Info IV
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2007 2157

HT501

1.

$A=\{1,2\}$ $\quad$ $A\cap B=\{2\}$

$B=\{2,4,6\}$ $\quad$ $S=\{1,2,3,4,5,6\}$

a) $P(A|B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{\frac{1}{6}}{\frac{1}{2}}=\dfrac{2}{6}=\dfrac{1}{3}$

$P(A\cap B)=\dfrac{1}{6}$

$P(B)=\dfrac{3}{6}=\dfrac{1}{2}$

b) $P(B|A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{\frac{1}{6}}{\frac{1}{3}}=\dfrac{3}{6}=\dfrac{1}{2}$

$P(A)=\dfrac{2}{6}=\dfrac{1}{3}$

2.

a)

$$P(10\Omega|A)=\frac{1}{6}\cdot\frac{500}{1000}+\frac{1}{6}\cdot\frac{0}{1000}+\frac{1}{6}\cdot\frac{200}{1000}+\frac{1}{6}\cdot\frac{800}{1600}+\frac{1}{6}\cdot\frac{1200}{2000}+\frac{1}{6}\cdot\frac{1000}{2000}$$

$$=\frac{1}{12}+0+\frac{1}{30}+\frac{1}{12}+\frac{1}{10}+\frac{1}{12}=\frac{23}{60}=0.3833$$

b) $P(3|10\Omega)=\dfrac{P(10\Omega|3)P(3)}{P(10\Omega)}=\dfrac{\frac{1}{5}\cdot\frac{1}{6}}{0.3833}=0.08696$

$P(10\Omega)=\sum_{i=1}^{6}P(10|B_i)P(3)=0.3833$

3.

$P(0)=0.2$ $\quad$ $P(1)=0.8$

$P(0|0)=0.9$ $\quad$ $P(1|0)=0.1$

$P(0|1)=0.2$ $\quad$ $P(1|1)=0.8$

a) $P(1|0)=1-P(0|0)=0.1$

$P(1|1)=1-P(0|1)=0.8$

b)

$P(0|A\varepsilon) = P(0|0)P(0) + P(0|1)P(1) = 0.18 + 0.16 = 0.34$ +

c)

$$P(0|1) = \frac{P(1|0)\,P(0)}{P(1)} \qquad P(1) = P(1|0)P(0) + P(1|1)P(1) = 0.66$$

$$= \frac{(0.1)(0.2)}{0.66} = 0.0303$$ +

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Fuente S_1 $\quad s_i^1 \in S_1 = \{ s_1^1, s_2^1, \ldots, s_{q_1}^1 \}$

$$P_1 = P(s_1^1),\ P_2 = P(s_2^1), \ldots, P_{q_1} = P(s_{q_1}^1)$$

Fuente S_2 $\quad s_i^2 \in S_2 = \{ s_1^2, s_2^2, \ldots, s_{q_2}^2 \}$

$$Q_1 = P(s_1^2),\ Q_2 = P(s_2^2), \ldots, Q_{q_2} = P(s_{q_2}^2)$$

$$H_1 = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \ldots + P_{q_1} \log \frac{1}{P_{q_1}}$$

$$H_2 = Q_1 \log \frac{1}{Q_1} + Q_2 \log \frac{1}{Q_2} + \ldots + Q_{q_2} \log \frac{1}{Q_{q_2}}$$

Fuente $S(\lambda)$ $\quad s_i^3 \in S_3 = \{ s_1^3, s_2^3, \ldots s_{q_1}^3, s_{q_1+1}^3, s_{q_1+2}^3, \ldots, s_{q_1+q_2}^3 \}$

$$P(s_1^3) = \lambda P_1 \qquad P(s_{q_1+1}^3) = (1-\lambda) Q_1$$

$$P(s_2^3) = \lambda P_2 \qquad P(s_{q_1+2}^3) = (1-\lambda) Q_2$$

$$\vdots \qquad \vdots$$

$$P(s_{q_1}^3) = \lambda P_{q_1} \qquad P(s_{q_1+q_2}^3) = (1-\lambda) Q_{q_2}$$

$$H[S(\lambda)] = \lambda P_1 \log \frac{1}{\lambda P_1} + \lambda P_2 \log \frac{1}{\lambda P_2} + \ldots + \lambda P_{q_1} \log \frac{1}{\lambda P_{q_1}} + (1-\lambda) Q_1 \log \frac{1}{(1-\lambda) Q_1}$$

