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martes, 23 de marzo de 2021 12:35

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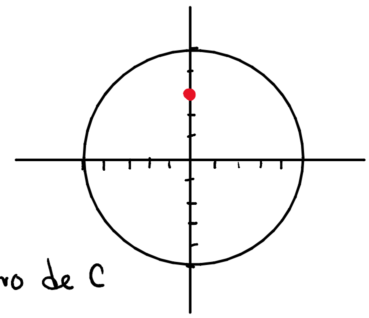
Matemática VII - AN

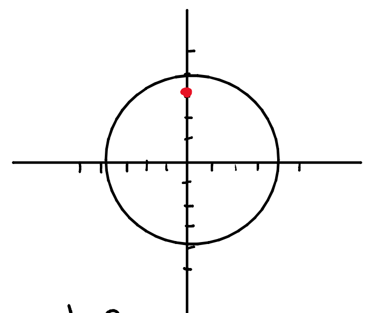
Evaluar las siguientes integrales a lo largo del contorno cerrado indicado.

1. $\oint_c \frac{4}{z-3i}dz \qquad |z| = 5$



C, es una curva cerrada simple
$f(z) = 4$, es holomorfa en y dentro de C
$z_0 = 3i$

$$\int_{|z|=5} \frac{4}{z-3i}\,dz = 2\pi i f(3i) = 8\pi i \;\#$$

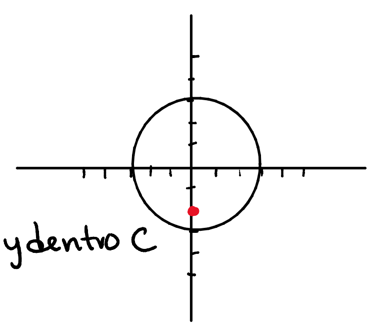


2. $\oint_c \frac{e^z}{z-\pi i}dz \qquad |z| = 4$

C, es una curva cerrada simple
$f(z) = e^z$, es holomorfa en y dentro de C
$z_0 = \pi i$

$$\int_{|z|=4} \frac{e^z}{z-\pi i}\,dz = 2\pi i f(\pi i) = 2\pi i e^{\pi i} \;\#$$

3. $\oint_c \frac{z^2-3z+4i}{z+2i}dz \qquad |z| = 3$



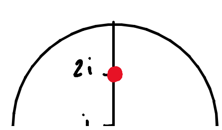
C, es una curva cerrada simple
$f(z) = z^2 - 3z + 4i$, holomorfa en y dentro C
$z_0 = \pi i$

$$\int_{|z|=3} \frac{z^2-3z+4i}{z+2i} = 2\pi i f(-2i) = 2\pi i \left[(-2i)^2 - 3(-2i) + 4i \right]$$

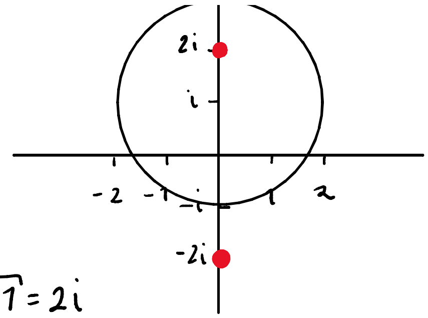
$$= 2\pi i \left[-4 + 6i + 4i \right] = 2\pi i (10i - 4)$$

$$= -20\pi - 8\pi i \;\#$$

4. $\oint_c \frac{z^2}{z^2+4}dz \qquad |z - i| = 2$



$\int_C \frac{\;}{z^2+4} dz \qquad |z-i|=2$



$$\int_c \frac{z^2}{z^2+4} dz = \int_c \frac{z^2}{z^2-(-4)}$$

$$\sqrt{-4} = \sqrt{4}\cdot\sqrt{-1} = 2i$$

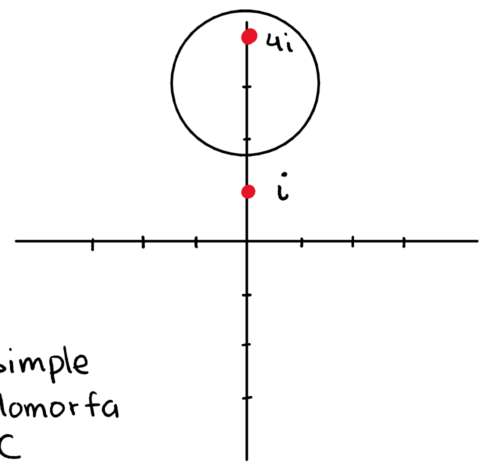
$$= \int_c \frac{z^2}{(z-2i)(z+2i)} dz$$

$$= \int_{|z-i|=2} \frac{z^2/(z+2i)}{z-2i} dz$$

C, es una curva cerrada simple
$f(z) = z^2/(z+2i)$ es holomorfa en y dentro de C
$z_0 = 2i$

$$= 2\pi i f(2i) = 2\pi i \left(\frac{(2i)^2}{2i+2i} \right) = 2\pi \cancel{i} \left(\frac{-\cancel{4}}{\cancel{4}\cancel{i}} \right) = -2\pi \quad \#$$

