

CS406: Compilers

Spring 2021

Week 5: Parsers

First Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{first}(S) = \{ \text{?} \}$

Think of all possible strings derivable from S .
Get the **first terminal symbol** in those strings
or λ if S derives λ

First Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{first}(S) = \{ x, y, c \}$

First Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{first}(S) = \{ x, y, c \}$
 $\text{first}(A) = \{ \text{?} \}$

First Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
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$\text{first}(A) = \{ x, y, c \}$

First Set - Example

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- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{first}(S) = \{ x, y, c \}$
 $\text{first}(A) = \{ x, y, c \}$
 $\text{first}(B) = \{ ? \}$

First Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{first}(S) = \{ x, y, c \}$
 $\text{first}(A) = \{ x, y, c \}$
 $\text{first}(B) = \{ b, \lambda \}$

Follow Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{follow}(S) = \{ \text{?} \}$

Think of all strings **possible in the language** having the form $\dots Sa \dots$. Get the **following terminal symbol** a after S in those strings or $\$$ if you get a string $\dots S\$$

Follow Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{follow}(S) = \{ \quad \}$

Follow Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{follow}(S) = \{ \quad \}$
 $\text{follow}(A) = \{ \text{?} \}$

Follow Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{follow}(S) = \{ \quad \}$

$\text{follow}(A) = \{ b, c \}$

e.g. $xaAbc\$$, $xaAc\$$

Follow Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{follow}(S) = \{ \quad \}$

$\text{follow}(A) = \{ b, c \}$ e.g. $xaAbc\$$, $xaAc\$$

What happens when you consider: $A \rightarrow xaA$ or $A \rightarrow yaA$?

Follow Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{follow}(S) = \{ \quad \}$

$\text{follow}(A) = \{ b, c \}$ e.g. $xaAbc\$$, $xaAc\$$

What happens when you consider: $A \rightarrow xaA$ or $A \rightarrow yaA$?

- You will get string of the form $A \Rightarrow^+ (xa)^+A$
- But we need strings of the form: $\dots Aa\dots$ or $\dots Ab\dots$ or $\dots Ac\dots$

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Follow Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{follow}(S) = \{ \quad \}$
 $\text{follow}(A) = \{ b, c \}$
 $\text{follow}(B) = \{ ? \}$

Follow Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

$\text{follow}(S) = \{ \quad \}$

$\text{follow}(A) = \{ b, c \}$

$\text{follow}(B) = \{ c \}$

Predict Set - Example

1) $S \rightarrow ABc\$$

2) $A \rightarrow xaA$

3) $A \rightarrow yaA$

4) $A \rightarrow c$

5) $B \rightarrow b$

6) $B \rightarrow \lambda$

$\text{Predict}(P) =$

$$\begin{cases} \text{First}(X_1 \dots X_m) & \text{if } \lambda \notin \text{First}(X_1 \dots X_m) \\ (\text{First}(X_1 \dots X_m) - \lambda) \cup \text{Follow}(A) & \text{otherwise} \end{cases}$$

$\text{Predict}(1) = \{ ? \} = \text{First}(ABc\$) \text{ if } \lambda \notin \text{First}(ABc\$)$

Predict Set - Example

- 1) S \rightarrow ABc\$
- 2) A \rightarrow xaA
- 3) A \rightarrow yaA
- 4) A \rightarrow c
- 5) B \rightarrow b
- 6) B \rightarrow λ

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | | | | | | |
| B | | | | | | |

Predict (1) = { x, y, c }

Predict Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | | | | | | |
| B | | | | | | |

Predict (1) = { x, y, c }

Predict (2) = { ? } = First(xaA) if $\lambda \notin \text{First}(xaA)$

Predict Set - Example

- 1) S \rightarrow ABc\$
- 2) A \rightarrow xaA
- 3) A \rightarrow yaA
- 4) A \rightarrow c
- 5) B \rightarrow b
- 6) B \rightarrow λ

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | | | | | |
| B | | | | | | |

