Assignment 12

Introduction

The task was to implement a 3-D n-body simulation. Although there are quite fast and clever algorithms for this problem like the Barnes-Hut-Algorithm, we decided to implement a naive version, which has a runtime complexity of $\mathcal{O}(n^2)$.
In the naive approach, every body compares its position directly to every other body, calculates the force between them and then updates its velocity and position accordingly.

Implementation

Each particle gets represented by a struct containing information about the mass, velocity and position of the body. Note that position and velocity consist of three components each (one for every dimension).

```
typedef struct body_struct {
    float position[3];
    float velocity[3];
    int mass;
} body;
```

Following functions which are necessary for the simulation are provided in `n_body.c`:

Initialization

Every body gets initialized with a random mass, a velocity of 0 and a random position in the predefined space.

```
void initialize_bodies(body* bodies, int size){
    srand(time(NULL));
    unsigned int seed = rand();
    for (int i = 0; i<size; ++i){
        for (int j = 0; j<3; ++j){
            bodies[i].position[j] = rand_r(&seed) % MAX_DIM;
            bodies[i].velocity[j] = 0;
        }
        bodies[i].mass = ((rand_r(&seed) % MAX_MASS) + 500) * 1000;
    }
}
```

Updating velocity

The velocities get updated as described in the introduction section. Note that the force on an axis gets set to 0 as soon as bodies get too close together, since we wanted to avoid absurdly high accelarations which can occur since we did not implement any collision model.

```
void update_velocities_naive(body* bodies, int size){
    for (int i = 0; i<size; ++i){
        for (int j = 0; j<size; ++j){
            if(i != j){
                float numerator = G * (bodies[i].mass + bodies[j].mass);
                for (int k = 0; k<3; ++k){
                    float radius = calculate_radius_squared(bodies[i].position[k],
bodies[j].position[k]);
                    float force;
                    if (fabs(radius) < 0.5){
                        force = 0;
                    } else{
                        force = numerator / radius;
                    }
                    bodies[i].velocity[k] += force/bodies[i].mass;
                }
            }
        }
    }
}
```

Updating position

The positions also get updated as described in the introduction section. Note that the velocity of a body gets set to 0 as soon as the simulation boundaries are reached.

```
void update_positions(body* bodies, int size){
    for (int i = 0; i<size; ++i){
        for (int j = 0; j<3; ++j){
            float update = bodies[i].position[j] + bodies[i].velocity[j];
            if (update > MAX_DIM){
                bodies[i].velocity[j] = 0;
            }else if(update<0){
                bodies[i].velocity[j] = 0;
            } else{
                bodies[i].position[j] = update;
            }
        }
    }
}
```

Saving the data to a file

In order to visualize the simulation it is necessary to write the position of each particle to a file. To achieve this, the following function is provided:

```
void print_bodies_to_file(body* bodies, int size){
    FILE* file = fopen (OUT_FILE, "a");
    for (int j = 0; j<3; ++j){
        for (int i = 0; i<size; ++i){
            fprintf(file, "%f ", bodies[i].position[j]);
        }
        fprintf(file, "\n");
    }
    fprintf(file, "\n");
    fclose(file);
}
```

