

## Summaries

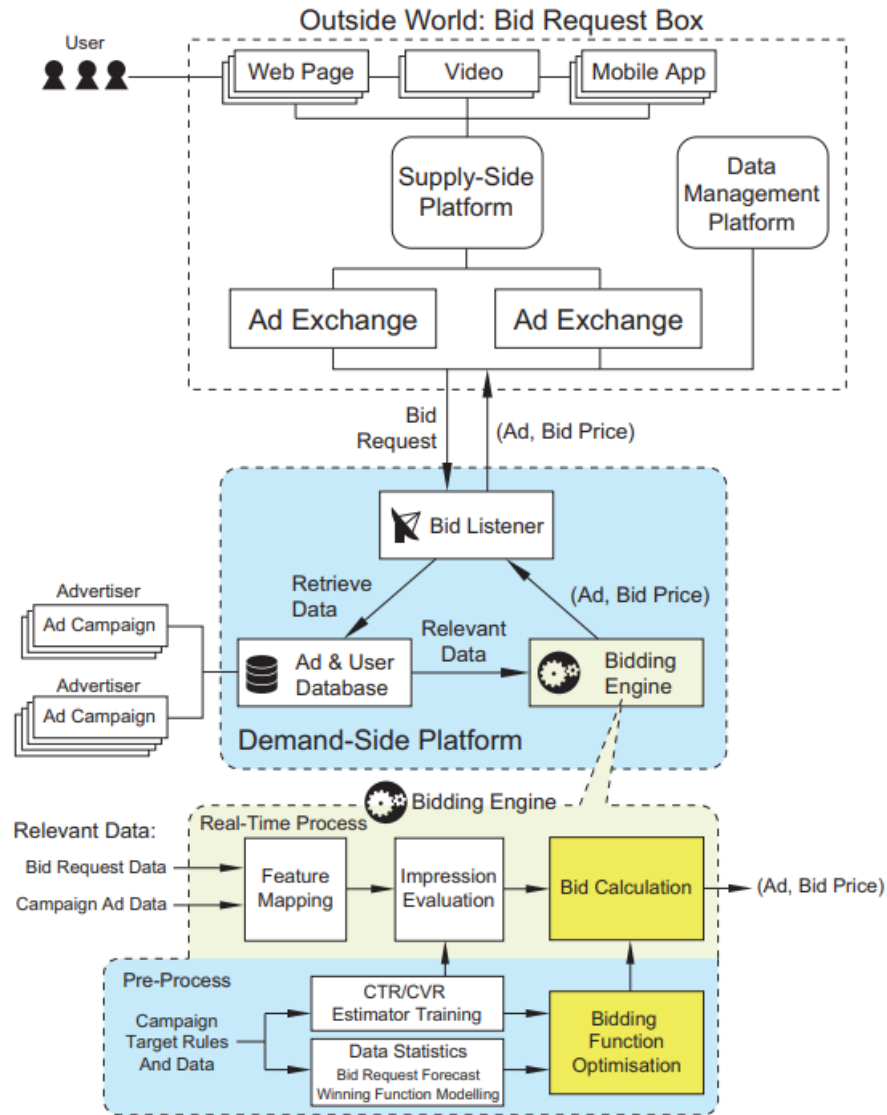
1. *Optimal Auctions* - Provide a theoretical base on how to set an optimal reserve price such that it maximizes the revenue for the publisher.
2. *Optimal Auction Design* - Provide mathematical proof on how to design an optimal auction mechanism that maximizes the revenue for the publisher.
3. *Yield Optimization of Display Advertising with Ad Exchange* - A practical implementation of the bid-policy algorithm based on Optimal Auctions and Optimal Auction Design.
4. *Reserve Prices in Internet Advertising Auctions: A Field Experiment* - A field study on Yahoo! sponsored search engine that shows the importance and practicality of setting an optimal reserve price in increasing the revenue.
5. *Pricing Analysis in Online Auctions using Clustering and Regression Tree Approach* - Show the improvements in predicting the end-price in auctions using clustering and regression trees.
6. *Multiple Linear Regression with Kalman Filter for Predicting End Prices of Online Auctions* - Show that MLRKF model effectively reduces training time and increases training accuracy in predicting end prices in auctions.
7. *An Empirical Study of Reserve Price Optimization in Real-Time Bidding* - Propose an algorithm OneShot for publisher revenue optimization in Real-Time Bidding (RTB) and compares its performance with several other commonly adopted algorithms.