Predict (1) = { x, y, c }

Predict (2) = { x }

Predict Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | | | | | |
| B | | | | | | |

Predict (1) = { x, y, c }

Predict (2) = { x }

Predict (3) = { ? } = First(yaA) if $\lambda \notin \text{First}(yaA)$

Predict Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | | |
| B | | | | | | |

Predict (1) = { x, y, c }

Predict (2) = { x }

Predict (3) = { y }

Predict Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | | |
| B | | | | | | |

Predict (1) = { x, y, c }

Predict (2) = { x }

Predict (3) = { y }

Predict (4) = { ? } = First(c) if $\lambda \notin \text{First}(c)$

Predict Set - Example

- 1) S \rightarrow ABc\$
- 2) A \rightarrow xaA
- 3) A \rightarrow yaA
- 4) A \rightarrow c
- 5) B \rightarrow b
- 6) B \rightarrow λ

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | | | |

- Predict (1) = { x, y, c }
- Predict (2) = { x }
- Predict (3) = { y }
- Predict (4) = { c }

Predict Set - Example

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | | | |

Predict (1) = { x, y, c }

Predict (2) = { x }

Predict (3) = { y }

Predict (4) = { c }

Predict (5) = { ? } = First(b) if $\lambda \notin \text{First}(b)$

Predict Set - Example

- 1) S \rightarrow ABc\$
- 2) A \rightarrow xaA
- 3) A \rightarrow yaA
- 4) A \rightarrow c
- 5) B \rightarrow b
- 6) B \rightarrow λ

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | | |

- Predict (1) = { x, y, c }
- Predict (2) = { x }
- Predict (3) = { y }
- Predict (4) = { c }
- Predict (5) = { b }

Predict Set - Example

- 1) S \rightarrow ABc\$
- 2) A \rightarrow xaA
- 3) A \rightarrow yaA
- 4) A \rightarrow c
- 5) B \rightarrow b
- 6) B \rightarrow λ

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | | |

Predict (1) = { x, y, c }

Predict (2) = { x }

Predict (3) = { y }

Predict (4) = { c }

Predict (5) = { b }

Predict (6) = { ? }

Predict(P) =

$$\begin{cases} \text{First}(X_1 \dots X_m) & \text{if } \lambda \notin \text{First}(X_1 \dots X_m) \\ (\text{First}(X_1 \dots X_m) - \lambda) \cup \text{Follow}(A) & \text{otherwise} \end{cases}$$

= First(λ) ?

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Predict Set - Example

- 1) S \rightarrow ABc\$
- 2) A \rightarrow xaA
- 3) A \rightarrow yaA
- 4) A \rightarrow c
- 5) B \rightarrow b
- 6) B \rightarrow λ

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | | |

Predict (1) = { x, y, c }

Predict (2) = { x }

Predict (3) = { y }

Predict (4) = { c }

Predict (5) = { b }

Predict (6) = { ? }

Predict(P) =

$$\begin{cases} \text{First}(X_1 \dots X_m) & \text{if } \lambda \notin \text{First}(X_1 \dots X_m) \\ (\text{First}(X_1 \dots X_m) - \lambda) \cup \text{Follow}(A) & \text{otherwise} \end{cases}$$

= ~~First(λ)~~ ? Follow(B)

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Predict Set - Example

- 1) S \rightarrow ABc\$
- 2) A \rightarrow xaA
- 3) A \rightarrow yaA
- 4) A \rightarrow c
- 5) B \rightarrow b
- 6) B \rightarrow λ

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

- Predict (1) = { x, y, c }
- Predict (2) = { x }
- Predict (3) = { y }
- Predict (4) = { c }
- Predict (5) = { b }
- Predict (6) = { c }

Computing Parse-Table

- 1) $S \rightarrow ABc\$$
- 2) $A \rightarrow xaA$
- 3) $A \rightarrow yaA$
- 4) $A \rightarrow c$
- 5) $B \rightarrow b$
- 6) $B \rightarrow \lambda$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

first (S) = {x, y, c}
first (A) = {x, y, c}
first(B) = {b, λ }

follow (S) = {}
follow (A) = {b, c}
follow(B) = {c}

P(1) = {x,y,c}
P(2) = {x}
P(3) = {y}
P(4) = {c}
P(5) = {b}
P(6) = {c}

Parsing using parse table and a stack-based model (non-recursive)

Top-Down Parsing - Example

string: xacc\$

| Stack | Rem. Input | Action |
|-------|------------|--------|
| ? | xacc\$ | ? |

What do you put on the stack?

