Mechael Nuñez 1800522-0 1N Mate 6

"HDT#6"

Serie 1:

1) $3te^{-9|t|} \to \mathcal{F}\{f(t)\} = 3i \frac{d}{dw}\left(18/(81+w^2)\right)$

$\frac{d}{dw} 18(81+w^2)^{-1} = -18(81+w^2)^{-2} 2w = -36w(81+w^2)^{-2}$

$\hat{f}(w) = -\frac{108iw}{(81+w^2)^2}$

2) $26H(t)te^{-2t} \to \mathcal{F}\{f(t)\} = 26i \frac{d}{dw}\left(\frac{1}{2+iw}\right)$

$\frac{d}{dw}(2+iw)^{-1} = -(2+iw)^{-2} i = -i(2+iw)^{-2}$

$\hat{f}(w) = \frac{26}{(2+iw)^2}$

3) $\frac{d}{dw}\left(H(t)e^{-3t}\right) \to \mathcal{F}\{f(t)\} = \frac{iw}{3+iw}$

Serie 4: $\int_{-\infty}^{\infty} |f(t)|^2 dt \qquad f(t) = H(t)e^{-2t}$

$\to \int_0^{\infty} e^{-4t} dt \to -\frac{e^{-4t}}{4}\Big|_0^{\infty \to 0} = \frac{1}{4}$

Sesgo 3: $y'' + 6y' + 5y = \delta(t-3)$

$\mathcal{F}\{y'' + 6y' + 5y\} = \mathcal{F}\{\delta(t-3)\}$

$\rightarrow -w^2 Y(w) + 6jw\, Y(w) + 5Y(w) = e^{-jw3}$

$\rightarrow Y(w)\left[-w^2 + 6jw + 5\right] = e^{-3jw}$

$\rightarrow Y(w) = -\dfrac{e^{-3jw}}{w^2 - 6jw - 5} \quad \left(\rightarrow Y(w) = -\dfrac{e^{-3jw}}{(w-j)(w-5j)}\right)$

$\rightarrow \dfrac{e^{-3jw}}{w^2 - 6jw + 3 - 8} \rightarrow \dfrac{e^{-3jw}}{(w-3j)^2 + 12 - 8} \rightarrow \dfrac{e^{-3jw}}{(w-3j)^2 + 4}$

$\rightarrow \dfrac{e^{-3jw}}{(w-3j)^2 + 2^2} \rightarrow \mathcal{F}^{-1}\{Y(w)\} = \mathcal{F}^{-1}\left\{\dfrac{e^{-3jw}}{(w-3j)^2 + 2^2}\right\}$

$\rightarrow t_0 = 3 \quad w_0 = 3j \rightarrow \dfrac{1}{w^2 + 2^2} \rightarrow a = 2 \rightarrow \dfrac{1}{4} e^{-2|t|}$

$\rightarrow \dfrac{1}{4} e^{-3t} e^{-2|t|} \quad t_0 = 3 \rightarrow \underline{\dfrac{1}{4} e^{-3t} e^{-2|(t-3)|}} \; //$

$$y = \frac{1}{4} e^{3j^2 t} e^{-2|(t-3)|} \; //$$

Serie 2:

1) $\frac{1}{(1+3w)^2} \rightarrow \frac{1}{(1+3w)} \cdot \frac{1}{(1+3w)}$ // $\mathcal{F}^{-1}$

$\rightarrow H(t)e^{-t} * H(t)e^{-t} \rightarrow \int_{-\infty}^{\infty} H(\tau)e^{-\tau} H(t-\tau)e^{-(t-\tau)} d\tau$

$H(\tau) \begin{cases} 0, & \tau < 0 \\ 1, & \tau \geq 0 \end{cases}$ $\quad H(t-\tau) \begin{cases} 0, & t < \tau \\ 1, & t \geq \tau \end{cases}$

$\rightarrow \int_{-\infty}^{\infty} H(\tau)H(t-\tau)e^{-t} d\tau \rightarrow e^{-t}\int_{0}^{\infty} d\tau$ con $\tau \geq 0 \wedge t \geq \tau$

$e^{-t}\left[\tau\right]_{0}^{\infty}$ con $\tau \geq 0 \rightarrow e^{-t}(\infty - 0)$ con $\begin{matrix}\tau \geq 0 \\ t \geq \tau\end{matrix}$ // ≈ 0

2) $\frac{\operatorname{sen}(3w)}{w(2+3w)} \rightarrow \frac{1}{2} \cdot \frac{2\operatorname{sen}(3w)}{w} \cdot \frac{1}{(2+3w)}$ // $\mathcal{F}^{-1}$

$\rightarrow \frac{1}{2}\left[H(t+3) - H(t-3)\right] * H(t)e^{-2t}$

$\rightarrow \frac{1}{2}\int_{-\infty}^{\infty}\left[H(\tau+3) - H(\tau-3)\right] H(t-\tau)e^{-2t+2\tau} d\tau$

$\left[H(\tau+3) - H(\tau-3)\right] \begin{cases} 0, & \text{otros} \\ 1, & -3 \leq \tau \leq 3 \end{cases}$ $\quad H(t-\tau) \begin{cases} 0, & t < \tau \\ 1, & t \geq \tau \end{cases}$

$\rightarrow \frac{1}{2}\int_{-3}^{3} e^{-2t}e^{2\tau} d\tau \rightarrow \frac{e^{-2t}}{2}\left[\frac{e^{2\tau}}{2}\right]_{-3}^{3}$ con $t \geq \tau$

$\rightarrow \frac{e^{-2t}}{4}(e^{6} - e^{-6})$ con $t \geq 3$ //