## Terminology

- **User attributes (Impression attributes, user information)** - a vector of attributes  $U_i \in \mathcal{U}$  where  $\mathcal{U}$  is some finite subset of  $\mathbb{R}^M$  that uniquely identify users/impressions. For instance, the web address, user's demographics, device, operating system, etc. [1]
- **Placement qualities** - some statistics that measure the quality of the impression. For instance, click-through rate (CTR).  $Q_{i,a}$  represents the quality advertiser  $a$  would receive if the impression  $i$  is assigned to her. [1]
- **Bid** - each advertiser submits a bid  $p$  to online bidding platform that represents their private valuation of the item. Usually, we assume that bids are symmetric (drawn from the same distribution) and independent conditional on user attributes disclosed. Assuming  $p$  is drawn from c.d.f.  $F(p; u)$ .
- **Reserve price** - the minimum price to enter the online bidding platform. Only bids that are higher than this price can get involved in the auction.
- **Pricing function** - a function  $p^*$  that quotes a reserve price based on bid and potentially user attributes. [1]

- **Revenue** - the amount earned. In our case, we care about the revenue for the publisher.

## Our problem -- Real-time Bidding [11]:



**Figure 1: An illustration of a demand-side platform and its bidding engine in RTB display advertising.**

## Breakdown & Pros and Cons

### Bid-Price Policy with Dynamic Pricing $\mu^B$ [1]

- Core idea:

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**Policy 2** Bid-Price Policy with Dynamic Pricing  $\mu^B$ .

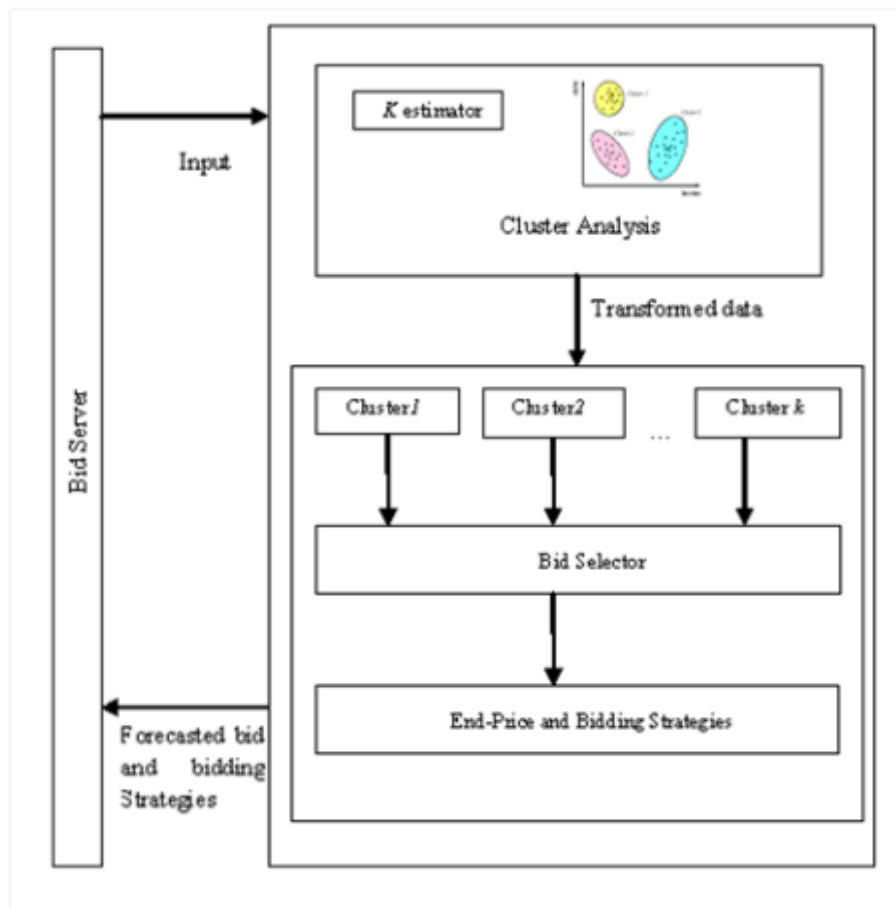
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- 1: Observe state  $(n, x)$ , and the vector of attributes  $u_n$ .
  - 2: Determine the vector of placement qualities  $q_n$ .
  - 3: Let  $\mathcal{A}_n = \{a \in \mathcal{A} : x_{n,a} > 0\}$  be the set of ads yet to be satisfied.
  - 4: **if**  $\sum_{a \in \mathcal{A}} x_{n,a} < n$  **then**
  - 5:   Let  $a_n^* = \arg \max_{a \in \mathcal{A}_n \cup \{0\}} \{\gamma q_{n,a} - v_a\}$ .
  - 6:   Submit to AdX with price  $p^*(\gamma q_{n,a_n^*} - v_{a_n^*}; u_n)$ .
  - 7:   **if** impression rejected by AdX and  $a_n^* \neq 0$  **then** assign to advertiser  $a_n^*$ , **else** discard.
  - 8: **else**
  - 9:   Assign to advertiser  $a_n^* = \arg \max_{a \in \mathcal{A}_n} \{\gamma q_{n,a} - v_a\}$ .
  - 10: **end if**
- 