Top-Down Parsing - Example

string: xacc\$

| Stack | Rem. Input | Action |
|-------|------------|--------|
| ? | xacc\$ | ? |

What do you put on the stack? – strings that you derive

Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|--------|------------|--------|
| S | xacc\$ | ? |

Top-down parsing. So, start with S.

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|--------|------------|--------|
| S | xacc\$ | ? |

Top-down parsing. So, start with S.

What action do you take when stack-top has symbol S and the string to be matched has terminal x in front?

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|--------|------------|----------------------|
| S | xacc\$ | Predict(1) S → ABc\$ |

Top-down parsing. So, start with S.

What action do you take when stack-top has symbol S and the string to be matched has terminal x in front? – consult parse table

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

Stack*

S
ABc\$

Rem. Input

xacc\$

Action

Predict(1) S → ABc\$

| | | | | | | |
|---|---|---|---|---|---|----|
| | x | y | a | b | c | \$ |
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|--------|------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| A Bc\$ | xacc\$ | |

What action do you take when stack-top has **symbol A** and the string to be matched has **terminal x** in front? – consult parse table

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|--------|------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |

What action do you take when stack-top has symbol A and the string to be matched has terminal x in front? – consult parse table

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|---------|------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | | |

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|-----------------|----------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| x aABc\$ | x acc\$ | ? |

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|-----------------|----------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| x aABc\$ | x acc\$ | match(x) |

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|---------|------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | xacc\$ | match(x) |
| aABc\$ | aacc\$ | match(a) |

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| Stack* | Rem. Input | Action |
|---------------|--------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | xacc\$ | match(x) |
| aABc\$ | acc\$ | match(a) |
| A Bc\$ | c c\$ | ? |

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

| Stack* | Rem. Input | Action |
|---------------------|--------------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | xacc\$ | match(x) |
| aABc\$ | acc\$ | match(a) |
| A ¹ Bc\$ | c ¹ c\$ | Predict(4) A->c |

* Stack top is on the left-side (first symbol) of the column

Top-Down Parsing - Example

string: xacc\$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

| Stack* | Rem. Input | Action |
|---------|------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | xacc\$ | match(x) |
| aABc\$ | acc\$ | match(a) |
| ABc\$ | cc\$ | Predict(4) A->c |
| cBc\$ | | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

| Stack* | Rem. Input | Action |
|---------|------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | xacc\$ | match(x) |
| aABc\$ | acc\$ | match(a) |
| ABc\$ | cc\$ | Predict(4) A->c |
| cBc\$ | cc\$ | ? |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

| Stack* | Rem. Input | Action |
|---------|------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | xacc\$ | match(x) |
| aABc\$ | acc\$ | match(a) |
| ABc\$ | cc\$ | Predict(4) A->c |
| cBc\$ | cc\$ | match(c) |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

| Stack* | Rem. Input | Action |
|--------------|-------------|---------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | xacc\$ | match(x) |
| aABc\$ | acc\$ | match(a) |
| ABc\$ | cc\$ | Predict(4) A->c |
| cBc\$ | cc\$ | match(c) |
| B c\$ | c \$ | ? |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

| Stack* | Rem. Input | Action |
|---------|------------|--------------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | xacc\$ | match(x) |
| aABc\$ | acc\$ | match(a) |
| ABc\$ | cc\$ | Predict(4) A->c |
| cBc\$ | cc\$ | match(c) |
| Bc\$ | c\$ | Predict(6) B-> λ |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

