CS406: Compilers

Spring 2021

Week 4: Parsers

Parsing – so far..

- Parsing involves:
  - identifying if a program has *syntax errors*
  - Identifying the *structure* of a valid program
- CFGs are formal notations for specifying the rules of the programming language
  - Has *symbols* (start, terminal(s), non-terminal(s)), and *productions/rules*
  - *Derivations* are a sequence of expansions of a string of symbols

  Left-most derivation and *Right-most derivation* are popular

  methods defining the order in the sequence

Parsing – so far..

- Parse trees are tree structures having terminals as leaves and non-terminals as nodes
  - The sequence involved in derivations define them
  - For a given string having *terminal symbols only,* there exists *only one parse tree* in an *unambiguous grammar*
    - A grammar is ambiguous if there exists some string for which different derivations result in more than one tree structure
- Ambiguity fixing in grammars
  - Manual rewriting of grammar
  - Hints to parser generators
- Error handling in parsers
  - Panic mode, error productions, and error recovery.

Top-down Parsing

- Idea: we know sentence has to start with initial symbol
- Build up partial derivations by *predicting* what rules are used to expand non-terminals
  - Often called *predictive parsers*
- If partial derivation has terminal characters, *match* them from the input stream

4

Top-down Parsing

- Also called recursive-descent parsing
- Equivalent to finding the left-derivation for an input string
  - Recall: expand the leftmost non-terminal in a parse tree
  - Expand the parse tree in pre-order i.e., identify parent nodes before children

5

Top-down Parsing

↑: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:      | a
```

Step	Input string	Parse tree
1	cad ↑	S

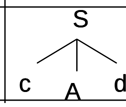
String: **cad**

Start with S

Top-down Parsing

: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:      | a
```

Step	Input string	Parse tree
1	cad	S
2	cad	

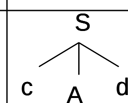
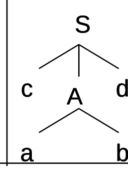
String: **cad**

Predict rule 1

Top-down Parsing

↑: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:     | a
```

Step	Input string	Parse tree
1	cad	S
2	cad	
3	cad	

String: **cad**

Predict rule 2

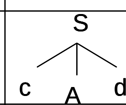
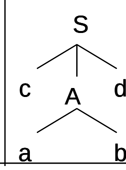
8

Top-down Parsing

↑: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:     | a
```

String: **cad**

Step	Input string	Parse tree
1	cad	S
2	cad	
3	cad	

No more non terminals!
String doesn't match.
Backtrack.

9

Top-down Parsing

↑: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:      | a
```

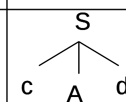
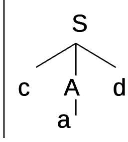
Step	Input string	Parse tree
1	cad	S
2	cad	S c A d

String: **cad**

Top-down Parsing

↑: next symbol to be read

```
1: S -> cAd
2: A -> ab
3:      | a
```

Step	Input string	Parse tree
1	cad	S
2	cad	
4	cad	

String: **cad**

Predict rule 3

Top-down Parsing – Table-driven Approach

