Hoja #3

Problema 1

$\omega_n = \frac{n\pi}{L}$

$0 = \sin(\omega_n x)_{n=1}^{\infty}$

$\langle \sin(\omega_n x) | \sin(\omega_m x) \rangle = 0$ Si $m \neq n$

$$= \int_0^L \sin(\omega_n x)\sin(\omega_m x)\,dx$$

$$= \frac{1}{2}\int_0^L \cos\left(\frac{\pi}{L}x(n-m)\right) - \cos\left(\frac{\pi}{L}x(n+m)\right) dx$$

$$= \frac{1}{2}\left[\frac{L}{\pi(n-m)}\sin\left(\frac{\pi}{L}x(n-m)\right) - \frac{L}{\pi(n+m)}\sin\left(\frac{\pi}{L}x(n+m)\right)\right]_0^L$$

$$= \frac{1}{2}\left[\frac{L}{\pi(n-m)}\cancel{\sin\left(\pi(n-m)\right)}^{0} - \frac{L}{\pi(n+m)}\cancel{\sin\left(\pi(n+m)\right)}^{0}\right]$$

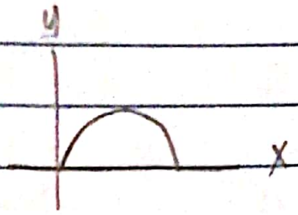
$= 0$

$\therefore$ Es ortogonal

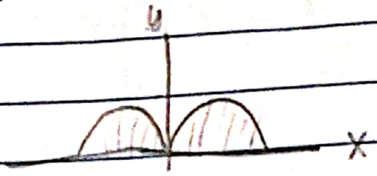
Problema 2

$f(t) = t - t^2 \qquad 0 < t < 1$

1)



2) $f_{even} = \begin{cases} f(t) & 0 < t < 1 \\ f(-t) & -1 < t < 0 \end{cases} = \begin{cases} t - t^2 & 0 < t < 1 \\ -t - t^2 & -1 < t < 1 \end{cases}$



3) $a_0 = \frac{1}{L} \int_0^1 (t - t^2)\, dt = \int_0^1 (t - t^2)\, dt =$

$= \frac{1}{2} t^2 - \frac{1}{3} t^3 \Big|_0^1 = \frac{1}{6}$

$a_n = \frac{2}{L} \int_0^1 (t - t^2) \cos\left(\frac{n\pi t}{L}\right) dt \rightarrow 2 \int_0^1 (t - t^2) \cos(n\pi t)\, dt$

$= 2 \int_0^1 t \cos(n\pi t) - t^2 \cos(n\pi t)\, dt$

$t \qquad \cos(n\pi t)$
$1 \qquad \sin(n\pi t)/n\pi$
$0 \qquad -\cos(n\pi t)/n^2\pi^2$

$t^2 \qquad \cos(n\pi t)$
$2t \qquad \sin(n\pi t)/n\pi$
$2 \qquad -\cos(n\pi t)/n^2\pi^2$
$0 \qquad -\sin(n\pi t)/n^3\pi^3$

$2 \left[\frac{t \cancelto{0}{\sin(n\pi t)}}{n\pi} + \frac{\cos(n\pi t)}{n^2\pi^2} - \frac{t^2 \cancelto{0}{\sin(n\pi t)}}{n\pi} - \frac{2t \cos(n\pi t)}{n^2\pi^2} + \frac{2 \cancelto{0}{\sin(n\pi t)}}{n^3\pi^3} \right.$

Escaneado con CamScanner

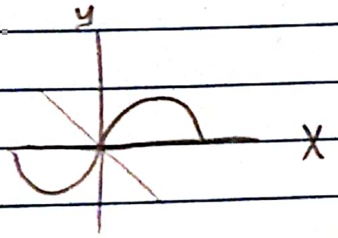
$$= 2\left[\frac{1}{n^2\pi^2}\left((-1)^n - 1 - 2(-1)^n\right)\right]$$

$$a_n = \frac{2}{n^2\pi^2}\left(-(-1)^n - 1\right) \rightarrow a_n = \frac{2}{n^2\pi^2}\left((-1)^{n+1} - 1\right)$$

$b_n = 0$

$$\hat{f}(t) = \frac{1}{6} + \sum_{n=1}^{\infty} \frac{2\left((-1)^{n+1} - 1\right)}{n^2\pi^2}\cos(\pi n t)$$

