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Hoja de trabajo No. 10.

Problema 1

Calcular espectro y vectores propios de $\mathbf{A} = \begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$.

$$\det(A - \lambda I) = \det \begin{pmatrix} -1-\lambda & 1 & 2 \\ 0 & -2-\lambda & 0 \\ 0 & 1 & 3-\lambda \end{pmatrix} = (-1-\lambda)(\lambda^2 - \lambda - 6) - 0$$
$$= -\lambda^3 + 7\lambda + 6$$

$$-\lambda^3 + 7\lambda + 6 = 0$$

$$-(\lambda+1)(\lambda+2)(\lambda-3) = 0$$

$\lambda_1 = -1$, $\lambda_2 = -2$, $\lambda_3 = -3$

$A = \{-1, -2, 3\}$

vectores propios

$\lambda^1 = -1 \Rightarrow \underline{x}^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda_2 = -2 \Rightarrow \underline{x}^2 = \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$

$\lambda_3 = 3 \Rightarrow x^3 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

Problema 2

Para $\mathbf{A} = \begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ y $\boldsymbol{q}_T(0) = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ calcular $\boldsymbol{q}_T(t)$.

Matriz modal

$$M = \begin{pmatrix} 1 & 3 & 1 \\ 0 & -5 & 0 \\ 0 & 1 & 2 \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{5} & 0 \\ 0 & \frac{1}{10} & \frac{1}{2} \end{pmatrix}$$

$$e^{\lambda t} = \begin{pmatrix} e^{-t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & e^{3t} \end{pmatrix}$$

$$e^{At} = M e^{\lambda t} M^{-1}$$

$$= \begin{pmatrix} 1 & 3 & 1 \\ 0 & -5 & 0 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} e^{-t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & e^{3t} \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{5} & 0 \\ 0 & \frac{1}{10} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-t} & 3e^{-2t} & e^{3t} \\ 0 & -5e^{-2t} & 0 \\ 0 & e^{-2t} & 2e^{3t} \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{5} & 0 \\ 0 & \frac{1}{10} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-t} & \frac{5e^{-t} - 6e^{-2t} + e^{3t}}{10} & \frac{e^{3t} - e^{-t}}{2} \\ 0 & e^{-2t} & 0 \\ 0 & \frac{-e^{-2t} + e^{3t}}{5} & e^{3t} \end{pmatrix}$$

$$x_T = e^{At} \underline{x}(0)$$

$$= \begin{pmatrix} e^{-t} & \frac{5e^{-t} - 6e^{-2t} + e^{3t}}{10} & \frac{e^{3t} - e^{-t}}{2} \\ 0 & e^{-2t} & 0 \\ 0 & \frac{-e^{-2t} + e^{3t}}{5} & e^{3t} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{5e^{-t} - 6e^{-2t} + e^{3t}}{10} + 2 \cdot \dfrac{e^{3t} - e^{-t}}{2} \\ e^{-2t} \\ \dfrac{-e^{-2t} + e^{3t}}{5} + 2e^{3t} \end{pmatrix}$$

$$x_T = \begin{pmatrix} \frac{1}{10}\left(11e^{4t} - 6e^{-t} + 5^{t}\right) \\ e^{-2t} \\ \frac{1}{5}\left(-e^{-2t} + 11e^{3t}\right) \end{pmatrix}$$

Problema 3

Dar ecuaciones de estado y de lectura en la forma $\mathbf{A}_*$ diagonal para los sistemas

3.1 $\dot{\boldsymbol{q}}(t) = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}\boldsymbol{q} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} y, \quad x = (1 \quad 0)\boldsymbol{q}$

$$\det(A - \lambda I) = \det \begin{pmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{pmatrix} = \lambda(3+\lambda) + 2$$
$$= \lambda^2 + 3\lambda + 2$$

$\lambda^2 + 3\lambda + 2 = 0$

$(\lambda + 1)(\lambda + 2) = 0$

$\lambda_1 = -1 \quad , \quad \lambda_2 = -2$

vectores propios

- λ_1

$$(A - \lambda_2 I) = \begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix} \xrightarrow{F_2 \leftarrow 2F_1 + F_2} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

