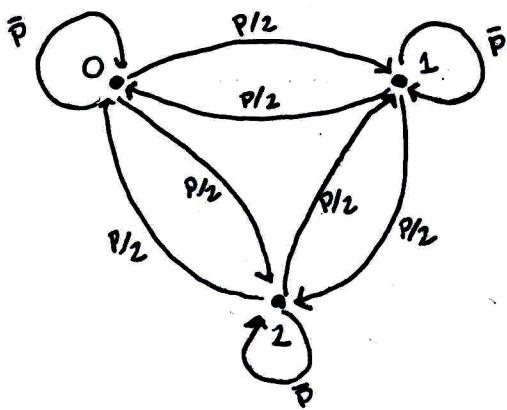


10

$$S = \{0, 1, 2\}$$

$$P(0) = P(1) = P(2) = \frac{1}{3}$$

$$\bar{P} = 1 - P$$



9)

### Informaciones Condicionadas

$$I(0|0) = \log \frac{1}{P(0|0)} = \log \frac{1}{\bar{P}}$$

$$I(0|1) = \log \frac{2}{P}$$

$$I(1|0) = \log \frac{2}{P}$$

$$I(1|1) = \log \frac{1}{P}$$

$$I(2|0) = \log \frac{2}{P}$$

$$I(2|1) = \log \frac{2}{P}$$

$$I(0|2) = \log \frac{2}{P}$$

$$I(1|2) = \log \frac{2}{P}$$

$$I(2|2) = \log \frac{1}{P}$$

### Entropías Condicionadas

$$H(S|0) = \bar{P} \log \frac{1}{\bar{P}} + \frac{P}{2} \log \frac{2}{P} + \frac{P}{2} \log \frac{2}{P} = \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P}$$

$$H(S|1) = \frac{P}{2} \log \frac{2}{P} + \bar{P} \log \frac{1}{\bar{P}} + \frac{P}{2} \log \frac{2}{P} = \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P}$$

$$H(S|2) = \frac{P}{2} \log \frac{2}{P} + \frac{P}{2} \log \frac{2}{P} + \bar{P} \log \frac{1}{\bar{P}} = \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P}$$

### Entropía de la Fuente

$$\begin{aligned}
 H(S) &= P(0) \left[ \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P} \right] + P(1) \left[ \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P} \right] + P(2) \left[ \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P} \right] \\
 &= \frac{1}{3} \left[ \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P} \right] + \frac{1}{3} \left[ \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P} \right] + \frac{1}{3} \left[ \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P} \right] \\
 &= \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P} = (1-P) \log \frac{1}{1-P} + P \log \frac{2}{P} \quad +
 \end{aligned}$$

Para  $P=0$

$$(1-0) \log \frac{1}{1-0} + (0) \log \frac{2}{(0)}$$

No aplica dado que la división entre "0" es indefinida

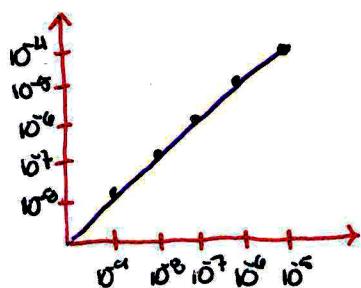
Para  $P=1$

$$(1-1) \log \frac{1}{1-1} + 1 \log \frac{2}{1}$$

No aplica dado que la división entre "0" es indefinida

b)  $P=\delta \rightarrow \delta \approx 0$

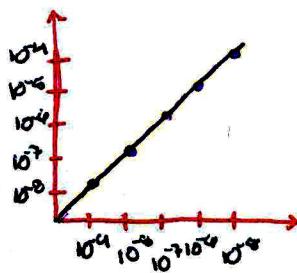
$\epsilon$	$H(\delta)$
$10^{-9}$	$3.23 * 10^{-8}$
$10^{-8}$	$2.902 * 10^{-7}$
$10^{-7}$	$2.57 * 10^{-6}$
$10^{-6}$	$2.23 * 10^{-5}$
$10^{-5}$	$1.905 * 10^{-4}$



c)  $P=1-\delta \rightarrow \delta \approx 0$

$$(1-P) = 1 - (1-\delta) = \delta$$

$\delta$	$H(\delta)$
$10^{-9}$	$3.09 * 10^{-8}$
$10^{-8}$	$2.758 * 10^{-7}$
$10^{-7}$	$2.425 * 10^{-6}$
$10^{-6}$	$2.093 * 10^{-5}$
$10^{-5}$	$1.761 * 10^{-4}$



d)  $P = \frac{1}{4}$

$$H(S|O) = H(S|1) = H(S|2) = \bar{P} \log \frac{1}{\bar{P}} + P \log \frac{2}{P}$$

$$= (1-\bar{P}) \log \frac{1}{1-\bar{P}} + \bar{P} \log \frac{2}{\bar{P}}$$

$$= (1 - \frac{1}{4}) \log \frac{1}{1 - \frac{1}{4}} + \frac{1}{4} \log \frac{2}{\frac{1}{4}}$$

$$= \frac{3}{4} \log \frac{4}{3} + \frac{1}{4} \log 8 = 1.061$$

+

20

$$P(0|0) = p \quad P(0|1) = q$$

$$P(1|0) = \bar{p} \quad P(1|1) = \bar{q}$$

a)

$$H(s|0) = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{1}{p} = (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p}$$

$$H(s|1) = q \log \frac{1}{q} + \bar{q} \log \frac{1}{\bar{q}} = q \log \frac{1}{q} + (1-q) \log \frac{1}{1-q}$$

