Funciones de transferencia

1. Un sistema se describe con la ecuación diferencial

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = 1;\; x(0) = 1 \text{ y } \frac{dx(0)}{dt} = -1$$

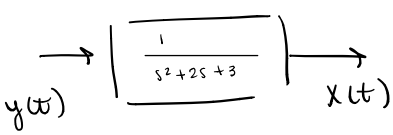
Describa al sistema utilizando una función de transferencia (escriba la función de transferencia, la señal de entrada y la señal de salida).

si asumimos $x(0) = 0$ y $\frac{dx}{dt}(0) = 0$ condiciones iniciales $= 0$

$$s^2 X(s) + 2s X(s) + 3X(s) = Y(s)$$

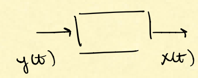
$$(s^2 + 2s + 3) X(s) = Y(s)$$

$$G(s) = \frac{X(s)}{Y(s)} = \frac{1}{s^2 + 2s + 3}$$

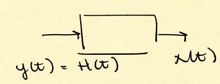


describimos el sistema por medio de una función de transferencia.

Ahora lo usaremos cuando las condiciones iniciales $!= 0$.



$\frac{dx}{dt} + 2x = y(t)$; $x(0) = x_0$



$x(t) =$ ______ , $t > 0$

$\frac{d^2x}{dt}x + 2\frac{d}{dt}x + 3x = H(t)$

entonces:

$$s^2 X(s) + 2s X(s) + 3X(s) = Y(s)$$

$$(s^2 X(s) - s + 1) + 2[s X(s) - 1] + 3X(s) = Y(s)$$

$$(s^2 + 2s + 3) X(s) = Y(s) + s + 1$$

$$X(s) = \frac{Y(s)}{s^2 + 2s + 3} + \frac{s+1}{s^2 + 2s + 3}$$

calcular $x(t)$ para $y(t) = H(t)$

$X(s) = \left[\frac{1}{s^2+2s+3}\right]\frac{1}{s} \;\; = X(s) = G(s) * Y(s)$

$x(t) = \mathcal{L}^{-1}\{X(s)\}$

condiciones iniciales iguales a 0

$\frac{df}{dt} \to sF(s) - f(0)$

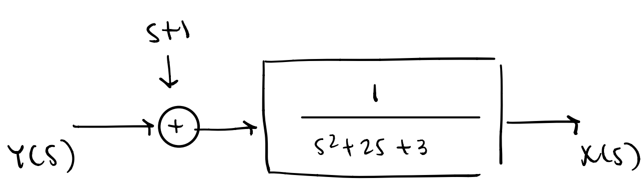
$\frac{d^2f}{dt^2} \to s^2F(s) - sf(0) - f'(0)$

$(s^2 X(s) - s + 1)$

$s^2X(s) - s\,x(0) - x'(0)$

condiciones iniciales $x(0) = 1$

$s^2 X(s) - s - (-1)$ *ver def. prob.

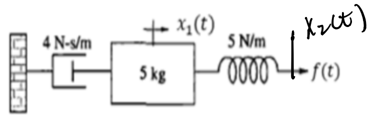


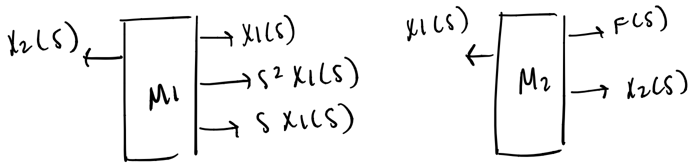
2. Encuentre la función de transferencia $G(s) = X_1(s)/F(s)$.

s^2 · inercia

s · viscoso

k · resorte





1) $X_1(s) + s^2 X_1(s) + s X_1(s) = X_2(s)$

$(s^2+s+1)X_1(s) - X_2(s) = 0$ ó $(s^2+s+1)X_1(s) = X_2(s)$

2) $F(s) + X_2(s) = X_1(s)$

$-X_1(s) + X_2(s) = F(s)$

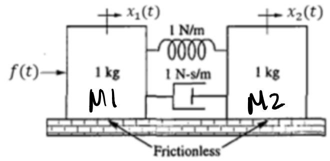
$\Rightarrow -X_1(s) + (s^2+s+1)X_1(s) = F(s)$

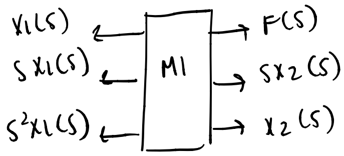
$(s^2+s \cancel{+1} \cancel{-1})X_1(s) = F(s)$

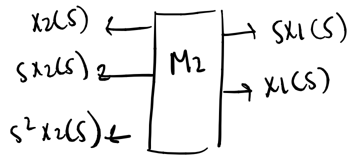
$(s^2+s)X_1(s) = F(s)$

$\frac{X_1(s)}{F(s)} = \frac{1}{s^2+s}$ □

3. Encuentre la función de transferencia $G(s) = X_2(s)/F(s)$.







1) $(s^2+s+1)\,X_1(s) - (s+1)\,X_2(s) = F(s)$

2) $-(s+1)\,X_1(s) + (s^2+s+1)\,X_2(s) = 0$

⇒ resolviendo:

