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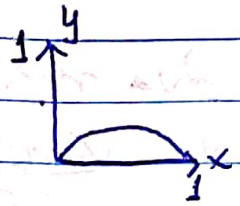
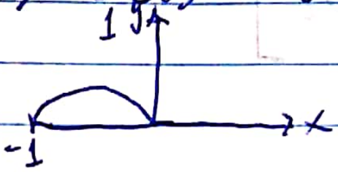
"HDT#3"

Problema 1: $f(t) = t - t^2, \quad 0 < t < 1$

1)



2) $f(t) = -t - t^2, \quad -1 < t < 0$



3) $f(t) = \begin{cases} -t - t^2, & -1 \leq t \leq 0 \\ t - t^2, & 0 \leq t \leq 1 \end{cases} \qquad f(t+2) = f(t) \Rightarrow L = 1$

$$a_0 = \frac{1}{2}\int_{-1}^{1} f(x)\,dx \rightarrow \frac{1}{2}\left[\int_{-1}^{0}(-t - t^2)\,dt + \int_{0}^{1}(t - t^2)\,dt\right]$$

$$\rightarrow \frac{1}{2}\left[\cancel{\left(-\frac{1}{2}t^2\right)\Big|_{-1}^{0}} - \left(\frac{1}{2}t^2 + \frac{1}{3}t^3\right)\Big|_{-1}^{0} + \left(\frac{1}{2}t^2 - \frac{1}{3}t^3\right)\Big|_{0}^{1}\right]$$

$$\rightarrow \frac{1}{2}\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{2} - \frac{1}{3}\right] \rightarrow \frac{1}{6} \rightarrow a_0 = \frac{1}{6}$$

$$a_n = \int_{-1}^{1} f(x)\cos(n\pi t)\,dt \rightarrow -\int_{-1}^{0}(t + t^2)\cos(n\pi t)\,dt + \int_{0}^{1}(t - t^2)\cos(n\pi t)\,dt$$

$$\rightarrow -\int_{-1}^{0} t\cos(n\pi t)\,dt - \int_{-1}^{0} t^2\cos(n\pi t)\,dt + \int_{0}^{1} t\cos(n\pi t)\,dt - \int_{0}^{1} t^2\cos(n\pi t)\,dt$$

$$\int t\cos(n\pi t)\,dt \qquad u = t \quad du = dt \qquad dv = \cos(n\pi t)\,dt \quad v = \frac{\sin(n\pi t)}{n\pi}$$

$$\rightarrow \frac{t\sin(n\pi t)}{n\pi}\Big| - \int \frac{\sin(n\pi t)}{n\pi}\,dt \rightarrow \frac{t\sin(n\pi t)}{n\pi}\Big| + \frac{\cos(n\pi t)}{n^2\pi^2}\Big|$$

$$\int t^2\cos(n\pi t)\,dt \qquad u=t^2 \qquad dv=\cos(n\pi t)\,dt$$
$$du=2t\,dt \qquad v=\frac{\sin(n\pi t)}{n\pi}$$

$$\to \frac{t^2\sin(n\pi t)}{n\pi}\Big| - 2\int t\,\frac{\sin(n\pi t)}{n\pi}\,dt \qquad u=t \qquad dv=\sin(n\pi t)\,dt$$
$$du=dt \qquad v=-\frac{\cos(n\pi t)}{n\pi}$$

$$\to \frac{t^2\sin(n\pi t)}{n\pi}\Big| - 2\left[-\frac{t\cos(n\pi t)}{n^2\pi^2}\Big| + \int\frac{\cos(n\pi t)}{n^2\pi^2}\,dt\right]$$

$$\to \frac{t^2\sin(n\pi t)}{n\pi}\Big| + \frac{2t\cos(n\pi t)}{n^2\pi^2}\Big| - \frac{2\sin(n\pi t)}{n^3\pi^3}\Big|$$

$$\cancel{\to -\left(\frac{t\sin(n\pi t)}{n\pi}\Big|^{0}\right)}$$

$$\to -\left(\cancel{\frac{t\sin(n\pi t)}{n\pi}}\Big|_{-1}^{0} + \frac{\cos(n\pi t)}{n^2\pi^2}\Big|_{-1}^{0}\right) - \left(\cancel{\frac{t^2\sin(n\pi t)}{n\pi}}\Big|_{-1}^{0} + \frac{2t\cos(n\pi t)}{n^2\pi^2}\Big|_{-1}^{0}\right.$$
$$\left. - \cancel{\frac{2\sin(n\pi t)}{n^3\pi^3}}\Big|_{-1}^{0}\right) + \left(\cancel{\frac{t\sin(n\pi t)}{n\pi}}\Big|_{0}^{1} + \frac{\cos(n\pi t)}{n^2\pi^2}\Big|_{0}^{1}\right)$$
$$- \left(\cancel{\frac{t^2\sin(n\pi t)}{n\pi}}\Big|_{0}^{1} + \frac{2t\cos(n\pi t)}{n^2\pi^2}\Big|_{0}^{1} - \cancel{\frac{2\sin(n\pi t)}{n^3\pi^3}}\Big|_{0}^{1}\right)$$

