Cálculo de la Probabilidad de Variables Discretas

Friday, July 13, 2018 4:04 PM

Sea ξ_i un resultado posible de un experimento aleatorio, con $(i = 1, 2, \ldots, N)$

Definición: El conjunto de todos los resultados posibles es el espacio de muestras $S = \{\xi_1, \xi_2, \ldots, \xi_N\}$

Definición: Un evento es un subconjunto de S.

Ejemplo

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{4, 5, 6\}$ $\quad P(A) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$

$C_1 = \{1\}$ $\quad P(C_1) = \frac{1}{6}$

Axiomas:

- $P(A) \geq 0$
- $P(S) = 1$
- si $A \cap B = \phi$ entonces $P(A \cup B) = P(A) + P(B)$

a) Sea A^c el complemento de A

$P(A^c) = 1 - P(A)$

b) Si $A \cap B \neq \phi$ entonces

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

c) Definición: los eventos A y B son independientes si

$P(A \cap B) = P(A) P(B)$

d) Probabilidad condicionada

$$P(B|A) \equiv \frac{P(B \cap A)}{P(A)}$$

e) Teorema de Probabilidad Total

$$P(B) = \sum_i P(A_i \cap B), \quad \underbrace{\cup A_i = S}_{\text{eventos exhaustivos}} \quad y \quad \underbrace{A_j \cap A_k = \phi}_{\text{eventos mutuamente excluyentes}}$$

$$\boxed{P(B) = \sum_i P(B|A_i) P(A_i)}$$

Forma práctica.

10Ω	0.5	0	0.2	0.5	0.6	0.5
	0.3	0.4	0.6	0.125	0.4	0
	0.2	0.6	0.2	0.375	0	0.5

Hoja de Trabajo 1

Friday, July 13, 2018 6:23 PM

1. $A=\{1,2\}$

$B=\{2,4,6\}$

Probabilidad de obtener 1 o 2 dado que se obtuvo un par.

$P(A|B)=\frac{1}{3}$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{6}}{\frac{3}{6}}=\frac{6}{18}=\frac{1}{3}$$

Probabilidad de obtener un número par dado que se obtuvo 1 o 2.

$P(B|A)=\frac{1}{2}$

$$P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{6}}{\frac{2}{6}}=\frac{6}{12}=\frac{1}{2}$$

	C_1	C_2	C_3	C_4	C_5	C_6
10Ω	0.5	0	0.2	0.5	0.6	0.5
100Ω	0.3	0.4	0.6	0.125	0.4	0
1000Ω	0.2	0.6	0.2	0.375	0	0.5

2. $B=\{r:10\Omega\}$

$A_1=\{comp.1\}$

$A_2=\{comp.2\}$

$A_3=\{comp.3\}$

$A_4=\{comp.4\}$

$A_5=\{comp.5\}$

$A_6=\{comp.6\}$

$$P(B)=\sum_{i=1}^{6}P(B|A_i)P(A_i)$$

$$=P(\{r:10\Omega\}|\{comp.1\})P(\{comp.1\})+P(\{r:10\Omega\}|\{comp.2\})P(\{comp.2\})+\dots$$

$$=0.5\tfrac{1}{6}+0+0.2\tfrac{1}{6}+0.5\tfrac{1}{6}+0.6\tfrac{1}{6}+0.5\tfrac{1}{6}=0.383$$

$$P(\{comp.3\}|\{r:10\Omega\})=\frac{P(\{r:10\Omega\}|\{comp.3\})P(\{comp.3\})}{P(\{r:10\Omega\})}=\frac{0.2\,\frac{1}{6}}{0.383}=0.08696$$

$P(\{r:10\Omega\})=0.383$

$S_1=\{c_1,c_2,\dots,c_6\}$

$A_1=\{c_1\}$

$A_2=\{c_2\}$

$\vdots$

$A_6=\{c_6\}$

$P(B|A_1)=$

$S_{C_1}=\{\underbrace{10\Omega,10\Omega,\dots,10\Omega}_{500},\underbrace{100\Omega,100\Omega,\dots,100\Omega}_{300},\underbrace{1000\Omega,1000\Omega,\dots,1000\Omega}_{200}\}$

$B_{C_1}=\{\underbrace{10\Omega,10\Omega,\dots,10\Omega}_{500}\}$

$P(B_{C_1}|A_1)=\frac{500}{1000}=\frac{1}{2}$

3. $S=\{\underbrace{0,0,\dots,0}_{n},\underbrace{1,1,\dots,1}_{4n}\}$

$A_1=\{\underbrace{0,0,\dots,0}_{n}\}$

$A_2=\{\underbrace{1,1,\dots,1}_{4n}\}$

$S_{A_1}=\{\underbrace{0,0,\dots,0}_{90\%},\underbrace{1,1,\dots,1}_{10\%}\}$ $S_{A_2}=\{\underbrace{0,0,\dots,0}_{20\%},\underbrace{1,1,\dots,1}_{80\%}\}$

$B_1=\{0,0,\dots,0\}$ 90% $C_1=\{0,0,\dots,0\}$ 20 %

$B_2=\{1,1,\dots,1\}$ 10% $C_2=\{1,1,\dots,1\}$ 80%

a) $P(B_2|S_{A_1})=\frac{10\%}{100\%}=0.1000$

$P(C_2|S_{A_2})=\frac{80\%}{100\%}=0.8000$

b) $P(0)=\sum P(0|A_i)P(A_i)$

$=P(0|A_1)P(A_1)+P(0|A_2)P(A_2)$

$=0.9(0.2)+0.2(0.8)=\boxed{0.3400}$

c) $P(A_1|1)=\frac{P(1|A_1)P(A_1)}{P(1)}=\frac{0.1(0.2)}{0.66}=\boxed{0.03030}$

$P(1)=\sum P(1|A_i)P(A_i)$

$=P(1|A_1)P(A_1)+P(1|A_2)P(A_2)$

$=0.1(0.2)+0.8(0.8)=0.66$

4. $\boxed{\text{fuente binaria}}\xrightarrow[P(s_1)=P(0)=0.2,\ P(s_2)=P(1)=0.8]{s_i\in S=\{s_1,s_2\}}\boxed{\text{canal}}\longrightarrow s'_i\in S'=\{s'_1,s'_2\},\ s'_1=0_s,\ s'_2=1_s$

$P(0_s|0_e)=0.9$

$P(0_s|1_e)=0.2$

$P(0_s|0_e)+P(1_s|0_e)=1$

entonces:

$P(1_s|0_e)=1-P(0_s|0_e)=0.1$

$P(0_s|1_e)+P(1_s|1_e)=1$

entonces:

$P(1_s|1_e)=1-P(0_s|1_e)=0.8$

1.

$\boxed{}\longrightarrow s_i\in S=\{1,2,3,4,5,6\}$

$P(1)=P(2)=P(3)=P(4)=P(5)=P(6)=\frac{1}{6}$

$A=\{1,2\}$ $P(A)=\frac{1}{3}$

$B=\{2,4,6\}$ $P(B)=\frac{1}{2}$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(2)}{1/2}=\frac{2}{6}=\frac{1}{3}$$

$$P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{P(2)}{1/3}=\frac{3}{6}=\frac{1}{2}$$

2.

$\boxed{}\longrightarrow s_i\in S=\{C_1,C_2,C_3,C_4,C_5,C_6\}$

$P(C_1)=P(C_2)=P(C_3)=P(C_4)=P(C_5)=P(C_6)=\frac{1}{6}$

$\boxed{}\longrightarrow s'_i\in S'=\{10\Omega,100\Omega,1000\Omega\}$

$P(10\Omega|C_1)=\frac{1}{2}$

$P(100\Omega|C_1)=\frac{3}{10}$

$P(1000\Omega|C_1)=\frac{1}{5}$

$\vdots$

$P(1000\Omega|C_6)=\frac{1}{2}$

3.

