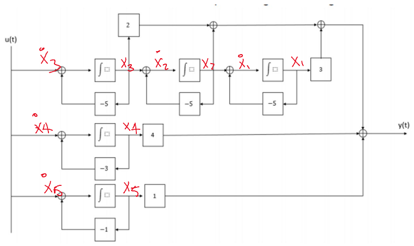
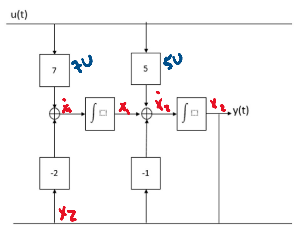
Ht 3

jueves, 29 de julio de 2021 17:22



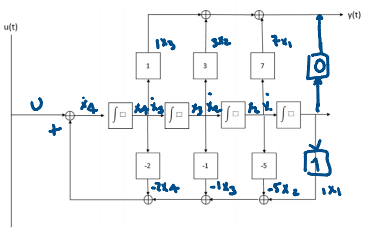
$$y = \begin{bmatrix} 3 & 1 & 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} -5 & 1 & 0 & 0 & 0 \\ 0 & -5 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} u$$



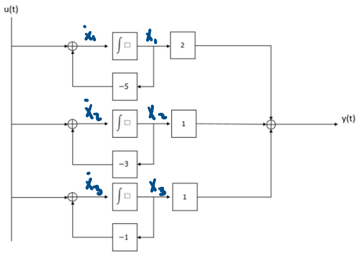
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 7 \\ 5 \end{bmatrix} u$$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -5 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 7 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_1\\ \dot{x}_3\end{bmatrix}\begin{bmatrix}-5 & 0 & 0\\ 0 & -3 & 0\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}U$$

$$Y=\begin{bmatrix}2 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\end{bmatrix}U$$

Parte #2

$$G(p)=\frac{3p+2}{(p+7)(p-2)} = \frac{3p+2}{(p+7)(p-2)}$$

$$\frac{3p+2}{(p+7)(p-2)} = \frac{A}{(p+7)} + \frac{B}{(p-2)}$$

$$3p+2 = \frac{A\cancel{(p+7)}(p-2)}{\cancel{(p+7)}} + \frac{B(p+7)\cancel{(p-2)}}{\cancel{(p-2)}}$$

$$3p+2 = A(p-2) + B(p+7)$$
$$3p+2 = Ap - 2A + Bp + 7B$$
$$3p+2 = p(A+B) + (-2A+7B)$$

$3p = p(A+B)$ | $2 = -2A + 7B$

$3 = A + B$

$A = 19/9$ $B = 8/9$

$$\frac{Y(p)}{U(p)} = \frac{19/9}{(p+7)} + \frac{8/9}{(p-2)} \quad // \; U(p)$$

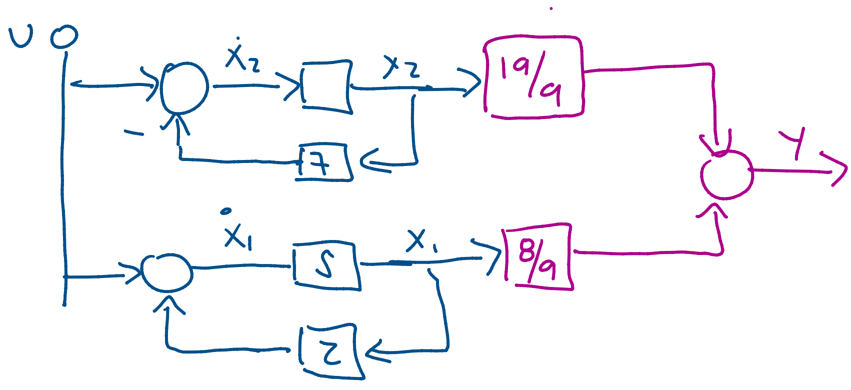
$$Y(p) = \underbrace{\frac{(19/9)U(p)}{(p+7)}}_{x_1} + \underbrace{\frac{(8/9)U(p)}{(p-2)}}_{x_2}$$