$x_1 + x_2 = 0 \qquad \underline{x} = \begin{pmatrix} -x_2 \\ x_2 \end{pmatrix} ; \; x_2 = 1 \qquad \underline{x}^2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$x_1 = -x_2$

- λ_2

$$(A - \lambda_1 I) = \begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix} \xrightarrow{f_2 \leftarrow f_1 + f_2} \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}$$

$2x_1 + x_2 = 0$ $\qquad \underline{x} = \begin{pmatrix} -x_2/2 \\ x_2 \end{pmatrix}$; $x_2 = 2$ $\qquad \underline{x}' = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$x_1 = \dfrac{-x_2}{2}$

Matrik Modal

$M = \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}$ $\qquad M^{-1} = \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}$

$$\dot{x}'(t) = (M^{-1}AM)\underline{x}' + (M^{-1}B)\underline{u}$$

$$= \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix}\underline{u}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}\underline{x}' + \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix}\underline{u}$$

$$\dot{x}(t) = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}\underline{x}' + \begin{pmatrix} -2 \\ 2 \end{pmatrix}\underline{u}$$

$$y(t) = (CM)\underline{x}' + D\underline{u}$$

$$= \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}\underline{x}'$$

$$= \begin{pmatrix} -1 & -1 \end{pmatrix}\underline{x}'$$

3.2 $\dot{\boldsymbol{q}}(t) = \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & -3 & 1 \end{pmatrix}\boldsymbol{q} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ -1 & 0 \end{pmatrix}\boldsymbol{y}, \quad \boldsymbol{x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}\boldsymbol{q}$

$$\det(A - \lambda I) = \det\begin{pmatrix} -\lambda & -3 & 1 \\ 1 & -4-\lambda & 1 \\ 0 & -3 & 1-\lambda \end{pmatrix} = \begin{aligned} &\lambda(4+\lambda)(1-\lambda) - 3 - 3\lambda + 3 - 3\lambda \\ &\lambda(4 - 3\lambda - \lambda^2 - 6) \\ &\lambda(-\lambda^2 - 3\lambda - 2) \end{aligned}$$

$$\lambda(-\lambda^2 - 3\lambda - 2) = 0$$

$$\lambda(\lambda+1)(\lambda+2) = 0$$

$$\lambda_1 = 0, \quad \lambda_2 = -1, \quad \lambda_3 = -2$$

$$(A-\lambda_1 I) = \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & -3 & 1 \end{pmatrix} \xrightarrow{F_3 \leftarrow -F_3 + F_1} \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_2 \leftarrow -F_1 + F_2}$$

$$\begin{pmatrix} 0 & -3 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} -3x_2 + x_3 &= 0 \\ x_2 &= \tfrac{1}{3} x_3 \end{aligned}$$

$$\underline{x} = \begin{pmatrix} x_1 \\ \frac{1}{3} x_3 \\ x_3 \end{pmatrix} ; \quad \begin{aligned} x_1 &= 1 \\ x_3 &= 3 \end{aligned} \qquad \underline{x}^1 = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$

$$(A-\lambda_2 I) = \begin{pmatrix} 1 & -3 & 1 \\ 1 & -3 & 1 \\ 0 & -3 & 0 \end{pmatrix} F_1 \leftarrow -F_2 + F_1 \begin{pmatrix} 0 & 0 & 0 \\ 1 & -3 & 1 \\ 0 & -3 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 0 \\ -3x_2 &= 0 \\ x_3 &= -x_1 \end{aligned} \qquad \underline{x} = \begin{pmatrix} x_3 \\ \frac{2}{3} x_3 \\ x_3 \end{pmatrix} ; \; x_3 = 3 \qquad \underline{x}^2 = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$$