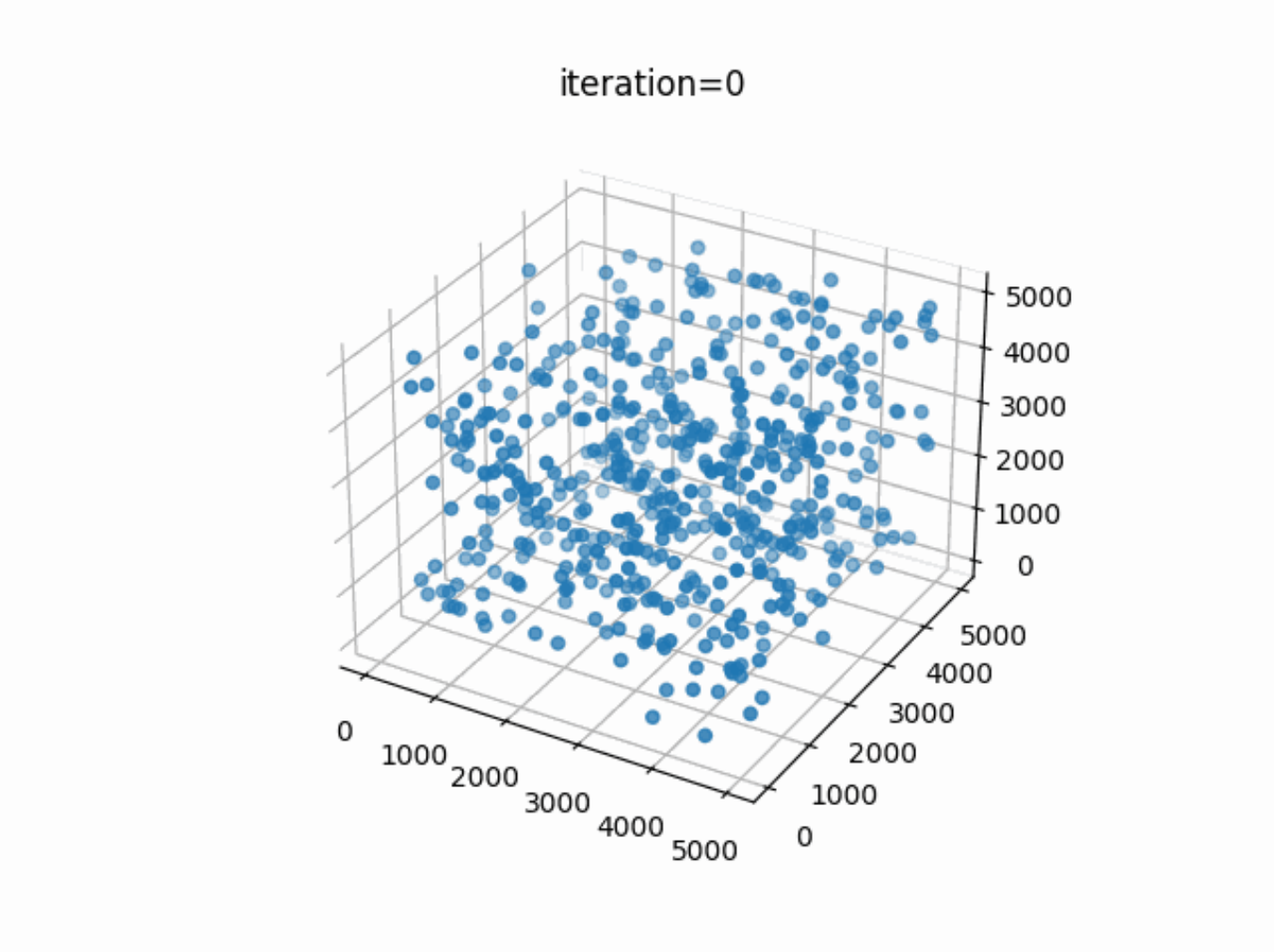
Putting it all together

The main simulation loop is defined in the main function in `simulation.c`. Note that nothing is written to the file if the constant `TIME_MES` is defined in the program.

```
for (int i = 0; i<steps; ++i){
        update_velocities_naive(bodies, size);
        update_positions(bodies, size);
        #ifndef TIME_MES
        print_bodies_to_file(bodies, size);
        #endif
    }
```

Visualization of the simulation

We visualized the simulation with a custom python script using matplotlib. In order to run the script, one needs to have matplotlib and numpy installed.
Here is an example of the generated plot:



Some observations

- Some points are very fast. This is due to some points being initialized with a very low mass and therefore getting accelerated quite violently.
- The simulation probably needs some fine tuning in terms of parameters (e.g. the masses of particles and the gravitational constant).
- A collision model would be very benefitial because we could avoid using workarounds (described in here) which currently lead to odd behaviour of the particles.

Parallel version

In order to parallelize the code, we added two simple statements in the most computationally intensive functions:

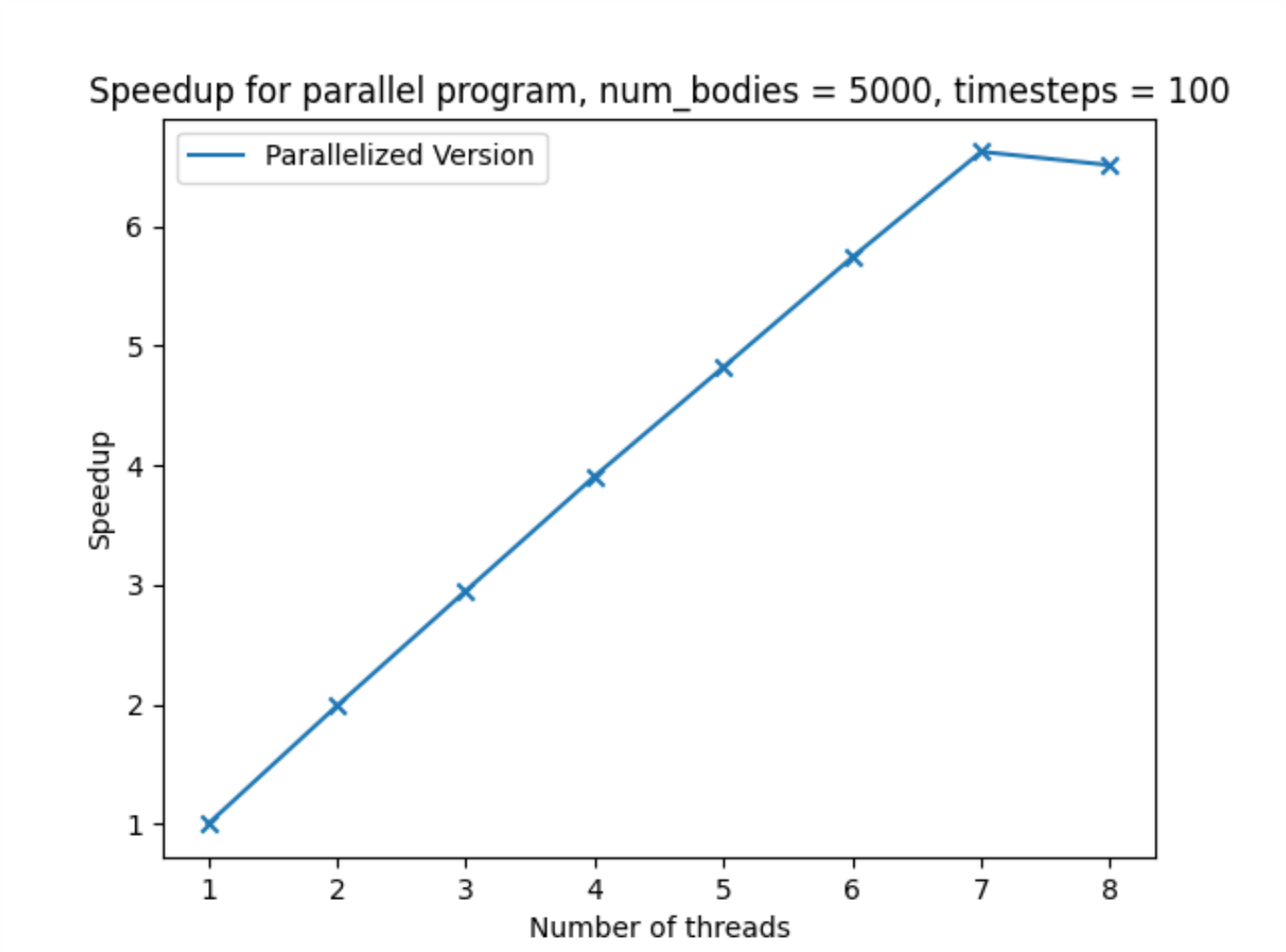
```
void update_positions(body* bodies, int size){
    #pragma omp parallel for collapse(2)
    for (int i = 0; i<size; ++i){
        for (int j = 0; j<3; ++j){
          ...
        }
    }
}
```

and

```
void update_velocities_naive(body* bodies, int size){
    #pragma omp parallel for collapse(2)
    for (int i = 0; i<size; ++i){
        for (int j = 0; j<size; ++j){
          ...
        }
    }
}
```

Benchmark

The program has been tested with `n_bodies` = 5000 and `timesteps`= 100.