1) $AX_1 - BX_2 = F$

2) $-CX_1 + DX_2 = 0$

3) $-CX_1 + DX_2 = 0$

$$CX_1 = DX_2$$

$$X_1 = \frac{D}{C}X_2$$

⇒ sustituyendo:

$$AX_1 - BX_2 = F$$

$$A\left(\frac{D}{C}\right)X_2 - BX_2 = F$$

$$\left(\frac{AD}{C} - B\right)X_2 = F$$

$$\left(\frac{AD-BC}{C}\right)X_2 = F$$

$$\frac{X_2}{F} = \frac{C}{AD-BC}$$

$(s^2+s+1)\,X_1(s) - (s+1)\,X_2(s) = F(s)$

$-(s+1)\,X_1(s) + (s^2+s+1)\,X_2(s) = 0$

$\Rightarrow$ reemplazando valores

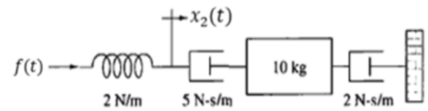
$$\frac{X_2(s)}{F(s)} = \frac{s+1}{(s^2+s+1)(s^2+s+1) - (s+1)(s+1)}$$

$$\frac{X_2(s)}{F(s)} = \frac{s+1}{s^4 \cancel{+s^3} \cancel{+s^2} \cancel{+s^3} \cancel{+s^2} \cancel{+s} \cancel{+s^2} \cancel{+s} +1 - s^2 - 2s - 1}$$

$$\frac{X_2(s)}{F(s)} = \frac{s+1}{s^4 +2s^3 +3s^2 \cancel{+2s} \cancel{+1} - s^2 \cancel{-2s} \cancel{-1}}$$

$$\frac{X_2(s)}{F(s)} = \frac{s+1}{s^4 +2s^3 +2s^2}$$

4. Encuentre la función de transferencia $G(s) = X_2(s)/F(s)$.



25. Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical system shown in Figure P2.11. (Hint: place a zero mass at $x_2(t)$.) [Section: 2.5]

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Control Solutions

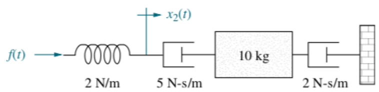

FIGURE P2.11

25.

Let $X_1(s)$ be the displacement of the left member of the spring and $X_3(s)$ be the displacement of the mass.

Writing the equations of motion

$$2x_1(s) - 2x_2(s) = F(s)$$
$$-2X_1(s) + (5s+2)X_2(s) - 5sX_3(s) = 0$$
$$-5sX_2(s) + (10s^2 + 7s)X_3(s) = 0$$

Solving for $X_2(s)$,

$$X_2(s) = \frac{\begin{vmatrix} 5s^2+10 & F(s) \\ -10 & 0 \end{vmatrix}}{\begin{vmatrix} 5s^2+10 & -10 \\ -10 & \frac{1}{5}s+10 \end{vmatrix}} = \frac{10F(s)}{s(s^2+50s+2)}$$

Thus, $\dfrac{X_2(s)}{F(s)} = \dfrac{1}{10}\dfrac{(10s+7)}{s(5s+1)}$

como hacer kramer? A partir de las ecuaciones de movimiento.

para el ing. X_3 es ↑aquí X_1

$\Rightarrow (10s^2 + 7s)\,X_1(s) - 5s\,X_2(s) = 0$

$-5\,X_1(s) + (5s+2)\,X_2(s) - 2\,X_3(s) = 0$

$-2\,X_2(s) + 2\,X_3(s) = F(s)$

entonces

$$\begin{bmatrix} 10s^2+7s & -5s & 0 \\ -5 & 5s+2 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F(s) \end{bmatrix}$$

Ahora revisemos que el determinante de la matriz no sea = '0'

paso 1: calcular det (A) por la place.

seleccionamos primera columna.

$$\det(A) = (-1)^{3+1}\,(0)\;\det\begin{bmatrix} -5s & 0 \\ 5s+2 & -2 \end{bmatrix} = 0$$

← columna

fila

número en posición

$$(-1)^{2+1}(-5s)\;\det\begin{bmatrix} -5s & 0 \\ -2 & 2 \end{bmatrix} = 5s\left(-10s\right) = -50s^2$$

$$(-1)^{1+1}(10s^2+7s)\;\det\begin{bmatrix} 5s+2 & -2 \\ -2 & 2 \end{bmatrix} = (10s^2+7s)(10s+\cancel{4}-\cancel{4})$$

$$(10s^2+7s)(10s)$$

$$= 100s^3+70s^2$$

$$\det(A) = 0 + (-50s^2) + (100s^3+70s^2)$$

$$= 100s^3 + 20s^2$$

$$= 20s^2(5s+1)$$

↓ reemplazo por el resultado.

paso 2:
$$X_2(s) = \frac{\det\begin{bmatrix} 10s^2+7s & 0 & 0 \\ -5s & 0 & -2 \\ 0 & F(s) & 2 \end{bmatrix}}{\det(A)}$$

$$\det \rightarrow (-1)^{1+1}(10s^2+7s)\;\det\begin{bmatrix} 0 & -2 \\ F(s) & 2 \end{bmatrix} = (10s^2+7s)(2F(s))$$

$$X_2(s) = \frac{(10s^2+7s)\,2F(s)}{100s^3+20s^2} = \frac{(20s^2+14s)\,F(s)}{100s^3+20s^2}$$

$$\frac{X_2(s)}{F(s)} = \frac{20s^2+14s}{100s^3+20s^2} = \frac{\cancel{2s}(10s+7)}{\overset{10}{\cancel{20}}s^2(5s+1)} = \frac{10s+7}{10s(5s+1)}$$