$$+ (1-\lambda) Q_2 \log \frac{1}{(1-\lambda) Q_2} + \ldots + (1-\lambda) Q_{q_2} \log \frac{1}{(1-\lambda) Q_{q_2}}$$

$$= \lambda P_1 \left[\log \frac{1}{\lambda} + \log \frac{1}{P_1} \right] + \lambda P_2 \left[\log \frac{1}{\lambda} + \log \frac{1}{P_1} \right] + \ldots + \lambda P_{q_1} \left[\log \frac{1}{\lambda} + \log \frac{1}{P_{q_1}} \right]$$

$$+ (1-\lambda) Q_1 \left[\log \frac{1}{1-\lambda} + \log \frac{1}{Q_1} \right] + (1-\lambda) Q_2 \left[\log \frac{1}{1-\lambda} + \log \frac{1}{Q_2} \right] + \ldots + (1-\lambda) Q_{q_2} \left[\log \frac{1}{1-\lambda} + \log \frac{1}{Q_{q_2}} \right]$$

$$= \lambda \log \frac{1}{\lambda} \left[\underbrace{P_1 + P_2 + \ldots + P_{q_1}}_{1} \right] + \lambda \left[\underbrace{P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \ldots + P_{q_1} \log \frac{1}{P_{q_1}}}_{H_1} \right]$$

$$+ (1-\lambda) \log \frac{1}{1-\lambda} \left[\underbrace{Q_1 + Q_2 + \ldots + Q_{q_2}}_{1} \right] + (1-\lambda) \left[\underbrace{Q_1 \log \frac{1}{Q_1} + Q_2 \log \frac{1}{Q_2} + \ldots + Q_{q_2} \log \frac{1}{Q_{q_2}}}_{H_2} \right]$$

$$= \underbrace{\lambda \log \frac{1}{\lambda} + \lambda H_1 + \tilde{\lambda} H_2 + (1-\lambda) \log \frac{1}{1-\lambda}}_{H(\lambda)}$$

$$= \lambda H_1 + \tilde{\lambda} H_2 + H(\lambda)$$

3.

Fuente S_i — $s_i \in S = \{s_1, s_2, \dots, s_q\}$

$$P_1 = P(s_1),\ P_2 = P(s_2), \dots, P_q = P(s_q)$$

$$H(S) = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \dots + P_q \log \frac{1}{P_q}$$

Fuente S_i' — $s_i' \in S' = \{s_1', s_2', \dots, s_q', s_{q+1}', s_{q+2}', \dots, s_{2q}'\}$

$$P_1' = (1-\varepsilon)P_1 \qquad P_{q+1}' = \varepsilon P_1$$

$$P_2' = (1-\varepsilon)P_2 \qquad P_{q+2}' = \varepsilon P_2$$

$$\vdots \qquad\qquad \vdots$$

$$P_q' = (1-\varepsilon)P_q \qquad P_{2q}' = \varepsilon P_q$$

$$H(S') = P_1' \log \frac{1}{P_1'} + P_2' \log \frac{1}{P_2'} + \dots + P_q' \log \frac{1}{P_q'} + P_{q+1}' \log \frac{1}{P_{q+1}} + P_{q+2}' \log \frac{1}{P_{q+2}} + \dots + P_{2q}' \log \frac{1}{P_{2q}}$$

$$= (1-\varepsilon)P_1 \log \frac{1}{(1-\varepsilon)P_1} + (1-\varepsilon)P_2 \log \frac{1}{(1-\varepsilon)P_2} + \dots + (1-\varepsilon)P_q \log \frac{1}{(1-\varepsilon)P_q} + \varepsilon P_1 \log \frac{1}{\varepsilon P_1} +$$
$$\varepsilon P_2 \log \frac{1}{\varepsilon P_2} + \dots + \varepsilon P_q \log \frac{1}{\varepsilon P_q}$$

