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Hoja de trabajo 6

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Clasificar las singularidades de las siguientes funciones:

1. $f(z) = \frac{e^{2z-1}}{z}$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

$f(z) = e^z$

$f'(z) = e^z$

$f''(z) = e^z$

$$\frac{e^0}{0!}(z)^0 + \frac{e^0}{1!}(z)^1 + \frac{e^0}{2!}(z)^2 + \dots$$

$$1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots = e^z$$

$$e^{2z-1} = 1 + (2z-1) + \frac{(2z-1)^2}{2!} + \frac{(2z-1)^3}{3!} + \dots$$

$$= 2z + \frac{4z^2 - 4z + 1}{2!} + \frac{8z^3 - 12z^2 + 6z - 1}{3!} + \dots$$

$$f(z) = \frac{e^{2z-1}}{z} = 2 + \frac{4z-4}{2!} + \frac{1}{2!(z)} + \frac{8z^2 - 12z + 6}{3!} - \frac{1}{3!(z)} + \dots$$

$$= \underbrace{\frac{1}{z}\left(\frac{1}{2!} - \frac{1}{3!} + \dots\right)}_{\text{Parte principal}} + 2 + \frac{4z-4}{2!} + \frac{8z^2-12z+6}{3!} + \dots$$

Como la parte principal tiene terminos finitos, $z=0$ es un polo y z^{-1} es el ultimo termino, es un polo de orden 1 //

2. $f(z) = \frac{1 - \cosh z}{z^4}$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(x_0)}{n!}(x-x_0)^n$$

$f(x) = \cosh(z)$ $\quad f(0) = 1$

$f'(x) = \sinh(z)$ $\quad f'(0) = 0$

! $\cosh(z) = \frac{e^z + e^{-z}}{2}$

$\sinh(z) = \frac{e^z - e^{-z}}{2}$

$$\cosh(z) = \frac{1}{0!}(z^0) + \frac{1}{2!}(z^2) + \frac{1}{4!}(z^4) + \dots$$

$$f''(x) = \cosh(z) \qquad f''(0) = 1 \qquad = 1 + \frac{1}{2!}z^2 + \frac{1}{4!}z^4 + \frac{1}{6!}z^6 + \dots$$

$$f(z) = \frac{1}{z^4}\left(\overset{0}{\cancel{1-1}} + \frac{1}{2!}z^2 + \frac{1}{4!}z^4 + \frac{1}{6!}z^6 + \dots\right)$$

$$= \underbrace{\frac{1}{2!(z^2)}}_{\text{parte principal}} + \frac{1}{4!} + \frac{1}{6!}z^2 + \dots$$

Como la parte principal tiene terminos finitos, $z=0$ es un polo y z^{-2} es el ultimo termino, es un polo de orden 2

3. $f(z) = \dfrac{1}{1-e^z}$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{si } |x| < 1$$

$$\frac{1}{1-e^z} = (e^z)^0 + (e^z)^1 + (e^z)^2 + (e^z)^3 + \dots$$

$$= 1 + e^z + e^{2z} + e^{3z} + \dots$$

$$= 1 + \left(1 + z + \frac{z^2}{2!} + \dots\right) + \left(1 + 2z + \frac{(2z)^2}{2!} + \dots\right) + \left(1 + 3z + \frac{(3z)^2}{2!} + \dots\right)$$

Como la parte principal es igual a 0, es singularidad removible. //

4. $f(z) = z^3 \sin\left(\dfrac{1}{z}\right)$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(x_0)}{n!}(x - x_0)^n$$

$$\begin{array}{ll}
f(z) = \sin(z) & f(0) = 0 \\
f'(z) = \cos(z) & f'(0) = 1 \\
f''(z) = -\sin(z) & f''(z) = 0 \\
f'''(z) = -\cos(z) & f'''(z) = \\
f''''(z) = \sin(z) & ''''(z) = 0
\end{array}$$

$$\sin(z) = \frac{0}{0!}z^0 + \frac{1}{1!}z + \frac{0}{2!}z^2 - \frac{1}{3!}z^3 + \frac{0}{4!}z^4 + \dots$$

$$= z - \frac{1}{3!}z^3 + \frac{1}{5!}z^5 - \frac{1}{7!}z^7 + \dots$$

$$\sin\left(\frac{1}{z}\right) = \frac{1}{z} - \frac{1}{3!(z)^3} + \frac{1}{5!(z)^5} - \frac{1}{7!(z)^7} + \dots$$

$$\sin\left(\frac{1}{z}\right) = \frac{1}{z} - \frac{1}{3!(z)^3} + \frac{1}{5!(z)^5} - \frac{1}{7!(z)^7} + \dots$$

$$f(z) = z^3 \sin\left(\frac{1}{z}\right) = z^2 - \frac{1}{3!} + \underbrace{\frac{1}{5!(z)^2} - \frac{1}{7!(z)^4}}_{\text{Parte principal}} + \dots$$

Como la parte principal tiene cantidad infinita de términos es singularidad esencial #

5. $f(z) = \dfrac{\sin(4z) - 4z}{z^2}$

$$\sin(z) = z - \frac{1}{3!}z^3 + \frac{1}{5!}z^5 - \frac{1}{7!}z^7 + \dots$$

$$\sin(4z) = 4z - \frac{1}{3!}(4z)^3 + \frac{1}{5!}(4z)^5 - \frac{1}{7!}(4z)^7 + \dots$$

$$f(z) = \frac{\sin(4z) - 4z}{z^2} = \frac{1}{z^2}\left(\cancel{4z - 4z}^{\,0} - \frac{1}{3!}(4z)^3 + \frac{1}{5!}(4z)^5 - \frac{1}{7!}(4z)^7 + \dots\right)$$

$$= -\frac{4^3}{3!}z + \frac{4^5}{5!}z^3 - \frac{4^7}{7!}z^5 + \dots$$

Como la parte principal es igual a 0, es singularidad removible. #

6. $f(z) = \dfrac{\sin z}{z^2 - z}$

$$\frac{\sin(z)}{z^2 - z} = \frac{1}{z(z-1)} \cdot \sin(z) = \frac{1}{z} \cdot \frac{1}{z-1} \cdot \sin(z) = \frac{1}{z} \cdot \sin(z) \cdot -\frac{1}{1-z}$$

$$\sin(z) = z - \frac{1}{3!}z^3 + \frac{1}{5!}z^5 - \frac{1}{7!}z^7 + \dots$$

$$-\frac{1}{1-z} = -\sum_{n=0}^{\infty} z^n = -(1 + z + z^2 + z^3 + \dots)$$

$$f(z) = \frac{1}{z}\sin(z) \cdot -\frac{1}{1-z} = \left(1 - \frac{1}{3!}z^2 + \frac{1}{5!}z^4 - \frac{1}{7!}z^6 + \dots\right) \cdot -(1 + z + z^2 + \dots)$$

Como la parte principal es igual a 0, es singularidad removible. #

7. $f(z) = \frac{e^z - 1}{z^2}$

$$1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots = e^z$$

$$f(z) = \frac{e^z - 1}{z^2} = \frac{1}{z^2}\left(\cancel{1 - 1}^{\,0} + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots\right)$$

$$= \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \ldots$$

Como la parte principal tiene terminos finitos, $z=0$ es un polo y z^{-1} es el ultimo termino, es un polo de orden 1