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Problema 1
Encuentre la serie de Fourier compleja de:

$$f(x) = \begin{cases} 0 & -1 < x < 0 \\ e^{-x} & 0 < x < 1 \end{cases}$$

$$\hat{f}(x) = \sum_{n=1}^{\infty} C_n e^{i\omega_n x} \qquad L = 1$$

$$C_n = \frac{1}{2L}\int_{-L}^{L} f(x) e^{-i\omega_n x} dx$$

$$= \frac{1}{2}\left[\cancel{\int_{-1}^{0} f(x) e^{-i\omega_n x}} dx + \int_{0}^{1} f(x) e^{-i\omega_n x} dx\right]$$

0

$$= \frac{1}{2}\int_0^1 e^{-x}\cdot e^{-i\omega_n x} dx = \frac{1}{2}\int_0^1 e^{-(1+i\omega_n)x} dx$$

$$= -\frac{1}{2(1+i\omega_n)} e^{-(1+i\omega_n)x}\Big|_0^1 = -\frac{1}{2(1+in\pi)}\left[e^{-(1+in\pi)} - 1\right]$$

$$\omega_n = \frac{n\pi}{L} = n\pi$$

$$\boxed{\hat{f}(x) = \sum_{n=-\infty}^{\infty}\left\{-\frac{1}{2(1+in\pi)}\left[e^{-(1+in\pi)} - 1\right]\right\} e^{in\pi x}}$$

Problema 2
Calcule la serie de Fourier compleja de:

$$f(x) = \sin x \quad 0 < x < \frac{\pi}{2}$$

$$\hat{f}(x) = \sum_{n=-\infty}^{\infty} C_n e^{i\omega_n x} \qquad L = \frac{\pi}{4}$$

$$C_n = \frac{1}{2L}\int_{-L}^{L} f(x) e^{-i\omega_n x} dx$$

$$\omega_n = \frac{n\pi}{L} = \frac{4n\cancel{\pi}}{\cancel{\pi}} = 4n$$

$$\frac{4}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(x)\, e^{-i\omega_n x} dx = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(x) e^{-i4nx} dx$$

$$u = \sin(x) \qquad dv = e^{-i4nx} dx$$

$$du = \cos(x) dx \qquad v = \frac{1}{-i4n} e^{-i4nx}$$

$$(\sin x)\,\frac{1}{-i4n}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - \frac{1}{-i4n}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-i4nx}\cos(x)\,dx$$

$$u=\cos(x) \qquad dv = e^{-i4nx}\,dx$$
$$du = -\sin(x)\,dx \qquad v = \frac{1}{-i4n}\,e^{-i4nx}$$

$$(\sin x)\,\frac{1}{-i4n}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - (\cos x)\,\frac{1}{(i4n)^2}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - \frac{1}{(i4n)^2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(x)\,e^{-i4nx}\,dx$$

$$\cancel{\frac{2}{\pi}}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(x)e^{-i4nx}\,dx = \cancel{\frac{2}{\pi}}\left\{(\sin x)\,\frac{1}{-i4n}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - (\cos x)\,\frac{1}{(i4n)^2}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - \frac{1}{(i4n)^2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(x)\,e^{-i4nx}\,dx\right\}$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(x)e^{-i4nx}\,dx\left[1+\frac{1}{(i4n)^2}\right] = (\sin x)\,\frac{1}{-i4n}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - (\cos x)\,\frac{1}{(i4n)^2}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(x)e^{-i4nx}\,dx\left[\frac{(i4n)^2+1}{(i4n)^2}\right] = (\sin x)\,\frac{1}{-i4n}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - (\cos x)\,\frac{1}{(i4n)^2}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin(x)e^{-i4nx}\,dx = \frac{(i4n)^{\cancel{2}}(\sin x)}{[(i4n)^2+1]}\,\frac{1}{(\cancel{-i4n})}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} - \frac{\cancel{(i4n)^2}(\cos x)}{[(i4n)^2+1]}\,\frac{1}{\cancel{(i4n)^2}}\,e^{-i4nx}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \frac{-i4n(\sin \pi/4)\,e^{-in\pi}}{(i4n)^2+1} + \frac{i4n(\sin -\pi/4)\,e^{in\pi}}{(i4n)^2+1} - \frac{\cos(\pi/4)\,e^{-in\pi}}{(i4n)^2+1} + \frac{\cos(\pi/4)\,e^{in\pi}}{(i4n)^2+1}$$

$$= \frac{-i4n(\sqrt{2}/2)\,e^{-in\pi}}{(i4n)^2+1} + \frac{i4n(-\sqrt{2}/2)\,e^{in\pi}}{(i4n)^2+1} - \frac{(\sqrt{2}/2)\,e^{-in\pi}}{(i4n)^2+1} + \frac{(\sqrt{2}/2)\,e^{in\pi}}{(i4n)^2+1}$$

$$= \frac{\sqrt{2}}{2[(i4n)^2+1]}\left[-i4ne^{-in\pi} - i4ne^{in\pi} - e^{-in\pi} + e^{in\pi}\right]$$

$$\frac{\sqrt{2}}{2[(i4n)^2+1]}\left[e^{in\pi}(1-i4n) - e^{-in\pi}(1+i4n)\right] = C_n$$