string: xacc\$

| | x | y | a | b | c | \$ |
|---|---|---|---|---|---|----|
| S | 1 | 1 | | | 1 | |
| A | 2 | 3 | | | 4 | |
| B | | | | 5 | 6 | |

| Stack* | Rem. Input | Action |
|---------|------------|--------------------------|
| S | xacc\$ | Predict(1) S->ABc\$ |
| ABc\$ | xacc\$ | Predict(2) A->xaA |
| xaABc\$ | xacc\$ | match(x) |
| aABc\$ | acc\$ | match(a) |
| ABc\$ | cc\$ | Predict(4) A->c |
| cBc\$ | cc\$ | match(c) |
| Bc\$ | c\$ | Predict(6) B-> λ |
| c\$ | | |

* Stack top is on the left-side (first symbol) of the column

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Top-Down Parsing - Example

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| c\$ | c\$ | ? |

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Top-Down Parsing - Example

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| cBc\$ | cc\$ | match(c) |
| Bc\$ | c\$ | Predict(6) B-> λ |
| c\$ | c\$ | match(c) |
| \$ | \$ | Done! |

* Stack top is on the left-side (first symbol) of the column

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Identifying LL(1) Grammar

- What we saw was an example of LL(1) Grammar
 - Scan input **L**eft-to-right, produce **L**eft-most derivation with 1 symbol look-ahead

Identifying LL(1) Grammar

- What we saw was an example of LL(1) Grammar
 - Scan input **Left-to-right**, produce **Left-most** derivation with 1 symbol look-ahead
- Not all Grammars are LL(1)

A Grammar is LL(1) iff for a production $A \rightarrow \alpha \mid \beta$, where α and β are distinct:

1. For no terminal a do both α and β derive strings beginning with a (i.e. no common prefix)
2. At most one of α and β can derive an empty string
3. If $\beta \xRightarrow{*} \epsilon$, then α does not derive any string beginning with a terminal in $\text{Follow}(A)$. If $\alpha \xRightarrow{*} \epsilon$, then β does not derive any string beginning with a terminal in $\text{Follow}(A)$

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Example (Left Factoring)

- Consider
 - $\langle \text{stmt} \rangle \rightarrow \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt list} \rangle \text{ endif}$
 - $\langle \text{stmt} \rangle \rightarrow \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt list} \rangle \text{ else } \langle \text{stmt list} \rangle \text{ endif}$
- This is not LL(1) (why?)
- We can turn this in to
 - $\langle \text{stmt} \rangle \rightarrow \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{stmt list} \rangle \langle \text{if suffix} \rangle$
 - $\langle \text{if suffix} \rangle \rightarrow \text{endif}$
 - $\langle \text{if suffix} \rangle \rightarrow \text{else } \langle \text{stmt list} \rangle \text{ endif}$

Example (Left Factoring)

- Consider

`<stmt> → if <expr> then <stmt list> endif`

`<stmt> → if <expr> then <stmt list> else <stmt list> endif`

- This is not LL(1) (why?)

- We can turn this in to

`<stmt> → if <expr> then <stmt list> <if suffix>`

`<if suffix> → endif`

`<if suffix> → else <stmt list> endif`

Left Factoring

$$A \rightarrow \alpha \beta \mid \alpha \mu$$

$$A \rightarrow \alpha N$$
$$N \rightarrow \beta$$
$$N \rightarrow \mu$$

Left recursion

- *Left recursion* is a problem for LL(1) parsers
 - LHS is also the first symbol of the RHS
- Consider:
$$E \rightarrow E + T$$
- What would happen with the stack-based algorithm?

Left recursion

- *Left recursion* is a problem for LL(1) parsers
 - LHS is also the first symbol of the RHS

- Consider:

$$E \rightarrow E + T$$

- What would happen with the stack-based algorithm?

E
E + T
E + T + T
E + T + T + T
...