$\boxed{}\xrightarrow[P(s_1)=P(0_e)=0.2,\ P(s_2)=P(1_e)=0.8]{s_i\in S=\{0_e,1_e\}}\boxed{\text{canal}}\xrightarrow[P(0_s|0_e)=0.9,\ P(0_s|1_e)=0.2]{s'_i\in S'=\{0_s,1_s\}}$

a) $P(1_s|0_e)=?$

$P(1_s|0_e)+P(0_s|0_e)=1$

$P(1_s|0_e)=1-P(0_s|0_e)=0.1$

$P(1_s|1_e)=?$

$P(0_s|1_e)+P(1_s|1_e)=1$

$P(1_s|1_e)=1-P(0_s|1_e)=0.8$

b) $P(0_s)=?$

Teorema de Probabilidad Total

$$P(0_s)=\sum_i P(0_s|A_i)P(A_i)=P(0_s|0_e)P(0_e)+P(0_s|1_e)P(1_e)$$

$$=0.9(0.2)+0.2(0.8)=0.34$$

a) $P(10\Omega)=?$

Teorema de Probabilidad Total

$$P(10)=\sum_{i=1}^{6}P(10\Omega|C_i)P(C_i)=P(10\Omega|C_1)P(C_1)+P(10\Omega|C_2)P(C_2)+P(10\Omega|C_3)P(C_3)+P(10\Omega|C_4)P(C_4)+P(10\Omega|C_5)P(C_5)+P(10\Omega|C_6)P(C_6)$$

$$=\frac{1}{6}\left(\frac{1}{2}+0+\frac{1}{5}+\frac{1}{2}+\frac{3}{5}+\frac{1}{2}\right)=0.3833$$

b) $P(C_3|10\Omega)=?$

Teorema de Bayes

$$P(C_3|10\Omega)=\frac{P(10\Omega|C_3)P(C_3)}{P(10\Omega)}=\frac{\frac{1}{5}\left(\frac{1}{6}\right)}{0.3833}=0.08696$$

c) $P(0_e | 1_s) = ?$

Teorema de Bayes

$$P(0_e | 1_s) = \frac{P(1_s | 0_e) P(0_e)}{P(1_s)}$$

$$= \frac{0.1(0.2)}{0.66} = 0.0303$$

Teorema de Probabilidad Total

$$P(1_s) = \sum_i P(1_s | A_i) P(A_i) = P(1_s | 0_e) P(0_e) + P(1_s | 1_e) P(1_e)$$

$$= 0.1(0.2) + 0.8(0.8) = 0.66$$

Fuentes Discretas de Información de Memoria Nula

Thursday, July 19, 2018 3:03 PM

Definición: Sea E un evento de un experimento aleatorio. Si el evento E ocurre, se recibe $I(E)$ unidades de información.

donde: $I(E) = \log_B \frac{1}{P(E)}$

Si $B=2$, las unidades de $I(E)$ son bits.

bit= binary unit

$\boxed{\text{fuente}} \rightarrow s_i \in S = \{s_1, s_2, \ldots, s_q\}$

$P_1 = P(s_1)$

$P_2 = P(s_2)$

$P_3 = P(s_3)$

$\vdots$

$P_q = P(s_q)$

Definición: Una fuente discreta aleatoria de memoria nula tiene probabilidad que cumpla:

$$P(s_i | s_k) = P(s_j)$$

promedio de las informaciones

entropía $H(S) = \sum_{i=1}^{q} P(s_i) I(s_i)$ bits

alfabeto de la fuente

$$\boxed{H(S') = e \log \frac{1}{e} + (1-e) \log \frac{1}{1-e} + H(S)}$$

Cálculo de la Entropía

Friday, July 20, 2018 4:03 PM

$s_i \in S = \{s_1, s_2, \ldots, s_q\}$

$P_1 = P(s_1)$

$P_2 = P(s_2)$

$\vdots$

$P_q = P(s_q)$

$s_i \in S' = \{s_1', s_2', \ldots, s_q'\}$

$P_1' = P(s_1') = (1-\varepsilon)P_1$ $\quad$ $P_{q+1}' = P(s_{q+1}') = \varepsilon P_1$

$P_2' = P(s_2') = (1-\varepsilon)P_2$ $\quad$ $P_{q+2}' = P(s_{q+2}') = \varepsilon P_2$

$\vdots$

$P_q' = P(s_q') = (1-\varepsilon)P_q$ $\quad$ $P_{2q}' = P(s_{2q}') = \varepsilon P_q$

$$H(S') = (1-\varepsilon)P_1 \log \frac{1}{(1-\varepsilon)P_1} + (1-\varepsilon)P_2 \log \frac{1}{(1-\varepsilon)P_2} + \ldots + (1-\varepsilon)P_q \log \frac{1}{(1-\varepsilon)P_q}$$
$$+ \varepsilon P_1 \log \frac{1}{\varepsilon P_1} + \varepsilon P_2 \log \frac{1}{\varepsilon P_2} + \ldots + \varepsilon P_q \log \frac{1}{\varepsilon P_q}$$

$$= (1-\varepsilon)P_1 \left[\log \frac{1}{1-\varepsilon} + \log \frac{1}{P_1}\right] + (1-\varepsilon)P_2 \left[\log \frac{1}{1-\varepsilon} + \log \frac{1}{P_2}\right] + \ldots + (1-\varepsilon)P_q \left[\log \frac{1}{1-\varepsilon} + \log \frac{1}{P_q}\right]$$
$$+ \varepsilon P_1 \left[\log \frac{1}{\varepsilon} + \log \frac{1}{P_1}\right] + \varepsilon P_2 \left[\log \frac{1}{\varepsilon} + \log \frac{1}{P_2}\right] + \ldots + \varepsilon P_q \left[\log \frac{1}{\varepsilon} + \log \frac{1}{P_q}\right]$$

$$= (\cancelto{1}{P_1 + P_2 + \ldots + P_q})(1-\varepsilon) \log \frac{1}{1-\varepsilon} + (\cancelto{1}{P_1 + P_2 + \ldots + P_q})\varepsilon \log \frac{1}{\varepsilon} + \underbrace{P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \ldots + P_q \log \frac{1}{P_q}}_{H(S)}$$

$$\boxed{= (1-\varepsilon) \log \frac{1}{1-\varepsilon} + \varepsilon \log \frac{1}{\varepsilon} + H(S)}$$

entropía: el promedio de las informaciones emitidas por la fuente si cada elemento es emitido una vez.

1 $s_i \in S = \{s_1, s_2, \ldots, s_{q_1}\}$

$P_1 = P(s_1)$

$P_2 = P(s_2)$

$\vdots$

$P_{q_1} = P(s_{q_1})$

$H_1(S_1)$

2 $s_i' \in S_2 = \{s_1', s_2', \ldots, s_{q_2}'\}$

$Q_1 = P(s_1)$

$Q_2 = P(s_2)$

$\vdots$

$Q_{q_2} = P(s_{q_2})$

$H_2(S_2)$

1 & 2 $s_i \in S(\lambda) = \{s_1, s_2, \ldots, s_{q_1}, s_1', s_2', \ldots, s_{q_2}'\}$

$R_1 = P(s_1) = \lambda P_1$ $\quad$ $R_{q_1+1} = P(s_1') = \bar{\lambda} Q_1$

$R_2 = P(s_2) = \lambda P_2$ $\quad$ $R_{q_1+2} = P(s_2') = \bar{\lambda} Q_2$

$\vdots$

$R_{q_1} = P(s_q) = \lambda P_{q_1}$ $\quad$ $R_{q_1+q_2} = P(s_{q_2}') = \bar{\lambda} Q_{q_2}$

$$H[S(\lambda)] = \lambda P_1 \log \frac{1}{\lambda P_1} + \lambda P_2 \log \frac{1}{\lambda P_2} + \ldots + \lambda P_{q_1} \log \frac{1}{\lambda P_{q_1}}$$
$$+ (1-\lambda) Q_1 \log \frac{1}{\bar{\lambda} Q_1} + (1-\lambda) Q_2 \log \frac{1}{\bar{\lambda} Q_2} + \ldots + (1-\lambda) Q_{q_2} \log \frac{1}{\bar{\lambda} Q_{q_2}}$$

$$= \lambda P_1 \left(\log \frac{1}{\lambda} + \log \frac{1}{P_1}\right) + \lambda P_2 \left(\log \frac{1}{\lambda} + \log \frac{1}{P_2}\right) + \ldots + \lambda P_{q_1} \left(\log \frac{1}{\lambda} + \log \frac{1}{P_{q_1}}\right)$$
$$+ (1-\lambda) Q_1 \left(\log \frac{1}{\bar{\lambda}} + \log \frac{1}{Q_1}\right) + (1-\lambda) Q_2 \left(\log \frac{1}{\bar{\lambda}} + \log \frac{1}{Q_2}\right) + \ldots + (1-\lambda) Q_{q_2} \left(\log \frac{1}{\bar{\lambda}} + \log \frac{1}{Q_{q_2}}\right)$$

$$= (\cancelto{1}{P_1 + P_2 + \ldots + P_{q_1}})\lambda \log \frac{1}{\lambda} + (\cancelto{1}{Q_1 + Q_2 + \ldots Q_{q_2}})(1-\lambda) \log \frac{1}{1-\lambda} + \lambda \underbrace{\left(P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \ldots + P_{q_1} \log \frac{1}{P_{q_1}}\right)}_{H_1}$$
$$\bar{\lambda} \underbrace{\left(Q_1 \log \frac{1}{Q_1} + Q_2 \log \frac{1}{Q_2} + \ldots + Q_{q_2} \log \frac{1}{Q_{q_2}}\right)}_{H_2}$$