$$H(s) = P(0) H(s|0) + P(1) H(s|1)$$

b)

$$p = 0.1 \quad q = 0.2$$

$$P(0) = \frac{q}{p+q} = \frac{0.2}{0.1+0.2} = \frac{0.2}{0.3}$$

$$P(1) = \frac{p}{p+q} = \frac{0.1}{0.1+0.2} = \frac{0.1}{0.3}$$

$$H(s) = P(0) H(s|0) + P(1) H(s|1)$$

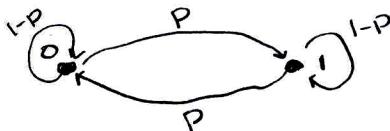
$$= \frac{q}{p+q} \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right] + \frac{p}{p+q} \left[ q \log \frac{1}{q} + (1-q) \log \frac{1}{1-q} \right]$$

$$= \frac{0.2}{0.3} \left[ 0.9 \log \frac{1}{0.9} + 0.1 \log \frac{1}{0.1} \right] + \frac{0.1}{0.3} \left[ 0.2 \log \frac{1}{0.2} + 0.8 \log \frac{1}{0.8} \right]$$

$$= 0.5533 \cancel{+}$$

c)

$$p = q$$

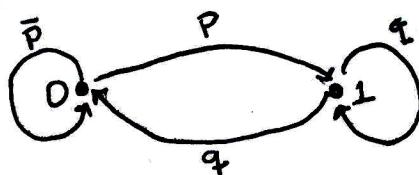


$$H(s) = \frac{p}{p+p} \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right] + \frac{p}{p+p} \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right]$$

$$= \frac{1}{2} \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right] + \frac{1}{2} \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right]$$

$$= (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \cancel{+}$$

3o



$$P(0) = \frac{q}{p+q} \quad P(1) = \frac{p}{p+q} \quad q=1 \rightarrow p \neq q$$



$$I(0|0) = \log \frac{1}{1-p}$$

$$I(0|1) = \log(1) = 0$$

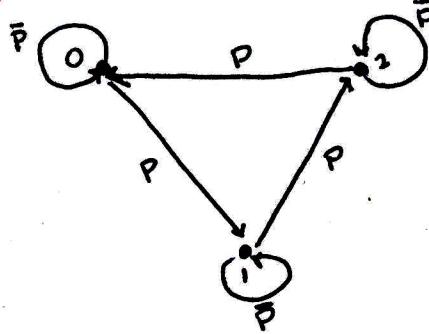
$$I(1|0) = \log \frac{1}{p}$$

$$H(S|0) = (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p}$$

$$H(S|1) = 0$$

$$H(S) = P(0) \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right] = \frac{1}{p+1} \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right] +$$

4o



$$S = \{0, 1, 2\}$$

$$P(0) = P(1) = P(2) = \frac{1}{3}$$

a)

$$\bar{p} = 1-p$$

$$I(0|0) = \log \frac{1}{1-p}$$

$$I(1|1) = \log \frac{1}{1-p}$$

$$I(0|2) = \log \frac{1}{p}$$

$$I(1|0) = \log \frac{1}{p}$$

$$I(2|1) = \log \frac{1}{p}$$

$$I(2|2) = \log \frac{1}{1-p}$$

$$H(S|0) = (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p}$$

$$H(S|1) = (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p}$$

$$H(S|2) = p \log \frac{1}{p} + (1-p) \log \frac{1}{1-p}$$

$H(S)$  no aplica para  $p=0$  y  $p=1$  porque la división entre "0" no es definida

$$H(S) = P(0) \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right] + P(1) \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right] + P(2) \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right]$$

$$= [P(0) + P(1) + P(2)] \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right] = \left[ \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \right] \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right]$$

$$= (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p}$$

$$b) H(S^2) = ?$$

$$s_0 = 0, s_1 = 1, s_2 = 2$$

$\sigma_i$	correspondencia	$P(\sigma_i)$	$I(\sigma_i)$
$\sigma_0$	$s_0/s_0$	$P(0 0) = 1-p$	$\log \frac{1}{1-p}$
$\sigma_1$	$s_0/s_1$	$P(1 0) = p$	$\log \frac{1}{p}$
$\sigma_2$	$s_0/s_2$	0	0
$\sigma_3$	$s_1/s_0$	0	0
$\sigma_4$	$s_1/s_1$	$1-p$	$\log \frac{1}{1-p}$
$\sigma_5$	$s_1/s_2$	$p$	$\log \frac{1}{p}$
$\sigma_6$	$s_2/s_0$	$p$	$\log \frac{1}{p}$
$\sigma_7$	$s_2/s_1$	0	0
$\sigma_8$	$s_2/s_2$	$1-p$	$\log \frac{1}{1-p}$

$$\begin{aligned} H(S^2) &= (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} + (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} + p \log \frac{1}{p} + (1-p) \log \frac{1}{1-p} \\ &= 3 \left[ (1-p) \log \frac{1}{1-p} + p \log \frac{1}{p} \right] \end{aligned}$$

$$= 3 H(S) \quad +$$

b)

a)