5. $\oint_c \frac{z^2+4}{z^2-5iz-4} dz \qquad |z-3i| = 1.3$



$z \quad -4i \qquad (z-4i)(z-i)$
$z \quad -1i$

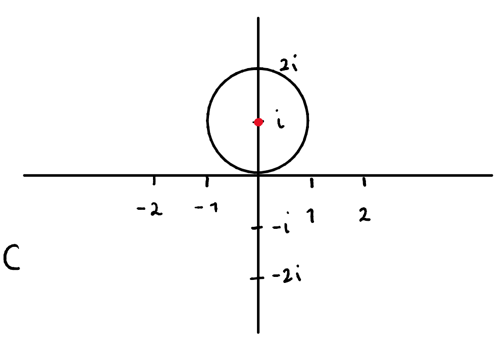
$$\int_c \frac{z^2+4}{(z-4i)(z-i)} dz =$$

$$\int_c \frac{z^2+4/z-i}{z-4i} dz =$$

C, es una curva cerrada simple
$f(z) = z^2+4/(z-i)$, holomorfa en y dentro de C
$z_0 = 4i$

$$2\pi i f(4i) = 2\pi i \left(\frac{(4i)^2+4}{4i-i} \right) = 2\pi i \left(\frac{-16+4}{3i} \right) = 2\pi \cancel{i} \left(\frac{-12}{3\cancel{i}} \right) = -8\pi \quad \#$$

6. $\oint_c \frac{e^{z^2}}{(z-i)^3} dz \qquad |z-i| = 1$



C, es una curva cerrada simple
$f(z) = e^{z^2}$ es holomorfa en y dentro de C
$z_0 = i$ dentro de C

$$\int_c \frac{e^{z^2}}{(z-i)^3} dz = \frac{2\pi i}{2!} f''(z_0) = \frac{2\pi i}{2} f''(z_0)$$

$$\int_c \frac{e^{z^2}}{(z-i)^3}\,dz = \frac{2\pi i}{2!} f''(z_0) = \frac{2\pi i}{2} f''(z_0)$$

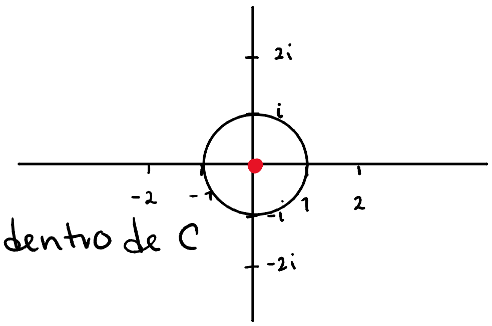
$f'(z) = e^{z^2}(2z)$

$f''(z) = 2e^{z^2} + 4z^2 e^{z^2}$

$$\Rightarrow \pi i\left[2e^{-1} - 4e^{-1}\right] \quad 2\pi i e^{-1} - 4\pi i e^{-1}$$

$$= -2\pi i e^{-1} \;\#$$

7. $\oint_c \frac{\cos 2z}{z^5}dz \qquad |z| = 1$



C es una curva cerrada simple
$f(z) = \cos(2z)$ holomorfa en y dentro de C
$z_0 = 0$ dentro de C

$$\int_c \frac{\cos(2z)}{(z-0)^5}\,dz = \frac{2\pi i}{4!} f''''(0)$$

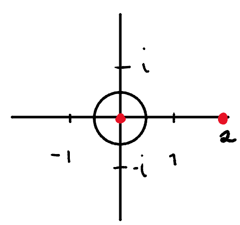
$f'(z) = -2\,\text{sen}(2z)$
$f''(z) = -4\cos(2z)$
$f'''(z) = 8\,\text{sen}(2z)$
$f''''(z) = 16\cos(2z)$

$$\Rightarrow \frac{2\pi i}{24}\, 16\,\overset{1}{\cancel{\cos(2(0))}} = \frac{4}{3}\pi i$$