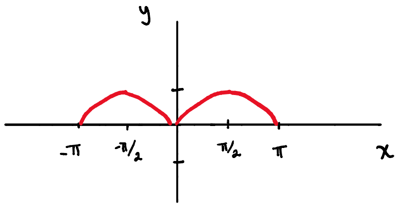
$$\boxed{\hat{f}(x) = \sum_{n=-\infty}^{\infty}\left\{\frac{\sqrt{2}}{2[(i4n)^2+1]}\left[e^{in\pi}(1-i4n) - e^{-in\pi}(1+i4n)\right]\right\}e^{i4nx}}$$

Problema 3

Para la función $f(x) = \sin x$ para $0 < x < \pi$.

1. Construya una extensión par de la función.
2. Calcule la serie de Fourier compleja de la extensión par.
3. De la serie de Fourier compleja calcule la serie de cosenos de la función original.

1.



$L = \pi$

2.

$$\sum_{n=-\infty}^{\infty} C_n e^{i\omega_n x} \qquad C_n = \frac{1}{2L}\int_{-L}^{L} f(x) e^{-i\omega_n x}\,dx \qquad \omega_n = \frac{n\pi}{L} = n$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin(x) e^{-inx}\,dx \quad \text{como es par} \Rightarrow \quad 2\cdot\frac{1}{2\pi}\int_0^{\pi} \sin(x) e^{-inx}\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin(x) e^{-inx}\,dx$$

$$\begin{array}{ll} U = \sin(x) & dv = e^{-inx}dx \\ du = \cos(x)dx & v = \frac{1}{-in} e^{-inx} \end{array} \Rightarrow (\sin x)\frac{1}{-in} e^{-inx}\Big|_0^{\pi} - \frac{1}{-in}\int_0^{\pi} \cos(x) e^{-inx}$$

$$\begin{array}{ll} U = \cos(x) & dv = e^{-inx} \\ du = -\sin(x) & v = \frac{1}{-in} e^{-inx} \end{array} \Rightarrow$$

$$(\sin x)\frac{1}{-in} e^{-inx}\Big|_0^{\pi} + \left(-\frac{1}{-in}\right)(\cos x)\left(\frac{1}{-in} e^{-inx}\right)\Big|_0^{\pi} + \left(-\frac{1}{-in}\right)\left(\frac{1}{-in}\right)\int_0^{\pi} \sin(x) e^{-inx}\,dx$$

$$(\sin x)\frac{1}{-in} e^{-inx}\Big|_0^{\pi} - \frac{1}{(in)^2}(\cos x) e^{-inx}\Big|_0^{\pi} - \frac{1}{(in)^2}\int_0^{\pi} \sin(x) e^{-inx}\,dx$$

$$\cancel{\frac{1}{\pi}}\int_0^{\pi} \sin(x) e^{-inx}\,dx = \cancel{\frac{1}{\pi}}\left\{(\sin x)\frac{1}{-in} e^{-inx}\Big|_0^{\pi} - \frac{1}{(in)^2}(\cos x) e^{-inx}\Big|_0^{\pi} - \frac{1}{(in)^2}\int_0^{\pi} \sin(x) e^{-inx}\,dx\right\}$$

$$\int_0^{\pi} \sin(x) e^{-inx}\,dx \left[1 + \frac{1}{(in)^2}\right] = (\sin x)\frac{1}{-in} e^{-inx}\Big|_0^{\pi} - \frac{1}{(in)^2}(\cos x) e^{-inx}\Big|_0^{\pi}$$

$$\Downarrow$$

$$\frac{(in)^2 + 1}{(in)^2}$$

$$\Downarrow$$

$$\frac{(in)^2+1}{(in)^2}$$

$$\int_0^\pi \sin(x)\, e^{-inx}\, dx = -\frac{(in)^{\cancel{2}}(\sin x)}{[(in)^2+1]}\frac{1}{\cancel{-in}} e^{-inx}\Big|_0^\pi - \frac{\cancel{(in)^2}}{[(in)^2+1]}\frac{1}{\cancel{(in)^2}}(\cos x)e^{-inx}\Big|_0^\pi$$

$$= -\frac{in}{(in)^2+1}\left[\overset{0}{\cancel{\sin(\pi)}}e^{-in\pi} - \overset{0}{\cancel{\sin(0)}}e^{-in(0)}\right] - \frac{1}{(in)^2+1}\left[\overset{-1}{\cancel{\cos(\pi)}}e^{-in\pi} - \overset{1}{\cancel{\cos(0)}}e^{-in\pi}\right]$$

$$= \frac{2e^{-in\pi}}{(in)^2+1} = C_n.$$

$$\boxed{\hat{f}(x) = \sum_{n=-\infty}^{\infty} \frac{2e^{-in\pi}}{(in)^2+1} e^{inx}}$$