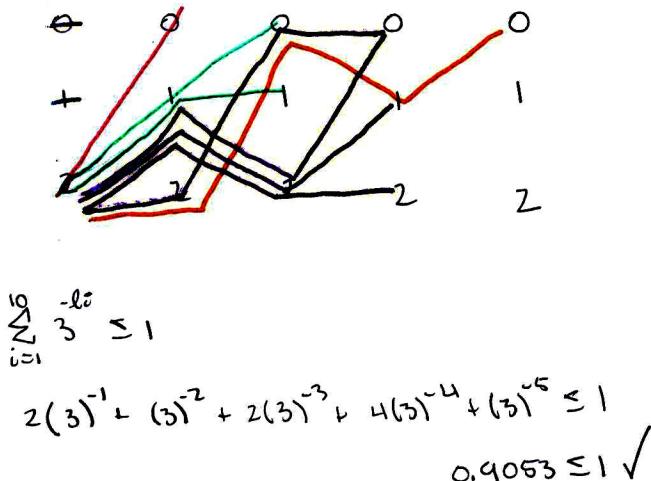
- Código A - Es univocamente decodificable pq todas las palabras son de la misma longitud.
- Código B - Es univocamente decodificable pq es un código coma.
- Código C - Es univocamente decodificable pq es un código coma.
- Código D - No es univocamente decodificable pq no es sintetizable.
- Código E - Es univocamente decodificable pq es sintetizable.
- Código F - No es univocamente decodificable pq no es sintetizable.

b)

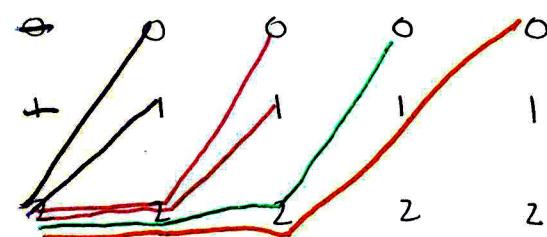
- Código A - Es instantáneo
- Código B - Es instantáneo
- Código C - Es instantáneo
- Código D - NO es instantáneo
- Código E - Es instantáneo
- Código F - NO es instantáneo

20

$s_i$	$l_i$	Código A
$s_1$	1	0
$s_2$	1	1
$s_3$	2	20
$s_4$	3	210
$s_5$	3	211
$s_6$	4	2120
$s_7$	4	2121
$s_8$	4	2122
$s_9$	4	2200
$s_{10}$	5	22010



$s_i$	$l_i$	Código B
$s_1$	1	0
$s_2$	1	1
$s_3$	2	20
$s_4$	2	21
$s_5$	3	220
$s_6$	3	221
$s_7$	4	2220
$s_8$	5	22210

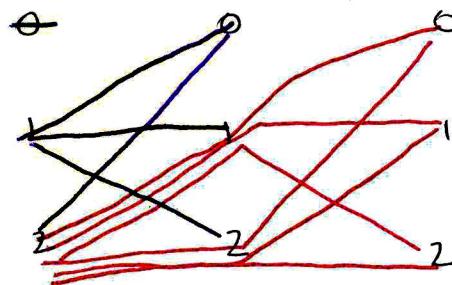


$$\sum_{i=1}^8 (3)^{-l_i} \leq 1$$

$$2(3)^{-1} + 2(3)^{-2} + 2(3)^{-3} + (3)^{-4} + (3)^{-5} \leq 1$$

$$0.4794 \leq 1 \quad \checkmark$$

$s_i$	$l_i$	Código C
$s_1$	1	0
$s_2$	2	10
$s_3$	2	11
$s_4$	2	12
$s_5$	2	20
$s_6$	3	210
$s_7$	3	211
$s_8$	3	212
$s_9$	3	220
$s_{10}$	3	221
$s_{11}$	3	222



$$\sum_{i=1}^{11} (3)^{-l_i} \leq 1$$

$$(3)^{-1} + 4(3)^{-2} + 6(3)^{-3} \leq 1$$

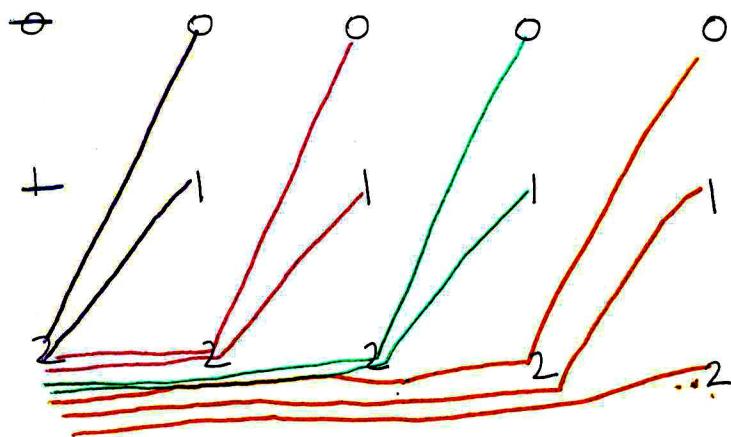
$$1 \leq 1 \quad \checkmark$$

$s_i$	$l_i$	Código D
$s_1$	1	0
$s_2$	1	1
$s_3$	2	20
$s_4$	2	21
$s_5$	3	220
$s_6$	3	221
$s_7$	4	2220
$s_8$	4	2221
$s_9$	5	22220
$s_{10}$	5	22221
$s_{11}$	5	22222

$$\sum_{i=1}^{\infty} (3)^{-l_i} \leq 1$$

$$2(3)^{-1} + 2(3)^{-2} + 2(3)^{-3} + 2(3)^{-4} + 3(3)^{-5} \leq 1$$

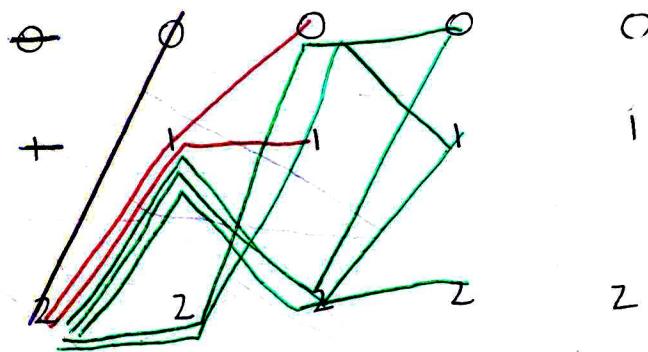
$$1 \leq 1 \checkmark$$



HT508

10

$s_i$	$l_i$	Código A
$s_1$	1	0
$s_2$	1	1
$s_3$	2	20
$s_4$	3	210
$s_5$	3	211
$s_6$	4	2120
$s_7$	4	2121
$s_8$	4	2122
$s_9$	4	2200
$s_{10}$	5	2201