$$\to -\left(\frac{\cos(n\pi t)}{n^2\pi^2}\Big|_{-1}^{0}\right) - \left(\frac{2t\cos(n\pi t)}{n^2\pi^2}\Big|_{-1}^{0}\right) + \left(\frac{\cos(n\pi t)}{n^2\pi^2}\Big|_{0}^{1}\right)$$
$$- \left(\frac{2t\cos(n\pi t)}{n^2\pi^2}\Big|_{0}^{1}\right)$$

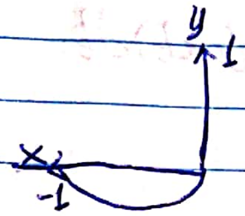
$$\to -\left(\frac{1}{n^2\pi^2} - \frac{(-1)^n}{n^2\pi^2}\right) + \cancel{\frac{2(-1)^n}{n^2\pi^2}} + \left(\frac{(-1)^n}{n^2\pi^2} - \frac{1}{n^2\pi^2}\right) - \cancel{\frac{2(-1)^n}{n^2\pi^2}}$$

$$\to \frac{2(-1)^n}{n^2\pi^2} - \frac{2}{n^2\pi^2} = a_n \;//$$

$$b_n = 0 \;//$$

$$\hat{f}(t) = \frac{1}{6} + \sum_{n=1}^{\infty}\left\{\left(\frac{2(-1)^n}{n^2\pi^2} - \frac{2}{n^2\pi^2}\right)\cos(n\pi t)\right\} \,\|$$

4) $f(t) = t + t^2, \quad -1 < t < 0$



$$f(t) = \begin{cases} t - t^2, & 0 \le t \le 1 \\ t + t^2, & -1 \le t \le 0 \end{cases}$$

$f(t+2) = f(t) \rightarrow L = 1$

$\underline{a_0 = 0 \,\|}$ $\quad$ $\underline{a_n = 0 \,\|}$

$$b_n = \int_{-1}^{1} f(t)\sin(n\pi t)\,dt \rightarrow \int_{-1}^{0}(t+t^2)\sin(n\pi t)\,dt + \int_{0}^{1}(t-t^2)\sin(n\pi t)\,dt$$

$$\rightarrow \int_{-1}^{0} t\sin(n\pi t)\,dt + \int_{-1}^{0} t^2\sin(n\pi t)\,dt + \int_{0}^{1} t\sin(n\pi t)\,dt - \int_{0}^{1} t^2\sin(n\pi t)\,dt$$

$$\int t\sin(n\pi t)\,dt \rightarrow -\frac{t\cos(n\pi t)}{n\pi}\Bigg| + \cancel{\frac{\sin(n\pi t)}{n^2\pi^2}}\Bigg| \rightarrow 0$$

$$\int t^2\sin(n\pi t)\,dt \rightarrow -\frac{t^2\cos(n\pi t)}{n\pi}\Bigg| + \cancel{\frac{2t\sin(n\pi t)}{n^2\pi^2}}\Bigg| - \frac{2\cos(n\pi t)}{n^3\pi^3}\Bigg|$$

$$\rightarrow -\frac{t\cos(n\pi t)}{n\pi}\Bigg|_{-1}^{0} - \left(\frac{t^2\cos(n\pi t)}{n\pi}\Bigg|_{-1}^{0} + \frac{2\cos(n\pi t)}{n^3\pi^3}\Bigg|_{-1}^{0}\right)$$

$$-\frac{t\cos(n\pi t)}{n\pi}\Bigg|_{0}^{1} + \left(\frac{t^2\cos(n\pi t)}{n\pi}\Bigg|_{0}^{1} + \frac{2\cos(n\pi t)}{n^3\pi^3}\Bigg|_{0}^{1}\right)$$

$$\rightarrow -\cancel{\frac{(-1)^n}{n\pi}} - \left(-\cancel{\frac{(-1)^n}{n\pi}} + \frac{2}{n^3\pi^3} - \frac{2(-1)^n}{n^3\pi^3}\right) - \cancel{\frac{(-1)^n}{n\pi}} + \left(\cancel{\frac{(-1)^n}{n\pi}} + \frac{2(-1)^n}{n^3\pi^3}\right)$$

$$\rightarrow \frac{4(-1)^n}{n^3\pi^3} - \frac{2}{n^3\pi^3} = b_n \,\|$$

$$\hat{f}(t) = \sum_{n=1}^{\infty}\left\{\left(\frac{4(-1)^n}{n^3\pi^3} - \frac{2}{n^3\pi^3}\right)\sin(n\pi t)\right\} \,\|$$