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Eliminating Left Recursion

$$A \rightarrow A\alpha \mid \beta$$

$$A \rightarrow NT$$
$$N \rightarrow \beta$$
$$T \rightarrow \alpha T$$
$$T \rightarrow \lambda$$

Eliminating Left Recursion

$E \rightarrow E + T$



$E \rightarrow E_1 \text{ Etail}$

$E_1 \rightarrow T$

$\text{Etail} \rightarrow + T \text{ Etail}$

$\text{Etail} \rightarrow \lambda$

LL(k) parsers

- Can look ahead more than one symbol at a time
 - k -symbol lookahead requires extending first and follow sets
 - 2-symbol lookahead can distinguish between more rules:
 $A \rightarrow ax \mid ay$
- More lookahead leads to more powerful parsers
- What are the downsides?

Are all grammars LL(k)?

- No! Consider the following grammar:

$$\begin{aligned} S &\rightarrow E \\ E &\rightarrow (E + E) \\ E &\rightarrow (E - E) \\ E &\rightarrow x \end{aligned}$$

- When parsing E, how do we know whether to use rule 2 or 3?
 - Potentially unbounded number of characters before the distinguishing '+' or '-' is found
 - No amount of lookahead will help!

LL(k)? - Example

string: ((x+x))\$

- 1) S -> E
- 2) E -> (E+E)
- 3) E -> (E-E)
- 4) E -> x

| Stack* | Rem. Input | Action |
|--------|------------|---------------------------|
| S | ((x+x))\$ | Predict(1) S->E |
| E | | Predict(2) or Predict(3)? |

LL(1)

| | (| + | - |) | x |
|---|-----|---|---|---|---|
| S | 1 | | | | 1 |
| E | 2,3 | | | | 4 |

LL(2)

| | ((| +(| -(|)\$ | (x |
|---|-----|----|----|-----|----|
| S | 1 | | | | 1 |
| E | 2,3 | | | | 4 |

In real languages?

- Consider the if-then-else problem
- if x then y else z
- Problem: else is optional
- if a then if b then c else d
 - Which if does the else belong to?
- This is analogous to a “bracket language”: $[^i]^j$ ($i \geq j$)

$S \rightarrow [S C$

$S \rightarrow \lambda$

$C \rightarrow]$

$C \rightarrow \lambda$

$[[]$ can be parsed: $SS\lambda C$ or $SSC\lambda$
(it's ambiguous!)

Solving the if-then-else problem

- The ambiguity exists at the language level. To fix, we need to define the semantics properly
 - “[” matches nearest unmatched “[”
 - This is the rule C uses for if-then-else
 - What if we try this?

$S \rightarrow [S$
 $S \rightarrow SI$
 $SI \rightarrow [SI]$
 $SI \rightarrow \lambda$

This grammar is still not LL(1)
(or LL(k) for any k!)

Two possible fixes

- If there is an ambiguity, prioritize one production over another
- e.g., if C is on the stack, always match "]" before matching "λ"

$$\begin{array}{ll} S & \rightarrow [S C \\ S & \rightarrow \lambda \\ C & \rightarrow] \\ C & \rightarrow \lambda \end{array}$$

- Another option: change the language!
- e.g., all if-statements need to be closed with an endif

$$\begin{array}{ll} S & \rightarrow \text{if } S \text{ E} \\ S & \rightarrow \text{other} \\ E & \rightarrow \text{else } S \text{ endif} \\ E & \rightarrow \text{endif} \end{array}$$

Parsing if-then-else

- What if we don't want to change the language?
 - C does not require { } to delimit single-statement blocks
- To parse if-then-else, *we need to be able to look ahead at the entire rhs of a production* before deciding which production to use
 - In other words, we need to determine how many "]" to match before we start matching "["s
- *LR parsers* can do this!