$n_1 \leq r$

$2 \leq 3 \checkmark$

$n_4 \leq (6-2)3$

$4 \leq 12 \checkmark$

$n_2 \leq (3-2)3$

$n_5 \leq (12-4)3$

$1 \leq 3 \checkmark$

$1 \leq 24 \checkmark$

$n_3 \leq (3-1)3$

$2 \leq 6 \checkmark$

$s_i$	$l_i$	Código B
$s_1$	1	0
$s_2$	1	1
$s_3$	2	20
$s_4$	2	21
$s_5$	3	220
$s_6$	3	221
$s_7$	4	2220
$s_8$	5	22210

$n_1 \leq 3$

$n_4 \leq (3-2)3$

$1 \leq 3 \checkmark$

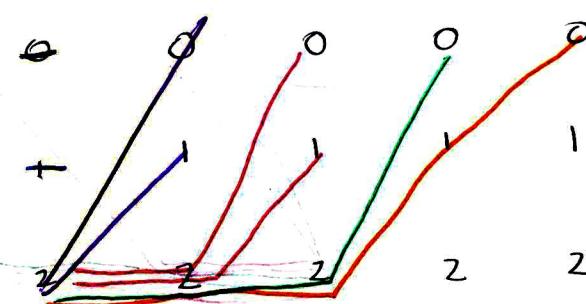
$n_2 \leq (3-2)3$

$n_5 \leq (3-1)3$

$1 \leq 6 \checkmark$

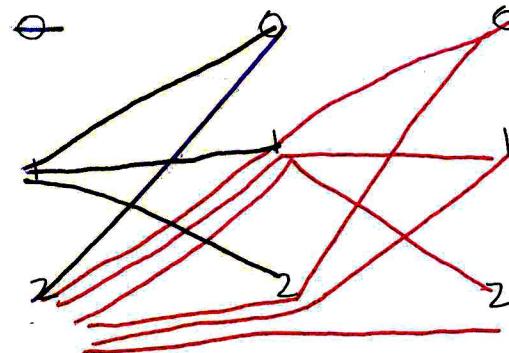
$n_3 \leq (3-1)3$

$2 \leq 3 \checkmark$



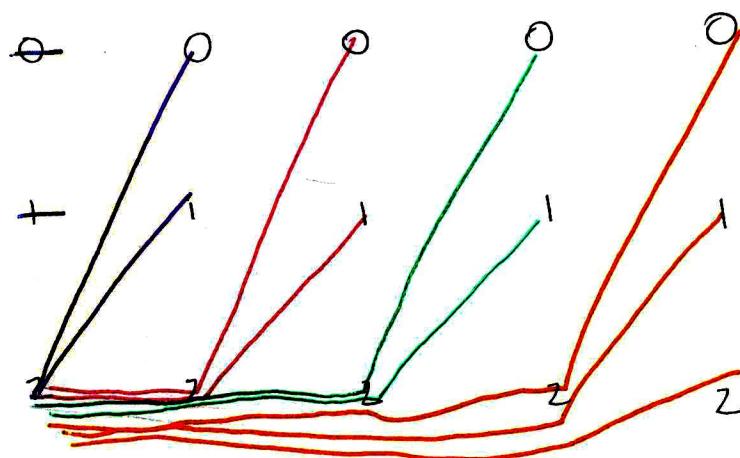
$s_i$	$l_i$	Código C
$s_1$	1	0
$s_2$	2	10
$s_3$	2	11
$s_4$	2	12
$s_5$	2	20
$s_6$	3	210
$s_7$	3	211
$s_8$	3	212
$s_9$	3	220
$s_{10}$	3	221
$s_{11}$	3	222

$$\begin{aligned}
 n_1 &\leq 3 & h_3 &\leq (6-4)3 \\
 1 &\leq 3 \checkmark & 6 &\leq 6 \checkmark \\
 n_2 &\leq (3-1)3 & \\
 4 &\leq 6 \checkmark
 \end{aligned}$$



$s_i$	$l_i$	Código D
$s_1$	1	0
$s_2$	1	1
$s_3$	2	20
$s_4$	2	21
$s_5$	3	220
$s_6$	3	221
$s_7$	4	2220
$s_8$	4	2221
$s_9$	5	22220
$s_{10}$	5	22221
$s_{11}$	6	22222

$$\begin{aligned}
 n_1 &\leq 3 & h_4 &\leq (3-2)3 \\
 2 &\leq 3 \checkmark & 2 &\leq 3 \checkmark \\
 n_2 &\leq (3-2)3 & n_5 &\leq (3-2)3 \\
 2 &\leq 3 \checkmark & 3 &\leq 3 \checkmark \\
 n_3 &\leq (3-2)3 & \\
 2 &\leq 3 \checkmark
 \end{aligned}$$