$x_1 = \dfrac{U(p)}{(p+7)}$ | $x_2 = \dfrac{U(p)}{(p-2)}$

$\dot{x}_1 = -7x_1 + U$ | $\dot{x}_2 = 2x_2 + U$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-7 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}U$$

$$Y = \begin{bmatrix}19/9 & 8/9\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\end{bmatrix}U$$



$$y = [19/9 \quad 8/9]\begin{bmatrix}x_1\\x_2\end{bmatrix} + [0]u$$

(b) $$G(p) = \frac{2p^2+4p+1}{(p+3)(p-2)(p+4)} = \frac{2p^2+4p+1}{(p+3)(p-2)(p+4)}$$

$$\frac{2p^2+4p+1}{(p+3)(p-2)(p+4)} = \frac{A}{(p+3)} + \frac{B}{(p-2)} + \frac{C}{(p+4)}$$

$$2p^2+4p+1 = \frac{A\cancel{(p+3)}(p-2)(p+4)}{\cancel{(p+3)}} + \frac{B(p+3)\cancel{(p-2)}(p+4)}{\cancel{(p-2)}} + \frac{C(p+3)(p-2)\cancel{(p+4)}}{\cancel{(p+4)}}$$

$$2p^2+4p+1 = A(p-2)(p+4) + B(p+3)(p+4) + C(p+3)(p-2)$$

$p = 2.$

$$2(2)^2+4(2)+1 = A\overset{0}{\cancel{(2-2)}}(2+4) + B(2+3)(2+4) + C(2+3)\cancel{(2-2)}$$

$$8+8+1 = B(5)(6)$$

$$17 = 30B$$

$$\frac{17}{30} = B$$

$p = (-4)$

$$2(-4)^2+4(-4)+1 = A(-4+2)\cancel{(-4+4)} + B(-4+3)\cancel{(-4+4)} + C(-4+3)(-4-2)$$

$$32-16+1 = C(-1)(-6)$$

$$17 = 6C$$

$$\frac{17}{6} = C$$

$p = -3$

$$2(-3)^2+4(-3)+1 = A(-3-2)(-3+4) + B\cancel{(-3+3)}(-3+4) + C\cancel{(-3+3)}(-3+2)$$

$$+18-12+1 = A(-5)(1)$$

$$7 = -5A$$

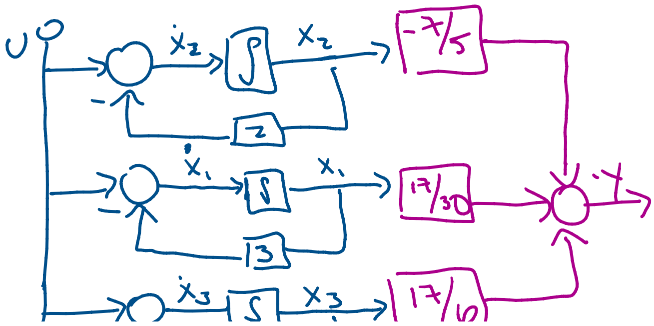
$$-\frac{7}{5} = A$$

$$\frac{Y(p)}{U(p)} = \frac{(-7/5)}{p+3} + \frac{(17/30)}{(p-2)} + \frac{(17/6)}{(p+4)} \qquad // \; U(p)$$

$$Y(p) = \frac{(-7/5)U(p)}{p+3} + \frac{(17/30)U(p)}{(p-2)} + \frac{(17/6)U(p)}{(p+4)}$$

$$\qquad\quad x_1 \qquad\qquad\qquad x_2 \qquad\qquad\qquad x_3$$

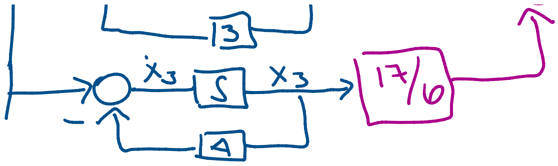
$x_1 = \dfrac{U(p)}{p+3}$	$x_2 = \dfrac{U(p)}{(p-2)}$	$x_3 = \dfrac{U(p)}{(p+4)}$
$\dot{x}_1 = -3x_1 + U$	$\dot{x}_2 = -2x_2 + U$	$\dot{x}_3 = -4x_3 + U$