Top-down vs. Bottom-up parsers

- Top-down parsers expand the parse tree in *pre-order*
 - Identify parent nodes before the children
- Bottom-up parsers expand the parse tree in *post-order*
 - Identify children before the parents
- Notation:
 - LL(1): Top-down derivation with 1 symbol lookahead
 - LL(k): Top-down derivation with k symbols lookahead
 - LR(1): Bottom-up derivation with 1 symbol lookahead

LR Parsers

- Parser which does a Left-to-right, Right-most derivation
- Rather than parse top-down, like LL parsers do, parse bottom-up, starting from leaves

Example:

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid id$

String: `id*id`

Demo

LR Parsers

- Basic idea: put tokens on a stack until an entire production is found
 - **shift** tokens onto the stack. At any step, keep the set of productions that could generate the read-in token
 - **reduce** the RHS of recognized productions to the corresponding non-terminal on the LHS of the production. Replace the RHS tokens on the stack with the LHS non-terminal.
- Issues:
 - Recognizing the endpoint of a production
 - Finding the length of a production (RHS)
 - Finding the corresponding nonterminal (the LHS of the production)

Data structures

- At each state, given the next token,
 - A *goto table* defines the successor state
 - An *action table* defines whether to
 - *shift* – put the next state and token on the stack
 - *reduce* – an RHS is found; process the production
 - *terminate* – parsing is complete

Simple example

1. $P \rightarrow S$
2. $S \rightarrow x ; S$
3. $S \rightarrow e$

| | | Symbol | | | | | Action |
|-------|---|--------|---|---|---|---|----------|
| | | x | ; | e | P | S | |
| State | 0 | 1 | | 3 | | 5 | Shift |
| | 1 | | 2 | | | | Shift |
| | 2 | 1 | | 3 | | 4 | Shift |
| | 3 | | | | | | Reduce 3 |
| | 4 | | | | | | Reduce 2 |
| | 5 | | | | | | Accept |

Parsing using an LR(0) parser

- Basic idea: parser keeps track, simultaneously, of all possible productions that *could be matched* given what it's seen so far. When it sees a full production, match it.
- Maintain a *parse stack* that tells you what state you're in
 - Start in state 0
- In each state, look up in action table whether to:
 - *shift*: consume a token off the input; look for next state in goto table; push next state onto stack
 - *reduce*: match a production; pop off as many symbols from state stack as seen in production; look up where to go according to non-terminal we just matched; push next state onto stack
 - *accept*: terminate parse

Example

- Parse “x ; x ; e”

| Step | Parse Stack | Remaining Input | Parser Action |
|------|-------------|-----------------|-------------------|
| 1 | 0 | x ; x ; e | Shift 1 |
| 2 | 0 1 | ; x ; e | Shift 2 |
| 3 | 0 1 2 | x ; e | Shift 1 |
| 4 | 0 1 2 1 | ; e | Shift 2 |
| 5 | 0 1 2 1 2 | e | Shift 3 |
| 6 | 0 1 2 1 2 3 | | Reduce 3 (goto 4) |
| 7 | 0 1 2 1 2 4 | | Reduce 2 (goto 4) |
| 8 | 0 1 2 4 | | Reduce 2 (goto 5) |
| 9 | 0 5 | | Accept |

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LR(k) parsers

- LR(0) parsers
 - No lookahead
 - Predict which action to take by looking only at the symbols currently on the stack
- LR(k) parsers
 - Can look ahead k symbols
 - Most powerful class of deterministic bottom-up parsers
 - LR(1) and variants are the most common parsers

Top-down vs. Bottom-up parsers

- Top-down parsers expand the parse tree in *pre-order*
 - Identify parent nodes before the children
- Bottom-up parsers expand the parse tree in *post-order*
 - Identify children before the parents
- Notation:
 - LL(1): Top-down derivation with 1 symbol lookahead
 - LL(k): Top-down derivation with k symbols lookahead
 - LR(1): Bottom-up derivation with 1 symbol lookahead

Abstract Syntax Trees

- Parsing recognizes a production from the grammar based on a sequence of tokens received from Lexer
- Rest of the compiler needs more info: a structural representation of the program construct
 - Abstract Syntax Tree or AST

Abstract Syntax Trees

- Are like parse trees but ignore certain details
- Example:

$E \rightarrow E + E \mid (E) \mid \text{int}$

String: 1 + (2 + 3)

Demo

Semantic Actions for Expressions

Review

- Scanners
 - Detect the presence of illegal tokens
- Parsers
 - Detect an ill-formed program
- Semantic actions
 - Last phase in the *front-end* of a compiler
 - Detect all other errors

What are these kind of errors?