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a)

s <sub>0</sub>	l <sub>0</sub>	Código
s <sub>1</sub>	1	
s <sub>2</sub>	2	
s <sub>3</sub>	2	
s <sub>4</sub>	2	
s <sub>5</sub>	2	
s <sub>6</sub>	2	
s <sub>7</sub>	3	
s <sub>8</sub>	3	
s <sub>9</sub>	3	
s <sub>10</sub>	3	

$$\sum_{i=1}^{10} (3)^{-l_i} \leq 1$$

$$(3)^{-1} + 5(3)^{-2} + 4(3)^{-3} \leq 1$$

$$1.037 \leq 1 \times$$

No es posible dado que las longitudes de las palabras de código no cumplen con la desigualdad de Kraft

b)

s <sub>0</sub>	l <sub>0</sub>	Código
s <sub>1</sub>	1	0
s <sub>2</sub>	2	10
s <sub>3</sub>	2	11
s <sub>4</sub>	2	12
s <sub>5</sub>	2	20
s <sub>6</sub>	3	210
s <sub>7</sub>	3	211
s <sub>8</sub>	3	212
s <sub>9</sub>	3	220
s <sub>10</sub>	4	2210

$$\sum_{i=1}^9 (3)^{-l_i} \leq 1$$

$$(3)^{-1} + 4(3)^{-2} + 4(3)^{-3} + (3)^{-4} \leq 1$$

$$0.9383 \leq 1 \checkmark$$

$$n_1 \leq 3$$

$$n_4 \leq (6-4)3$$

$$1 \leq 3 \checkmark$$

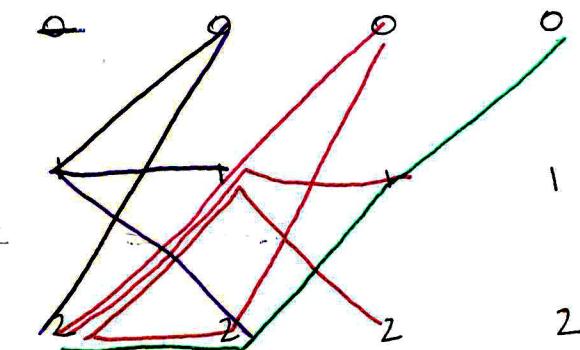
$$1 \leq 6$$

$$n_2 \leq (3-1)3$$

$$4 \leq 6 \checkmark$$

$$n_3 \leq (6-4)3$$

$$4 \leq 6 \checkmark$$



30

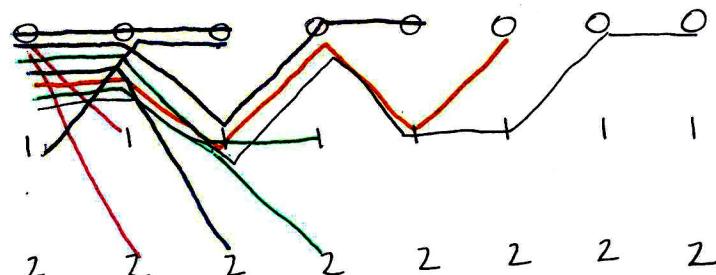
a)

$s_i$	$l_i$	Código
$s_1$	3	000
$s_2$	5	00100
$s_3$	2	01
$s_4$	4	0011
$s_5$	3	002
$s_6$	6	001010
$s_7$	4	0012
$s_8$	8	00101100
$s_9$	2	02
$s_{10}$	3	100

$$\sum_{i=1}^{10} (3)^{-l_i} \leq 1$$

$$3(3)^{-3} + (3)^{-5} + 2(3)^{-2} + 2(3)^{-4} + (3)^{-6} + (3)^{-8} \leq 1$$

$$0.3637 \leq 1 \checkmark$$



b)

$$\sum_{i=1}^q r^{-l_i} \leq 1$$

Definimos  $n_i$  como el número de palabras de longitud " $i$ ";  $n_2$  las de longitud " $2$ ", etc. si la más larga de las  $l_i$  es igual a 1, tendremos

$$\sum_{i=1}^q n_i = q$$

al introducir en la primera expresión

$$\sum_{i=1}^l n_i r^{-l_i} \leq 1 \quad ||* r^l$$

$$\sum_{i=1}^l n_i r^{-l_i} r^l \leq r^l$$

$$\sum_{i=1}^l n_i r^{l-i} \leq r^l$$

$$n_1 r^{l-1} + n_2 r^{l-2} + \dots + n_l r^0 \leq r^l$$

$$n_1 \leq r^{l-1} - n_2 r^{l-2} - n_3 r^{l-3} - \dots - n_{l-1} r \quad ||*(r)^{-1}$$

$$n_{l-1} \leq r^{l-2} - n_2 r^{l-3} - n_3 r^{l-4} - \dots - n_{l-2} r$$

$$n_2 \leq r^3 - n_1 r^2 - n_3 r$$

$$n_3 \leq r^2 - n_1 r$$

$$n_1 \leq r$$

Puesto que se pueden elegir  $n_1$  símbolos de código arbitrariamente, quedan entonces  $r-n_1$  prefijos de longitud 1 permitidos. Pueden formarse hasta  $(r-n_1)r = r^2 - nr$  palabras de longitud 2. Esta ecuación asegura que el número de palabras de longitud 2 no deben exceder de esta cantidad, quedan entonces