$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{bmatrix} = \begin{bmatrix}-3&0&0\\0&-2&0\\0&0&-4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix}U$$

$$Y = [-7/5 \quad 17/30 \quad 17/6][x_1 \ldots$$

$$Y = \begin{bmatrix} -7/5 & 17/30 & 17/6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} U$$



c) $G = \dfrac{3p+2}{(p+7)^2(p+1)} = \dfrac{3p+2}{(p+7)^2(p+1)}$

$$\frac{3p+2}{(p+7)^2(p+1)} = \frac{A}{(p+7)} + \frac{B}{(p+7)^2} + \frac{C}{(p+1)}$$

$$3p+2 = \frac{A(p+7)^{\cancel{2}}(p+1)}{\cancel{(p+7)}} + \frac{B\cancel{(p+7)^2}(p+1)}{\cancel{(p+7)^2}} + \frac{C(p+7)^2 + \cancel{(p+1)}}{\cancel{(p+1)}}$$

$$3p+2 = A(p+7)(p+1) + B(p+1) + C(p+7)^2$$

$p = -7$

$$3(-7)+2 = \cancel{A(0)} + B(-7+1) + \cancel{C(0)}$$
$$-21+2 = -6B$$
$$-19 = -6B$$
$$\frac{19}{6} = B$$

$p = -1$

$$3(-1)+2 = A(-1+7)\cancel{(-1+1)} + B\cancel{(-1+1)} + C(-1+7)^2$$
$$-3+2 = 36C$$
$$\frac{-1}{36} = C$$

$$3p+2 = A(p+7)(p+1) + \frac{19}{6}(p+1) - \frac{1}{36}(p+7)^2$$

$(p+7)^2$

$p^2 + 2(p)(7) + 7^2$

$p^2 + 14p + 49.$

$$3p+2 = A(p^2 + p + 7p + 7) + \frac{19}{6}p + \frac{19}{6} - \frac{1}{36}(p^2 + 14p + 49)$$

$$3p+2 = \cancel{Ap^2} + 8Ap + \cancel{7A} + \cancel{\frac{19}{6}}p + \cancel{\frac{19}{6}} - \cancel{\frac{1}{36}}p^2 - \cancel{\frac{7}{18}}p - \cancel{\frac{49}{36}}$$

$$3p+2 = \cancel{Ap^2} - \cancel{\frac{1}{36}p^2} + \cancel{8Ap} + \cancel{7A} + \cancel{\frac{25}{9}}p + \cancel{\frac{65}{36}}$$

$$3p+2 = p^2\left(A - \frac{1}{36}\right) + p\left(8A + \frac{25}{9}\right) + \left(\frac{65}{36} + 7A\right)$$

$$0 = p^2\left(A - \frac{1}{36}\right)$$

$$2 = \frac{65}{36} + 7A \longrightarrow \frac{7}{36} = 7A \rightarrow \frac{1}{36} = A$$

$$\frac{Y(p)}{U(p)} = \frac{(1/36)}{(p+7)} + \frac{(19/6)}{(p+7)^2} + \frac{(-1/36)}{(p+1)} \qquad // * \; U(p)$$

$$Y(p) = \underbrace{\frac{(1/36)U(p)}{(p+7)}}_{X_2} + \underbrace{\frac{(19/6)U(p)}{(p+7)^2}}_{X_1} + \underbrace{\frac{(-1/36)U(p)}{(p+1)}}_{X_3}$$

$$X_2 = \frac{U(p)}{(p+7)} \qquad \dot{x}_2 = -7x_1 + U$$

$$X_1 = \frac{U(p)}{(p+7)^2} = \frac{1}{(p+7)} * \frac{U(p)}{(p+7)}$$

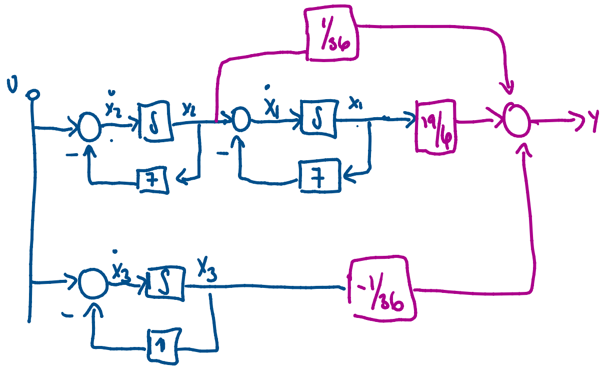
$$X_1 = \frac{1}{(p+7)} * X_2$$

$$\dot{x}_1 = -7x_1 + x_2$$

$$X_3 = \frac{U(p)}{(p+1)} \qquad \dot{x}_3 = -1x_3 + U$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}\begin{bmatrix}-7 & 1 & 0\\ 0 & -7 & 0\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}U$$