8. $\oint_c \frac{2z+5}{z^2-2z}dz \qquad$ **(a)** $|z| = \frac{1}{2} \qquad$ **(b)w** $|z+1| = 2$

$$\int_c \frac{2z+5}{z(z-2)}\,dz$$

a) $C: |z| = \frac{1}{2}$

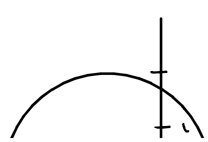


$$\int_c \frac{(2z+5)/(z-2)}{z-0}\,dz$$

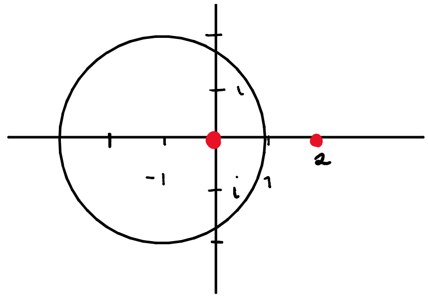
C es una curva cerrada simple
$f(z) = \frac{2z+5}{z-2}$, homorfa en y dentro de C
$z_0 = 0$ dentro de C

$$\int_{|z|=\frac{1}{2}} \frac{(2z+5)/(z-2)}{z-0}\,dz = 2\pi i f(0) = 2\pi i\left(\frac{2(0)+5}{0-2}\right) = 2\pi i\left(-\frac{5}{2}\right) = -5\pi i \;\#$$

b) $C: |z+1| = 2$



b) $C: |z+1| = 2$



$$\int_{|z+1|=2} \frac{(2z+5)/(z-2)}{z-0}\, dz$$

C es una curva cerroda simple

$f(z) = \dfrac{2z+5}{z-2}$, homorfa en y dentro de C

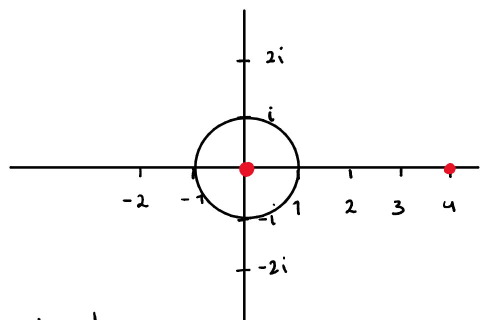
$z_0 = 0$ dentro de C

y por teorema de deformación $\int_C f = \int_{\hat{C}} f$

$$\int_{|z+1|=2} \frac{(2z+5)/(z-2)}{z-0} = -5\pi i \quad \#$$

9. $\oint_c \frac{1}{z^3(z-4)} dz$ **(a)** $|z| = 1$ **(b)** $|z-2| = 1$

a: $|z| = 1$



$$\int_c \frac{1/(z-4)}{(z-0)^3}\, dz$$

C es una curva cerrada simple

$f(z) = \dfrac{1}{(z-4)}$ es holomorfa en y dentro C

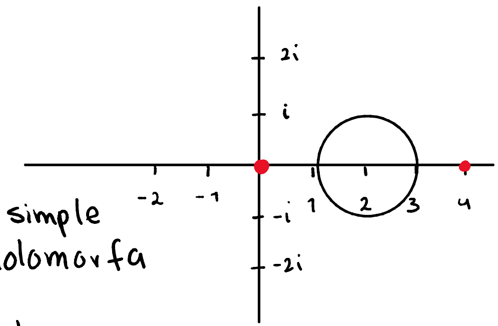
$z_0 = 0$, dentro de C

$$\int_{|z|=1} \frac{1/(z-4)}{(z-0)^3}\, dz = \frac{2\pi i}{2!} f''(0)$$

$$f'(z) = (z-4)^{-1'} = -(z-4)^{-2}(1) = -\frac{1}{(z-4)^2} \Rightarrow \pi i \left(\frac{2}{(0-4)^3} \right) = -\frac{\pi i}{32} \quad \#$$

$$f''(z) = -(z-4)^{-2'} = 2(z-4)^{-3} = \frac{2}{(z-4)^3}$$

b) $|z-2|=1$



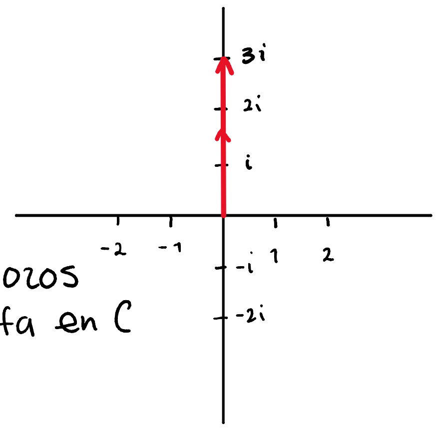
C_1 es una curva cerrada simple
$f(z) = \dfrac{1/(z-4)}{z^3}$, holomorfa
en y dentro de C

Por teorema de Cauchy

$$\int_{|z-2|=1} \frac{1/(z-4)}{(z-0)^3} = 0 \quad \#$$

10. $\oint_c \sin(z)dz$ c = recta de 0 a 3i

$$\int_c \sin(z)\,dz$$



C es una curva suave a trozos
$f(z) = \sin(z)$ es holomorfa en C

Teorema Fundamental

$$-\cos(z)\Big|_0^{3i} = -\cos(3i) + \cos(0) = 1 - \cos(3i) \quad \#$$