Propiedades de Entropía

Thursday, July 26, 2018 2:07 PM

ACM

$$H(s)=\sum_{i=1}^{q} P(s_i)\log\frac{1}{P(s_i)}$$

$$\vdots$$

$$\sum_{i=1}^{q} x_i\log\frac{1}{x_i} \leq \sum_{i=1}^{q} x_i\log\frac{1}{y_i}$$

$$\sum_{i=1}^{q} x_i\log\frac{y_i}{x_i} = \frac{1}{\ln 2}\sum_{i=1}^{q} x_i\ln\frac{y_i}{x_i}$$

$$\frac{1}{\ln 2}\sum_{i=1}^{q} x_i\ln\frac{y_i}{x_i} < \frac{1}{\ln 2}\sum_{i=1}^{q} x_i f\left(\frac{y_i}{x_i}\right)$$

si $f\left(\frac{y_i}{x_i}\right)=\frac{y_i}{x_i}-1$ entonces:

$$\frac{1}{\ln 2}\sum_{i=1}^{q} x_i\ln\frac{y_i}{x_i} \leq \frac{1}{\ln 2}\sum_{i=1}^{q} x_i\left[\frac{y_i}{x_i}-1\right]$$

como:

$$\frac{1}{\ln 2}\sum_{i=1}^{q}(y_i-x_i)=\frac{1}{\ln 2}\left(\sum_{i=1}^{q} y_i-\sum_{i=1}^{q} x_i\right)=\emptyset$$

entonces:

$$\frac{1}{\ln 2}\sum_{i=1}^{q} x_i\ln\frac{y_i}{x_i}\leq\emptyset$$

$$\sum_{i=1}^{q} x_i\log\frac{y_i}{x_i}=\sum_{i=1}^{q} x_i\log\frac{1}{x_i}-\sum_{i=1}^{q} x_i\log\frac{1}{y_i}$$

$$\sum_{i=1}^{q} x_i\log\frac{1}{x_i}-\sum_{i=1}^{q} x_i\log\frac{1}{y_i}\leq\emptyset$$

$$\boxed{\therefore \sum_{i=1}^{q} x_i\log\frac{1}{x_i}\leq\sum_{i=1}^{q} x_i\log\frac{1}{y_i}}$$

$$\sum_{i=1}^{q} P(s_i)[\log(1)-\log(P(s_i))]$$

$$\sum_{i=1}^{q} p(s_i)\log\frac{1-p(s_i)}{p(s_i)}=\sum_{i=1}^{q} p(s_i)[\log(1-p(s_i))-\log p(s_i)]$$

$$=\sum_{i=1}^{q} p(s_i)\log[1-p(s_i)]-\sum_{i=1}^{q} p(s_i)\log p(s_i)$$

$$=p(s_i)\left\{\sum_{i=1}^{q}\log[1-p(s_i)]-\sum_{i=1}^{q}\log p(s_i)\right\}$$

$$=p(s_i)\left\{-\sum_{i=1}^{q}\log\left[\frac{1}{1-p(s_i)}\right]+\sum_{i=1}^{q}\log\frac{1}{p(s_i)}\right\}$$

$$=x_i\left\{\sum_{i=1}^{q}\log\frac{1}{x_i}-\sum_{i=1}^{q}\log\frac{1}{y_i}\right\}$$

$$=x_i\sum_{i=1}^{q}\log\frac{1}{x_i}-x_i\sum_{i=1}^{q}\log\frac{1}{y_i}$$

Leer el texto y estudiar la demostración de

$H(s)\leq\log(q)$

$$\downarrow$$

$$H(s)=\sum_{i=1}^{q} P_i\log\frac{1}{P_i}$$

$$\sum_{i=1}^{q} P_i=1\Rightarrow\log q=\log q\sum_{i=1}^{q} P_i=\sum_{i=1}^{q} P_i\log q$$

$$\log q-H(s)=\sum_{i=1}^{q} P_i\log q-\sum_{i=1}^{q} P_i\log\frac{1}{P_i}$$

$$=\sum_{i=1}^{q} P_i\left(\log q-\log\frac{1}{P_i}\right)$$

$$=\sum_{i=1}^{q} P_i\log(qP_i)$$

$$=\log e\sum_{i=1}^{q} P_i\ln(qP_i)$$

como:

$$\ln|x|\leq x-1$$

$$-\ln|x|\geq 1-x$$

$$\ln\left|\frac{1}{x}\right|\geq 1-x$$

entonces:

$$\log e\sum_{i=1}^{q} P_i\ln(qP_i)\geq\log e\sum_{i=1}^{q} P_i\left(1-\frac{1}{qP_i}\right)$$

$$\geq\log e\left(\sum_{i=1}^{q} P_i-\sum_{i=1}^{q}\frac{1}{q}\right)=\log e\Big[1-\big(\underbrace{\tfrac{1}{q}+\tfrac{1}{q}+\dots+\tfrac{1}{q}}_{q}\big)\Big]=\emptyset$$

$$\log q-H(s)\geq\emptyset\Rightarrow\boxed{H(s)\leq\log q}$$

n-ava Extensión de una Fuente

Thursday, August 2, 2018 2:45 PM

fuente $\longrightarrow$ $s_i \in S = \{s_0, s_1\}$

$P(s_0) = P(0)$

$P(s_1) = P(1)$

fuente procesamiento $\longrightarrow$ $\sigma_i \in S^2 = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}$

$P(\sigma_0) = P(0)P(0)$

$P(\sigma_1) = P(0)P(1)$

$P(\sigma_2) = P(1)P(0)$

$P(\sigma_3) = P(1)P(1)$

$$H(S^2) = P(\sigma_0)\log\frac{1}{P(\sigma_0)} + \dots + P(\sigma_3)\log\frac{1}{P(\sigma_3)}$$

$$\boxed{H(S^n) = nH(S)}$$

$$\sum_{S^n} P(\sigma_i)\log\frac{1}{P_{i1}} + \sum_{S^n} P(\sigma_i)\log\frac{1}{P_{i2}} + \dots + \sum_{S^n} P(\sigma_i)\log\frac{1}{P_{in}}$$

$$= \sum_{S^n} P(\sigma_i)\log\left(\frac{1}{P_{i1}P_{i2}\dots P_{in}}\right) = H(S^n)$$

$$\sum_{S^n} P(\sigma_i)\log\frac{1}{P_{i2}} = P_1P_1P_1\log\frac{1}{P_1} + P_1P_1P_2\log\frac{1}{P_1} + P_1P_2P_1\log\frac{1}{P_1}$$

$$+ P_1P_2P_2\log\frac{1}{P_1} + P_2P_1P_1\log\frac{1}{P_2} + P_2P_1P_2\log\frac{1}{P_2}$$

$$+ P_2P_2P_1\log\frac{1}{P_2} + P_2P_2P_2\log\frac{1}{P_2}$$

$$= (P_1P_1 + P_1P_2 + P_2P_1 + P_2P_2)P_2\log\frac{1}{P_2} + (P_1P_1 + P_1P_2 + P_2P_1 + P_2P_2)\log\frac{1}{P_1}$$

$$= [P_1(P_1+P_2) + P_2(P_1+P_2)]\left(P_2\log\frac{1}{P_2} + P_1\log\frac{1}{P_1}\right)$$

$$= P_2\log\frac{1}{P_2} + P_1\log\frac{1}{P_1} = H(S)$$

Hoja de Trabajo 2

Sunday, August 19, 2018 7:14 PM

2.