```
1: S -> F
2: S -> (S + F)
3: F -> a
```

string: `(a+a)`

string': `(a+a)$`

	(	)	a	+	$
S	2	-	1	-	-
F	-	-	3	-	-

Assume that the table is given.

- Table-driven (Parse Table) approach doesn't require backtracking

But how do we construct such a table?

Important Concepts: First Sets and Follow Sets

13

Concepts for analyzing the grammar

First and follow sets

- First(α): the set of terminals (and/or λ) that begin all strings that can be derived from α
  - First(A) = {x, y, λ}
  - First(xaA) = {x}
  - First (AB) = {x, y, b}
- Follow(A): the set of terminals (and/ or $, but no λs) that can appear immediately after A in some partial derivation
  - Follow(A) = {b}

$S \rightarrow A\ B\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow \lambda$

$B \rightarrow b$

First and follow sets

- $\text{First}(\alpha) = \{a \in V_t \mid \alpha \Rightarrow^* a\beta\} \cup \{\lambda \mid \text{if } \alpha \Rightarrow^* \lambda\}$
- $\text{Follow}(A) = \{a \in V_t \mid S \Rightarrow^+ \ldots Aa \ldots\} \cup \{\$ \mid \text{if } S \Rightarrow^+ \ldots A\ \$\}$

S:	start symbol
a:	a terminal symbol
A:	a non-terminal symbol
α,β:	a string composed of terminals and non-terminals (typically, α is the RHS of a production

$\Rightarrow$:	derived in 1 step
$\Rightarrow^*$:	derived in 0 or more steps
$\Rightarrow^+$:	derived in 1 or more steps

15

Computing first sets

- Terminal: First(a) = {a}
- Non-terminal: First(A)
  - Look at all productions for A

    $A \rightarrow X_1 X_2 \ldots X_k$
  - First(A) $\supseteq$ (First(X_1) - λ)
  - If $\lambda \in$ First(X_1), First(A) $\supseteq$ (First(X_2) - λ)
  - If λ is in First(X_i) for all i, then $\lambda \in$ First(A)
- Computing First(α): similar procedure to computing First(A)

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  $x\ a\ c\ c\ \$$

A simple example

S → A B c $

A → x a A

A → y a A

A → c

B → b

B → λ

special "end of input" symbol

- A sentence in the grammar:

  x a c c $

18

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: S

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: A B c $

Predict rule

A simple example

$S \rightarrow A\ B\ c\ \$$

Choose based on *first set* of rules

$\boxed{\begin{array}{l} A \rightarrow x\ a\ A \\ A \rightarrow y\ a\ A \\ A \rightarrow c \end{array}}$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a A B c $

Predict rule *based on next token*

21

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a A B c $

Match token

A simple example

S → A B c $

A → x a A

A → y a A

A → c

B → b

B → λ

- A sentence in the grammar:

  x a c c $

Current derivation: x a A B c $

Match token

23

A simple example

$S \rightarrow A\ B\ c\ \$$

Choose based on *first set* of rules

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a c B c $

Predict rule *based on next token*

A simple example

S → A B c $

A → x a A

A → y a A

A → c

B → b

B → λ

- A sentence in the grammar:

  x a c c $

Current derivation: x a c B c $

Match token

25

A simple example

$\boxed{S \rightarrow A\ B\ c\ \$}$

$A \rightarrow x\ a\ A$

Choose based on *follow set*

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a c λ c $

Predict rule *based on next token*

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a c c $

Match token

27

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a c c $

Match token

Towards parser generators

- Key problem: as we read the source program, we need to decide what productions to use
- Step 1: find the tokens that can tell which production P (of the form $A \rightarrow X_1X_2 \ldots X_m$) applies

$$\text{Predict}(P) =$$

$$\begin{cases} \text{First}(X_1 \ldots X_m) & \text{if } \lambda \notin \text{First}(X_1 \ldots X_m) \\ (\text{First}(X_1 \ldots X_m) - \lambda) \cup \text{Follow}(A) & \text{otherwise} \end{cases}$$

- If next token is in Predict(P), then we should choose this production

29

Computing Parse-Table

```
1) S -> Ac$
2) A -> xaA
3) A -> yaA
4) A -> c
5) B -> b
6) B -> λ
```

	x	y	a	b	c	$
S	1	1			1	
A	2	3			4	
B				5	6	