- Pros:
  - **Optimal**: asymptotically optimal (28) in the sense that it maximizes the revenue for the publisher.
- Cons:
  - **Input**: need to know user attributes, placement qualities, pricing functions in advance.
  - **Complexity**: have a very complicated tie-breaking rule (18). Have complicated math. Need to use advanced methods such as SAA (Sample Average Approximation) and SDM (sub-gradient descent method) in practice (24).
  - **Assumptions**: have a lot of assumptions. For example, the bids are drawn i.i.d. from  $F$ .
  - **Hyper-parameter tuning**: have to tune the hyper-parameter  $\gamma$ .

## Clustering and Regression [3]

- Core idea:



- Pros:
  - **Complexity:** clustering and regression can be implemented relatively easy with existing tools such as sklearn.
- Cons:
  - **Input:** clustering is done based on attributes for each auction (4).
  - **Relevance:** the article mainly talks about how to predict the end-price from a set of auctions, whereas in our problem, there is one auction and many bidders.
  - **Ambiguity:** it is not clear how the bid selector works and how it nominates the cluster (6). It is also not clear (to me) how the autonomous agent based system is developed.
  - **Optimality:** the optimal strategies mentioned in the article is optimal relative to other possible bidding strategies produced by trees. It is not rigorous whether it is the absolute optimal strategy.

## Multiple Linear Regression with Kalman Filter (MLRKF) [4]

- Core idea:

