HDT-4

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HDT - 4

Problema 1

Encuentre la serie de Fourier compleja de:

$$f(x) = \begin{cases} 0 & -1 < x < 0 \\ e^{-x} & 0 < x < 1 \end{cases}$$

$T = 1 \qquad \omega_n = \frac{n\pi}{L} \Rightarrow n\pi$

$$C_n = \frac{1}{2L}\int_{-L}^{L} f(x)\, e^{-i\omega_n x}\, dx = \frac{1}{2}\left[\cancel{\int_{-1}^{0} f(x) e^{-i\omega_n x}} dx + \int_0^1 f(x)\, e^{-i\omega_n x}\, dx\right]$$

$$= \frac{1}{2}\left[\int_0^1 e^{-x} e^{-i\omega_n x} dx\right] = \frac{1}{2}\left[\int_0^1 e^{-(i\omega_n + 1)x} dx\right]$$

$$= \frac{1}{2}\left[\frac{-e^{-i\omega_n - 1}}{i\omega_n + 1}\right]_0^1 = \frac{1}{2}\left[\frac{-e^{-i\omega_n - 1}}{i\omega_n + 1} + \frac{1}{i\omega_n + 1}\right]$$

$$= \frac{1}{2}\left[\frac{1 - e^{-i\omega_n - 1}}{i\omega_n + 1}\right] \quad // \ \omega_n = \frac{n\pi}{L}$$

$\omega_n = n\pi$

$$= \frac{1 - e^{-in\pi - 1}}{in\pi + 1}$$

$$\boxed{\hat{f}(x) = \sum_{n=-\infty}^{\infty} \frac{1 - e^{-in\pi - 1}}{in\pi + 1} \cdot e^{in\pi x}}$$

Problema 2

Calcule la serie de Fourier compleja de:

$$f(x) = \sin x \quad 0 < x < \frac{\pi}{2}$$

$\hat{c} \to \cos(\ldots)$

$\hat{s} \to \sin(\ldots)$

$$C_n = \frac{1}{2L}\int_{-L}^{L} e^{-i\omega_n x} f(x)\, dx = \frac{1}{2\left(\frac{\pi}{2}\right)}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-i\omega x} f(x)\, dx$$

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$$= \frac{1}{\pi}\left[\cancel{\int_{-\frac{\pi}{2}}^{0} e^{-i\omega x} \cdot 0\, dx} \cdot \int_0^{\pi/2} e^{-i\omega x} \hat{s}\, x\, dx\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/2} e^{-i\omega x}\, \hat{s}\, x\, dx\right] \cdot \quad u = \sin x \qquad dv = e^{-i\omega x} dx$$

$$du = \cos x\, dx \qquad v = \frac{-e^{-i\omega x}}{i\omega x}$$

$$v = -\frac{e^{-i\omega x}}{i\omega x}$$

$$= \frac{1}{\pi}\left[-\hat{S}x\,\frac{e^{-i\omega x}}{i\omega x} + \frac{1}{i\omega}\int_0^{\frac{\pi}{2}} e^{-i\omega n x}\,\hat{C}x\,dx\right] \qquad u = \hat{C}x \quad v = -\frac{e^{-i\omega x}}{i\omega_n}\,dv$$

$$du = -\hat{S}x \qquad dv = -\frac{e^{-i\omega x}}{i\omega_n^2}$$

$$= \frac{1}{\pi}\left[-\hat{S}x\,\frac{e^{-i\omega x}}{i\omega x} + \frac{1}{i\omega}\left[-\hat{C}x\,\frac{e^{i\omega x}}{i\omega^2} - \frac{1}{i\omega^2}\int_0^{\frac{\pi}{2}} \hat{S}x\,e^{-i\omega x}\,dx\right]\right]$$

$$\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \hat{S}x\,e^{-i\omega x}\,dx = -\hat{S}x\,\frac{e^{-i\omega x}}{i\omega x} + \hat{C}x\,\frac{e^{i\omega x}}{i\omega^3} + \frac{1}{i\omega^3}\int_0^{\frac{\pi}{2}} \hat{S}x\,e^{-i\omega x}\,dx$$

$$\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \hat{S}x\,e^{-i\omega x}\,dx - \frac{1}{i\omega^3}\int_0^{\frac{\pi}{2}} \hat{S}x\,e^{-i\omega x}\,dx = -\hat{S}x\,\frac{e^{-i\omega x}}{i\omega x} + \hat{C}x\,\frac{e^{i\omega x}}{i\omega^3}$$