$$(r^2 - nr) - n_2$$

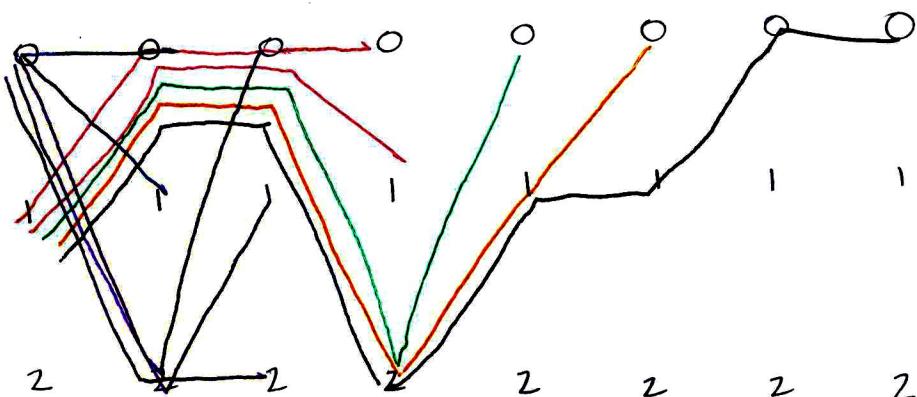
por lo que las palabras de longitud 3 deben cumplir

$$n_3 \leq [(r^2 - nr) - n_2]r$$

y así sucesivamente, en cada etapa queda un número suficiente de prefijos

### Ejemplo

<u>Si</u>	<u>l<sub>i</sub></u>	<u>Código</u>	$n_1 \leq 3$	$n_2 \leq (54-2)3$
s <sub>1</sub>	3	020	$0 \leq 3$	$1 \leq 154$
s <sub>2</sub>	5	10020	$n_2 \leq (3-0)3$	$n_3 \leq (154-1)3$
s <sub>3</sub>	2	00	$2 \leq 9$	$1 \leq 465$
s <sub>4</sub>	4	1000	$n_3 \leq (9-2)3$	$n_4 \leq (465-1)3$
s <sub>5</sub>	3	021	$3 \leq 21$	$0 \leq 1392$
s <sub>6</sub>	6	100210	$n_4 \leq (21-3)3$	$n_5 \leq (1392-0)3$
s <sub>7</sub>	4	1001	$2 \leq 54$	$1 \leq 4176$
s <sub>8</sub>	8	10021100		
s <sub>9</sub>	2	01		
s <sub>10</sub>	3	022		



## HT509-2

 10  
 a)

Demostrar si  $l_i = \log_r (\frac{1}{p_i})$  y  $l_i$  es un entero entonces  $L = H_r(S)$

$L$  longitud promedio

$$H(S) = \sum_{i=1}^q p_i \log \frac{1}{p_i}$$

$$\sum_{i=1}^q x_i \log \frac{1}{x_i} \leq \sum_{i=1}^q x_i \log \frac{1}{l_i}$$

$$\sum_{i=1}^q p_i \log \frac{1}{p_i} \leq \sum_{i=1}^q p_i \log \frac{1}{l_i}$$

$$H(S) \leq \sum_{i=1}^q p_i \log \frac{1}{l_i}$$

$$\text{Si } Q_i = \frac{r^{-l_i}}{\sum_{j=1}^q r^{-l_j}} \text{ entonces}$$

$$Q_1 + Q_2 + \dots + Q_q = \frac{r^{-l_1}}{\sum l_j} + \frac{r^{-l_2}}{\sum l_j} + \dots + \frac{r^{-l_q}}{\sum l_j} = \frac{\sum l_j}{\sum l_j} = 1$$

$$\sum_{i=1}^q p_i \log \frac{1}{Q_i} = - \sum_{i=1}^q p_i \log Q_i$$

$$= - \sum_{i=1}^q p_i \log (r^{-l_i}) + \sum_{i=1}^q p_i \log \left( \sum_{j=1}^q r^{-l_j} \right)$$

$$= \sum_{i=1}^q p_i \log (r^{-l_i}) + \left[ \log \left( \sum_{j=1}^q r^{-l_j} \right) \right] \sum_{i=1}^q p_i$$

$$\text{Si } r=2 \text{ entonces}$$

$$\sum_{i=1}^q p_i \log \frac{1}{Q_i} = L + \log \left( \sum_{j=1}^q r^{-l_j} \right) \leq L$$

$$H(S) \leq L + \log \left( \sum_{j=1}^q r^{-l_j} \right) \leq L$$

$$\text{Si } l_i = \log \frac{1}{p_i} \text{ y } l_i \text{ es un entero}$$

$$H(S) = \sum_{i=1}^q p_i \log \frac{1}{p_i} \quad L = \sum_{i=1}^q p_i l_i$$

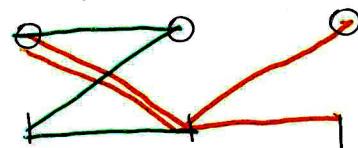
$$= \sum_{i=1}^q p_i l_i$$

$$H(S) = L$$

b)

$$S = \{s_1, s_2, \dots, s_5\} \quad L = H(S)$$

$s_i$	$P(s_i)$	$l_i$	Código
$s_1$	$\frac{1}{4}$	2	00
$s_2$	$\frac{1}{8}$	3	010
$s_3$	$\frac{1}{8}$	3	011
$s_4$	$\frac{1}{4}$	2	10
$s_5$	$\frac{1}{4}$	2	11



2o

Está demostrado en el problema 1

3o

a)