$\boxed{S_1} \longrightarrow s_i \in S_1 = \{s_1, s_2, \ldots, s_{q_1}\}$, $P(s_i) = P_i$

$$H_1 = \sum_{i=1}^{q_1} P_i \log \frac{1}{P_i}$$

$\boxed{S_2} \longrightarrow s_i' \in S_2 = \{s_1', s_2', \ldots, s_{q_2}'\}$, $P(s_i') = Q_i$

$$H_2 = \sum_{i=1}^{q_2} Q_i \log \frac{1}{Q_i}$$

$\boxed{S(\lambda)} \longrightarrow \xi_i \in S(\lambda) = \{s_1, s_2, \ldots, s_{q_1}, s_1', s_2', \ldots, s_{q_2}'\}$

$P(s_1) = \lambda P_1$	$P(s_1') = \bar{\lambda} Q_1$
$P(s_2) = \lambda P_2$	$P(s_2') = \bar{\lambda} Q_2$
$\vdots$	$\vdots$
$P(s_{q_1}) = \lambda P_{q_1}$	$P(s_{q_2}') = \bar{\lambda} Q_{q_2}$

Demostrar:

$$H[S(\lambda)] = \lambda H_1 + \bar{\lambda} H_2 + H(\lambda)$$

$$H[S(\lambda)] = \sum_{i=1}^{q_1+q_2} P(\xi_i) \log \frac{1}{P(\xi_i)} = \lambda P_1 \log \frac{1}{\lambda P_1} + \lambda P_2 \log \frac{1}{\lambda P_2} + \cdots + \lambda P_{q_1} \log \frac{1}{\lambda P_{q_1}}$$
$$+ (1-\lambda) Q_1 \log \frac{1}{(1-\lambda) Q_1} + (1-\lambda) Q_2 \log \frac{1}{(1-\lambda) Q_2} + \cdots + (1-\lambda) Q_{q_2} \log \frac{1}{(1-\lambda) Q_{q_2}}$$

$$= \lambda P_1 \left(\log \frac{1}{\lambda} + \log \frac{1}{P_1}\right) + \lambda P_2 \left(\log \frac{1}{\lambda} + \log \frac{1}{P_2}\right) + \cdots + \lambda P_{q_1} \left(\log \frac{1}{\lambda} + \log \frac{1}{P_{q_1}}\right)$$
$$+ (1-\lambda) Q_1 \left(\log \frac{1}{1-\lambda} + \log \frac{1}{Q_1}\right) + (1-\lambda) Q_2 \left(\log \frac{1}{1-\lambda} + \log \frac{1}{Q_2}\right) + \cdots + (1-\lambda) Q_{q_2} \left(\log \frac{1}{1-\lambda} + \log \frac{1}{Q_{q_2}}\right)$$

$$= \lambda \log \frac{1}{\lambda} (P_1 + P_2 + \cdots + P_{q_1}) + (1-\lambda) \log \frac{1}{1-\lambda} (Q_1 + Q_2 + \cdots + Q_{q_2})$$

$$+ \lambda \left(P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \cdots + P_{q_1} \log \frac{1}{P_{q_1}}\right) + (1-\lambda) \left(Q_1 \log \frac{1}{Q_1} + Q_2 \log \frac{1}{Q_2} + \cdots + Q_{q_2} \log \frac{1}{Q_{q_2}}\right)$$

$$= \lambda \log \frac{1}{\lambda} + (1-\lambda) \log \frac{1}{1-\lambda} + \lambda H_1 + (1-\lambda) H_2$$
$$= \lambda H_1 + \bar{\lambda} H_2 + H(\lambda)$$

Si una fuente compuesta por S_1 y S_2 asigna un peso λ a S_1 y un peso $\bar{\lambda}$ a S_2, su entropía es equivalente a la suma de las entropías de S_1 y S_2 por su peso y la entropía del peso.

3.

$\boxed{S} \longrightarrow s_i \in S = \{s_1, s_2, \ldots, s_q\}$

$P(s_1) = P_1$
$P(s_2) = P_2$
$\vdots$
$P(s_q) = P_q$

$\boxed{S'} \longrightarrow s_i \in S' = \{s_1', s_2', \ldots, s_{2q}'\}$

$P(s_1') = (1-\varepsilon) P_1$	$P(s_{q+1}') = \varepsilon P_1$
$P(s_2') = (1-\varepsilon) P_2$	$P(s_{q+2}') = \varepsilon P_2$
$\vdots$	$\vdots$
$P(s_q') = (1-\varepsilon) P_q$	$P(s_{2q}') = \varepsilon P_q$

$$H(S') = \sum_{i=1}^{2q} P(s_i') \log \frac{1}{P(s_i')} = (1-\varepsilon) P_1 \log \frac{1}{(1-\varepsilon) P_1} + (1-\varepsilon) P_2 \log \frac{1}{(1-\varepsilon) P_2} + \cdots + (1-\varepsilon) P_q \log \frac{1}{(1-\varepsilon) P_q}$$
$$+ \varepsilon P_1 \log \frac{1}{\varepsilon P_1} + \varepsilon P_2 \log \frac{1}{\varepsilon P_2} + \cdots + \varepsilon P_q \log \frac{1}{\varepsilon P_q}$$

$$= (1-\varepsilon) P_1 \left(\log \frac{1}{1-\varepsilon} + \log \frac{1}{P_1}\right) + (1-\varepsilon) P_2 \left(\log \frac{1}{(1-\varepsilon)} + \log \frac{1}{P_2}\right) + \cdots + (1-\varepsilon) P_q \left(\log \frac{1}{1-\varepsilon} + \log \frac{1}{P_q}\right)$$
$$+ \varepsilon P_1 \left(\log \frac{1}{\varepsilon} + \log \frac{1}{P_1}\right) + \varepsilon P_2 \left(\log \frac{1}{\varepsilon} + \log \frac{1}{P_2}\right) + \cdots + \varepsilon P_q \left(\log \frac{1}{\varepsilon} + \log \frac{1}{P_q}\right)$$

$$= (1-\varepsilon) \log \frac{1}{1-\varepsilon} (P_1 + P_2 + \cdots + P_q) + \varepsilon \log \frac{1}{\varepsilon} (P_1 + P_2 + \cdots P_q) + P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \cdots + P_q \log \frac{1}{P_q}$$

$$= H(\varepsilon) + H(S)$$

Hoja de Trabajo 3

Sunday, August 19, 2018 9:09 PM

a) $q=2$

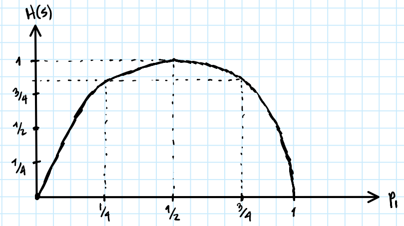
$\boxed{S} \xrightarrow{} \quad s_i \in S = \{s_1, s_2\}$

$P(s_1) = P_1$

$P(s_2) = P_2 = 1 - P_1$

$$H(S) = \sum_{i=1}^{2} P(s_i)\log\frac{1}{P(s_i)} = P_1\log\frac{1}{P_1} + P_2\log\frac{1}{P_2}$$

$$= P_1\log\frac{1}{P_1} + (1-P_1)\log\frac{1}{1-P_1}$$



$q=3$

$\boxed{S} \xrightarrow{} \quad s_i \in S = \{s_1, s_2, s_3\}$

$P(s_1) = P_1$

$P(s_2) = P_2$

$P(s_3) = P_3 = 1 - P_1 - P_2$

$$H(S) = \sum_{i=1}^{3} P(s_i)\log\frac{1}{P(s_i)} = P_1\log\frac{1}{P_1} + P_2\log\frac{1}{P_2} + P_3\log\frac{1}{P_3}$$

$$= P_1\log\frac{1}{P_1} + P_2\log\frac{1}{P_2} + (1-P_1-P_2)\log\frac{1}{1-P_1-P_2}, \quad P_1 + P_2 \leq 1$$

b) Demostrar:

$$H(S_1) = H(S) + P_q H(S_2)$$

$$H(S_1) = \sum_{i=1}^{q+1} P(s_i)\log\frac{1}{P(s_i)} = P_1\log\frac{1}{P_1} + P_2\log\frac{1}{P_2} + \cdots + P_{q-1}\log\frac{1}{P_{q-1}} + \alpha P_q\log\frac{1}{\alpha P_q} + \bar{\alpha} P_q\log\frac{1}{\bar{\alpha} P_q}$$

$$= P_1\log\frac{1}{P_1} + P_2\log\frac{1}{P_2} + \cdots + P_{q-1}\log\frac{1}{P_{q-1}} + \alpha P_q\left(\log\frac{1}{\alpha} + \frac{1}{P_q}\right) + (1-\alpha)P_q\left(\log\frac{1}{1-\alpha} + \log\frac{1}{P_q}\right)$$

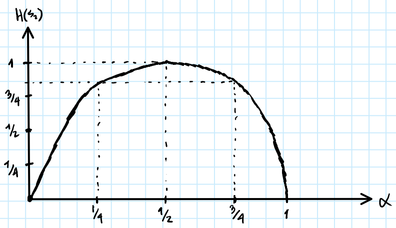
$$= P_1\log\frac{1}{P_1} + P_2\log\frac{1}{P_2} + \cdots + P_{q-1}\log\frac{1}{P_{q-1}} + \alpha P_q\log\frac{1}{\alpha} + (1-\alpha)P_q\log\frac{1}{1-\alpha} + P_q\log\frac{1}{P_q}$$

$$= \sum_{i=1}^{q} P_i\log\frac{1}{P_i} + P_q\left(\alpha\log\frac{1}{\alpha} + \bar{\alpha}\log\frac{1}{\bar{\alpha}}\right)$$

$$= H(S) + P_q H(S_2)$$

c)

$$H(S_2) = \sum_{i=1}^{2} P(s_i)\log\frac{1}{P(s_i)} = \alpha\log\frac{1}{\alpha} + \bar{\alpha}\log\frac{1}{\bar{\alpha}}$$