$$= (1-\varepsilon)P_1\left[\log \frac{1}{1-\varepsilon} + \log \frac{1}{P_1}\right] + (1-\varepsilon)P_2\left[\log \frac{1}{1-\varepsilon} + \log \frac{1}{P_1}\right] + \dots + (1-\varepsilon)P_q\left[\log \frac{1}{1-\varepsilon} + \log \frac{1}{P_q}\right]$$
$$+ \varepsilon P_1\left[\log \frac{1}{\varepsilon} + \log \frac{1}{P_1}\right] + \varepsilon P_2\left[\log \frac{1}{\varepsilon} + \log \frac{1}{P_2}\right] + \dots + \varepsilon P_q\left[\log \frac{1}{\varepsilon} + \log \frac{1}{P_q}\right]$$

$$= (1-\varepsilon)\left[\cancel{P_1 + P_2 + \dots + P_q}\right]^{1} \log \frac{1}{1-\varepsilon} + (1-\varepsilon)\left[P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \dots + P_q \log \frac{1}{P_q}\right]$$
$$+ \varepsilon \log \frac{1}{\varepsilon}\left[\cancel{P_1 + P_2 + \dots + P_q}\right]^{1} + \varepsilon\left[P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \dots + P_q \log \frac{1}{P_q}\right]$$

$$= (1-\varepsilon) H(S) + \varepsilon H(S) + (1-\varepsilon) \log \frac{1}{1-\varepsilon} + \varepsilon \log \frac{1}{\varepsilon}$$

$$= H(S)(\cancel{1-\varepsilon+\varepsilon})^{1} + H(\varepsilon) = H(S) + H(\varepsilon)$$

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b)

$S = \{S_1, S_2, \ldots, S_{q-1}, S_q\}$

$P(S_1) = P_1, P(S_2) = P_2, \ldots, P(S_{q-1}) = P_{q-1}, P(S_q) = P_q$

$S_1 = \{S_1, S_2, \ldots, S_{q-1}, S_q, S_{q+1}\}$

$P(S_1) = P_1, P(S_2) = P_2, \ldots, P(S_{q-1}) = P_{q-1}, P(S_q) = \alpha P_q, P(S_{q+1}) = \bar{\alpha} P_q$

$\bar{\alpha} = 1 - \alpha$

$S_2 = \{S_1, S_2\}$

$P(S_1) = \alpha, P(S_2) = \bar{\alpha}$

a)

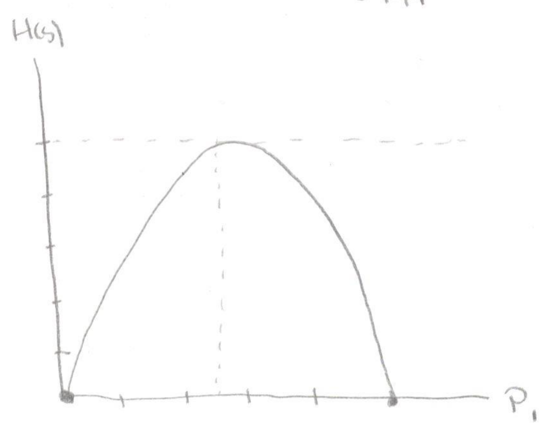
$H(S) = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \ldots + P_{q-1} \log \frac{1}{P_{q-1}} + P_q \log \frac{1}{P_q}$

Para $q=2$

$H(S) = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2}$

$P_2 = 1 - P_1$

$H(S) = P_1 \log \frac{1}{P_1} + (1-P_1) \log \frac{1}{1-P_1}$



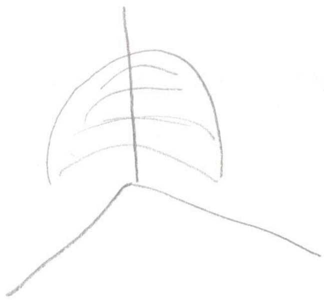
Para $q=2$

$H(S)$ es simetrica

Para $q=3$

$$H(S) = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + P_3 \log \frac{1}{P_3}$$