What we cannot express using CFGs

- Examples:
 - Identifiers declared before their use (scope)
 - Types in an expression must be consistent
 - Number of formal and actual parameters of a function must match
 - Reserved keywords cannot be used as identifiers
 - etc.

Depends on the language..



Semantic Records

- Data structures produced by semantic actions
- Associated with both non-terminals (code structures) and terminals (tokens/symbols)
- Build up semantic records by performing a bottom-up walk of the abstract syntax tree

Scope

- Scope of an identifier is the part of the program where the identifier is accessible
- Multiple scopes for same identifier name possible
- Static vs. Dynamic scope

exercise: what are the different scopes in Micro?

Types

- Static vs. Dynamic
- Type checking
- Type inference

Referencing identifiers

- What do we return when we see an identifier?
 - Check if it is ⁱⁿ symbol table
 - Create new [^]AST node with pointer to symbol table entry
 - Note: may want to directly store type information in AST (or could look up in symbol table each time)





Expressions Example

$$x + y + 5$$

Expressions Example

$x + y + 5$

identifier "x"

Expressions Example

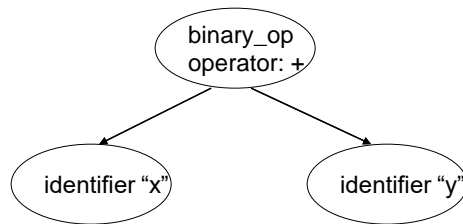
$x + y + 5$

identifier "x"

identifier "y"

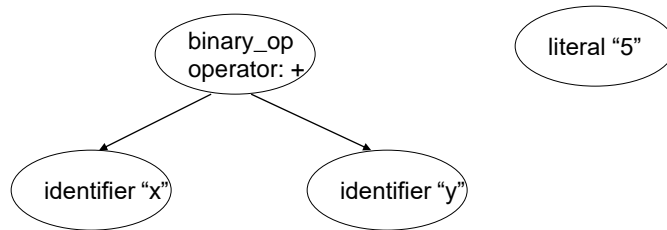
Expressions Example

x + y + 5



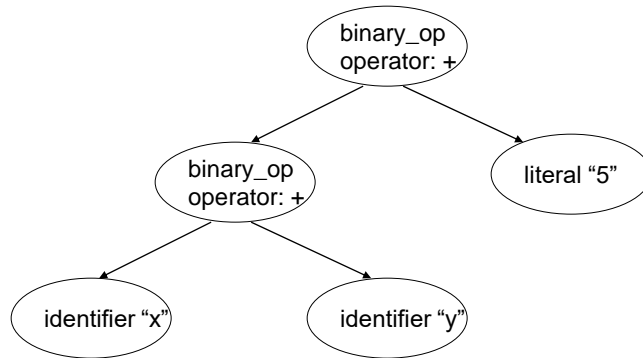
Expressions Example

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Expressions Example

x + y + 5



Suggested Reading

- Alfred V. Aho, Monica S. Lam, Ravi Sethi and Jeffrey D. Ullman:
Compilers: Principles, Techniques, and Tools, 2/E, AddisonWesley
2007
 - Chapter 4 (4.5, 4.6 (introduction)). Chapter 5 (5.3), Chapter 6 (6.1)
- Fisher and LeBlanc: Crafting a Compiler with C
 - Chapter 8 (Sections 8.1 to 8.3), Chapter 9 (9.1, 9.2.1 – 9.2.3)

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- Fisher and LeBlanc: Crafting a Compiler with C
 - Chapter 4, Chapter 5(Sections 5.1 to 5.5, 5.9)