$$(A-\lambda_3 I) = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 0 & -3 & 3 \end{pmatrix} \xrightarrow[F_2 \rightleftarrows F_3]{F_2 \leftarrow F_2 - \frac{1}{2} F_1} \begin{pmatrix} 2 & -3 & 1 \\ 0 & -3 & 3 \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \xrightarrow{F_3 \leftarrow F_3 - \frac{1}{6} F_2}$$

$$\begin{pmatrix} 2 & -3 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow[F_1 \leftarrow F_1 + 3F_2]{F_2 \leftarrow -\frac{1}{3} F_2} \begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_1 \leftarrow \frac{1}{2} F_1}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} x_1 - x_3 &= 0 \\ x_2 - x_3 &= 0. \end{aligned} \qquad \begin{aligned} x_1 &= x_3 \\ x_2 &= x_3 \end{aligned}$$

$$\underline{x} = \begin{pmatrix} x_3 \\ x_3 \\ x_3 \end{pmatrix} ; x_3 = 1 \qquad x^3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Matriz Modal

$$M = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix}$$

$$M^{-1} = \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 3 & 3 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[F2 \leftarrow F2 - \frac{1}{3}F1]{F3 \leftrightarrow F1} \left(\begin{array}{ccc|ccc} 3 & 3 & 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{2}{3} & 0 & 1 & -\frac{1}{3} \\ 1 & 3 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$\xrightarrow[F2 \leftrightarrow F3]{F3 \leftarrow F3 - \frac{1}{3}F1} \left(\begin{array}{ccc|ccc} 3 & 3 & 1 & 0 & 0 & 1 \\ 0 & 2 & \frac{2}{3} & 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{3} & 0 & 1 & -\frac{1}{3} \end{array}\right) \xrightarrow[F3 \leftarrow 3F3]{F3 \leftarrow F3 - \frac{1}{2}F2} \left(\begin{array}{ccc|ccc} 3 & 3 & 1 & 0 & 0 & 1 \\ 0 & 2 & \frac{2}{3} & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{3}{2} & 3 & -\frac{1}{2} \end{array}\right)$$

$$\xrightarrow[F1 \leftarrow F1 - F3]{F2 \leftarrow F2 - \frac{2}{3}F3} \left(\begin{array}{ccc|ccc} 3 & 3 & 0 & \frac{3}{2} & -3 & \frac{3}{2} \\ 0 & 2 & 0 & 2 & -2 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 3 & -\frac{1}{2} \end{array}\right) \xrightarrow[F1 \leftarrow F1 - 3F2]{F2 \leftarrow \frac{1}{2}F2} \left(\begin{array}{ccc|ccc} 3 & 0 & 0 & -\frac{3}{2} & 0 & \frac{3}{2} \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 3 & -\frac{1}{2} \end{array}\right)$$

$$\xrightarrow{F1 \leftarrow \frac{1}{3}F1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 3 & -\frac{1}{2} \end{array}\right)$$

$$M^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 1 & -1 & 0 \\ -\frac{3}{2} & 3 & -\frac{1}{2} \end{pmatrix}$$

$$\dot{x}' = (M^{-1}AM)\,\underline{x}' + (M^{-1}B)\,\underline{w}$$

$$= \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 1 & -1 & 0 \\ -\frac{3}{2} & 3 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 1 & -1 & 0 \\ -\frac{3}{2} & 3 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -6 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 1 & -1 & 0 \\ -\frac{3}{2} & 3 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ -1 & 0 \end{pmatrix}$$

$$\underline{\dot{x}}(t) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \underline{x}' + \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ 1 & 1 \\ -\frac{5}{2} & -\frac{3}{2} \end{pmatrix} \underline{w}$$

$$\underline{y}(t) = (CM)\,\underline{x}' + 0\,\underline{w}$$

3.2 $\dot{q}(t) = \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & -3 & 1 \end{pmatrix} q + \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ -1 & 0 \end{pmatrix} y, \quad x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} q$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix} \underline{x}'$$

$$y(t) = \begin{pmatrix} 1 & 3 & 1 \\ 4 & 5 & 2 \end{pmatrix} \underline{x}'$$

Problema 4

Para el sistema dado por la función de transferencia

$$G(p) = \frac{3}{(p+1)^2(p+2)}$$

dar ecuaciones de estado y de lectura con $\mathbf{A}_*$ diagonal.