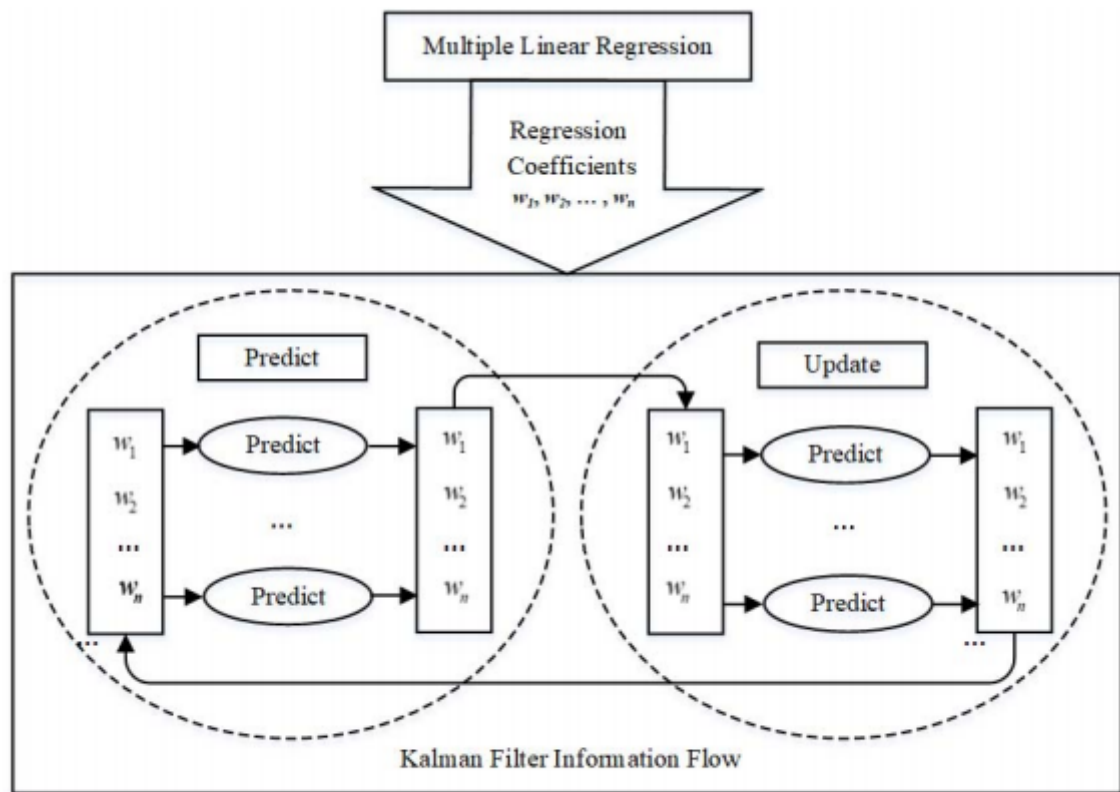


Figure 2. The framework of MLRKF prediction.

- Pros:
  - **Performance:** produce highly accurate results with less training data and lower time cost (182). It also reduces over-fitting.
  - **Dynamically update coefficient:** the MLRKFP model can dynamically optimize regression coefficients to produce more accurate results (183).
  - **Implementation:** multiple-linear regression and Kalman filter is relatively easy to implement using existing tools such as sklearn and pyfilter.
  - **Input:** only need to know  $x_1, x_2, \dots, x_n$  as descriptive variables, coefficients can be learned by regression (184).
- Cons:
  - **Complexity:** needs to have a very good understanding of multi-dimensional Kalman filter.

## OneShot [10]

- Core Idea:

$$\begin{cases} \alpha(t+1) = (1 - \epsilon^t \lambda_h) \alpha(t) & \text{if } \alpha(t) > b_1(t) \\ \alpha(t+1) = (1 + \epsilon^t \lambda_e) \alpha(t) & \text{if } b_1(t) \geq \alpha(t) \geq b_2(t) \\ \alpha(t+1) = (1 + \epsilon^t \lambda_l) \alpha(t) & \text{if } b_2(t) > \alpha(t) \end{cases}$$

- Pros:
  - **Complexity:** the algorithm is easy with only three lines of mathematical equations.
  - **Performance:** out-performed than a variety of other algorithms (9).
  - **Relevance:** it is an algorithm for real-time bidding (RTB), which closely relates to our problem.
- Cons:
  - **Hyper-parameter tuning:** in practice, one needs to tune 4 parameters based on the training data.
  - **Assumptions:** needs to assume the distribution of bids, which can lead to poor performance (9).

## Comparison

ALGORITHMS	PARAMETERS	UPDATE BETWEEN AUCTIONS
Bid-Price Policy with Dynamic Pricing $\mu^B$ [1]	user attributes, placement qualities, pricing functions	No
Clustering and Regression [3]	$k$ , Bid Selector, attributes for an auction	No
Multiple Linear Regression with Kalman Filter (MLRKf) [4]	data features (188)	Yes
OneShot [10]	$\epsilon, \lambda_h, \lambda_e, \lambda_l$ (5)	Yes

## Literature Review

### Optimal Auctions

Goals:

1. what form does the competition among the few buyers take under the most common auction procedures? How is the sale price determined?
2. by what means can the seller best exploit his monopoly position? For example, would it be more profitable for the seller to require payment not only by the highest bidder, but also by those with lower ranked bids?

Assumptions:

- A **single seller** with reservation value  $v$ , faces  **$n$  potential buyers**, where buyer  $i$  holds reservation value  $v_i$ ,  $i = 1, \dots, n$  (381).
- The reservation values of the parties are **independent and identically distributed**, drawn from the **common** distribution  $F$  (381).
- There is a common equilibrium bidding strategy in which each buyer makes a bid  $b$ , which is a strictly **increasing function** of his reservation value  $u$  (383).
- A **single indivisible good** is to be sold to the highest bidder (389).
- The greater a bidder's reservation value the more he will bid for the good (389).
- Buyer roles are symmetrical (i.e., buyer values are drawn from a common distribution) and each buyer is **risk neutral** (389).
- Buyer values are independent (389).
- Each buyer is informed of his own reservation price and, more important, that his price conveys no information about any other buyer's value (390).

