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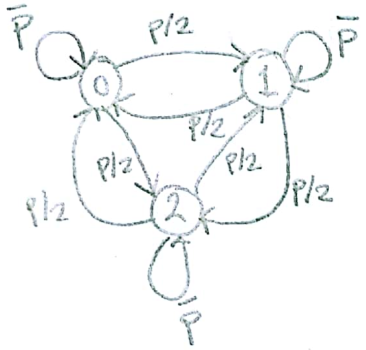
Informática IV

Hoja de Trabajo 6
Markov

1. $S = \{0, 1, 2\}$ $\quad P(0) = P(1) = P(2) = 1/3 \qquad \bar{p} = 1 - p$



a) • Informaciones condicionadas

$I(0|0) = \log \frac{1}{\bar{p}}$

$I(1|0) = \log \frac{1}{(p/2)} = \log \frac{2}{p}$

$I(2|0) = \log \frac{1}{(p/2)} = \log \frac{2}{p}$

$I(0|1) = \log \frac{1}{(p/2)} = \log \frac{2}{p}$

$I(1|1) = \log \frac{1}{\bar{p}}$

$I(2|1) = \log \frac{1}{(p/2)} = \log \frac{2}{p}$

$I(0|2) = \log \frac{1}{(p/2)} = \log \frac{2}{p}$

$I(1|2) = \log \frac{1}{(p/2)} = \log \frac{2}{p}$

$I(2|2) = \log \frac{1}{\bar{p}}$

• Promedio de las Informaciones

$H(S|0) = \bar{p} \log \frac{1}{\bar{p}} + \left[\frac{p}{2} \log \frac{2}{p} + \frac{p}{2} \log \frac{2}{p}\right] = p \log \frac{2}{p} + \bar{p} \log \frac{1}{\bar{p}}$

$H(S|1) = \left[\frac{p}{2} \log \frac{2}{p}\right] + \bar{p} \log \frac{1}{\bar{p}} + \left[\frac{p}{2} \log \frac{2}{p}\right] = p \log \frac{2}{p} + \bar{p} \log \frac{1}{\bar{p}}$

$H(S|2) = \left[\frac{p}{2} \log \frac{2}{p} + \frac{p}{2} \log \frac{2}{p}\right] + \bar{p} \log \frac{1}{\bar{p}} = p \log \frac{2}{p} + \bar{p} \log \frac{1}{\bar{p}}$

• Entropía de la fuente

$H(S) = P(0)[H(S|0)] + P(1)[H(S|1)] + P(2)[H(S|2)]$

$H(S) = \frac{1}{3}\left[\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}\right] + \frac{1}{3}\left[\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}\right] + \frac{1}{3}\left[\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}\right]$

$H(S) = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}$

$= (1-p) \log \frac{1}{(1-p)} + p \log \frac{2}{p}$

a) Cuando $p = 0$

$$H(S) = (1-0)\log\frac{1}{(1-0)} + 0\log\boxed{\frac{1}{0}}$$

• Incorrecta ya que hay división entre 0 lo cual es indefinido

Cuando $P = 1$

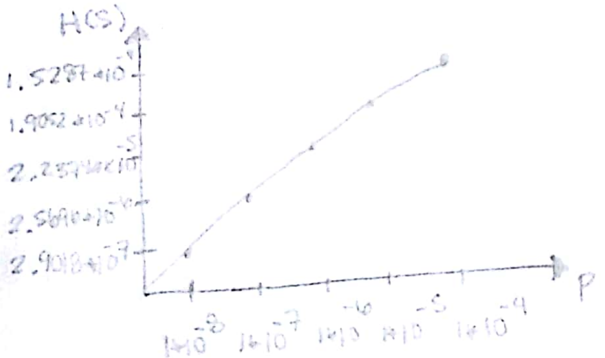
$$H(S) = (1-1)\log\boxed{\frac{1}{\underbrace{(1-1)}_{=0}}} + 1\log\frac{1}{1}$$

• Incorrecto ya que hay division entre 0 lo cual es indefinido.

