Hoja de trabajo 5

viernes, 18 de septiembre de 2020 22:57

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Serie 1

Encuentre la Transformada de Fourier de las siguientes funciones

1. $\begin{cases} 1 & \text{para } 0 \leq t \leq 1 \\ -1 & \text{para } -1 \leq t < 0 \\ 0 & \text{otro caso} \end{cases}$
2. $f(t) = e^{3it}[H(t+1) - H(t-1)]$
3. $f(t) = 4H(t-2)e^{-3t}$

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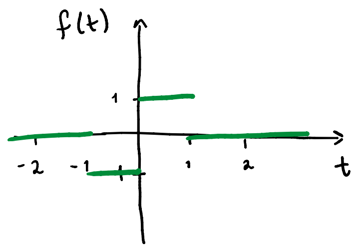
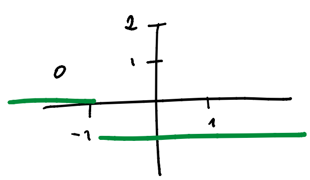
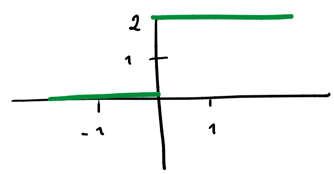


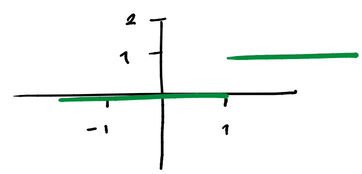
$-H(t+1)$



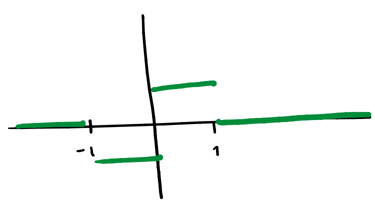
$2H(t-0)$



$H(t-1)$



$$f(t) = 2H(t-0) - H(t+1) - H(t-1)$$



$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt = \int_{-1}^{0} -1 e^{-iwt} dt + \int_{0}^{1} 1 e^{-iwt} dt$$

$$= \frac{1}{iw} e^{-iwt} \Big|_{-1}^{0} - \frac{1}{iw} e^{-iwt} \Big|_{0}^{1}$$

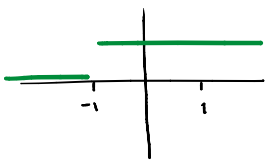
$$= \left[\frac{1}{iw} - \frac{1}{iw} e^{iw} \right] - \left[\frac{1}{iw} e^{-iw} - \frac{1}{iw} \right] = \frac{1}{iw} - \frac{1}{iw} e^{iw} - \frac{1}{iw} e^{-iw} + \frac{1}{iw}$$

$$= \frac{2}{iw} - \frac{e^{iw} + e^{-iw}}{iw} \cdot \frac{2}{2} = \frac{2}{iw} - \frac{2}{iw} \left(\frac{e^{iw} + e^{-iw}}{2} \right) = \frac{2}{iw} \left[1 - \cos(w) \right]$$

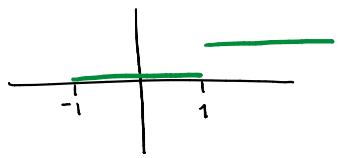
$$\boxed{\mathcal{F}\left\{2H(t)-[H(t+1)+H(t-1)]\right\} = \frac{2}{i\omega}\left[1-\cos(\omega)\right]}$$

2. $f(t) = e^{3it}\left[H(t+1)-H(t-1)\right]$

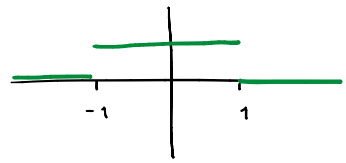
$H(t+1)$



$H(t-1)$



$H(t+1) - H(t-1)$



$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt = \int_{-1}^{1} e^{3it} \cdot e^{-i\omega t}\, dt$$

$$= \int_{-1}^{1} e^{3it-i\omega t}\, dt = \int_{-1}^{1} e^{it(3-\omega)}\, dt = \frac{1}{i(3-\omega)} e^{it(3-\omega)}\Big|_{-1}^{1}$$