2. $$\sum_{i=1}^{q} x_i\log\frac{1}{x_i} \leq \sum_{i=1}^{q} x_i\log\frac{1}{y_i}$$

$x_i = P(s_i)$

$y_i = P(s_i)$

$$\sum_{i=1}^{q} x_i\log\frac{y_i}{x_i} = \frac{1}{\ln 2}\sum_{i=1}^{q} x_i \ln\frac{y_i}{x_i}$$

$$\frac{1}{\ln 2}\sum_{i=1}^{q} x_i\ln\frac{y_i}{x_i} \leq \frac{1}{\ln 2}\sum_{i=1}^{q} x_i f\left(\frac{y_i}{x_i}\right)$$

$\ln\left|\frac{y_i}{x_i}\right| \leq \frac{y_i}{x_i} - 1$, entonces:

$$\frac{1}{\ln 2}\sum_{i=1}^{q} x_i\ln\frac{y_i}{x_i} \leq \frac{1}{\ln 2}\sum_{i=1}^{q} x_i\left(\frac{y_i}{x_i} - 1\right) = \frac{1}{\ln 2}\sum_{i=1}^{q}(y_i - x_i)$$

$$= \frac{1}{\ln 2}\sum_{i=1}^{q} y_i - \frac{1}{\ln 2}\sum_{i=1}^{q} x_i = 0$$

$$\frac{1}{\ln 2}\sum_{i=1}^{q} x_i\ln\frac{y_i}{x_i} = \sum_{i=1}^{q} x_i\left(\log\frac{1}{x_i} - \log\frac{1}{y_i}\right)$$

$$= \sum_{i=1}^{q} x_i\log\frac{1}{x_i} - \sum_{i=1}^{q} x_i\log\frac{1}{y_i} \leq 0$$

$$\therefore \sum_{i=1}^{q} x_i\log\frac{1}{x_i} \leq \sum_{i=1}^{q} x_i\log\frac{1}{y_i}$$

3. Demostrar:

$$\log(q) - H(S) \geq 0$$

$$\log(q) = \log(q)\sum_{i=1}^{q} P_i = \sum_{i=1}^{q} P_i\log(q)$$

$$\log(q) - H(S) = \sum_{i=1}^{q} P_i\log(q) - \sum_{i=1}^{q} P_i\log\left(\frac{1}{P_i}\right)$$

$$= \sum_{i=1}^{q} P_i\left[\log(q) - \log\left(\frac{1}{P_i}\right)\right]$$

$$= \sum_{i=1}^{q} P_i\log(qP_i) = \frac{1}{\ln(2)}\sum_{i=1}^{q} P_i\ln(qP_i)$$

$\ln|x| \leq x - 1$

$-\ln|x| \geq 1 - x$

$\ln\left|\frac{1}{x}\right| \geq 1 - x$, entonces:

$$\frac{1}{\ln(2)}\sum_{i=1}^{q} P_i\ln(qP_i) \geq \frac{1}{\ln(2)}\sum_{i=1}^{q} P_i\left(1 - \frac{1}{qP_i}\right)$$

$$= \frac{1}{\ln(2)}\sum_{i=1}^{q} P_i - \frac{1}{\ln(2)}\sum_{i=1}^{q}\frac{1}{q} = 0$$

$$\frac{1}{\ln(2)}\sum_{i=1}^{q} P_i\ln(qP_i) \geq 0$$

$$\log(q) - H(S) \geq 0$$

Hoja de Trabajo 4

Thursday, August 9, 2018 2:06 PM

1.

fuente binaria $\rightarrow s_i \in S = \{0, 1\}$

$P(0) = 0.2$

$P(1) = 0.8$

tercera extensión

i	j	k
0	0	0
1	1	1

```
for(int i=0; i < S.length; i++){
    for(int j=0; j < S.length; j++){
        for(int k=0; k < S.length; k++){
            system.out.println(S[i]S[j]S[k]);
        }
    }
}
```

0 0 0	σ_0
0 0 1	σ_1
0 1 0	σ_2
0 1 1	σ_3
1 0 0	σ_4
1 0 1	σ_5
1 1 0	σ_6
1 1 1	σ_7

3era extensión $\rightarrow \sigma_i \in S^3 = \{\sigma_0, \sigma_1, \dots, \sigma_7\}$

$P(\sigma_0) = P(000) = P(0)^3 = 0.2^3$

$P(\sigma_1) = P(001) = P(0)^2 P(1)$

$\vdots$

$P(\sigma_7) = P(1)^3$

$S' \rightarrow s_i \in S' = \{0, 1, 2, 3\}$

$P'(0) = 0.8^3$

$P'(1) = P(\sigma^3) + P(\sigma^5) + P(\sigma^6) = 3P(0)P(1)^2$

$P'(2) = P(\sigma_2) + P(\sigma_3) + P(\sigma_4) = 3P(1)P(0)^2$

$P'(3) = 0.2^3$

$H(S^3) = 3H(S) = 3\left[0.2 \log \frac{1}{0.2} + 0.8 \log \frac{1}{0.8}\right]$

$= 3(0.46 + 0.26) = \boxed{2.166}$

$H(S') = 0.8^3 \log \frac{1}{0.8^3} + 3(0.2)(0.8)^2 \log \frac{1}{3(0.2)(0.8)^2} + 3(0.2)^2(0.8) \log \frac{1}{3(0.2)^2(0.8)} + 0.2^3 \log \frac{1}{0.2^3}$

$= 0.4945 + 0.5302 + 0.3246 + 0.05573 = \boxed{1.405}$

$2.1666 - 1.405 = \boxed{0.7610}$

2.

fuente binaria $\rightarrow s_i \in S = \{s_1, s_2\}$

$P(s_1) = P_1$

$P(s_2) = P_2$

segunda extensión:

i	j		
s_1	s_1	$s_1 s_1$	σ_1
s_2	s_2	$s_1 s_2$	σ_2
		$s_2 s_1$	σ_3
		$s_2 s_2$	σ_4

segunda extensión $\rightarrow \sigma_i \in S^2 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$

$P(\sigma_1) = P_1^2$

$P(\sigma_2) = P_1 P_2$

$P(\sigma_3) = P_2 P_1$

$P(\sigma_4) = P_2^2$

Demostrar $H(S^2) = 2H(S)$

$H(S^2) = P(\sigma_1) \log \frac{1}{P(\sigma_1)} + P(\sigma_2) \log \frac{1}{P(\sigma_2)} + P(\sigma_3) \log \frac{1}{P(\sigma_3)} + P(\sigma_4) \log \frac{1}{P(\sigma_4)}$

$= P_1^2 \log \frac{1}{P_1^2} + P_1 P_2 \log \frac{1}{P_1 P_2} + P_2 P_1 \log \frac{1}{P_2 P_1} + P_2^2 \log \frac{1}{P^2}$

$= P_1 P_1 \left(\log \frac{1}{P_1} + \log \frac{1}{P_1}\right) + P_1 P_2 \left(\log \frac{1}{P_1} + \log \frac{1}{P_2}\right) + P_2 P_1 \left(\log \frac{1}{P_2} + \log \frac{1}{P_1}\right) + P_2 P_2 \left(\log \frac{1}{P_2} + \frac{1}{P_2}\right)$

$= 2 P_1 P_1 \log \frac{1}{P_1} + 2 P_1 P_2 \log \frac{1}{P_1} + 2 P_1 P_2 \log \frac{1}{P_2} + 2 P_2 P_2 \log \frac{1}{P_2}$

$= 2\left[\log \frac{1}{P_1} (P_1 P_1 + P_1 P_2) + \log \frac{1}{P_2} (P_1 P_2 + P_2 P_2)\right]$

$= 2\left[\log\left(\frac{1}{P_1}\right) P_1 + \log\left(\frac{1}{P_2}\right) P_2\right] = \boxed{2H(S)}$

1.

fuente binaria $\rightarrow s'_i \in S' = \{0, 1\}$

$P(0) = 0.2$

$P(1) = 0.8$

$S^3 \xrightarrow{\sigma_i \in S_0 = \{\sigma_1, \sigma_2, \dots, \sigma_8\}} S$

$P(\sigma_1) = P(0)P(0)P(0) = 0.2^3$

$P(\sigma_2) = P(0)P(0)P(1) = 0.2^2 0.8$

$\vdots$

$P(\sigma_8) = P(1)P(1)P(1) = 0.8^3$

$S \rightarrow s_i \in S = \{0, 1, 2, 3\}$

$P(s_1) = 0.8^3$

$P(s_2) = 3(0.8^2)0.2$

$P(s_3) = 3(0.8)0.2^2$

$P(s_4) = 0.2^3$

tercera extensión

σ_i	
σ_1	000
σ_2	001
σ_3	010
σ_4	011
σ_5	100
σ_6	101
σ_7	110
σ_8	111

a)

$H(S_0) = 3H(S') = 3 \sum_{i=1}^{2} P(s'_i) \log \frac{1}{P(s'_i)} = 3\left(0.2 \log \frac{1}{0.2} + 0.8 \log \frac{1}{0.8}\right) = 2.166$

$H(S) = \sum_{i=1}^{4} P(s_i) \log \frac{1}{P(s_i)} = 0.8^3 \log \frac{1}{0.8^3} + 3(0.8^2)0.2 \log \frac{1}{3(0.8^2)0.2} + 3(0.8)0.2^2 \log \frac{1}{3(0.8)0.2^2} + 0.2^3 \log \frac{1}{0.2^3}$

$= 1.405$

b)

$H(S_0) - H(S) = 0.7608$

Dado que la resta entre la entropía de la tercera extensión y la entropía de S es positiva, se concluye que el valor medio de incertidumbre de un observador antes de conocer la salida es mayor en la tercera extensión.

