SOLUCIÓN HOJA DE TRABAJO No. 4 - Regla de L'Hospital

1.

$$\lim_{x\to -1}\frac{x^2-1}{x+1}=\lim_{x\to -1}\frac{(x+1)(x-1)}{x+1}=\lim_{x\to -1}(x-1)=-2$$

2.

$$\lim_{x\to 1}\frac{x^a-1}{x^b-1}\overset{\mathrm{H}}{=}\lim_{x\to 1}\frac{ax^{a-1}}{bx^{b-1}}=\frac{a}{b}$$

3.

$$\lim_{x\to 0}\frac{x+\tan x}{\sin x}\overset{\mathrm{H}}{=}\lim_{x\to 0}\frac{1+\sec^2 x}{\cos x}=\frac{1+1^2}{1}=2$$

4.

$$\lim_{x\to \pi}\frac{\tan x}{x}=\frac{\tan\pi}{\pi}=\frac{0}{\pi}=0$$

5.

$$\lim_{x\to \infty}\frac{e^x}{x}\overset{\mathrm{H}}{=}\lim_{x\to \infty}\frac{e^x}{1}=\lim_{x\to \infty}e^x=\infty$$

6.

$$\lim_{t\to 0}\frac{5^t-3^t}{t}\overset{\mathrm{H}}{=}\lim_{t\to 0}\frac{5^t\ln 5-3^t\ln 3}{1}=\ln 5-\ln 3=\ln\tfrac{5}{3}$$

7.

$$\lim_{x\to 0}\frac{e^x-1-x-x^2/2}{x^3}\overset{\mathrm{H}}{=}\lim_{x\to 0}\frac{e^x-1-x}{3x^2}\overset{\mathrm{H}}{=}\lim_{x\to 0}\frac{e^x-1}{6x}\overset{\mathrm{H}}{=}\lim_{x\to 0}\frac{e^x}{6}=\frac{1}{6}$$

8.

$$\lim_{x\to \infty}\frac{(\ln x)^3}{x^2}\overset{\mathrm{H}}{=}\lim_{x\to \infty}\frac{3(\ln x)^2(1/x)}{2x}=\lim_{x\to \infty}\frac{3(\ln x)^2}{2x^2}\overset{\mathrm{H}}{=}\lim_{x\to \infty}\frac{6(\ln x)(1/x)}{4x}$$

$$=\lim_{x\to \infty}\frac{3\ln x}{2x^2}\overset{\mathrm{H}}{=}\lim_{x\to \infty}\frac{3/x}{4x}=\lim_{x\to \infty}\frac{3}{4x^2}=0$$

9.

$$\lim_{x\to 0}\frac{\cos mx-\cos nx}{x^2}\overset{\mathrm{H}}{=}\lim_{x\to 0}\frac{-m\sin mx+n\sin nx}{2x}\overset{\mathrm{H}}{=}\lim_{x\to 0}\frac{-m^2\cos mx+n^2\cos nx}{2}=\tfrac{1}{2}\left(n^2-m^2\right)$$

10.

$$\lim_{x\to 0}\frac{1-e^{-2x}}{\sec x}=\frac{1-1}{1}=0.$$

11.

$$\lim_{x\to 0^+}\sqrt{x}\ln x=\lim_{x\to 0^+}\frac{\ln x}{x^{-1/2}}\overset{\mathrm{H}}{=}\lim_{x\to 0^+}\frac{1/x}{-\frac{1}{2}x^{-3/2}}=\lim_{x\to 0^+}\left(-2\sqrt{x}\right)=0$$

12.

$$\lim_{x\to -\infty}x^2e^x=\lim_{x\to -\infty}\frac{x^2}{e^{-x}}\overset{\mathrm{H}}{=}\lim_{x\to -\infty}\frac{2x}{-e^{-x}}\overset{\mathrm{H}}{=}\lim_{x\to -\infty}\frac{2}{e^{-x}}=\lim_{x\to -\infty}2e^x=0$$

13.

$$\lim_{x\to(\pi/2)^-} \sec 7x \cos 3x = \lim_{x\to(\pi/2)^-} \frac{\cos 3x}{\cos 7x} = \lim_{x\to(\pi/2)^-} \frac{-3\sin 3x}{-7\sin 7x} = \frac{3(-1)}{7(-1)} = \frac{3}{7}$$

14.

$$\lim_{x\to\pi} (x-\pi)\cot x = \lim_{x\to\pi} \frac{x-\pi}{\tan x} \overset{\text{H}}{=} \lim_{x\to\pi} \frac{1}{\sec^2 x} = \frac{1}{(-1)^2} = 1$$

15.

$$\lim_{x\to 0} (\csc x - \cot x) = \lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right) = \lim_{x\to 0} \frac{1-\cos x}{\sin x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{\sin x}{\cos x} = 0$$

16.

$$\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right) = \lim_{x\to 1} \frac{x-1-\ln x}{(x-1)\ln x} \overset{\text{H}}{=} \lim_{x\to 1} \frac{1-1/x}{\ln x + (x-1)(1/x)}$$

$$= \lim_{x\to 1} \frac{x-1}{x\ln x + x - 1} \overset{\text{H}}{=} \lim_{x\to 1} \frac{1}{\ln x + 1 + 1} = \frac{1}{0+2} = \frac{1}{2}$$

17.

$y = x^{\sin x} \quad \Rightarrow \quad \ln y = \sin x \ln x$, so

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \sin x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\csc x} \overset{\text{H}}{=} \lim_{x\to 0^+} \frac{1/x}{-\csc x \cot x} = -\left(\lim_{x\to 0^+} \frac{\sin x}{x}\right)\left(\lim_{x\to 0^+} \tan x\right)$$

$$= -1 \cdot 0 = 0 \quad \Rightarrow$$

$$\lim_{x\to 0^+} x^{\sin x} = \lim_{x\to 0^+} e^{\ln y} = e^0 = 1.$$

18.

$y = (1-2x)^{1/x} \quad \Rightarrow \quad \ln y = \frac{1}{x}\ln(1-2x)$, so $\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\ln(1-2x)}{x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{-2/(1-2x)}{1} = -2 \quad \Rightarrow$

$\lim_{x\to 0} (1-2x)^{1/x} = \lim_{x\to 0} e^{\ln y} = e^{-2}.$