```
first (S) = {x, y, c}
first (A) = {x, y, c}
first(B) = {b, λ}
```

```
follow (S) = {}
follow (A) = {b, c}
follow(B) = {c}
```

```
P(1) = {x,y,c}
P(2) = {x}
P(3) = {y}
P(4) = {c}
P(5) = {b}
P(6) = {c}
```

Parsing using stack-based model (non-recursive) of a predictive parser

31

Computing Parse-Table

string: xacc$

Stack*	Remaining Input	Action
S	xacc$	Predict(1) S->ABc$
ABc$	xacc$	Predict(2) A->xaA
xaABc$	xacc$	match(x)
aABc$	acc$	match(a)
ABc$	cc$	Predict(4) A->c
cBc$	cc$	match(c)
Bc$	c$	Predict(6) B->λ
c$	c$	match(c)
c$	c$	Done!

* Stack top is on the left-side (first character) of the column

Identifying LL(1) Grammar

- What we saw was an example of LL(1) Parser
- Not all Grammars are LL(1)

A Grammar is LL(1) iff for a production A -> α **|** β, where α and β are distinct:

1. For no terminal a do both α and β derive strings beginning with a
2. At most one of α and β can derive an empty string
3. If $\beta \overset{*}{\Rightarrow} \epsilon$, then α does not derive any string beginning with a terminal in `Follow(A)`. If $\alpha \overset{*}{\Rightarrow} \epsilon$, then does not derive any string beginning with a terminal in `Follow(A)`

33

Left recursion

- *Left recursion* is a problem for LL(1) parsers
  - LHS is also the first symbol of the RHS
- Consider:

  $E \rightarrow E + T$
- What would happen with the stack-based algorithm?

Example (Left Factoring)

- Consider

  <stmt> → if <expr> then <stmt list> endif

  <stmt> → if <expr> then <stmt list> else <stmt list> endif

- This is not LL(1) (why?)
- We can turn this in to

  <stmt> → if <expr> then <stmt list> <if suffix>

  <if suffix> → endif

  <if suffix> → else <stmt list> endif

Eliminating Left Recursion

$A \rightarrow A\ \alpha \mid \beta$

$A \rightarrow \beta A'$
$A' \rightarrow \alpha A' \mid \lambda$

36

LL(k) parsers

- Can look ahead more than one symbol at a time
    - *k*-symbol lookahead requires extending first and follow sets
    - 2-symbol lookahead can distinguish between more rules:

      $A \rightarrow ax \mid ay$
- More lookahead leads to more powerful parsers
- What are the downsides?

Are all grammars LL(k)?

- No! Consider the following grammar:

$$
\begin{aligned}
S &\rightarrow E \\
E &\rightarrow (E + E) \\
E &\rightarrow (E - E) \\
E &\rightarrow x
\end{aligned}
$$

- When parsing E, how do we know whether to use rule 2 or 3?
  - Potentially unbounded number of characters before the distinguishing '+' or '–' is found
  - No amount of lookahead will help!

In real languages?

- Consider the if-then-else problem
- `if x then y else z`
- Problem: else is optional
- `if a then if b then c else d`
    - Which `if` does the `else` belong to?
- This is analogous to a "bracket language": $[^i\ ]^j\ (i \geq j)$

$$
\begin{aligned}
S &\to [\ S\ C \\
S &\to \lambda \\
C &\to\ ] \\
C &\to \lambda
\end{aligned}
$$

[[ ] can be parsed: SSλC or SSCλ
(it's ambiguous!)

39

Solving the if-then-else problem

- The ambiguity exists at the language level. To fix, we need to define the semantics properly
    - "] matches nearest unmatched ["
    - This is the rule C uses for if-then-else
    - What if we try this?

S → [S
S → SI
SI → [SI]
SI → λ

This grammar is still not LL(1) (or LL(k) for any k!)

Two possible fixes

- If there is an ambiguity, prioritize one production over another
  - e.g., if C is on the stack, always match "]" before matching "λ"

$$
\begin{aligned}
S &\to [\ S\ C \\
S &\to \lambda \\
C &\to\ ] \\
C &\to \lambda
\end{aligned}
$$

- Another option: change the language!
  - e.g., all if-statements need to be closed with an endif

$$
\begin{aligned}
S &\to \text{if } S\ E \\
S &\to \text{other} \\
E &\to \text{else } S \text{ endif} \\
E &\to \text{endif}
\end{aligned}
$$

Parsing if-then-else

- What if we don't want to change the language?
  - C does not require { } to delimit single-statement blocks
- To parse if-then-else, *we need to be able to look ahead at the entire rhs of a production* before deciding which production to use
  - In other words, we need to determine how many "]" to match before we start matching "["s
- *LR parsers* can do this!

Top-down vs. Bottom-up parsers

- Top-down parsers expand the parse tree in *pre-order*
  - Identify parent nodes before the children
- Bottom-up parsers expand the parse tree in *post-order*
  - Identify children before the parents
- Notation:
  - LL(1): Top-down derivation with 1 symbol lookahead
  - LL(k): Top-down derivation with k symbols lookahead
  - LR(1): Bottom-up derivation with 1 symbol lookahead

LR Parsers

- Parser which does a Left-to-right, Right-most derivation
  - Rather than parse top-down, like LL parsers do, parse bottom-up, starting from leaves

Example:

```
E -> E + T | T
T -> T * F | F
F -> (E) | id
```

String: `id*id`

Demo

44

LR Parsers

- Basic idea: put tokens on a stack until an entire production is found
  - **shift** tokens onto the stack. At any step, keep the set of productions that could generate the read-in token
    - **reduce** the RHS of recognized productions to the corresponding non-terminal on the LHS of the production. Replace the RHS tokens on the stack with the LHS non-terminal.
- Issues:
  - Recognizing the endpoint of a production
  - Finding the length of a production (RHS)
  - Finding the corresponding nonterminal (the LHS of the production)

45

Data structures

- At each state, given the next token,
  - A *goto table* defines the successor state
  - An *action table* defines whether to
    - *shift* – put the next state and token on the stack
    - *reduce* – an RHS is found; process the production
    - *terminate* – parsing is complete

Simple example

1. P → S
2. S → x ; S
3. S → e

		Symbol					Action
		x	;	e	P	S	
State	0	1		3		5	Shift
	1		2				Shift
	2	1		3		4	Shift
	3						Reduce 3
	4						Reduce 2
	5						Accept

Parsing using an LR(0) parser

- Basic idea: parser keeps track, simultaneously, of all possible productions that *could be matched* given what it's seen so far. When it sees a full production, match it.
- Maintain a *parse stack* that tells you what state you're in
  - Start in state 0
- In each state, look up in action table whether to:
  - *shift*: consume a token off the input; look for next state in goto table; push next state onto stack
  - *reduce*: match a production; pop off as many symbols from state stack as seen in production; look up where to go according to non-terminal we just matched; push next state onto stack
  - *accept*: terminate parse

Example

- Parse “x ; x ; e”

Step	Parse Stack	Remaining Input	Parser Action
1	0	x ; x ; e	Shift 1
2	0 1	; x ; e	Shift 2
3	0 1 2	x ; e	Shift 1
4	0 1 2 1	; e	Shift 2
5	0 1 2 1 2	e	Shift 3
6	0 1 2 1 2 3		Reduce 3 (goto 4)
7	0 1 2 1 2 4		Reduce 2 (goto 4)
8	0 1 2 4		Reduce 2 (goto 5)
9	0 5		Accept

LR(k) parsers

- LR(0) parsers
  - No lookahead
  - Predict which action to take by looking only at the symbols currently on the stack
- LR(k) parsers
  - Can look ahead *k* symbols
  - Most powerful class of deterministic bottom-up parsers
  - LR(1) and variants are the most common parsers

50

Top-down vs. Bottom-up parsers

- Top-down parsers expand the parse tree in *pre-order*
  - Identify parent nodes before the children
- Bottom-up parsers expand the parse tree in *post-order*
  - Identify children before the parents
- Notation:
  - LL(1): Top-down derivation with 1 symbol lookahead
  - LL(k): Top-down derivation with k symbols lookahead
  - LR(1): Bottom-up derivation with 1 symbol lookahead

Abstract Syntax Trees

- Parsing recognizes a production from the grammar based on a sequence of tokens received from Lexer
- Rest of the compiler needs more info: a structural representation of the program construct
  - Abstract Syntax Tree or AST

52

Abstract Syntax Trees

- Are like parse trees but ignore certain details
- Example:

E -> E + E | (E) | int

String: 1 + (2 + 3)

Demo

Semantic Actions for Expressions

54

Review

- Scanners
  - Detect the presence of illegal tokens
- Parsers
  - Detect an ill-formed program
- Semantic actions
  - Last phase in the *front-end* of a compiler
  - Detect all other errors

What are these kind of errors?

55

What we cannot express using CFGs

- Examples:
  - Identifiers declared before their use (scope)
  - Types in an expression must be consistent
  - Number of formal and actual parameters of a function must match
  - Reserved keywords cannot be used as identifiers
  - etc.

  Depends on the language..

56

57

Semantic Records

- Data structures produced by semantic actions
- Associated with both non-terminals (code structures) and terminals (tokens/symbols)
- Build up semantic records by performing a bottom-up walk of the abstract syntax tree

58

Scope

- *Scope* of an identifier is the part of the program where the identifier is accessible
- Multiple scopes for same identifier name possible
- Static vs. Dynamic scope

exercise: what are the different scopes in Micro?

59

Types

- Static vs. Dynamic
- Type checking
- Type inference

60

Referencing identifiers

- What do we return when we see an identifier?
  - Check if it is in symbol table
  - Create new AST node with pointer to symbol table entry
  - Note: may want to directly store type information in AST (or could look up in symbol table each time)

62

Expressions Example