$$= P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + (1-P_1-P_2) \log \frac{1}{1-P_1-P_2}$$



b)

$$H(S_1) = H(S) + P_q H(S_2)$$

$$H(S_1) = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \dots + P_{q-1} \log \frac{1}{P_{q-1}} + \alpha P_q \log \frac{1}{\alpha P_q} + \bar{\alpha} P_q \log \frac{1}{\bar{\alpha} P_q}$$

$$= P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \dots + P_{q-1} \log \frac{1}{P_{q-1}} + \alpha P_q \left[\log \frac{1}{\alpha} + \log \frac{1}{P_q} \right] + \bar{\alpha} P_q \left[\log \frac{1}{\bar{\alpha}} + \log \frac{1}{P_q} \right]$$

$$= P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \dots + P_{q-1} \log \frac{1}{P_{q-1}} + P_q \left[\alpha \log \frac{1}{\alpha} + \bar{\alpha} \log \frac{1}{\bar{\alpha}} \right] + \cancel{(\alpha + \bar{\alpha})} P_q \log \frac{1}{P_q}$$

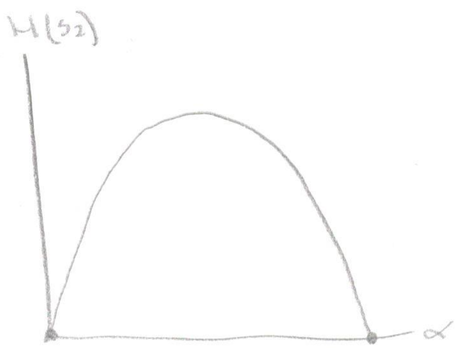
$$= P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \dots + P_{q-1} \log \frac{1}{P_{q-1}} + P_q \log \frac{1}{P_q} + P_q \underbrace{\left[\alpha \log \frac{1}{\alpha} + \bar{\alpha} \log \frac{1}{\bar{\alpha}} \right]}_{H(S_2)}$$

$$= H(S) + P_q H(S_2)$$

c)

$$H(S_2) = \alpha \log \frac{1}{\alpha} + \bar{\alpha} \log \frac{1}{\bar{\alpha}}$$

$$= \alpha \log \frac{1}{\alpha} + (1-\alpha) \log \frac{1}{(1-\alpha)}$$



Si es continua

20 $$\sum_{i=1}^{q} x_i \log\left(\frac{1}{x_i}\right) \leq \sum_{i=1}^{q} x_i \log\left(\frac{1}{y_i}\right)$$

$\boxed{\text{Fuente A}}$ — $s_i \in S = \{s_1, s_2, \dots, s_q\}$

$P(s_i) = x_i \qquad H(s_i) = \sum_{i=1}^{q} x_i \log \frac{1}{x_i}$

$\boxed{\text{Fuente B}}$ — $t_i \in T = \{t_1, t_2, \dots, t_q\}$

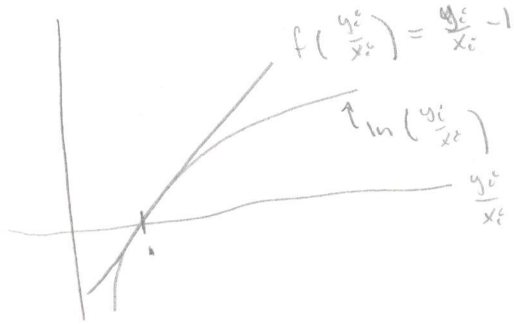
$P(t_i) = y_i$

$H(T) = \sum_{i=1}^{q} y_i \log \frac{1}{y_i}$

$$H(S) = \sum_{i=1}^{q} P_i \log \frac{1}{P_i}$$

$$\sum_{i=1}^{q} x_i \log\left(\frac{y_i}{x_i}\right) = \frac{1}{\ln 2} \sum_{i=1}^{q} x_i \ln\left(\frac{y_i}{x_i}\right) \longrightarrow \sum_{i=1}^{q} x_i \log\left(\frac{y_i}{x_i}\right) = \frac{1}{\ln 2} \sum_{i=1}^{q} x_i \ln\left(\frac{y_i}{x_i}\right)$$