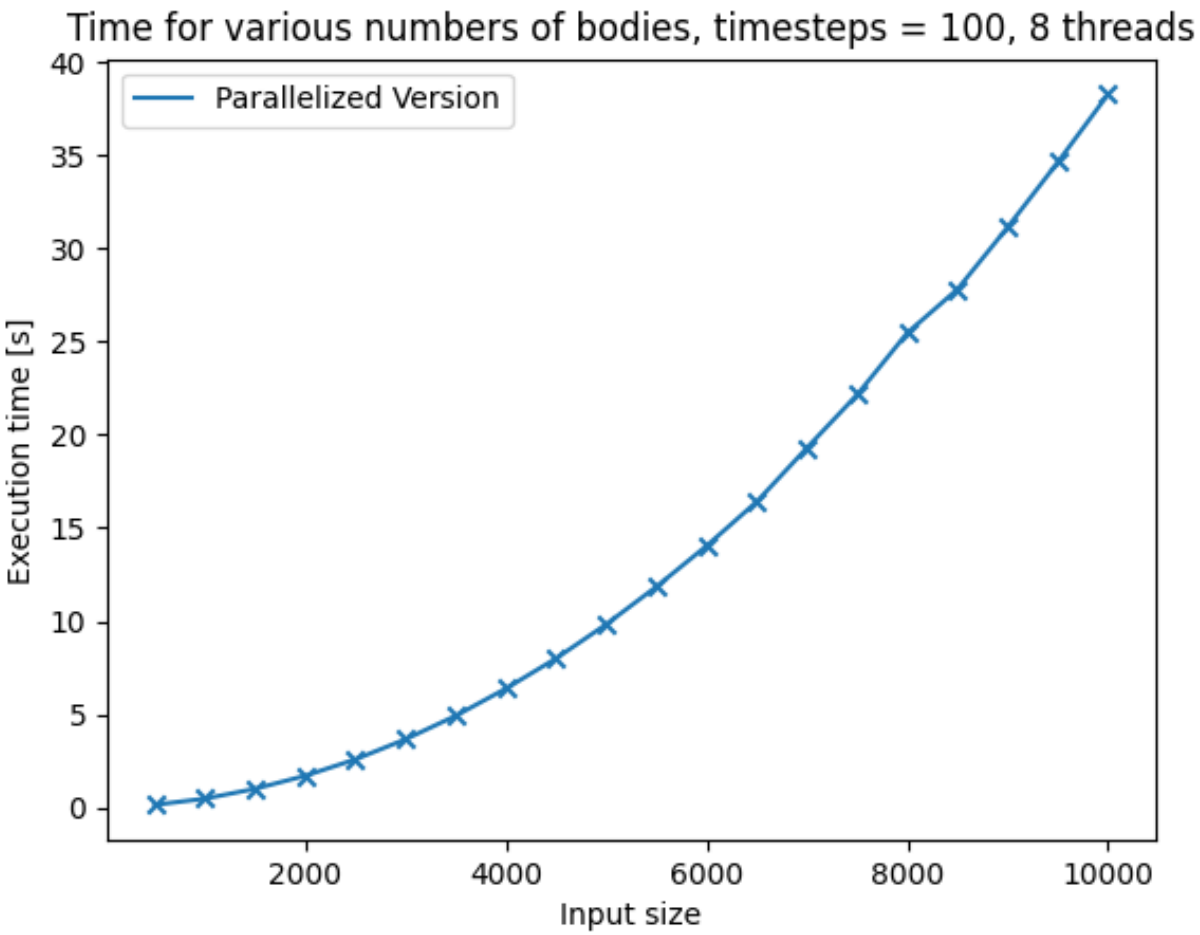
Number of threads	Parallelized Version	Reference
1	74.01	74.0
2	37.25	74.06
3	25.08	73.98
4	18.93	73.99
5	15.34	73.97
6	12.89	74.0
7	11.18	74.05
8	11.36	73.97

We are quite happy with this (nearly linear) speedup and especially with the effort to speedup ratio.

Time measurements for various inputs

Varying the number of bodies

Measurements have been taken with 100 timesteps and 8 threads for various numbers of bodies.