$$Y = \begin{bmatrix}19/6 & 1/36 & -1/36\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}U$$



d) $G = \frac{2}{(p+2)^3\,(p+1)} = \frac{2}{(p+2)^3(p+1)}$

$$\frac{2}{(p+2)^3(p+1)} = \frac{A}{(p+2)} + \frac{B}{(p+2)^2} + \frac{C}{(p+2)^3} + \frac{D}{(p+1)}$$

$$2 = \frac{A(p+2)^3(p+1)}{\cancel{(p+2)}} + \frac{B(p+2)^3(p+1)}{\cancel{(p+2)^2}} + \frac{C\cancel{(p+2)^3}(p+1)}{\cancel{(p+2)^3}} + \frac{D(p+2)^3\cancel{(p+1)}}{\cancel{(p+1)}}$$

$$2 = A(p+2)^2(p+1) + B(p+2)(p+1) + C(p+1) + D(p+2)^3$$

$p = -2$

$$2 = A\cancel{(-2+2)^2}(-2+1) + B\cancel{(-2+2)}(-2+1) + C(-2+1) + D\cancel{(-2+2)^3}$$

$$2 = -C$$

$$-2 = C$$

$p = -1$

$$2 = A(-1+2)^2\cancel{(-1+1)} + B(-1+2)\cancel{(-1+1)} + C\cancel{(-1+1)} + D(-1+2)^3$$

$$2 = D$$

$$\begin{array}{r} p^2+4p+4\\ p+1\\ \hline p^2+4p+4\\ p^3+4p^2+4p \end{array} \qquad \begin{array}{r} p^2+4p+4\\ p+2\\ \hline 2p^2+8p+8\\ p^3+4p^2+4p\\ \hline p^3+6p^2+12p\;\;8 \end{array}$$

$$2 = A(p+2)^2(p+1) + B(p+2)(p+1) - 2(p+1) + 2(p+2)^3$$

$$2 = A(p^2+4p+4)(p+1) + B(p^2+p+2p+2) - 2p - 2 + 2(p^2+4p+4)(p+2)$$

$$2 = A(p^3+5p^2+8p+4) + B(p^2+p+2p+2) - 2p - 2 + 2(p^3+6p^2+12p+8)$$

$$2 = A(p^3+5p^2+8p+4) + B(p^2+3p+2) - 2p - 2 + 2p^3 + 12p^2 + 24p + 16$$

$$0 = p^3(A+2) \longrightarrow -2 = A$$

$$0 = p^2(5A+B+12) \longrightarrow -12 = 5A + B$$

$$0 = p(8A+3B+22) \longrightarrow -22 = 8A + 3B$$

$$2 = 4A + 2B + 14 \longrightarrow -12 = 4A + 2B$$

$$A = -2$$
$$B = -2$$
$$C = -2$$
$$D = 2$$

$$\frac{Y(p)}{U(p)} = \frac{-2}{(p+2)} + \frac{-2}{(p+2)^2} + \frac{-2}{(p+2)^3} + \frac{2}{(p+1)} \quad // * U(p)$$

$$Y(p) = \underbrace{\frac{-2\,U(p)}{(p+2)}}_{X_3} + \underbrace{\frac{-2\,U(p)}{(p+2)^2}}_{X_2} + \underbrace{\frac{-2\,U(p)}{(p+2)^3}}_{X_1} + \underbrace{\frac{2\,U(p)}{(p+1)}}_{X_4}$$

$$x_3 = \frac{U(p)}{(p+2)}$$

$$\dot{x}_3 = -2x_3 + U$$

$$x_2 = \frac{U(p)}{(p+2)^2} = \frac{1}{(p+2)} * \frac{U(p)}{(p+2)}$$

$$x_2 = \frac{1}{(p+2)} X_3$$

$$\dot{x}_2 = -2x_2 + x_3 + U$$

$$x_1 = \frac{U(p)}{(p+2)^3} = \frac{1}{(p+2)^2} * \frac{U(p)}{(p+2)}$$

$$x_1 = \frac{1}{(p+2)} x_2$$

$$\dot{x}_1 = -2x_1 + x_2 + U$$

$$x_4 = \frac{U(p)}{(p+1)}$$

$$\dot{x}_4 = -x_4 + U$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} -2 & -2 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]\,U$$