b) $\varepsilon \approx 0$

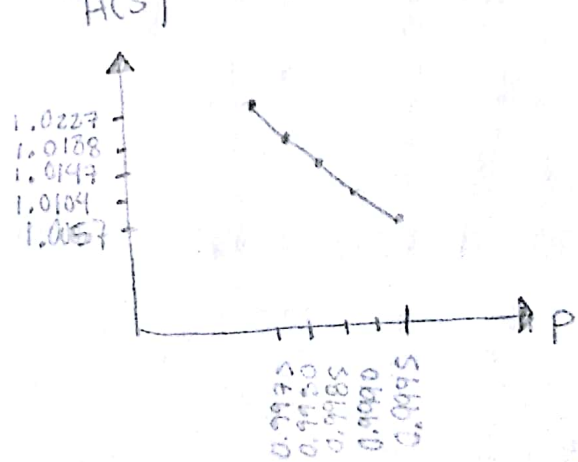
$$H(S) = (1-P)\log\frac{1}{(1-P)} + P\log\frac{2}{P}$$

$P = \varepsilon$	$H(S)$
0.0001	$1.5287 * 10^{-3}$
0.00001	$1.9052 * 10^{-4}$
0.000001	$2.2374 * 10^{-5}$
0.0000001	$2.5696 * 10^{-6}$
0.00000001	$2.9018 * 10^{-7}$



c) $\delta \approx 0$

$P = 1-\delta$	$H(S)$
0.9995	1.0057
0.9990	1.0104
0.9985	1.0147
0.9980	1.0188
0.9975	1.0227



d.

$$H(S|0) = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}$$

$$H(S|1) = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}$$

$$H(S|2) = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}$$

$$H(S) = \frac{1}{\cancel{3}}\left[\cancel{3}\left(\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}\right)\right]$$

$$= (1-p) \log \frac{1}{(1-p)} + p \log \frac{2}{p}$$

$$= (1-1/4) \log \frac{1}{(1-1/4)} + 1/4 \log \frac{2}{(1/4)}$$

$$\boxed{H(S) = 1.0612} //$$

2.

a) • Informaciones Condicionales

$$I(0|0) = \log \frac{1}{P(0|0)} = \log \frac{1}{\bar{p}}$$

$$I(1|0) = \log \frac{1}{P(1|0)} = \log \frac{1}{p}$$

Sup: $\bar{p} = 1-p$

$\bar{q} = 1-q$

$$I(0|1) = \log \frac{1}{P(0|1)} = \log \frac{1}{q}$$

$$I(1|1) = \log \frac{1}{P(1|1)} = \log \frac{1}{\bar{q}}$$

$$H(S|0) = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{1}{p} = (1-p) \log \frac{1}{(1-p)} + p \log \frac{1}{p}$$

$$H(S|1) = q \log \frac{1}{q} + \bar{q} \log \frac{1}{\bar{q}} = q \log \frac{1}{q} + (1-q) \log \frac{1}{(1-q)}$$

$$\boxed{H(S) = P(0) H(S|0) + P(1) H(S|1)}$$

b) $p = 0.1$ $q = 0.2$

$$P(0) = \frac{0.2}{0.1+0.2} = 0.6666 \qquad P(1) = \frac{0.1}{0.1+0.2} = 0.3333$$

$$H(S) = 0.3126 + 0.2406 = \boxed{0.5532 = H(S)}$$

Scanned by CamScanner

Cuando $p = q$

C. $H(S|0) = (1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}$

$H(S|1) = (1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}$

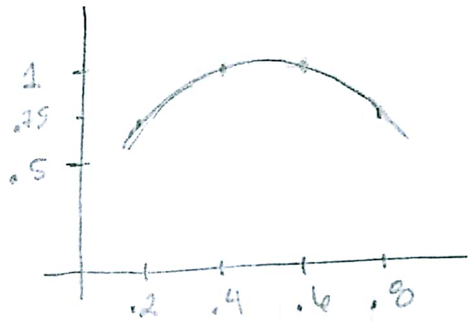
$$H(S) = \frac{p}{p+p}\left[(1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}\right] + \frac{p}{p+p}\left[(1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}\right]$$