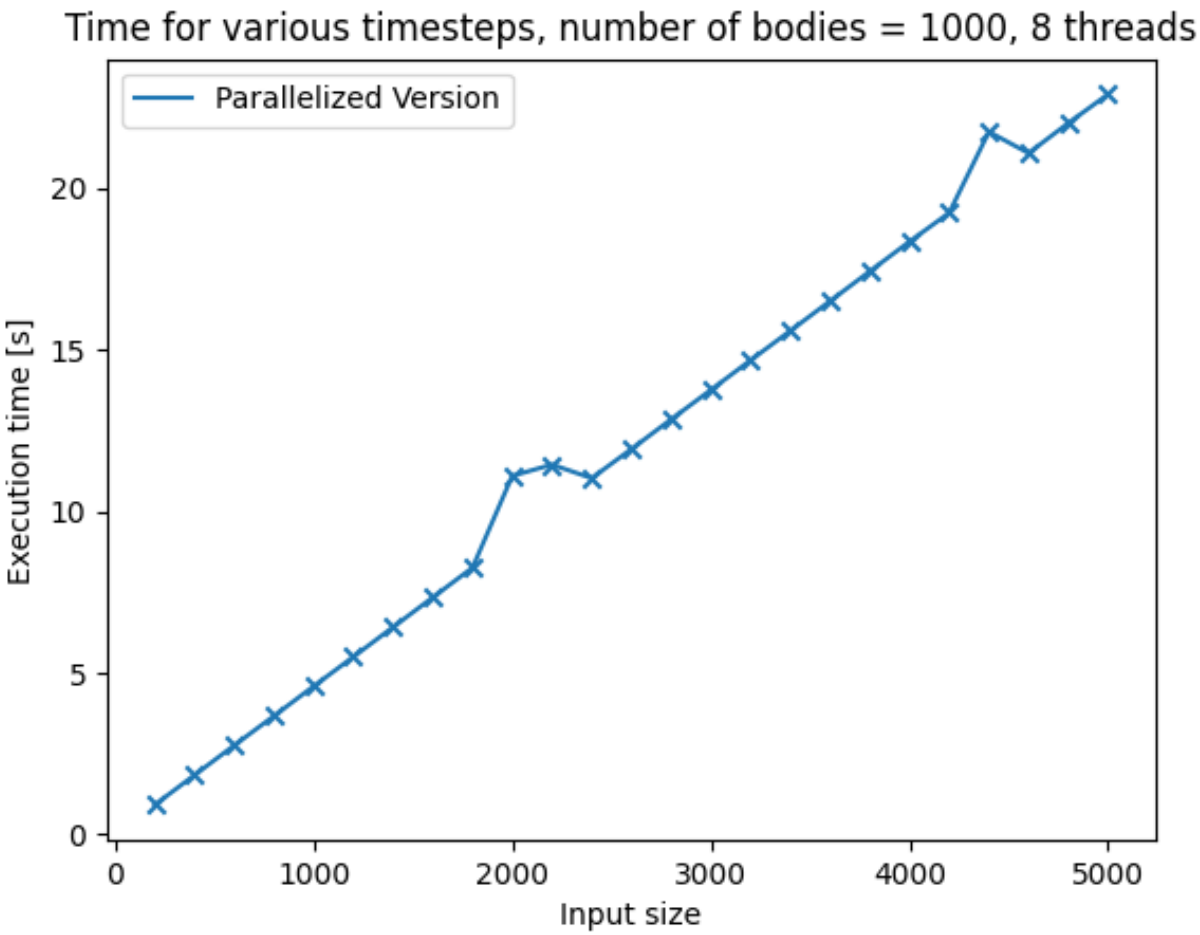

Time for various numbers of bodies, timesteps = 100, 8 threads
Parallelized Version
Execution time [s]
Input size
0
5
10
15
20
25
30
35
40
2000
4000
6000
8000
10000

Number of bodies	Execution time
500	0.14
1000	0.47
1500	1.0
2000	1.7
2500	2.58
3000	3.66
3500	4.92
4000	6.38
4500	8.0
5000	9.83
5500	11.85
6000	14.04
6500	16.41
7000	19.3
7500	22.15
8000	25.47
8500	27.81
9000	31.14
9500	34.63
10000	38.27

One can clearly see the exponentially rising runtime (which fits our asymptotic complexity of $\mathcal{O}(n^2)$.

Varriyng number of timesteps

Measurements have been taken with 1000 bodies and 8 threads for various numbers of timesteps.


Time for various timesteps, number of bodies = 1000, 8 threads
Parallelized Version
Execution time [s]
0
5
10
15
20
0
1000
2000
3000
4000
5000
Input size

Input size	Parallelized Version
200	0.92
400	1.84
600	2.76
800	3.67
1000	4.59
1200	5.5
1400	6.42
1600	7.34
1800	8.26
2000	11.1
2200	11.45
2400	11.03
2600	11.93
2800	12.85
3000	13.76
3200	14.68
3400	15.59
3600	16.51
3800	17.43
4000	18.35
4200	19.24
4400	21.75
4600	21.09
4800	22.02
5000	22.91

This curve also fullfills our expectations, since the progam has a linear runtime complexity ($\mathcal{O}(n)$) in the number of timesteps.