Assignment 10

The program was analyzed twice: once with the --line flag, and onces without it.

Function based analysis

Without the --line flag, the program gets analyzed on a function level. The output first shows how much total time the program spent per function.

```
Each sample counts as 0.01 seconds.
 % cumulative self
                             self
                                     total
time seconds seconds calls ms/call ms/call name
49.78
        8.59 8.59
                       147 58.45 63.08 resid
                        168 22.86 26.87 psinv
22.25
        12.43 3.84
 8.40 13.88 1.45
                       147 9.87
                                     9.87 interp
     15.31 1.43 147 9.73 13.73 rprj3
16.55 1.24 131072 0.01 0.01 vranlc
 8.29
 7.19
 4.06
       17.26
               0.70
                              1.44
                                     4.00 norm2u3
                       485
 0.06
       17.27
               0.01
                                           mg3P.constprop.2
 0.00
       17.27 0.00 131642 0.00 0.00 randle
              0.00
                               0.00
 0.00
       17.27
                                      0.00 wtime_
                          4
```

The top time consuming function looks like this:

```
static void resid(void *ou, void *ov, void *or, int n1, int n2, int n3,
                  double a[4], int k)
 double (*u)[n2][n1] = (double (*)[n2][n1])ou;
 double (*v)[n2][n1] = (double (*)[n2][n1])ov;
 double (*r)[n2][n1] = (double (*)[n2][n1])or;
 int i3, i2, i1;
 double u1[M], u2[M];
 if (timeron) timer_start(T_resid);
 for (i3 = 1; i3 < n3-1; i3++) {
   for (i2 = 1; i2 < n2-1; i2++) {
     for (i1 = 0; i1 < n1; i1++) {
       u1[i1] = u[i3][i2-1][i1] + u[i3][i2+1][i1]
              + u[i3-1][i2][i1] + u[i3+1][i2][i1];
       u2[i1] = u[i3-1][i2-1][i1] + u[i3-1][i2+1][i1]
              + u[i3+1][i2-1][i1] + u[i3+1][i2+1][i1];
     }
     for (i1 = 1; i1 < n1-1; i1++) {
       r[i3][i2][i1] = v[i3][i2][i1]
                      - a[0] * u[i3][i2][i1]
                      - a[2] * (u2[i1] + u1[i1-1] + u1[i1+1])
                      -a[3] * (u2[i1-1] + u2[i1+1]);
     }
   }
 if (timeron) timer_stop(T_resid);
```

```
comm3(r, n1, n2, n3, k);

if (debug_vec[0] >= 1) {
    rep_nrm(r, n1, n2, n3, " resid", k);
}

if (debug_vec[2] >= k) {
    showall(r, n1, n2, n3);
}
}
```

There are tree nested for loops in this function. This is the part which (probably) is the most time consuming. To verify this, we can look at the linewise analysis output.

Line based analysis

The line based analysis produced output that is way too large to put in here, so there is only the relevant part included.

% c	umulative	self		self	total			
time	seconds	seconds	calls	Ts/call	Ts/call	name		
8.78	1.52	1.52				resid	(real.c:526	@
40411a)								
3.88	2.19	0.67				resid	(real.c:524	@
4040c2)								
3.56	2.80	0.62				psinv	(real.c:464	@
404e10)								
3.54	3.41	0.61				resid	(real.c:529	@
40435c)								
2.90	3.91	0.50				resid	(real.c:523	@
40408f)								
2.87	4.41	0.50				resid	(real.c:525	@
40413f)								
2.75	4.88	0.48				psinv	(real.c:471	@
405069)								
2.72	5.35	0.47				resid	(real.c:538	@
404354)								
2.35	5.76	0.41				resid	(real.c:529	a
404315)	6 16	0 41					<i>(</i>] [27	
2.35	6.16	0.41				resid	(real.c:537	a
40431b) 2.17	C F4	0.38					(222) 21525	0
4040ee)	6.54	0.38				resta	(real.c:525	ų.
2.06	6.89	0.36				rocid	(real.c:537	a
40432e)	0.03	0.30				16314	(Γεατ. C. 337	Œ.
1.94	7.23	0.34				resid	(real.c:524	a
4040ea)	7.23	0.54				1 03 10	(. cu : . c : 524	
1.85	7.55	0.32				resid	(real.c:525	a.
404109)								
1.80	7.86	0.31				psinv	(real.c:462	@
404dbd)								
1.65	8.14	0.29				rprj3	(real.c:605	@
4059a0)						-		

The lines printed above are the top 15 time consuming lines in the program. As we can clearly see, most of those lines are in the function resid at the loop nest.