Core ideas:

- if the assumptions hold, then the **reserve price** for the seller should be set as (385):

**PROPOSITION 3:** *If the IID assumption holds and buyers are risk neutral, the members of the family  $\mathcal{Q}$  of auction rules, which maximize the expected gain of the seller are those for which the reservation value  $v_*$ , below which it is not worthwhile bidding, satisfies*

$$v_* = v_0 + 1 - F(v_*)/F'(v_*)$$

*independent of the number of buyers.*

- The high and second bid auctions with reserve price  $b_0 = v_*$  are both optimal in the family of auctions.
- In asymmetrical model (where the reservation prices of the buyers are no longer drawn from a **common** distribution  $F$ ), The assignment of the good and the appropriate buyer payment will depend not only on the list of offers, but also on

the **identities** of the buyers who submit the bids. In short, an optimal auction under asymmetric conditions violates the principle of buyer anonymity (389).

- Under the assumption of risk neutrality, the seller can do no better than to employ the second bid auction with an optimal reserve price. In this auction, buyers will have no difficulty formulating an optimal bidding strategy. Nor need they know the form of the distribution function  $F(u)$ . Against any distribution of opponents' bids, each buyer's dominant strategy is to **bid his reservation value** (389).
- The optimal reserve price does not depend on the number of buyers (390).
- Assuming that the reservation values (buyers' valuation of the good) follow a uniform distribution and that the seller's valuation of the object  $v_0 = 0$ , then the optimal price is  $1/2$ . (386).

Limitations:

- can't deal with multiple goods (389).
- can't deal with a single divisible good (389).
- The derivation of the class of optimal auctions relied explicitly on the existence of a common equilibrium bidding strategy. Without this, these propositions no longer hold (389).

## Optimal Auction Design

Goal:

- Construct optimal auctions for a wide class of sellers' auction design problems and prove that no other auction mechanism can give higher expected utility to the user (58).

Assumptions:

- one seller, a single object to sell,  $n$  bidders (59).
- for each bidder, there is some quantity  $t_i$  which is  $i$ 's **value estimate** for the object (59).
- the seller's uncertainty about the value estimate of bidder  $i$  can be described by a continuous probability distribution over a finite interval (59).
- the value estimates of the  $n$  bidders are stochastically independent random variables (59).
- the seller has no private information about the object, so that  $t_0$  is known to all the bidders (59).

Core ideas:



- **preference uncertainty vs. quality uncertainty:** if there are quality uncertainties, then bidder  $i$  might tend to revise his valuation of the object after learning about other bidders' value estimates. That is, if  $i$  learned that  $t_j$  was very low, suggesting that  $j$  had received discouraging information about the quality of the object, then  $i$  might honestly revise downward his assessment of how much he should be willing to pay for the object (60).
- **with quality uncertainty:** the optimal reserve price can be set as:

$$c_i(t_i) = t_i - e_i(t_i) - \frac{1 - F_i(t_i)}{f_i(t_i)}$$

- **optimal price in the general case (70):**

That is, the seller should offer to sell the object at the price

$$\bar{c}_1^{-1}(t_0) = \min\{s_1 \mid \bar{c}_1(s_1) \geq t_0\},$$

and he should keep the object if the bidder is unwilling to pay this price.

## Relationships with Optimal Auctions

- **Optimal Auctions** assumes **pure preference uncertainty**, while **Optimal Auction Design** also adds in **quality uncertainty**, which gives rise to the **revision effect functions**  $e_i$ . Without revision effect functions, two are essentially the same.
- **Optimal Auctions** focuses more on determining the reserve price, while **Optimal Auction Design** focuses on mathematical proof of designing optimal auction mechanisms.

## Yield Optimization of Display Advertising with Ad Exchange

Goal:

- Formalize this combined optimization problem as a multi-objective stochastic control problem and derive an efficient policy for online ad allocation in settings with general joint distribution over placement quality and exchange prices (1).