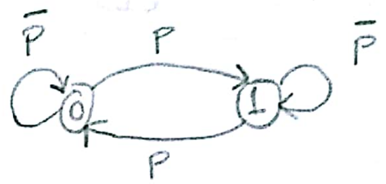
$$= 2\left[\frac{p}{p+p}\left((1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}\right)\right]$$

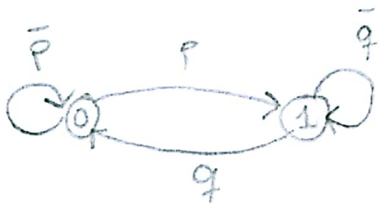
$$= \cancel{2}\left[\frac{\cancel{p}}{\cancel{2}\cancel{p}}(1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}\right]$$

$$= (1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}$$

P	H(S)
0.20	0.72
0.40	0.97
0.60	0.97
0.80	0.72





3. 

$q = 1 \Rightarrow p \neq q$

$P(0) = \frac{q}{p+q}$ $\quad P(1) = \frac{p}{p+q}$

- Informaciones Condicionadas

$I(0|0) = \log\frac{1}{1-p}$ $\quad I(1|0) = \log\frac{1}{p}$

$I(0|1) = \log\frac{1}{q}$ $\quad I(1|1) = \log\frac{1}{1-q}$

- Promedio de las Informaciones

$H(S|0) = (1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}$

$H(S|1) = q\log\frac{1}{q} + (1-q)\log\frac{1}{(1-q)}$. para $q = 1$ entonces

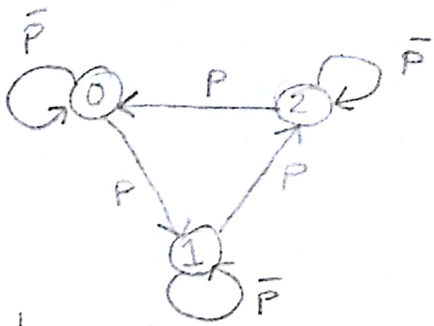
$H(S|1) = \underbrace{1 \log \frac{1}{1}}_{0} + 0 \log \frac{1}{0}$

$H(S|1) = 0$

$$H(S) = \frac{1}{p+1}\left[(1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}\right] \quad //$$

4. $S = \{0,1,2\}$

$P(0) = P(1) = P(2) = 1/3$



• Información Condicionada

$I(0|0) = \log \frac{1}{\bar{p}}$

$I(1|0) = \log \frac{1}{p}$

$I(1|1) = \log \frac{1}{\bar{p}}$

$I(2|1) = \log \frac{1}{p}$

$I(2|2) = \log \frac{1}{\bar{p}}$

$I(0|2) = \log \frac{1}{p}$

• Promedio de las Informaciones

$H(S|0) = \bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}$

$H(S|1) = \bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}$

$H(S|2) = \bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}$

$$H(S) = \frac{1}{\cancel{3}}\left[\cancel{3}\left(\bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}\right)\right] = \bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}$$

$$H(S) = (1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p} \quad //$$

Cuando $p = 0$

$H(S) = \underbrace{1\log\frac{1}{1}}_{0} + 0\log\boxed{\frac{1}{0}}$ • División dentro de 0 indefinido

Cuando $p = 1$

$H(S) = 0\log\boxed{\frac{1}{0}} + \underbrace{1\log\frac{1}{1}}_{0}$ • División dentro de 0 indefinido

b. $H(S^2)$

sabemos que $H(S^n) = nH(S)$ entonces

$S = \{0,1,2\}$

$s_0 = 0$

$s_1 = 1$

$s_2 = 2$

$H(S^2) = 2H(S)$

$$H(S^2) = 2\left[(1-p)\log\frac{1}{(1-p)} + p\log\frac{1}{p}\right] \quad //$$