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Demostrear que una fuente de información discreta, aleatoria,
y de memoria nula,

$$\log(q) \geq H(s) \rightarrow \log(q) - H(s) \geq 0.$$

$$\ln(x) \geq 1 - \frac{1}{x} \quad // \times (-1).$$

$$-\ln(x) \leq \frac{1}{x} - 1$$

$$\ln\left(\frac{1}{x}\right) \leq \frac{1}{x} - 1$$

$$\log(q) - H(s) = \log(q) - \sum_{i=1}^q p_i \log\left(\frac{1}{p_i}\right)$$

$$= \left(\sum_{i=1}^q p_i\right) \log(q) - \sum_{i=1}^q p_i \log\left(\frac{1}{p_i}\right)$$

$$\sum_{i=1}^q p_i \left[\log(q) - \log\left(\frac{1}{p_i}\right) \right] \quad // \quad \log\left(\frac{x}{y}\right) = \log(x) - \log(y)$$
$$\frac{q}{1/p_i} = q p_i$$

$$\sum_{i=1}^q p_i \log(q p_i).$$

$$\frac{1}{\ln(2)} \sum_{i=1}^q p_i \ln(q p_i)$$

Por lo demostrado sabemos que

$$\frac{1}{\ln(2)} \sum_{i=1}^q p_i \ln(q p_i) \geq \frac{1}{\ln(2)} \sum_{i=1}^q p_i \left[1 - \frac{1}{q p_i} \right]$$

$$\rightarrow \frac{1}{\ln(2)} \sum_{i=1}^q p_i - \frac{1}{q} = \frac{1}{\ln(2)} \left[\sum_{i=1}^q p_i - \sum_{i=1}^q \frac{1}{q} \right] = 0 \quad q \cdot 1/q = 1$$

$$\frac{1}{\ln(2)} \sum_{i=1}^q p_i \log(q p_i) \geq 0.$$

$$\log(q) - H(s) \geq 0.$$

$$\log(q) \geq \underline{\underline{H(s)}}$$