How to parallelize the program

To parallelize this program, we would suggest to first parallelize the loop nest in the resid function. If this does not yield enough speedup, one can also look at the loop nest in the psinv function.

Other information provided by the analysis

The analysis did also produce a call tree which indicates how much time of a function call has been spent in 'child' function calls.

We did not consider this information for our conclusion, but we are sure that it is useful for other people. Here is an example for a call tree given by gprof:

index	% time	self	children	n called		
F4.7	100.0	0.01	17.00		<pre><spontaneous> 2 513</spontaneous></pre>	
ГΤ]	100.0				mg3P.constprop.2 [1]	
			0.68		resid [2]	
				168/168	psinv [3]	
				147/147		
				147/147 	interp [6]	
		8.59	0.68	147/147	mg3P.constprop.2 [1]	
[2]	53.7	8.59	0.68	147	resid [2]	
				170/485	norm2u3 [5]	
				168/168		
[3]	26.1	3.84	0.67	168	psinv [3]	
				168/485	norm2u3 [5]	
					mg3P.constprop.2 [1]	
[4]	11.7			147		
					norm2u3 [5]	
				8	 norm2u3 [5]	
		0.21	0.38	147/485		
				168/485		
				170/485		
[5]	11.2			485+8		
					vranlc [7]	
					randlc [8]	
				8	norm2u3 [5]	
		1.45	0.00	147/147	mg3P.constprop.2 [1]	
[6]	8.4	1.45	0.00	147	interp [6]	
				131072/131072	 norm2u3 [5]	
[7]		1.24	0.00	131072	vranlc [7]	
				131642/131642	 norm2u3 [5]	
[8]	0.0	0.00	0.00	131642	randlc [8]	
				2/4	timer_start [24]	
					timer_stop [25]	
[9]	0.0			4		

Task 2

Like we have seen in task 1, we want to optimize the methods psinv, resid and item because those 3 function take the most execution time.

First Attempts

At first, we focused on the **resid** function, because it has the longest execution time and has nested construct of 4 for-loops. So we tried to parallelize those or rather to optimize the whole nested for-loop construction.

But every try of collapsing the loops failed, because the calculations were wrong afterwards. Example of one attempt:

```
#pragma omp parallel
#pragma omp for collapse(3)
for (i3 = 1; i3 < n3-1; i3++) {
for (i2 = 1; i2 < n2-1; i2++) {
for (i1 = 0; i1 < n1; i1++) {
u1[i1] = u[i3][i2-1][i1] + u[i3][i2+1][i1] + u[i3-1][i2][i1] + u[i3+1][i2][i1];
u2[i1] = u[i3-1][i2-1][i1] + u[i3-1][i2+1][i1] + u[i3+1][i2-1][i1] + u[i3+1]
[i2+1][i1];
}
}
}
#pragma omp for collapse(3)
for (i3 = 1; i3 < n3-1; i3++) {
for (i2 = 1; i2 < n2-1; i2++) {
for (i1 = 1; i1 < n1-1; i1++) {
r[i3][i2][i1] = v[i3][i2][i1] - a[0] _ u[i3][i2][i1]
//----
// Assume a[1] = 0 (Enable 2 lines below if a[1] not= 0)
// - a[1] _ ( u[i3][i2][i1-1] + u[i3][i2][i1+1]
// + u1[i1]
//----- a[2] _ (
u2[i1] + u1[i1-1] + u1[i1+1]) - a[3] _ ( u2[i1-1] + u2[i1+1] );
}
}
}
}
```

We also tried a lot of different adjustments to the pragmas, but even without collapse it calculated the wrong solution. The only thing that worked for us, is shown in the following part.

Parallelized Code

nested for-loops from function resid:

```
#pragma omp parallel for private(u1, u2) schedule(dynamic)
for (i3 = 1; i3 < n3-1; i3++) {
  for (i2 = 1; i2 < n2-1; i2++) {
   for (i1 = 0; i1 < n1; i1++) {
     u1[i1] = u[i3][i2-1][i1] + u[i3][i2+1][i1]</pre>
```

```
+ u[i3-1][i2][i1] + u[i3+1][i2][i1];
     u2[i1] = u[i3-1][i2-1][i1] + u[i3-1][i2+1][i1]
           + u[i3+1][i2-1][i1] + u[i3+1][i2+1][i1];
   }
   for (i1 = 1; i1 < n1-1; i1++) {
     r[i3][i2][i1] = v[i3][i2][i1]
                 - a[0] * u[i3][i2][i1]
     // Assume a[1] = 0 (Enable 2 lines below if a[1] not= 0)
     //-----
                 - a[1] * ( u[i3][i2][i1-1] + u[i3][i2][i1+1]
     //
                       + u1[i1] )
                 - a[2] * (u2[i1] + u1[i1-1] + u1[i1+1])
                 -a[3] * (u2[i1-1] + u2[i1+1]);
   }
 }
}
```