Fracciones parciales:

$$\frac{3}{(p+1)^2(p+2)} = \frac{a_0}{p+1} + \frac{a_1}{(p+1)^2} + \frac{a_2}{p+2}$$

$$\frac{3\cancel{(p+1)^2}\cancel{(p+2)}}{\cancel{(p+1)^2}\cancel{(p+2)}} = \frac{a_0 (p+1)^2(p+2)}{\cancel{(p+1)}} + \frac{a_1 (p+2)\cancel{(p+1)^2}}{\cancel{(p+1)^2}} + \frac{a_2 (p+1)^2\cancel{(p+2)}}{\cancel{(p+2)}}$$

$$3 = a_0(p+1)(p+2) + a_1(p+2) + a_2(p+1)^2$$

$p = -1$

$$3 = a_0\,(\cancel{-1+1})(-1+2) + a_1(-1+2) + a_2(\cancel{-1+1})^2$$

$$3 = a_1$$

$p = -2$

$$3 = a_0\,(\cancel{-2+1)(-2+2}) + \cancel{3(-2+2)} + a_2(-2+1)^2$$

$$3 = a_2$$

$$3 = a_0(p+1)(p+2) + 3(p+2) + 3(p+1)^2$$

$$3 = a_0(p^2+3p+2) + 3p+6 + 3p^2+6p+3$$

$$3 = p^2a_0 + 3pa_0 + 2a_0 + 3p^2 + 9p + 9$$

$$3 = p^2(a_0+3) + p(3a_0+9) + (2a_0+9)$$

$$3 = 2a_0 + 9$$

$$a_0 = -3$$

$$\frac{3}{(p+1)^2(p+2)} = \frac{a_0}{p+1} + \frac{a_1}{(p+1)^2} + \frac{a_2}{p+2}$$

$$= \frac{3}{(p+1)^2} - \frac{3}{(p+1)} + \frac{3}{(p+2)}$$

$$x_1(p) = \frac{u(p)}{(p+1)^2}$$

$$\frac{x_1(p)}{x_2(p)} = \frac{\cancel{3u(p)}}{(p+1)^2} * \frac{\cancel{(p+1)}}{\cancel{-3u(p)}}$$

$$\frac{x_1(p)}{x_2(p)} = \frac{1}{(p+1)}$$

$$x_1(p+1) = x_2$$

$$+\dot{x}_1 + x_1 = x_2$$

$$x_2(p) = \frac{u(p)}{p+1}$$

$$x_2(p+1) = u$$

$$\dot{x}_2 + x_2 = u$$

$$\dot{x}_2 = -x_2 \quad u$$

$$x_3(p) = \frac{u}{(p+2)}$$

$$x_3(p+2) = u$$

$$\dot{x}_3 + 2x_3 = u$$

$$\dot{x}_3 = -2x_3 + u$$

$\dot{x}_1 + x_1 = x_2$

$\dot{x}_1 = -x_1 + x_2$

$$\underline{\dot{x}}(t) = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \underline{u}$$

$$\underline{y}(t) = \begin{bmatrix} 3 & -3 & 3 \end{bmatrix} \underline{x}' + [0]\, \underline{u}$$

Problema 5

¿Son los eigenvalores de una matriz invariantes respecto de una transformación lineal? ¡Demostrar!

$\det(\lambda I - P^{-1}AP) = 0$

$\det(\lambda P^{-1} I P - P^{-1}AP) = 0$

$\det(P^{-1}(\lambda I P - AP)) = 0$

$\det(P^{-1}(\lambda I - A)P) = 0$

$\det(P^{-1}) \cdot \det(\lambda I - A) \cdot \det(P) = 0$

$\det(\lambda I - A) = 0$

Problema 6

Dar $\mathbf{A}_*$ diagonal para $\mathbf{A} = \begin{pmatrix} 0 & 0 & -4 \\ 1 & 0 & -6 \\ 0 & 1 & -4 \end{pmatrix}$.

