Repaso

Ejercicio No. 25 capitulo 2

25. Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical system shown in Figure P2.11. (Hint: place a zero mass at $x_2(t)$.) [Section: 2.5]

WileyPLUS
WPCS
Control Solutions

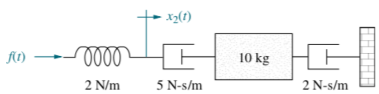

FIGURE P2.11

25.

Let $X_1(s)$ be the displacement of the left member of the spring and $X_3(s)$ be the displacement of the mass.

Writing the equations of motion

$$2x_1(s) - 2x_2(s) = F(s)$$
$$-2X_1(s) + (5s+2)X_2(s) - 5sX_3(s) = 0$$
$$-5sX_2(s) + (10s^2 + 7s)X_3(s) = 0$$

Solving for $X_2(s)$,

$$X_2(s) = \frac{\begin{vmatrix} 5s^2+10 & F(s) \\ -10 & 0 \end{vmatrix}}{\begin{vmatrix} 5s^2+10 & -10 \\ -10 & \frac{1}{5}s+10 \end{vmatrix}} = \frac{10F(s)}{s(s^2+50s+2)}$$

Thus, $\dfrac{X_2(s)}{F(s)} = \dfrac{1}{10}\dfrac{(10s+7)}{s(5s+1)}$

como hacer Kramer? A partir de las ecuaciones de movimiento.

para el ing. X_3 es ↑aquí X_1

$$\Rightarrow (10s^2 + 7s)\,X_1(s) - 5s\,X_2(s) = 0$$
$$-5\,X_1(s) + (5s+2)\,X_2(s) - 2\,X_3(s) = 0$$
$$-2\,X_2(s) + 2\,X_3(s) = F(s)$$

entonces

$$\begin{bmatrix} 10s^2+7s & -5s & 0 \\ -5 & 5s+2 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F(s) \end{bmatrix}$$

Ahora revisemos que el determinante de la matriz no sea = '0'

paso 1: calcular det (A) por la place.

seleccionamos primera columna.

$$\det(A) = \underset{\text{fila}}{(-1)^{3+1 \leftarrow \text{columna}}} \underset{\uparrow\ \text{numero en posición}}{(0)} \det\begin{bmatrix} -5s & 0 \\ 5s+2 & -2 \end{bmatrix} = 0$$

$$(-1)^{2+1}(-5s)\det\begin{bmatrix} -5s & 0 \\ -2 & 2 \end{bmatrix} = 5s\left(-10s\right) = -50s^2$$

$$(-1)^{1+1}(10s^2+7s)\det\begin{bmatrix} 5s+2 & -2 \\ -2 & 2 \end{bmatrix} = (10s^2+7s)(10s+\cancel{4}-\cancel{4})$$

$$(10s^2+7s)(10s)$$

$$= 100s^3+70s^2$$

$$\det(A) = 0 + (-50s^2) + (100s^3+70s^2)$$

$$= 100s^3 + 20s^2$$

$$= 20s^2(5s+1)$$

$\downarrow$ reemplazo por el resultado.

paso 2:

$$X_2(s) = \frac{\det\begin{bmatrix} 10s^2+7s & 0 & 0 \\ -5s & 0 & -2 \\ 0 & F(s) & 2 \end{bmatrix}}{\det(A)}$$

$$\det \nearrow (-1)^{1+1}(10s^2+7s)\det\begin{bmatrix} 0 & -2 \\ F(s) & 2 \end{bmatrix} = (10s^2+7s)(2F(s))$$

$$X_2(s) = \frac{(10s^2+7s)\,2F(s)}{100s^3+20s^2} = \frac{(20s^2+14s)\,F(s)}{100s^3+20s^2}$$

$$\frac{X_2(s)}{F(s)} = \frac{20s^2+14s}{100s^3+20s^2} = \frac{\cancel{2s}(10s+7)}{\overset{10}{\cancel{20}}s^2(5s+1)} = \frac{10s+7}{10s(5s+1)}$$

TAREA

¿De que sirve conocer la función de transferencia?

Encontrar respuesta del sistema $x_2(t)$ y graficarla

¿Qué se puede concluir?

hint: $X_2(s) = \left[\dfrac{10s+7}{10s(5s+1)}\right]$ (1) $\Rightarrow$ esto salió de $G(s) = \dfrac{X_2(s)}{F(s)}$

$$\downarrow$$

$$x_2(t) = \mathcal{L}^{-1}\{X_2(s)\}$$

$$\rightarrow \frac{10s+7}{10s(5s+1)} = \frac{A}{10s} + \frac{B}{5s+1}$$

$$\frac{(10s+7)\cancel{(10s)(5s+1)}}{\cancel{10s(5s+1)}} = \frac{A\cancel{(10s)}(5s+1)}{\cancel{10s}} + \frac{B\cancel{(10s)}\cancel{(5s+1)}}{10\cancel{(5s+1)}}$$

$$10s+7 = A(5s+1) + Bs$$

$$10s+7 = A + (5A+B)s$$

$$7 = A$$

$$10 = 5A + B$$

$$\Rightarrow 10 = 35 + B$$

$$10 - 35 = B$$

$$-25 = B$$

$$\Rightarrow \frac{10s+7}{10s(5s+1)} = \frac{7}{10s} - \frac{5}{2(5s+1)}$$

$$\mathcal{L}^{-1}\left\{\frac{7}{10s} - \frac{5}{2(5s+1)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{7}{10s}\right\} - \mathcal{L}^{-1}\left\{\frac{5}{2(5s+1)}\right\}$$

$$\frac{1}{10}\mathcal{L}^{-1}\left\{\frac{7}{s}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{5}{5s+1}\right\} \quad // \div \frac{1}{5}$$

$$\frac{1}{10}\cdot 7u(t) - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{5}}\right\}$$

$$x_2(t) = \left(\frac{7}{10} - \frac{1}{2}e^{-\frac{1}{5}t}\right)u(t)$$

Nos sirve saber la funcion de transferencia ya que con eso conoceremos la relacion de entrada y salida del sistema.