```
x + y + 5
```

Expressions Example

```
x + y + 5
```

identifier "x"

65

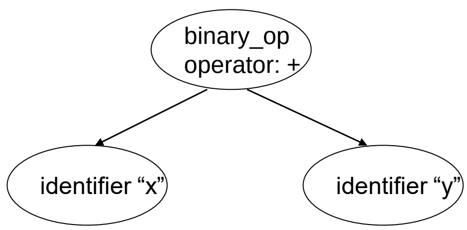
Expressions Example

x + y + 5

66

Expressions Example

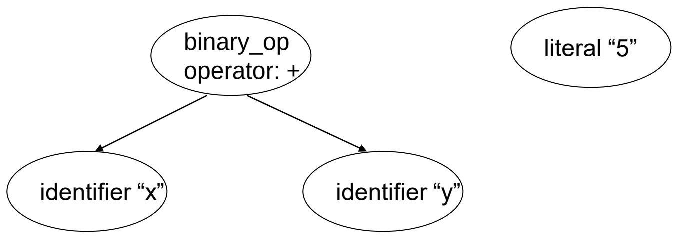
x + y + 5



67

Expressions Example

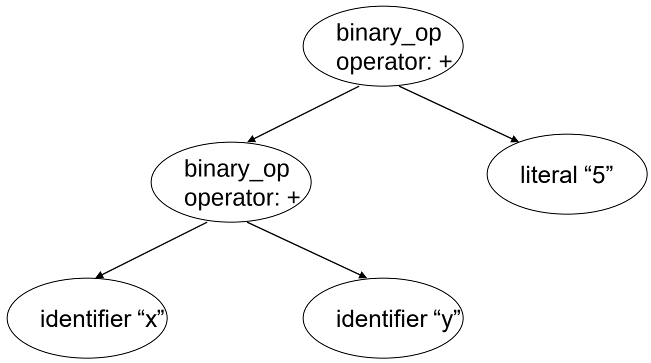
x + y + 5



68

Expressions Example

x + y + 5



69

Suggested Reading

- Alfred V. Aho, Monica S. Lam, Ravi Sethi and Jeffrey D.Ullman: Compilers: Principles, Techniques, and Tools, 2/E, AddisonWesley 2007
  - Chapter 4 (4.5, 4.6 (introduction)). Chapter 5 (5.3), Chapter 6 (6.1)
- Fisher and LeBlanc: Crafting a Compiler with C
  - Chapter 8 (Sections 8.1 to 8.3), Chapter 9 (9.1, 9.2.1 – 9.2.3)

Suggested Reading

- Alfred V. Aho, Monica S. Lam, Ravi Sethi and Jeffrey D.Ullman: Compilers: Principles, Techniques, and Tools, 2/E, AddisonWesley 2007
  - Chapter 4 (Sections: 4.1 to 4.4)
- Fisher and LeBlanc: Crafting a Compiler with C
  - Chapter 4, Chapter 5(Sections 5.1 to 5.5, 5.9)