2.

$S \rightarrow s_i \in S = \{s_1, s_2\}$

$P(s_1) = P_1$

$P(s_2) = P_2$

Segunda extensión

σ_i	
σ_1	$s_1 s_1$
σ_2	$s_1 s_2$
σ_3	$s_2 s_1$
σ_4	$s_2 s_2$

$S^2 \rightarrow \sigma_i \in S^2 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$

$P(\sigma_1) = P_1 P_1$

$P(\sigma_2) = P_1 P_2$

$P(\sigma_3) = P_2 P_1$

$P(\sigma_4) = P_2 P_2$

Demostrar:

$H(S^2) = 2H(S)$

$H(S^2) = \sum_{i=1}^{4} P(\sigma_i) \log \frac{1}{P(\sigma_i)} = P(\sigma_1) \log \frac{1}{P(\sigma_1)} + P(\sigma_2) \log \frac{1}{P(\sigma_2)} + P(\sigma_3) \log \frac{1}{P(\sigma_3)} + P(\sigma_4) \log \frac{1}{P(\sigma_4)}$

$= P_1 P_1 \log \frac{1}{P_1 P_1} + P_1 P_2 \log \frac{1}{P_1 P_2} + P_2 P_1 \log \frac{1}{P_2 P_1} + P_2 P_2 \log \frac{1}{P_2 P_2}$

$= P_1 P_1 \left(\log \frac{1}{P_1} + \log \frac{1}{P_1}\right) + P_1 P_2 \left(\log \frac{1}{P_1} + \log \frac{1}{P_2}\right) + P_2 P_1 \left(\log \frac{1}{P_2} + \log \frac{1}{P_1}\right) + P_2 P_2 \left(\log \frac{1}{P_2} + \log \frac{1}{P_2}\right)$

$= (P_1 P_1 + P_1 P_1 + P_1 P_2 + P_2 P_1) \log \frac{1}{P_1} + (P_1 P_2 + P_2 P_1 + P_2 P_2 + P_2 P_2) \log \frac{1}{P_2}$

$= (2P_1 P_1 + 2P_1 P_2) \log \frac{1}{P_1} + (2P_1 P_2 + 2P_2 P_2) \log \frac{1}{P_2}$

$= 2\left[P_1(P_1 + P_2)\right] \log \frac{1}{P_1} + 2\left[P_2(P_1 + P_2)\right] \log \frac{1}{P_2}$

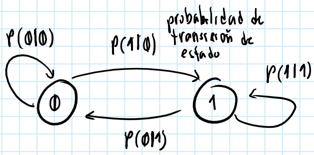
$= 2P_1 \log \frac{1}{P_1} + 2P_2 \log \frac{1}{P_2} = 2 \sum_{i=1}^{2} P_i \log \frac{1}{P_i} = 2H(S)$

Fuente de Markov Bin 1

Thursday, August 9, 2018 2:47 PM

orden: número de símbolos utilizados para estimar la probabilidad del siguiente símbolo.

estado: toda fuente de Markov
se debe esperar a que se emitan n símbolos para definir el estado.



$P(0|0) + P(1|0) = 1$

Fuente Markov →

$$I(0|0) = \log \frac{1}{P(0|0)} \qquad I(1|0) = \log \frac{1}{P(1|0)}$$

entonces:

$$\boxed{H(S|0) = P(0|0) \log \frac{1}{P(0|0)} + P(1|0) \log \frac{1}{P(1|0)}}$$

$$I(0|1) = \log \frac{1}{P(0|1)} \qquad I(1|1) = \log \frac{1}{P(1|1)}$$

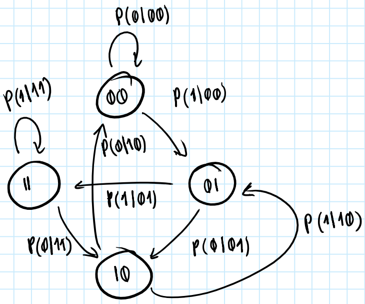
entonces:

$$\boxed{H(S|1) = P(0|1) \log \frac{1}{P(0|1)} + P(1|1) \log \frac{1}{P(1|1)}}$$

finalmente:

$$\boxed{H(S) = P(0)\, H(S|0) + P(1)\, H(S|1)}$$

probabilidad de estado (estacionaria)



$$I(0|00) = \log \frac{1}{P(0|00)} \qquad I(1|00) = \log \frac{1}{P(1|00)}$$

entonces:

$$H(S|00) = P(0|00) \log \frac{1}{P(0|00)} + P(1|00) \frac{1}{P(1|00)}$$

$$I(0|01) = \log \frac{1}{P(0|01)} \qquad I(1|01) = \log \frac{1}{P(1|01)}$$

entonces:

$$H(S|01) = P(0|01) \log \frac{1}{P(0|01)} + P(1|01) \frac{1}{P(1|01)}$$

$$I(0|10) = \log \frac{1}{P(0|10)} \qquad I(1|10) = \log \frac{1}{P(1|10)}$$

entonces:

$$H(S|10) = P(0|10) \log \frac{1}{P(0|10)} + P(1|10) \frac{1}{P(1|10)}$$

$$I(0|11) = \log \frac{1}{P(0|11)} \qquad I(1|11) = \log \frac{1}{P(1|11)}$$

entonces:

$$H(S|11) = P(0|11) \log \frac{1}{P(0|11)} + P(1|11) \frac{1}{P(1|11)}$$

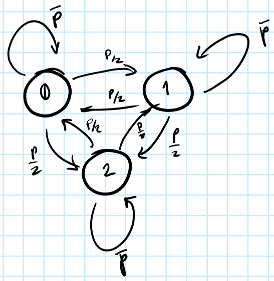
Finalmente

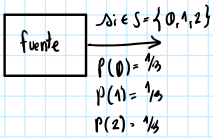
$$\boxed{H(S) = P(00)\, H(S|00) + P(01)\, H(S|01) + P(10)\, H(S|10) + P(11)\, H(S|11)}$$

Hoja de Trabajo 6

Friday, August 10, 2018 5:01 PM

1.





a.

$I(0|0) = \log \frac{1}{P(0|0)} = \log \frac{1}{\bar{p}}$

$I(1|0) = \log \frac{1}{P(1|0)} = \log \frac{2}{p}$

$I(2|0) = \log \frac{1}{P(2|0)} = \log \frac{2}{p}$

entonces:

$H(S|0) = P(0|0) \log \frac{1}{\bar{p}} + P(1|0) \log \frac{2}{p} + P(2|0) \log \frac{2}{p}$

$= \bar{p} \log \frac{1}{\bar{p}} + \frac{p}{2} \log \frac{2}{p} + \frac{p}{2} \log \frac{2}{p} = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}$

$I(0|1) = \log \frac{1}{P(0|1)} = \log \frac{2}{p}$

$I(1|1) = \log \frac{1}{P(1|1)} = \log \frac{1}{\bar{p}}$

$I(2|1) = \log \frac{1}{P(2|1)} = \log \frac{2}{p}$

entonces:

$H(S|1) = P(0|1) \log \frac{2}{p} + P(1|1) \log \frac{1}{\bar{p}} + P(2|1) \log \frac{2}{p}$

$= \frac{p}{2} \log \frac{2}{p} + \bar{p} \log \frac{1}{\bar{p}} + \frac{p}{2} \log \frac{2}{p} = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}$

$I(0|2) = \log \frac{1}{P(0|2)} = \log \frac{2}{p}$

$I(1|2) = \log \frac{1}{P(1|2)} = \log \frac{2}{p}$

$I(2|2) = \log \frac{1}{P(2|2)} = \log \frac{1}{\bar{p}}$

entonces:

$H(S|2) = P(0|2) \log \frac{2}{p} + P(1|2) \log \frac{2}{p} + P(2|2) \log \frac{1}{\bar{p}}$

$= \frac{p}{2} \log \frac{2}{p} + \frac{p}{2} \log \frac{2}{p} + \bar{p} \log \frac{1}{\bar{p}} = \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}$

$H(S) = P(0) H(S|0) + P(1) H(S|1) + P(2) H(S|2) = \frac{1}{3}\left(\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}\right) + \frac{1}{3}\left(\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}\right) + \frac{1}{3}\left(\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}\right)$

$= \boxed{\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{2}{p}}$

La respuesta es correcta para $p = 0$ y para $p = 1$ porque $0 * \infty = 0$

b.