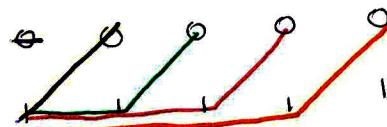
$s_i$	$P(s_i)$	Código	$l_i$
$s_1$	0.5	0	1
$s_2$	0.25	10	2
$s_3$	0.125	110	3
$s_4$	0.1	1110	4
$s_5$	0.025	11110	5

$$H_2(S) = L \rightarrow H_2(S) = 1.8$$

$$\sum_{i=1}^5 (2)^{-l_i} \leq 1$$

$$(2)^{-1} + (2)^{-2} + (2)^{-3} + (2)^{-4} + (2)^{-5} \leq 1$$

$$0.9684 \leq 1 \checkmark$$



b)

$$\log \frac{1}{P_i} \leq l_i < \log \frac{1}{P_i} + 1$$

$$P_1 = 0.5$$

$$1 \leq l_1 < 2 \rightarrow l_1 = 1$$

$$P_4 = 0.1$$

$$3.322 \leq l_4 < 4.322 \rightarrow l_4 = 4$$

$$P_2 = 0.25$$

$$2 \leq l_2 < 3 \rightarrow l_2 = 2$$

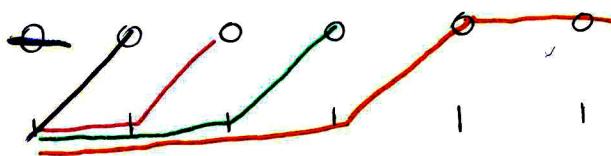
$$P_5 = 0.025$$

$$5.322 \leq l_5 < 6.322 \rightarrow l_5 = 6$$

$$P_3 = 0.125$$

$$3 \leq l_3 < 4 \rightarrow l_3 = 3$$

$s_i$	$l_i$	$P(s_i)$	Código
$s_1$	1	0.5	0
$s_2$	2	0.25	10
$s_3$	3	0.125	110
$s_4$	4	0.1	1110
$s_5$	6	0.025	111100



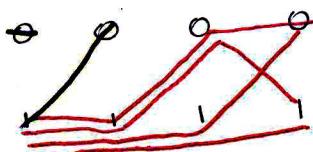
40

$s_i$	$P(s_i)$	$l_i$	Código
$s_1$	0.4	1	0
$s_2$	0.3	2	10
$s_3$	0.1	4	1100
$s_4$	0.1	4	1101
$s_5$	0.06	4	1110
$s_6$	0.04	4	1111

$$\sum_{i=1}^4 (2)^{-l_i} \leq 1$$

$$(2)^{-1} + (2)^{-2} + 4(2)^{-4} \leq 1$$

$$1 \leq 1 \checkmark$$



$$H(S) = \sum_{i=1}^6 P_i \log \frac{1}{P_i} = 0.4 \log \frac{1}{0.4} + 0.3 \log \frac{1}{0.3} + 0.1 \log \frac{1}{0.1} + 0.1 \log \frac{1}{0.1} + 0.06 \log \frac{1}{0.06} + 0.04 \log \frac{1}{0.04}$$

$$H(S) = 2.144 \text{ bits/mensaje} \checkmark$$

$$L = \sum_{i=1}^6 P_i l_i = (0.4)(1) + (0.3)(2) + 4(0.1 + 0.1 + 0.06 + 0.04) = 2.2 \text{ bits/mensaje} \checkmark$$

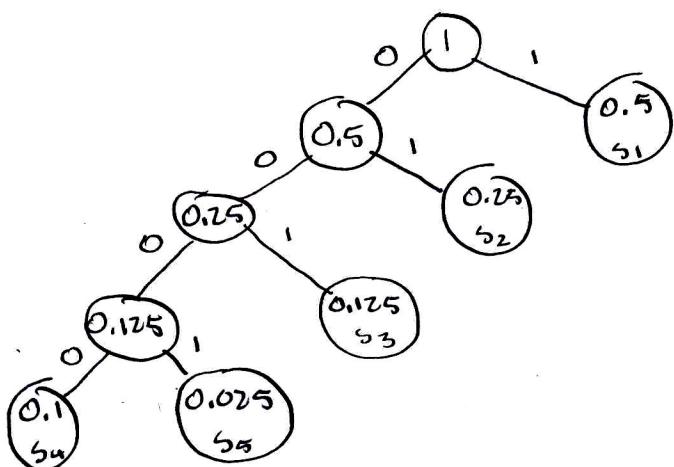
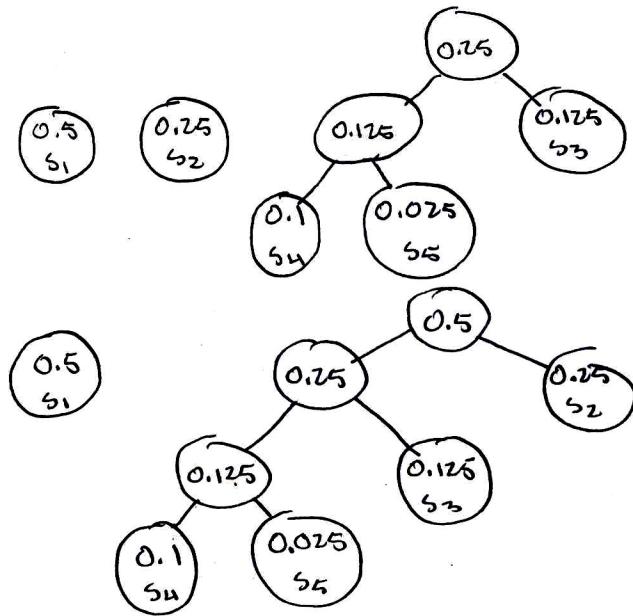
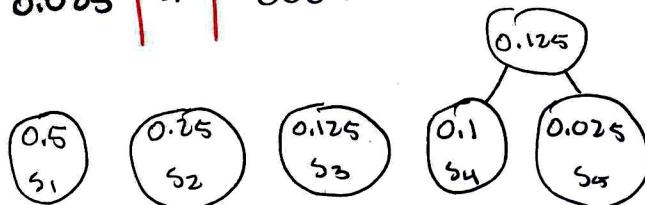
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$s_i$	$P(s_i)$	$l_i$	Código
$s_1$	0.5	1	1
$s_2$	0.25	2	01
$s_3$	0.125	3	001
$s_4$	0.1	4	0000
$s_5$	0.025	4	0001