$$\left(\frac{1}{\pi} - \frac{1}{i\omega^3}\right)\int_0^{\frac{\pi}{2}} \hat{S}x\,e^{-i\omega x}\,dx = -\hat{S}x\,\frac{e^{-i\omega x}}{i\omega x} + \hat{C}x\,\frac{e^{i\omega x}}{i\omega^3}$$

$$\frac{i\omega^3 - \pi}{\pi i\omega^3}\int_0^{\frac{\pi}{2}} \hat{S}x\,e^{-i\omega x}\,dx = -\hat{S}x\,\frac{e^{-i\omega x}}{i\omega x} + \hat{C}x\,\frac{e^{i\omega x}}{i\omega^3}$$

$$\int_0^{\frac{\pi}{2}} \hat{S}x^{-i\omega x}\,dx = \left[-\hat{S}x\,\frac{e^{-i\omega x}}{i\omega}\cdot\frac{\pi i\omega^3}{i\omega^3 - \pi} + \hat{C}x\,\frac{e^{-i\omega x}}{i\omega^2}\cdot\frac{\pi i\omega^3}{i\omega^3 - \pi}\right]_0^{\pi/2}$$

$$= -\frac{e^{i\omega\frac{\pi}{2}}}{i\omega}\cdot\frac{\pi\omega^3}{i\omega^3 - \pi} - \frac{\pi i\omega^3}{i\omega^3(i\omega^3 - \pi)} \qquad \omega = \frac{n\pi}{L} \; ; \quad L = \pi/2$$

$$\omega = \frac{2n\pi}{\pi} = 2n$$

$$= -\frac{e^{i2n\frac{\pi}{2}}}{i2n}\cdot\frac{\pi^3}{i8n^3 - \pi} - \frac{\pi i^3}{i8n^3(i8n^3 - \pi)}$$

$$\Rightarrow \hat{f}(x) = \sum_{n=-\infty}^{\infty}\left[-\frac{e^{i2n\frac{\pi}{2}}\,\pi i 8n^3}{i2n(i8n^3 - \pi)} - \frac{\pi i 8n^3}{i8n^3(i8n^3 - \pi)}\right]e^{i2nx}$$

Problema 3

Para la función $f(x) = \sin x$ para $0 < x < \pi$.

1. Construya una extensión par de la función.
2. Calcule la serie de Fourier compleja de la extensión par.
3. De la serie de Fourier compleja calcule la serie de cosenos de la función original.

1)

$$f(x) = \begin{cases} \hat{S}x & 0 < x < \pi \\ f(-x) & -\pi < x < 0 \end{cases} = \begin{cases} \hat{S}x & 0 < x < \pi \\ -\hat{S}x & -\pi < x < 0 \end{cases}$$

2)
$$C_n = \frac{1}{2L}\int_{-L}^{L} e^{-i\omega_n x} f(x)\, dx$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{0} e^{-i\omega x}(-\hat{S}x)\, dx + \int_{0}^{\pi} e^{-i\omega x}(\hat{S}x)\, dx$$

$$\int_{0}^{\pi} \hat{S}x\, e^{-i\omega x} = -\sin x \,\frac{e^{-i\omega x}}{i\omega_n} + \hat{C}x\,\frac{e^{-i\omega x}}{i\omega^3} + \frac{1}{i\omega^3}\int_{0}^{\pi} \hat{S}x\; e^{-i\omega x} dx$$ Demostrada anteriormente

3) $$\hat{S}ax \cdot \hat{C}bx = \frac{1}{2}\left[\hat{S}(a-b)x + \hat{S}(a+b)x\right]$$

$$\frac{2}{\pi}\int_{0}^{\pi} \frac{1}{2}\left[\hat{S}(1-n)x + \sin(1+n)x\right] dx$$

$$= \frac{1}{\pi}\left[\int_{0}^{\pi} \hat{S}(1-n)x\,dx + \int_{0}^{\pi} \sin(1+n)x\, dx\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{1-n} + \frac{1}{1+n} + \frac{1}{1-n} + \frac{1}{1+n}\right]$$

$$= \frac{1}{\pi}\left[\frac{2}{1-n} + \frac{2}{1+n}\right] = \frac{1}{\pi}\left[\frac{2+2n+2-2n}{(1-n)^2}\right] = \frac{4}{\pi(1-n)^2}$$

$$b_n = 0$$

$$f(x) = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{4}{\pi(1-n)^2}\;\hat{C}nx$$