$$\frac{1}{\ln 2} \sum_{i=1}^{q} x_i \ln\left(\frac{y_i}{x_i}\right) < \frac{1}{\ln 2} \sum_{i=1}^{q} x_i f\left(\frac{y_i}{x_i}\right) \rightarrow \frac{1}{\ln 2} \sum_{i=1}^{q} x_i \ln\left(\frac{y_i}{x_i}\right) < \frac{1}{\ln 2} \sum_{i=1}^{q} x_i f\left(\frac{y_i}{x_i}\right)$$



$$\frac{1}{\ln 2} \sum_{i=1}^{q} x_i \ln\left(\frac{y_i}{x_i}\right) \leq \frac{1}{\ln 2} \sum_{i=1}^{q} x_i \left[\frac{y_i}{x_i} - 1\right] \longrightarrow \frac{1}{\ln 2} \sum_{i=1}^{q} x_i \ln\left(\frac{x_i}{y_i}\right) \leq \frac{1}{\ln 2} \sum_{i=1}^{q} x_i \left[\frac{y_i}{x_i} - 1\right]$$

$$\frac{1}{\ln 2} \sum_{i=1}^{q} x_i \left[\frac{y_i}{x_i} - 1\right] = \frac{1}{\ln 2} \sum_{i=1}^{q} y_i - x_i = \frac{1}{\ln 2} \left[\cancel{\sum_{i=1}^{q} y_i}^{1} - \cancel{\sum_{i=1}^{q} x_i}^{1}\right] = \emptyset$$

entonces $\sum_{i=1}^{q} x_i \log\left(\frac{y_i}{x_i}\right) \le 0$

$$\sum_{i=1}^{q} x_i \log\left(\frac{y_i}{x_i}\right) = \sum_{i=1}^{q} x_i \left[\log(y_i) + \log\left(\frac{1}{x_i}\right)\right]$$

$$= \sum_{i=1}^{q} x_i \left[-\log\left(\frac{1}{y_i}\right) + \log\left(\frac{1}{x_i}\right)\right]$$

$$\sum_{i=1}^{q} x_i \left[-\log\left(\frac{1}{y_i}\right) + \log\left(\frac{1}{x_i}\right)\right] \le 0$$

$$\sum_{i=1}^{q} x_i \log\frac{1}{x_i} \le \sum_{i=1}^{q} x_i \log\frac{1}{y_i}$$

3o

$\log(q) - H(S) \ge 0$

$$H(S) = \sum_{i=1}^{q} P(s_i) \log\frac{1}{P(s_i)}$$

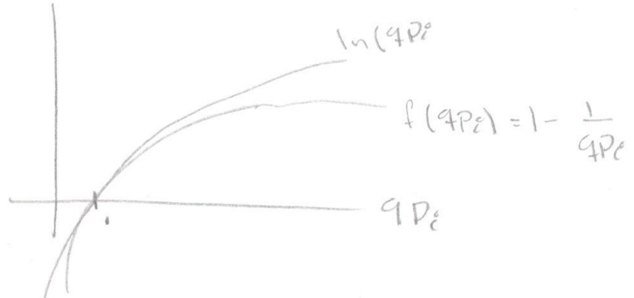
$q - \sum_{i=1}^{q} P(s_i) \log\frac{1}{P(s_i)}$ comparar dos numeros

$$\sum_{i=1}^{q} P(s_i)\log(q) - \sum_{i=1}^{q} P(s_i)\log\frac{1}{P(s_i)} = \sum_{i=1}^{q} P(s_i)\log[q\,P(s_i)]$$