Core ideas:

- **trade-off:** in presence of Ad Exchanges, publishers face the multi-objective problem of maximizing the overall placement quality of the impressions assigned to the reservations together with the total revenue obtained from AdX, while complying with the contractual obligations (2).

- **workflow:**

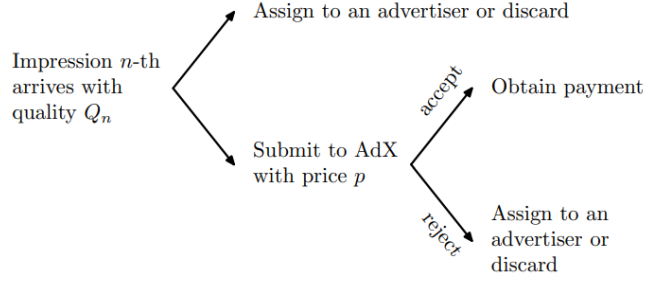


Figure 1 Publisher's decision tree for a new impression. In the absence of a fixed cost for accessing the exchange, the upper branch is never taken because the publisher is always better off testing AdX with a sufficiently high price that compensates for the yield loss of not assigning the impression.

- **objective:**  $\text{yield} = \text{revenue}(\text{AdX}) + \gamma \cdot \text{quality}(\text{advertisers})$ .
- **user information:** in practice, bids from the exchange may be correlated with the user information disclosed, and publishers maintain different estimates of the bids' distribution as a function of the attributes. We assume that conditional on the user information  $u \in \mathcal{U}$  bids are independent across impressions, and identically distributed according to a c.d.f.  $F(\cdot; u)$ . Hence, when the publisher discloses some information  $u$  the impression is accepted with probability  $1 - F(p; u) = \bar{F}(p; u)$  (9).
- **opportunity cost:** suppose the publisher has computed an opportunity cost  $c$  for selling this inventory in the exchange; that is, the publisher stands to gain  $c$  if the impression is given to a reservation advertiser (10).
- **objective function:**  $R(c; u) = \max_{p \geq 0} \bar{F}(p; u)p + F(p; u)c$
- **stochastic algorithm:**

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**Policy 2** Bid-Price Policy with Dynamic Pricing  $\mu^B$ .

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  - 10: **end if**
- 

- **tie-breaking:**

Given a vector of dual variables  $v \in \mathbb{R}^A$  the tie-breaking probabilities are obtained by solving the following assignment problem

$$\sum_{S \subseteq \mathcal{A}_0: a \in S} \mathbb{P}(S\text{-tie}) p_a(S) = \rho_a, \quad \forall a \in \mathcal{A}, \quad (5a)$$

- **sample-average approximation (SAA)**: approximating the underlying stochastic program via sampling, and then solving the approximate problem via a Sub-gradient Descent Method (SDM) (24).

Results:

- Interestingly, starting from the baseline case that disregards AdX ( $\gamma = \infty$ ), we observe that the revenue from AdX can be substantially increased by sacrificing a small fraction of the overall quality of the impressions assigned (27).
- Conversely, starting from the case that disregards the advertiser's quality ( $\gamma = 0$ ), the publisher can raise placement quality by a large amount at the expense of a small decrease in AdX's revenue.
- the Greedy Policy under-performs in the given instances with losses in yield of up to 70% (28).
- The Static Price Policy tends to under perform when the trade-off parameter  $\gamma$  is close zero, that is, when the publisher strives to maximize the revenue extracted from AdX. If the publisher fails to dynamically adjust the auctions' reserve price to take into account the opportunity cost of the impression, she can incur losses in yield of up to 69% (28).
- When the trade-off parameter  $\gamma$  is large, the exchange's revenue contribution to the yield is negligible, and the Static Price Policy is nearly optimal.

Limitations:

- learning in the case of unknown distributions.
- stochastic approximation of the deterministic problem.

## Reserve Prices in Internet Advertising Auctions: A Field Experiment\*

Goal:

- We present the results of a large field experiment on setting reserve prices in auctions for online advertisements, guided by the theory of optimal auction design suitably adapted to the sponsored search setting. Consistent with the theory, revenues increased substantially after the new reserve prices were introduced (1).

Core ideas:

- For the case with symmetric bidders, the answer is particularly elegant: the optimal mechanism can be implemented by a second-price auction with an

appropriately chosen reserve price (2).