$$L = \sum_{i=1}^5 P_i l_i$$

$$L = (0.5)(1) + (0.25)(2) + (0.125)(3) + (0.1 + 0.025)(4)$$

$$L = 1.875 \text{ bits/message}$$



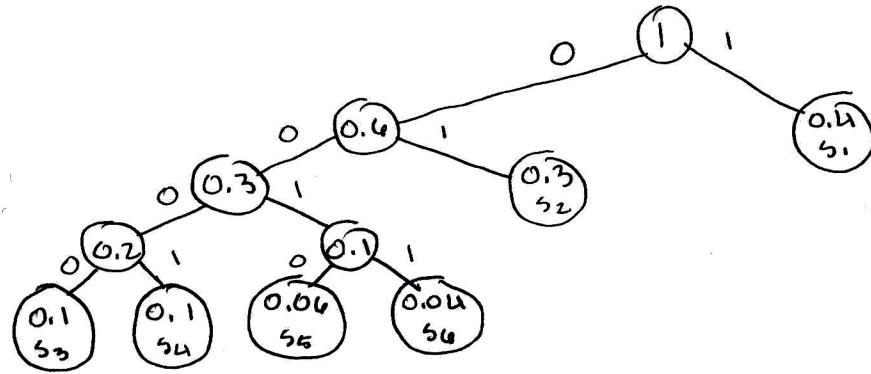
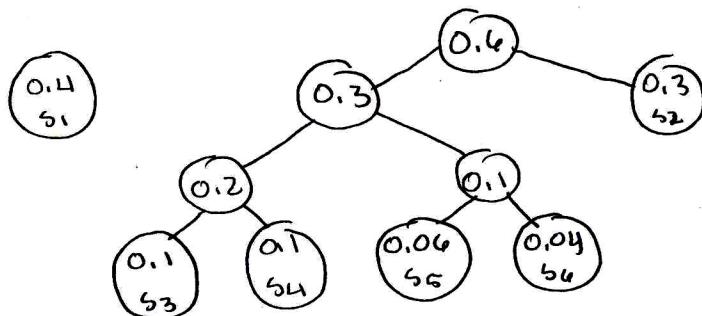
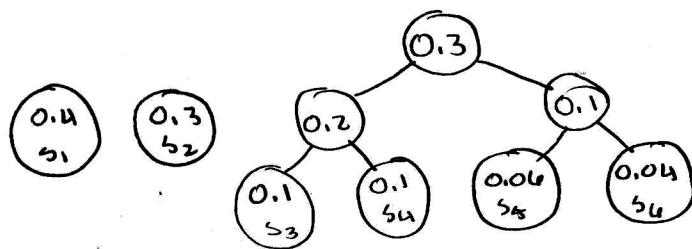
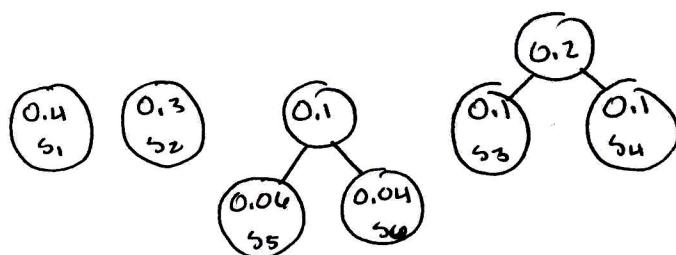
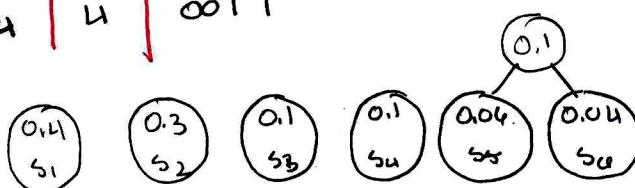
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<u>s<sub>i</sub></u>	<u>P(s<sub>i</sub>)</u>	<u>l<sub>i</sub></u>	<u>Código</u>
s <sub>1</sub>	0.4	1	1
s <sub>2</sub>	0.3	2	01
s <sub>3</sub>	0.1	4	0000
s <sub>4</sub>	0.1	4	0001
s <sub>5</sub>	0.06	4	0010
s <sub>6</sub>	0.04	4	0011

$$L = \sum_{i=1}^6 P_i l_i$$

$$L = (0.4)(1) + (0.3)(2) + (0.1 + 0.1 + 0.06 + 0.04)(4)$$

L = 2.2 bits/message +



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$s_i$	$P(s_i)$	Código A	Código B
$s_1$	$\frac{1}{3}$	00	1
$s_2$	$\frac{1}{3}$	01	2
$s_3$	$\frac{1}{9}$	100	01
$s_4$	$\frac{1}{9}$	101	02
$s_5$	$\frac{1}{27}$	111	006
$s_6$	$\frac{1}{27}$	1106	001
$s_7$	$\frac{1}{27}$	1101	002

a)