4)



$$f_{odd}\begin{cases} f(t) & 0<t<1 \\ -f(-t) & -1<t<0 \end{cases} = f_{odd}\begin{cases} t - t^2 & 0<t<1 \\ -(-t-t^2) & -1<t<0 \end{cases}$$

$a_0 = 0$; $a_n = 0$

$$b_n = \frac{2}{L}\int_0^1 (t - t^2)\sin\left(\frac{n\pi t}{L}\right)dt \rightarrow 2\int_0^1 t\sin(n\pi t) - t^2\sin(n\pi t)\,dt$$

$$\begin{array}{lll} t & & \sin(n\pi t) \\ 1 & & -\cos(n\pi t)/n\pi \\ 0 & & -\sin(n\pi t)/n^2\pi^2 \end{array} \qquad \begin{array}{lll} t^2 & \oplus & \sin(n\pi t) \\ 2t & \ominus & -\cos(n\pi t)/n\pi \\ 2 & \oplus & -\sin(n\pi t)/n^2\pi^2 \\ 0 & & \cos(n\pi t)/n^3\pi^3 \end{array}$$

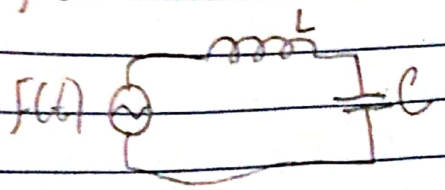
$$2\left[\overset{0}{\cancel{\frac{\sin(n\pi t)}{n^2\pi^2}}} - \overset{0}{\cancel{\frac{t\cos(n\pi t)}{n\pi}}} + \overset{0}{\cancel{\frac{t^2\cos(n\pi t)}{n\pi}}} - \overset{0}{\cancel{\frac{2t\sin(n\pi t)}{n^2\pi^2}}} - \frac{2\cos(n\pi t)}{n^3\pi^3}\right]_0^1$$

$$= 2\left[\frac{-2}{n^3\pi^3}\left((-1)^n - 1\right)\right] \rightarrow \frac{-4}{n^3\pi^3}\left((-1)^n - 1\right)$$

$$\hat{f}(x) = -\frac{4}{\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^3}\sin(\pi n t)$$

serie 3

$f(t) = 1 - |t| \quad -1 < t < 1 \qquad f(t+2) = f(t)$



$V_L = L\frac{di}{dt} = L\frac{d^2q}{dt^2}$

$V_C = \frac{q}{C}$

$L\frac{d^2q}{dt^2} + \frac{q}{C} = f(t) \rightarrow \frac{d^2q}{dt^2} + \frac{q}{LC} = \frac{f(t)}{L}$

f(t) es periodica y par

$\frac{d^2q}{dt^2} + \frac{q}{LC} = \left(a_0 + \sum_{n=1}^{\infty} a_n\cos(w_n t)\right)\frac{1}{L}$

Por Super Posición

$\boxed{\begin{array}{l} \frac{d^2q}{dt^2} + \frac{q}{LC} = \frac{a_0}{L} \\ q_p = A \\ \frac{A}{\cancel{L}C} = \frac{a_0}{\cancel{L}} \rightarrow A = a_0 C \qquad q_p = a_0 C \end{array}}$

$\frac{d^2q}{dt^2} + \frac{q}{LC} = \frac{a_n\cos(W_n t)}{L}$

$q_{p_n} = A\cos(w_n t)$

$-Aw_n^2\cos(w_n t) + \frac{A\cos(w_n t)}{LC} = \frac{a_n\cos(w_n t)}{L}$

$-Aw_n^2 + \frac{A}{LC} = \frac{a_n}{L} \rightarrow A\left(\frac{1}{LC} - W_n^2\right) = \frac{a_n}{L}$

$A\left(\frac{1 - LCW_n^2}{LC}\right) = \frac{a_n}{L} \rightarrow A = \frac{a_n C}{1 - LCW_n^2}$

$$q_p(t) = a_0 C + \sum_{n=1}^{\infty} \frac{a_n C}{1 - LC\omega_n^2} \cos(\omega_n t)$$

4) Realizada en clase

Solución

$$u(x,t) = \sum_{n=1}^{\infty} \frac{4(1-(-1)^n)}{n^3 \pi} \sin(nx)\, e^{-kn^2 t}$$