$A_* = M^{-1} A M$

$$\det(A - \lambda I) = \det \begin{pmatrix} -\lambda & 0 & -4 \\ 1 & -\lambda & -6 \\ 0 & 1 & -4-\lambda \end{pmatrix} = -\lambda(4\lambda + \lambda^2 + 6) - 0 \cdot (-\lambda - 4) - 4$$

$-\lambda^3 - 4\lambda^2 - 6\lambda - 4$

$$-\lambda^3 - 4\lambda^2 - 6\lambda - 4 = 0$$

$$-(\lambda+2)(\lambda^2+2\lambda+2) = 0$$

$$\lambda_1 = -2$$

$$\lambda_2, \lambda_3 = \frac{-2 \pm \sqrt{4-8}}{2} = -1+i, -1-i$$

$$\lambda_1 = -2, \quad \lambda_2 = -1+i, \quad \lambda_3 = -1-i$$

$$(A-\lambda_1) = \begin{pmatrix} 2 & 0 & -4 \\ 1 & 2 & -4 \\ 0 & 1 & -2 \end{pmatrix} \xrightarrow[F_3 \leftarrow F_3 - \frac{1}{2}F_2]{F_2 \leftarrow F_2 - \frac{1}{2}F_1} \begin{pmatrix} 2 & 0 & -4 \\ 0 & 2 & -4 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow[F_1 \leftarrow \frac{1}{2}F_1]{F_2 \leftarrow \frac{1}{2}F_2}$$

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} \qquad x_1 - 2x_3 = 0 \qquad x_1 = 2x_3$$

$$x_2 - 2x_3 = 0$$

$$x_2 = 2x_3$$

$$\underline{x} = \begin{pmatrix} 2x_3 \\ 2x_3 \\ x_3 \end{pmatrix} \quad x_3 = 1 \quad x^1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$(A - \lambda_2 I) = \begin{pmatrix} 1-i & 0 & -4 \\ 1 & 1-i & -4 \\ 0 & 1 & -3-i \end{pmatrix} \xrightarrow[F_3 \leftarrow F_3 - \frac{1}{2}(1+i)F_2]{F_2 \leftarrow F_2 - \frac{1}{2}(1+i)F_1} \begin{pmatrix} 1-i & 0 & -4 \\ 0 & 1-i & -4+2i \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{F_2 \leftarrow \frac{1}{2}(1+i)F_2} \begin{pmatrix} 1-i & 0 & -4 \\ 0 & 1 & -3-i \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_1 \leftarrow \frac{1}{2}(1+i)F_1} \begin{pmatrix} 1 & 0 & -2-2i \\ 0 & 1 & -3-i \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_1 + x_3(-2-2i) = 0 \Rightarrow x_1 = -(-2-2i)x_3$$

$$x_2 + x_3(-3-i) = 0 \Rightarrow x_2 = -(-3-i)x_3$$

$$x = \begin{pmatrix} -(-2-2i)x_3 \\ -(-3-i)x_3 \\ x_3 \end{pmatrix} \quad x_3 = 1 \quad \underline{x}^2 = \begin{pmatrix} 2+2i \\ 3+i \\ 1 \end{pmatrix}$$

$$(A-\lambda_3 I) = \begin{pmatrix} 1+i & 0 & -4 \\ 1 & 1+i & -4 \\ 0 & 1 & -3+i \end{pmatrix} \xrightarrow[F_3 \leftarrow F_3 - \frac{1}{2}(1-i)F_2]{F_2 \leftarrow F_2 - \frac{1}{2}(1-i)F_1} \begin{pmatrix} 1+i & 0 & -4 \\ 0 & 1+i & -4+2i \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{F_2 \leftarrow \frac{1}{2}(1-i)F_2} \begin{pmatrix} 1+i & 0 & -4 \\ 0 & 1 & -3+i \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_1 \leftarrow \frac{1}{2}(1-i)F_1} \begin{pmatrix} 1 & 0 & -2+2i \\ 0 & 1 & -3+i \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_1 + x_3(-2+2i) = 0 \Rightarrow x_1 = -(-2+2i)x_3$$

$$x_2 + x_3(-3+i) = 0 \Rightarrow x_2 = -(-3+i)x_3$$

$$x = \begin{pmatrix} -(-2+2i)x_3 \\ -(-3+i)x_3 \\ x_3 \end{pmatrix} \qquad x_3 = 1 \qquad \underline{x}^3 = \begin{pmatrix} 2-2i \\ 3-i \\ 1 \end{pmatrix}$$