Problema 2: $f(t) = 1 - |t|, \quad -1 < t < 1$ Es par

$$f(t+2) = f(t) \rightarrow L = 1$$

$$a_0 = \int_0^1 (1-|t|)\,dt \rightarrow \left(t - \left|\frac{1}{2}t^2\right|\right)\Big|_0^1 \rightarrow \frac{1}{2} \rightarrow a_0 = \frac{1}{2}$$

$$a_n = \int_{-1}^{1} (1-|t|)\cos(n\pi t)\,dt \rightarrow \int_{-1}^{1} \cos(n\pi t)\,dt + \int_{-1}^{1} |t| \cos(n\pi t)\,dt$$

$$\rightarrow 2\left(\cancel{\int_0^1 \cos(n\pi t)\,dt} \rightarrow 0 + \int_0^1 t\cos(n\pi t)\,dt\right)$$

$$\int t\cos(n\pi t)\,dt = \cancel{\frac{t\,\mathrm{sen}(n\pi t)}{n\pi}}\Big|^{0} + \frac{\cos(n\pi t)}{n^2\pi^2}\Big|$$

$$\rightarrow 2\left[\frac{\cos(n\pi t)}{n^2\pi^2}\right]_0^1 \rightarrow \frac{2(-1)^n}{n^2\pi^2} - \frac{2}{n^2\pi^2} \rightarrow \frac{2(-1)^n - 2}{n^2\pi^2}$$

$$\hat{f}(t) = \frac{1}{2} + \sum_{n=1}^{\infty}\left\{\left(\frac{2(-1)^n - 2}{n^2\pi^2}\right)\cos(n\pi t)\right\}$$

$$Lq'' + \frac{q}{C} = \frac{1}{2} + \sum_{n=1}^{\infty}\left\{\frac{2(-1)^n - 2}{n^2\pi^2}\cos(n\pi t)\right\}$$

$$q = A + \sum_{n=1}^{\infty}\left\{B_n \cos(n\pi t)\right\} \rightarrow \cancel{q' = -\sum_{n=1}^{\infty}\left\{B_n\,\mathrm{sen}(n\pi t)\right\}}$$

$$\cancel{\rightarrow q'' = -\sum_{n=1}^{\infty}\left\{B_n\right.}$$

$$\cancel{q' = -\frac{1}{n\pi}\sum_{n=1}^{\infty}\left\{B_n\,\mathrm{sen}(n\pi t)\right\} \rightarrow q'' = -\frac{}{n}}$$

$$\cancel{q' = -\frac{1}{\pi}\sum_{n=1}^{\infty}\left\{\frac{B_n\,\mathrm{sen}(n\pi t)}{n}\right\} \rightarrow q'' = -\frac{1}{\pi^2}\sum_{n=1}^{\infty}\left\{\frac{B_n\cos(n\pi t)}{n^2}\right\}}$$

$$q' = -\sum_{n=1}^{\infty}\left\{n\pi B_n\,\mathrm{sen}(n\pi t)\right\} \rightarrow q'' = -\sum_{n=1}^{\infty}\left\{n^2\pi^2 B_n \cos(n\pi t)\right\}$$

$$-L\sum_{n=1}^{\infty}\left\{n^2\pi^2 B_n \cos(n\pi t)\right\} + \frac{A}{C} + \frac{1}{C}\sum_{n=1}^{\infty}\left\{B_n \cos(n\pi t)\right\}$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty}\left\{\frac{2(-1)^n-2}{n^2\pi^2}\cos(n\pi t)\right\}$$

$$\rightarrow \frac{A}{C} + \sum_{n=1}^{\infty}\left\{\left(\frac{1}{C}-Ln^2\pi^2\right)B_n\cos(n\pi t)\right\} = \frac{1}{2} + \sum_{n=1}^{\infty}\left\{\frac{2(-1)^n-2}{n^2\pi^2}\cos(n\pi t)\right\}$$

$$\rightarrow \left(\frac{1}{C}-Ln^2\pi^2\right)B_n = \frac{2(-1)^n-2}{n^2\pi^2} \rightarrow B_n = \frac{2(-1)^n-2}{n^2\pi^2}\left(C-\frac{1}{Ln^2\pi^2}\right)$$

$$\rightarrow \frac{A}{C} = \frac{1}{2} \rightarrow A = \frac{C}{2}$$

$$\underline{q_p = \frac{C}{2} + \sum_{n=1}^{\infty}\left\{\frac{2(-1)^n-2}{n^2\pi^2}\left(C-\frac{1}{Ln^2\pi^2}\right)\cos(n\pi t)\right\}} \;//$$

<u>Problema 3:</u> $\frac{du}{dt} = K\frac{d^2u}{dx^2}$

sujeta a: $u(x,0) = f(x)$; $\frac{du}{dt}(0,t) = 0$; $\frac{du}{dx}(L,t) = 0$

$M(t) = K\,N''(x)$