$P(s_i) = P_i$

$$\sum_{i=1}^{q} P_i \log(q) - \sum_{i=1}^{q} P_i \log\frac{1}{P_i} = \sum_{i=1}^{q} P_i \log(qP_i)$$

$$= \frac{1}{\ln 2}\sum_{i=1}^{q} P_i \ln(qP_i)$$



$$\frac{1}{\ln 2}\sum_{i=1}^{q} P_i \ln(qP_i) > \frac{1}{\ln 2}\sum_{i=1}^{q} P_i f(qP_i)$$

$$\frac{1}{\ln 2}\sum_{i=1}^{q} P_i \ln(qP_i) \ge \frac{1}{\ln 2}\sum_{i=1}^{q} P_i\left[1 - \frac{1}{qP_i}\right]$$

$$\frac{1}{\ln 2}\sum_{i=1}^{q} P_i\left[1 - \frac{1}{qP_i}\right] = \frac{1}{\ln 2}\left[\sum_{i=1}^{q} P_i - \sum_{i=1}^{q}\frac{1}{q}\right] = 0$$

Nota:

$$\sum_{i=1}^{q} \frac{1}{q} = \frac{1}{q} \sum_{i=1}^{q} 1 = \frac{1}{q} [\overbrace{1+1+1+\dots+1}^{q}] = 1$$

Conclusión:

$$\sum_{i=1}^{q} P_i \log(q) - \sum_{i=1}^{q} P_i \log \frac{1}{P_i} \geq 0$$

$$\log(q) - H(S) \geq 0$$

$$\log(q) \geq H(S)$$

HTSO42

1.

$P(0) = 0{,}2$

a)

$H(S_0)$ y $H(S)$

$\boxed{S_0} \xrightarrow{\sigma_i \in S^3}$

$S' = \{S_0, S_1\} = \{0, 1\}$

$S_0 = 0 \qquad P(S_0) = 0{,}2$

$S_1 = 1 \qquad P(S_1) = 1 - P(S_0) = 0{,}8$

Para tercera extensión

σ_i	Correspondencia en S'	$P(\sigma_i)$
σ_0	$S_0 S_0 S_0$	$P(\sigma_0) = P(S_0 S_0 S_0) = P(S_0)P(S_0)P(S_0)$
σ_1	$S_0 S_0 S_1$	$P(\sigma_1) = P(S_0)P(S_0)P(S_1)$
σ_2	$S_0 S_1 S_0$	$P(\sigma_2) = P(S_0)P(S_1)P(S_0)$
σ_3	$S_0 S_1 S_1$	$P(\sigma_3) = P(S_0)P(S_1)P(S_1)$
σ_4	$S_1 S_0 S_0$	$P(\sigma_4) = P(S_1)P(S_0)P(S_0)$
σ_5	$S_1 S_0 S_1$	$P(\sigma_5) = P(S_1)P(S_0)P(S_1)$
σ_6	$S_1 S_1 S_0$	$P(\sigma_6) = P(S_1)P(S_1)P(S_0)$
σ_7	$S_1 S_1 S_1$	$P(\sigma_7) = P(S_1)P(S_1)P(S_1)$

$$H(S_0) = H(S'^3) = 3H(S')$$

$$H(S_0) = 3\left[P(S_0)\log\frac{1}{P(S_0)} + P(S_1)\log\frac{1}{P(S_1)}\right]$$

$$= 3\left[0{.}2\log\frac{1}{0{.}2} + 0{.}8\log\frac{1}{0{.}8}\right] = 2{.}166$$

b)

Eventos Posibles

Fuente So emite un "1", Fuente S emite un "0"

$P(0|1) = P\sigma_7 = P(S_1S_1S_1) = (0.8)(0.8)(0.8) = 0.512$

Fuente So emite un "0", Fuente S emite un "1"

$P(1|0) = P(\sigma_3) + P(\sigma_5) + P(\sigma_6)$

$= (0.2)(0.8)(0.8) + (0.8)(0.2)(0.8) + (0.8)(0.8)(0.2) = 0.384$

Fuente S_0 emite dos "Ø", fuente S emite un "2"

$$P(2|00) = P(\sigma_1) + P(\sigma_2) + P(P\sigma_4)$$
$$= (0.2)(0.2)(0.8) + (0.2)(0.8)(0.2) + (0.8)(0.2)(0.2) = 0.096$$