Matriz Modal

$$M = \begin{pmatrix} 2 & 2+2i & 2-2i \\ 2 & 3+i & 3-i \\ 1 & 1 & 1 \end{pmatrix}$$

$$M^{-1} = \left(\begin{array}{ccc|ccc} 2 & 2+2i & 2-2i & 1 & 0 & 0 \\ 2 & 3+i & 3-i & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[F_3 \leftarrow F_3 - \frac{1}{2}F_1]{F_2 \leftarrow F_2 - F_1} \left(\begin{array}{ccc|ccc} 2 & 2+2i & 2-2i & 1 & 0 & 0 \\ 0 & 1-i & 1+i & -1 & 1 & 0 \\ 0 & -i & i & -\frac{1}{2} & 0 & 1 \end{array}\right)$$

$$\xrightarrow[F_3 \leftarrow F_3 - (1+i)F_2]{F_2 \leftrightarrow F_3} \left(\begin{array}{ccc|ccc} 2 & 2+2i & 2-2i & 1 & 0 & 0 \\ 0 & -i & i & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & 2 & \frac{1}{2}(-1+i) & 1 & -1-i \end{array}\right) \xrightarrow[F_2 \leftarrow F_2 - iF_3]{F_3 \leftarrow \frac{1}{2}F_3} \left(\begin{array}{ccc|ccc} 2 & 2+2i & 2-2i & 1 & 0 & 0 \\ 0 & -i & 0 & \frac{1}{4}(-1-i) & -\frac{i}{2} & \frac{1}{2}(1+i) \\ 0 & 0 & 1 & \frac{1}{4}(-1+i) & \frac{1}{2} & \frac{1}{2}(-1-i) \end{array}\right)$$

$$\xrightarrow[F_1 = \frac{1}{2}F_1]{F_1 \leftarrow F_1 - (2-2i)F_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -1 & 2 \\ 0 & 1 & 0 & \frac{1}{4}(-1-i) & \frac{1}{2} & \frac{1}{2}(-1+i) \\ 0 & 0 & 1 & \frac{1}{4}(-1+i) & \frac{1}{2} & \frac{1}{2}(-1-i) \end{array}\right)$$

$$M^{-1} = \begin{pmatrix} \frac{1}{2} & -1 & 2 \\ \frac{1}{4}(-1-i) & \frac{1}{2} & \frac{1}{2}(-1+i) \\ \frac{1}{4}(-1+i) & \frac{1}{2} & \frac{1}{2}(-1-i) \end{pmatrix}$$

$$A_* = M^{-1} A M$$

$$= \begin{pmatrix} \frac{1}{2} & -1 & 2 \\ \frac{1}{4}(-1-i) & \frac{1}{2} & \frac{1}{2}(-1+i) \\ \frac{1}{4}(-1+i) & \frac{1}{2} & \frac{1}{2}(-1-i) \end{pmatrix} \begin{pmatrix} 0 & 0 & -4 \\ 1 & 0 & -4 \\ 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} 2 & 2+2i & 2-2i \\ 2 & 3+i & 3-i \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 & -4 \\ \frac{1}{2} & \frac{1}{2}(-1+i) & -i \\ \frac{1}{2} & \frac{1}{2}(-1-i) & i \end{pmatrix} \begin{pmatrix} 2 & 2+2i & 2-2i \\ 2 & 3+i & 3-i \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1+i & 0 \\ 0 & 0 & -1-i \end{pmatrix}$$