$\lim_{\varepsilon \to 0} (1-\varepsilon) \log \frac{1}{1-\varepsilon} + \varepsilon \log \frac{2}{\varepsilon} = 1(0) + 0 = 0$

c.

$\lim_{\delta \to 0} (1-1+\delta) \log \frac{1}{1-1+\delta} + (1-\delta) \log \frac{2}{1-\delta} = 0(\infty) + 1(\log 2) = \log 2$

d.

$I(0|0) = \log \frac{1}{P(0|0)}$

$I(1|0) = \log \frac{1}{P(1|0)}$

$I(2|0) = \log \frac{1}{P(2|0)}$

entonces: $H(S|0) = P(0|0) \log \frac{1}{P(0|0)} + P(1|0) \log \frac{1}{P(1|0)} + P(2|0) \log \frac{1}{P(2|0)}$

$= \frac{3}{4} \log \frac{4}{3} + \frac{1}{8} \log 8 + \frac{1}{8} \log 8 = \frac{3}{4} \log \frac{4}{3} + \frac{1}{4} \log 8$

$H(S|1) = \frac{3}{4} \log \frac{4}{3} + \frac{1}{4} \log 8$

$H(S|2) = \frac{3}{4} \log \frac{4}{3} + \frac{1}{4} \log 8$

$H(S) = P(0) H(S|0) + P(1) H(S|1) + P(2) H(S|2)$

$\boxed{= \frac{3}{4} \log \frac{4}{3} + \frac{1}{4} \log 8}$

2.

$H(S|0) = P(0|0) \log \frac{1}{P(0|0)} + P(1|0) \log \frac{1}{P(1|0)}$

$= \bar{p} \log \frac{1}{\bar{p}} + p \log \frac{1}{p}$

$H(S|1) = P(0|1) \log \frac{1}{P(0|1)} + P(1|1) \log \frac{1}{P(1|1)}$

$= q \log \frac{1}{q} + \bar{q} \log \frac{1}{\bar{q}}$

$H(S) = P(0)\left(\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{1}{p}\right) + P(1)\left(q \log \frac{1}{q} + \bar{q} \log \frac{1}{\bar{q}}\right)$

a.

b.

$H(S) = P(0)\left(\bar{p} \log \frac{1}{\bar{p}} + p \log \frac{1}{p}\right) + P(1)\left(q \log \frac{1}{q} + \bar{q} \log \frac{1}{\bar{q}}\right)$

a.

$$\boxed{\begin{aligned} P(0) &= \sum_i P(0|A_i)P(A_i) \\ P(1) &= 1 - P(0) \end{aligned}}$$

comprobación:

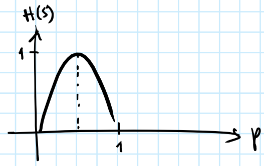
$$P(0) = P(0|0)P(0) + P(0|1)P(1)$$
$$= \bar{p}P(0) + q(1 - P(0))$$
$$= P(0) - P(0)p + q - P(0)q$$
$$0 = q - P(0)(p+q)$$
$$P(0) = \frac{q}{p+q}$$

b.

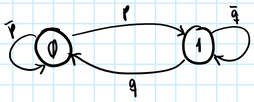
$$H(S) = P(0)\left(\bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}\right) + P(1)\left(q\log\frac{1}{q} + \bar{q}\log\frac{1}{\bar{q}}\right)$$
$$= \frac{0.2}{0.3}\left(0.9\log\frac{1}{0.9} + 0.1\log 10\right) + \frac{0.1}{0.3}\left(0.2\log 5 + 0.8\log\frac{1}{0.8}\right)$$
$$\boxed{= 0.5533 \text{ bits}}$$

c.

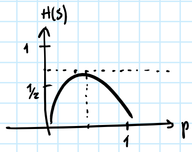
$$H(S) = \frac{p}{2p}\left(\bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}\right) + \frac{p}{2p}\left(\bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}\right)$$
$$= \bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}$$



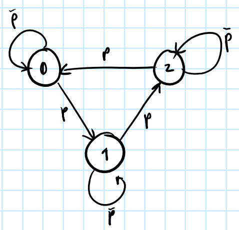
3.



$$H(S) = P(0)\left[P(0|0)\log\frac{1}{P(0|0)} + P(1|0)\log\frac{1}{P(1|0)}\right] + P(1)\left[P(0|1)\log\frac{1}{P(0|1)} + P(1|1)\log\frac{1}{P(1|1)}\right]$$
$$= \frac{1}{1+p}\left(\bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}\right) + \frac{p}{1+p}\left(\cancel{\log(1)}^{0} + 0\,\cancel{\log\frac{1}{0}}^{0}\right)$$
$$\boxed{= \frac{1}{1+p}\left(\bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}\right)}$$



4.



a.

$$H(S|0) = P(0|0)\log\frac{1}{P(0|0)} + P(1|0)\log\frac{1}{P(1|0)} + P(2|0)\log\frac{1}{P(2|0)}$$
$$= \bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p} + 0\,\cancel{\log\frac{1}{0}}^{0}$$

$$H(S|1) = P(0|1)\log\frac{1}{P(0|1)} + P(1|1)\log\frac{1}{P(1|1)} + P(2|1)\log\frac{1}{P(2|1)}$$
$$= 0\,\cancel{\log\frac{1}{0}}^{0} + \bar{p}\log\frac{1}{\bar{p}} + p\log\frac{1}{p}$$

$$H(S|2) = P(0|2)\log\frac{1}{P(0|2)} + P(1|2)\log\frac{1}{P(1|2)} + P(2|2)\log\frac{1}{P(2|2)}$$
$$= p\log\frac{1}{p} + 0\,\cancel{\log\frac{1}{0}}^{0} + \bar{p}\log\frac{1}{\bar{p}}$$

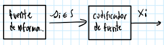
$$H(S) = P(0)H(S|0) + P(1)H(S|1) + P(2)H(S|2)$$
$$= \frac{1}{3}\left(p\log\frac{1}{p} + \bar{p}\log\frac{1}{\bar{p}}\right) + \frac{1}{3}\left(p\log\frac{1}{p} + \bar{p}\log\frac{1}{\bar{p}}\right) + \frac{1}{3}\left(p\log\frac{1}{p} + \bar{p}\log\frac{1}{\bar{p}}\right)$$
$$\boxed{= p\log\frac{1}{p} + \bar{p}\log\frac{1}{\bar{p}}}$$

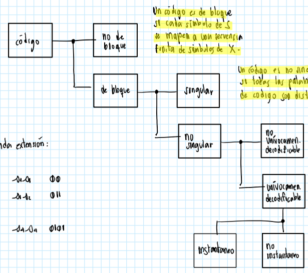
b.

$$\boxed{H(S^2) = 2H(S) = 2\left(p\log\frac{1}{p} + \bar{p}\log\frac{1}{\bar{p}}\right)}$$

Propiedades de Códigos

Thursday, August 16, 2018 2:09 PM





Un código es de bloque si cada símbolo de S se mapea a una secuencia finita de símbolos de X.

Un código es no singular si todas las palabras de código son distintas

código

un mapeo de una secuencia de símbolos de S a una secuencia de símbolos de X.

$S = \{s_1, s_2, \dots, s_q\}$

$X = \{x_1, x_2, \dots, x_r\}$

↑ alfabeto del código

ejemplo

s_i	código A	código B	código C
s_1	0	00	0
s_2	11	01	10
s_3	00	10	110
s_4	01	11	1110

segunda extensión:

σ_i		
σ_1	$s_1 s_1$	00
σ_2	$s_1 s_2$	011
⋮		
σ_{16}	$s_4 s_4$	0101

no es unívocamente decodificable, contraejemplo:

000 se puede interpretar como $s_1 s_3$ ó $s_1 s_1 s_1$

Un código es unívocamente decodificable si se cumple cualquiera de las siguientes afirmaciones:

1. Todas las palabras de código tienen la misma longitud.
2. El código dado es un código "coma". 0 10 110 1110
3. El código es sintetizable.