nested for-loops from function psinv:

```
#pragma omp parallel for private(r1, r2) schedule(dynamic)
 for (i3 = 1; i3 < n3-1; i3++) {
   for (i2 = 1; i2 < n2-1; i2++) {
     for (i1 = 0; i1 < n1; i1++) {
      r1[i1] = r[i3][i2-1][i1] + r[i3][i2+1][i1]
            + r[i3-1][i2][i1] + r[i3+1][i2][i1];
      r2[i1] = r[i3-1][i2-1][i1] + r[i3-1][i2+1][i1]
            + r[i3+1][i2-1][i1] + r[i3+1][i2+1][i1];
     }
     for (i1 = 1; i1 < n1-1; i1++) {
      u[i3][i2][i1] = u[i3][i2][i1]
                  + c[0] * r[i3][i2][i1]
                  + c[1] * ( r[i3][i2][i1-1] + r[i3][i2][i1+1]
                          + r1[i1] )
                  + c[2] * (r2[i1] + r1[i1-1] + r1[i1+1]);
      // Assume c[3] = 0 (Enable line below if c[3] not= 0)
      //-----
                  + c[3] * (r2[i1-1] + r2[i1+1])
                        _____
    }
   }
 }
```

nested for-loops from function interp:

```
#pragma omp parallel for private(z1, z2, z3) schedule(dynamic)
for (i3 = 0; i3 < mm3-1; i3++) {
   for (i2 = 0; i2 < mm2-1; i2++) {
     for (i1 = 0; i1 < mm1; i1++) {
        z1[i1] = z[i3][i2+1][i1] + z[i3][i2][i1];
        z2[i1] = z[i3+1][i2][i1] + z[i3][i2][i1];
        z3[i1] = z[i3+1][i2+1][i1] + z[i3+1][i2][i1] + z1[i1];
}</pre>
```

```
for (i1 = 0; i1 < mm1-1; i1++) {
      u[2*i3][2*i2][2*i1] = u[2*i3][2*i2][2*i1]
                          + z[i3][i2][i1];
      u[2*i3][2*i2][2*i1+1] = u[2*i3][2*i2][2*i1+1]
                            + 0.5 * (z[i3][i2][i1+1] + z[i3][i2][i1]);
    }
    for (i1 = 0; i1 < mm1-1; i1++) {
      u[2*i3][2*i2+1][2*i1] = u[2*i3][2*i2+1][2*i1]
                            + 0.5 * z1[i1];
      u[2*i3][2*i2+1][2*i1+1] = u[2*i3][2*i2+1][2*i1+1]
                              + 0.25 * (z1[i1] + z1[i1+1]);
    }
    for (i1 = 0; i1 < mm1-1; i1++) {
      u[2*i3+1][2*i2][2*i1] = u[2*i3+1][2*i2][2*i1]
                              + 0.5 * z2[i1];
      u[2*i3+1][2*i2][2*i1+1] = u[2*i3+1][2*i2][2*i1+1]
                              + 0.25 * (z2[i1] + z2[i1+1]);
    for (i1 = 0; i1 < mm1-1; i1++) {
      u[2*i3+1][2*i2+1][2*i1] = u[2*i3+1][2*i2+1][2*i1]
                              + 0.25 * z3[i1];
      u[2*i3+1][2*i2+1][2*i1+1] = u[2*i3+1][2*i2+1][2*i1+1]
                                + 0.125 * (z3[i1] + z3[i1+1]);
    }
  }
}
```

With those modifications, the calculations are still correct and we achieve a maximum speed-up of 1.6 using 8 threads. That is almost half the time from the beginning with only 3 additional lines of code.

Results

In the table below are the measured execution times for different amounts of threads.

Reference shows the time for the given base version and Parallelized version shows the times for the optimized code.

Number of threads	Reference	Parallelized version
0	17.61	17.63
1	17.61	12.45
2	17.61	12.59
3	17.61	12.55
4	17.61	12.14
5	17.61	11.74
6	17.61	11.52
7	17.61	11.27

This results in following speed-up values:

Speedup for vectorized program, various inputs