- The revenues in the treatment group have increased substantially relative to the control group, showing that reserve prices in auctions can play an important role and that theory provides a useful guide for setting them (2).
- Remarkably, the optimal reserve price depends neither on the number of bidders nor on the number of available positions. The impact of reserve prices on revenues, however, depends critically on these parameters (6).
- In fact, in single-object auctions, reserve prices only play an important role if the number of bidders is small (6).
- In order to estimate optimal reserve prices for the experiment, it was assumed that bidders' values are drawn from log-normal distributions (7).

Results:

- Even with six bidders, reserve prices remain very important: the optimal reserve price improves the revenues by 45% (7).
- Similar to the example with the uniform distribution of values, the impact of optimal reserve prices on revenues in the GSP auction is substantial: with six bidders, setting the reserve price at zero instead of the optimal level results in the loss of 25% of revenues; and even with ten bidders, the loss is noticeable: 9% (7).
- The results of the experiment described in this paper show that setting appropriate reserve prices can lead to substantial increases in auction revenues (19).
- These results also show (to the best of our knowledge, for the first time) that the theory of optimal auction design is directly applicable in practice (19).

## Pricing Analysis in Online Auctions using Clustering and Regression Tree Approach

Goal:

- Show the improvements in the **end price prediction** using clustering and regression tree approach (1).

Core Ideas:

- Forecast bid amounts of buyers using historical data (2).
- Use a price forecasting agent (PFA) to forecast the end-price of an online auction for autonomous agent based system (2).
- The input auctions are clustered into groups of similar auctions based on their characteristics using k-means algorithm (2).

- Features are predefined attributes from the vast feature space of online auctions (5).
- K is estimated using ANOVA (2).
- Bid selector nominates the cluster after clustering (2).
- Finally, regression trees are employed to forecast the end-price and to design the bidding strategies (2).
- Optimal bidding strategies are designed by employing regression trees on each cluster and evaluating the model by test-sample cross validation approach (10).
- Forecasting is seen as a time-series problem and is solved using moving averages and exponential smoothing (3).
- Authors have used sequence mining to improve the decision mechanism (3).

Results:

- The improvement in root mean square error after clustering for each of the five clusters are 16.43%, 29.47%, 20.76%, 31.17% and 64.91% respectively. This indicated that the proposed price forecasting agent can improve the accuracy of the forecasted price for online auctions (8).

## Multiple Linear Regression with Kalman Filter for Predicting End Prices of Online Auctions

Goal:

- Show that a hybrid algorithm with multiple linear regression and Kalman filter can produce highly accurate results with less training data and lower time cost.

Core ideas:

- Multiple linear regression:  $y$  is the end price of an online auction i.  $x_1, x_2, \dots, x_i$  are the available explanatory variables, and they may be StartingBid, HitCount, AuctionAvgHitCount, and SellerItemAvg (184).
- Kalman filter is used to optimize weight coefficients, which is a parameter optimization problem (185).
- Let  $x$  denote the  $n$ -dimensional weight vector of a multiple linear regression algorithm. We assume that  $x$  follows Gaussian distribution with mean  $x_b$  and covariance  $P_b$ .
- More details are on page (186).

Results:

- MLRKf has the lowest training time compared to random forest regressor, SVM, RNN, Lasso (188).
- Table IV shows that the proposed model has the smallest MAE, RMSE, and MAPE values compared to MLRP, MLRRP, Lasso, Random Forest, SVM and RNN models (188).
- MLRKf prediction is better fitted for true end prices with dataset1 (189).

## An Empirical Study of Reserve Price Optimization in Real-Time Bidding

Goal:

- Report the first empirical study and live test of the reserve price optimization problem in the context of Real-Time Bidding (RTB) display advertising from an operational environment and show that the proposed game theory based OneShot algorithm performed the best.

Core Ideas:

- A reserve price defines the minimum that a publisher would accept from bidders. It reflects the publisher's private valuation of the inventory: bids will be discarded if they are below the reserve price (1).
- One iteration usually finishes in less than 100ms to achieve users' satisfactory (2).
- In practice, there are drawbacks of the optimal auction theory mostly due to the difficulty of learning bidders' private values, e.g.,  $F(x)$ . Firstly, a bidder could have a complex private value distribution for impressions.

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