Fuente S_0 emite tres "Ø", fuente S emite un "3"

$$P(3|00) = P(\sigma_0) = (0.2)(0.2)(0.2) = 0.008$$

$$H(S) = P(0|1)\log\frac{1}{P(0|1)} + P(1|0)\log\frac{1}{P(1|0)} + P(2|00)\log\frac{1}{P(2|00)} + P(3|000)\log\frac{1}{P(3|000)}$$

$$= 0.512\log\frac{1}{0.512} + 0.384\log\frac{1}{0.384} + 0.096\log\frac{1}{0.096} + 0.008\log\frac{1}{0.008}$$

$$H(S) = 1.2105$$

2.

$$H(S) = P(s_1)\log\frac{1}{P(s_1)} + P(s_2)\log\frac{1}{P(s_2)}$$

σ_i	Correspondencia de S	
σ_1	$s_1 s_1$	$P(\sigma_1) = P(s_1)P(s_1)$
σ_2	$s_1 s_2$	$P(\sigma_2) = P(s_1)P(s_2)$
σ_3	$s_2 s_1$	$P(\sigma_3) = P(s_2)P(s_1)$
σ_4	$s_2 s_2$	$P(\sigma_4) = P(s_2)P(s_2)$

$$H(S^2) = P(\sigma_1)\log\frac{1}{P(\sigma_1)} + P(\sigma_2)\log\frac{1}{P(\sigma_2)} + P(\sigma_3)\log\frac{1}{P(\sigma_3)} + P(\sigma_4)\log\frac{1}{P(\sigma_4)}$$

$$= P(s_1)P(s_1)\log\frac{1}{P(s_1)P(s_1)} + P(s_1)P(s_2)\log\frac{1}{P(s_1)P(s_2)} + P(s_2)P(s_1)\log\frac{1}{P(s_2)P(s_1)} + P(s_2)P(s_2)\log\frac{1}{P(s_2)P(s_2)}$$

$$= P(s_1)P(s_1)\left[\log\frac{1}{P(s_1)} + \log\frac{1}{P(s_1)}\right] + P(s_1)P(s_2)\left[\log\frac{1}{P(s_1)} + \log\frac{1}{P(s_2)}\right] + P(s_2)P(s_1)\left[\log\frac{1}{P(s_2)} + \log\frac{1}{P(s_1)}\right]$$
$$+ P(s_2)P(s_2)\left[\log\frac{1}{P(s_2)} + \log\frac{1}{P(s_2)}\right]$$

$$= P(s_1)P(s_1)\log\frac{1}{P(s_1)} + P(s_1)P(s_1)\log\frac{1}{P(s_1)} + P(s_1)P(s_2)\log\frac{1}{P(s_1)} + P(s_1)P(s_2)\log\frac{1}{P(s_2)}$$
$$+ P(s_2)P(s_1)\log\frac{1}{P(s_2)} + P(s_2)P(s_1)\log\frac{1}{P(s_1)} + P(s_2)P(s_2)\log\frac{1}{P(s_2)} + P(s_2)P(s_2)\log\frac{1}{P(s_2)}$$

$$= P(s_1)\left[P(s_1)\log\frac{1}{P(s_1)} + P(s_2)\log\frac{1}{P(s_2)}\right] + P(s_2)\left[P(s_1)\log\frac{1}{P(s_1)} + P(s_2)\log\frac{1}{P(s_2)}\right]$$
$$+ P(s_1)\left[P(s_1)\log\frac{1}{P(s_1)} + P(s_2)\log\frac{1}{P(s_2)}\right] + P(s_2)\left[P(s_1)\log\frac{1}{P(s_1)} + P(s_2)\log\frac{1}{P(s_2)}\right]$$

$$= 2P(s_1)\underbrace{\left[P(s_1)\log\frac{1}{P(s_1)} + P(s_2)\log\frac{1}{P(s_2)}\right]}_{H(S)} + 2P(s_2)\underbrace{\left[P(s_1)\log\frac{1}{P(s_1)} + P(s_2)\log\frac{1}{P(s_2)}\right]}_{H(S)}$$

$$= 2H(S)\left[\cancel{P(s_1) + P(s_2)}\right]$$

$$= 2H(S)$$