $a$  (i.e. no common prefix)
  2. At most one of  $\alpha$  and  $\beta$  can derive an empty string
  3. If  $\beta \xRightarrow{*} \epsilon$ , then  $\alpha$  does not derive any string beginning with a terminal in  $\text{Follow}(A)$ . If  $\alpha \xRightarrow{*} \epsilon$ , then  $\beta$  does not derive any string beginning with a terminal in  $\text{Follow}(A)$

# Example (Left Factoring)

- Consider

$\langle \text{stmt} \rangle \rightarrow \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt list} \rangle \text{ endif}$

$\langle \text{stmt} \rangle \rightarrow \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt list} \rangle \text{ else } \langle \text{stmt list} \rangle \text{ endif}$

- This is not LL(1) (why?)
- We can turn this in to

$\langle \text{stmt} \rangle \rightarrow \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt list} \rangle \langle \text{if suffix} \rangle$

$\langle \text{if suffix} \rangle \rightarrow \text{endif}$

$\langle \text{if suffix} \rangle \rightarrow \text{else } \langle \text{stmt list} \rangle \text{ endif}$

# Example (Left Factoring)

- Consider

$\langle \text{stmt} \rangle \rightarrow \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt list} \rangle \text{ endif}$

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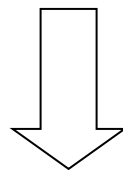
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$\langle \text{if suffix} \rangle \rightarrow \text{else } \langle \text{stmt list} \rangle \text{ endif}$

# Left Factoring

$$A \rightarrow \alpha \beta \mid \alpha \mu$$

$$A \rightarrow \alpha N$$
$$N \rightarrow \beta$$
$$N \rightarrow \mu$$

# Left recursion

- *Left recursion* is a problem for LL(1) parsers
  - LHS is also the first symbol of the RHS
- Consider:  
$$E \rightarrow E + T$$
- What would happen with the stack-based algorithm?

# Left recursion

- *Left recursion* is a problem for LL(1) parsers
  - LHS is also the first symbol of the RHS

- Consider:

$$E \rightarrow E + T$$

- What would happen with the stack-based algorithm?

E

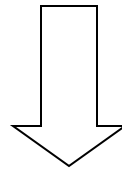
E + T

E + T + T

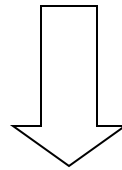
E + T + T + T



# Eliminating Left Recursion

$$A \rightarrow A\alpha \mid \beta$$

$$A \rightarrow NT$$
$$N \rightarrow \beta$$
$$T \rightarrow \alpha T$$
$$T \rightarrow \lambda$$

# Eliminating Left Recursion

$$E \rightarrow E + T \mid T$$

$$E \rightarrow E_1 \text{ Etail}$$
$$E_1 \rightarrow T$$
$$\text{Etail} \rightarrow + T \text{ Etail}$$
$$\text{Etail} \rightarrow \lambda$$

# LL(k) parsers

- Can look ahead more than one symbol at a time
  - $k$ -symbol lookahead requires extending first and follow sets
  - 2-symbol lookahead can distinguish between more rules:  
$$A \rightarrow ax \mid ay$$
- More lookahead leads to more powerful parsers
- What are the downsides?

# LL(k)? - Example

string: ((x+x))\$

- 1)  $S \rightarrow E$
- 2)  $E \rightarrow (E+E)$
- 3)  $E \rightarrow (E-E)$
- 4)  $E \rightarrow x$

Stack*	Rem. Input	Action
S	((x+x))\$	Predict(1) $S \rightarrow E$
E		Predict(2) or Predict(3)?

LL(1)

	(	+ -	)	x
S	1			1
E	2,3			4

LL(2)

	((	+(	-(	)\$	(x
S	1				1
E	2,3				4

# Are all grammars LL(k)?

- No! Consider the following grammar:

$$\begin{array}{lcl} S & \rightarrow & E \\ E & \rightarrow & (E + E) \\ E & \rightarrow & (E - E) \\ E & \rightarrow & x \end{array}$$

- When parsing E, how do we know whether to use rule 2 or 3?
  - Potentially unbounded number of characters before the distinguishing '+' or '-' is found
  - No amount of lookahead will help!

# In real languages?

- Consider the if-then-else problem
- `if x then y else z`
- Problem: else is optional
- `if a then if b then c else d`
  - Which if does the else belong to?
- This is analogous to a “bracket language”:  $[^i ]^j$  ( $i \geq j$ )

$S \rightarrow [ S C$

$S \rightarrow \lambda$

$C \rightarrow ]$

$C \rightarrow \lambda$

$[ [ ]$  can be parsed:  $SS\lambda C$  or  $SSC\lambda$   
(it's ambiguous!)

# Solving the if-then-else problem

- The ambiguity exists at the language level. To fix, we need to define the semantics properly
  - “[” matches nearest unmatched “[”
  - This is the rule C uses for if-then-else
  - What if we try this?

$$\begin{aligned} S &\rightarrow [ S \\ S &\rightarrow SI \\ SI &\rightarrow [ SI ] \\ SI &\rightarrow \lambda \end{aligned}$$

This grammar is still not LL(1)  
(or LL(k) for any k!)

# Two possible fixes

- If there is an ambiguity, prioritize one production over another
- e.g., if C is on the stack, always match “]” before matching “ $\lambda$ ”

$$\begin{array}{lcl} S & \rightarrow & [ S C \\ S & \rightarrow & \lambda \\ C & \rightarrow & ] \\ C & \rightarrow & \lambda \end{array}$$

- Another option: change the language!
- e.g., all if-statements need to be closed with an endif

$$\begin{array}{lcl} S & \rightarrow & \text{if } S \text{ E} \\ S & \rightarrow & \text{other} \\ E & \rightarrow & \text{else } S \text{ endif} \\ E & \rightarrow & \text{endif} \end{array}$$



# Parsing if-then-else

- What if we don't want to change the language?
  - C does not require { } to delimit single-statement blocks
- To parse if-then-else, we *need to be able to look ahead at the entire rhs of a production* before deciding which production to use
  - In other words, we need to determine how many “]” to match before we start matching “[”s
- *LR parsers* can do this!

# Bottom-up Parsing

- More general than top-down parsing
- Used in most parser-generator tools
- Need not have left-factored grammars (i.e. can have left recursion)
- E.g. can work with the bracket language