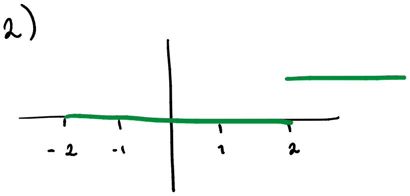
$$= \frac{1}{(3-\omega)i}\left[e^{(3-\omega)i} - e^{-(3-\omega)i}\right] \cdot \frac{2}{2}$$

$$= \frac{2}{3-\omega}\left[\frac{e^{(3-\omega)i} - e^{-(3-\omega)i}}{2i}\right] = \text{Si } \alpha = 1 \text{ y } \omega = 3-\omega \Rightarrow \frac{2}{3-\omega}\sin(3-\omega)$$

$$\boxed{\mathcal{F}\left\{e^{3it}\left[H(t+1)-H(t-1)\right]\right\} = \frac{2}{3-\omega}\sin(3-\omega)}$$

3. $f(t) = 4H(t-2)e^{-3t}$

$H(t-2)$



$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt = 4\int_{2}^{\infty} e^{-3t} \cdot e^{-i\omega t}\, dt$$

$$\int_{-\infty} \qquad \int_{2}$$

$$= 4\int_2^{\infty} e^{-3t-i\omega t}\,dt = 4\int_2^{\infty} e^{-t(3+i\omega)}\,dt = \frac{4}{3+i\omega} e^{-t(3+i\omega)}\Big|_2^{\infty}$$

$$= \frac{4}{3+i\omega}\left[e^{-2(3+i\omega)} - \cancel{e^{-\infty(3+i\omega)}}^{\,0}\right] = \frac{4}{3+i\omega} e^{-2(3+i\omega)}$$

$$\boxed{\mathcal{F}\{4H(t-2)e^{-3t}\} = \frac{4}{3+i\omega} e^{-2(3+i\omega)}}$$

Serie 2

Encuentre la Transformada Inversa de Fourier de las siguientes funciones

1. $\hat{f}(w) = \frac{1}{3+i\omega}$
2. $\hat{f}(w) = \frac{10\sin(3\omega)}{\omega+\pi}$

1. $\hat{f}(\omega) = \frac{1}{3+i\omega}$ $\qquad\qquad$ $\mathcal{F}\{H(t)e^{-at}\} = \frac{1}{a+i\omega}$

$$\mathcal{F}^{-1}\left\{\frac{1}{3+i\omega}\right\} = H(t)e^{-3t}$$

2. $\hat{f}(\omega) = \frac{10\,\text{sen}(3\omega)}{\omega+\pi} = \frac{10}{\omega+\pi}\,\text{sen}(3(\omega-\pi+\pi))$

$$\boxed{\begin{array}{l}\mathcal{F}\{k[H(t+a)-H(t-a)]\} = \frac{2k}{\omega}\,\text{sen}(a\omega) \\ \mathcal{F}\{e^{i\omega_0 t}f(t)\} = \hat{f}(\omega-\omega_0)\end{array}}$$

$$= \frac{10}{\omega+\pi}\,\text{sen}(3(\omega+\pi)-3\pi)$$

$$\mathcal{F}^{-1}\left\{\frac{10}{\omega+\pi}\,\text{sen}(\overbrace{3(\omega+\pi)}^{\alpha}-\overbrace{3\pi}^{B})\right\}$$

$$\boxed{\text{sen}(\alpha-B) = \text{sen}(\alpha)\cos(B) - \text{sen}(B)\cos(\alpha)}$$

$$\mathcal{F}^{-1}\left\{\frac{10}{\omega+\pi}\left[\text{sen}(3(\omega+\pi))\underbrace{\cancel{\cos(3\pi)}}_{-1} - \overbrace{\cancel{\text{sen}(3\pi)}}^{0}\cos(3(\omega+\pi))\right]\right\}$$

$$-(1)\mathcal{F}^{-1}\left\{\frac{10}{\omega-\pi}\,\text{sen}(3(\omega+\pi))\right\}$$

$$\mathcal{F}^{-1}\left\{\frac{2\cdot 5}{\omega}\,\text{sen}(3\omega)\right\} = f(t) = 5[H(t+3) - H(t-3)]$$

$$\mathcal{F}^{-1}\left\{\frac{2\cdot 5}{\omega}\operatorname{sen}(3\omega)\right\} = f(t) = 5\left[H(t+3) - H(t-3)\right]$$

$\omega_0 = -\pi$

$$\Rightarrow (-1)\cdot e^{-i\pi t}\cdot 5\left[H(t+3) - H(t-3)\right]$$

$$\boxed{\mathcal{F}^{-1}\left\{\frac{10\operatorname{sen}(3\omega)}{\omega+\pi}\right\} = (-1)\cdot e^{-i\pi t}\cdot 5\left[H(t+3) - H(t-3)\right]}$$