0 01 011 0111 01111

0110101111

↑ se puede confirmar la palabra en el primer símbolo de la siguiente palabra

Un código unívocamente decodificable es instantáneo si ninguna palabra de código es un prefijo de otra palabra de código.

s_i	código A	l_i
s_1	00	2
s_2	01	2
s_3	10	2
s_4	110	3
s_5	111	3

$$\sum_{i=1}^{q} r^{-l_i} = \frac{3}{4} + \frac{2}{8} = \frac{8}{8} \checkmark$$

Desigualdad de Kraft

$$\sum_{i=1}^{q} r^{-l_i} \leq 1$$

Si no se cumple no existe un código instantáneo con esas longitudes.

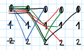
q = # de símbolos en S.

r = # de símbolos en X.

l_i = longitud de la i-ava palabra de código.

$X = \{0, 1, 2\}$

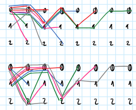
s_i	l_i	código A
s_1	3	000
s_2	4	0010
s_3	1	1
s_4	4	0011
s_5	1	2
s_6	4	0012
s_7	2	01
s_8	4	0020
s_9	5	00210
s_{10}	3	020



$$\sum_{i=1}^{10} 3^{-l_i} = \frac{2}{3} + \frac{1}{9} + \frac{2}{27} + \frac{4}{81} + \frac{1}{243} = 0.9053 \leq 1$$

se cumple la desigualdad de Kraft

s_i	l_i		
s_1	3	000	020
s_2	5	00100	10020
s_3	2	01	00
s_4	4	0011	1000
s_5	3	002	021
s_6	6	001010	100210
s_7	4	0012	1001
s_8	8	00101100	10021100
s_9	2	02	01
s_{10}	3	100	022



$0 = n_1 \leq r = 3 \checkmark$

$2 = n_2 \leq r^2 - \cancel{n_1} r = 9 \checkmark$

$3 = n_3 \leq r^3 - \cancel{n_1 r^2} - n_2 r = 21 \checkmark$

$2 = n_4 \leq r^4 - \cancel{n_1} r^3 - n_2 r^2 - n_3 r = 54 \checkmark$

$1 = n_5 \leq r^5 - \cancel{n_1} r^4 - n_2 r^3 - n_3 r^2 - n_4 r = 156 \checkmark$

$1 = n_6 \leq r^6 - \cancel{n_1} r^5 - n_2 r^4 - n_3 r^3 - n_4 r^2 - n_5 r = 465 \checkmark$

$0 = n_7 \leq r^7 - \cancel{n_1} r^6 - n_2 r^5 - n_3 r^4 - n_4 r^3 - n_5 r^2 - \cancel{n_6} r = 1{,}392 \checkmark$

$1 = n_8 \leq r^8 - \cancel{n_1 r^7} - n_2 r^6 - n_3 r^5 - n_4 r^4 - n_5 r^3 - \cancel{n_6} r^2 - \cancel{n_7} r = 4176 \checkmark$

$n_1 \leq r$, r cardinalidad del alfabeto

$n_2 \leq r^2 - n_1 r$

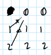
$3^2 - 1 * 3$

$r^3 - n_1 r^2 - n_2 r$

$r(r^2 - n_1 r - n_2)$

$27 - 9 - 6$

12



1 2 3
4 5 6
7 8 9
10 11 12

$r(r^3 - n_1 r^2 - n_2 r - n_3)$

$$\boxed{n_{i} = r^i - \sum_{j=1}^{i-1} n_j r^{i-j}}$$

Codificación de Fuentes de Información

Thursday, September 6, 2018 2:06 PM

$$Q_i = \frac{r^{-l_i}}{\sum_{j=1}^{q} r^{-l_j}}$$

entonces:

$$H(S) \leq -\sum_{i=1}^{q} P_i \log(Q_i) = -\sum_{i=1}^{q} P_i \log\left[\frac{r^{-l_i}}{\sum_{j=1}^{q} r^{-l_j}}\right]$$

$$= -\sum_{i=1}^{q} P_i\left[\log(r^{-l_i}) - \log\left(\sum_{j=1}^{q} r^{-l_j}\right)\right]$$

$$= \sum_{i=1}^{q} P_i l_i \log(r) + \log\left(\sum_{j=1}^{q} r^{-l_j}\right)$$

$$= \log(r) L + \log\left(\sum_{j=1}^{q} r^{-l_i}\right), \quad L = \sum_{i=1}^{q} P_i l_i$$

$$\boxed{H(S) \leq L}$$

Cuando $H(S) = L$ la fuente es especial

Primer Teorema de Shannon

Existe siempre un entero en el intervalo

$$\log \frac{1}{P_i} \leq l_i < \log \frac{1}{P_i} + 1 \quad \| \ \forall P_i$$

$$P_i \log \frac{1}{P_i} \leq P_i l_i < P_i \log \frac{1}{P_i} + P_i \quad \| \sum_{i=1}^{q} (\cdot)$$

$$\sum_{i=1}^{q} P_i \log \frac{1}{P_i} \leq \sum_{i=1}^{q} P_i l_i < \sum_{i=1}^{q} P_i \log \frac{1}{P_i} + \sum_{i=1}^{q} P_i$$

$$\boxed{H(S) \leq L < H(S) + 1}$$

Definición:

Un código es compacto si su longitud promedio es menor o igual que L_{ave}.

$$L_{ave} = \frac{L_1 + L_2 + \dots + L_n}{n}$$

promedio de longitudes promedio

$$H(S) = \sum_{i=1}^{5} P_i \log \frac{1}{P_i} = 0.5 + 0.5 + 0.375 + 0.3322 + 0.1330$$

$$\approx 1.84$$

$$\boxed{\text{como } H(S) > L \text{ entonces el código no existe.}}$$

$$\log \frac{1}{0.5} = 1$$

$$\log \frac{1}{0.25} = 2$$

$$\log \frac{1}{0.125} = 3$$

$$\log \frac{1}{\cdot} = 3.32$$

s_1	0.5	1	0
s_2	0.25	2	10
s_3	0.125	3	110
s_4	0.1	4	1110
s_5	0.025	6	111100

$\log \frac{1}{0.125}$

s_5 0.025 6 111100

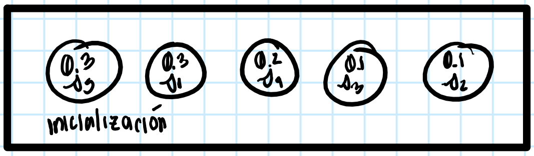
$\log \frac{1}{0.1} = 3.32$
4

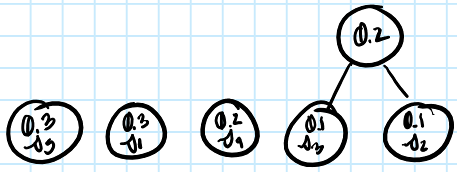
$\log \frac{1}{0.025} = 5.322$
6

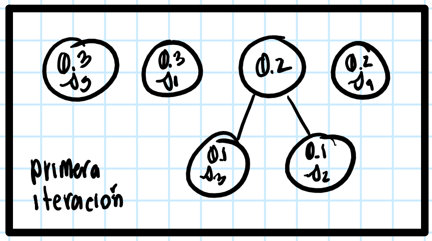
Códigos de Redundancia Mínima

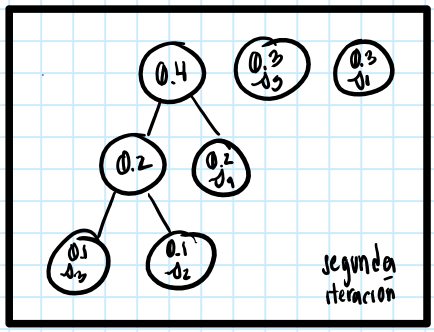
Thursday, September 13, 2018 2:13 PM

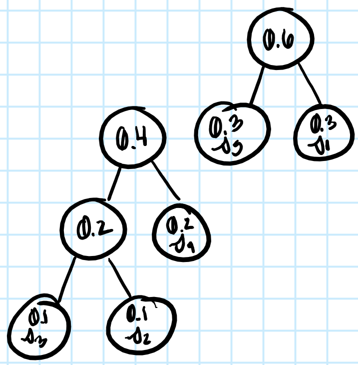
s_i	$P(s_i)$	código de Hoffman binario
s_1	0.3	01
s_2	0.1	101
s_3	0.1	100
s_4	0.2	11
s_5	0.3	00

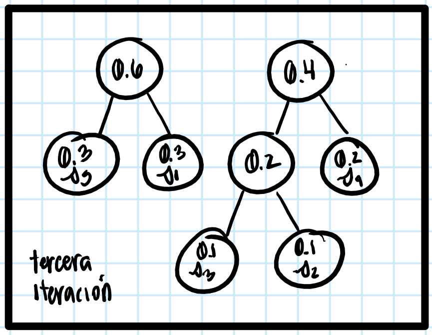


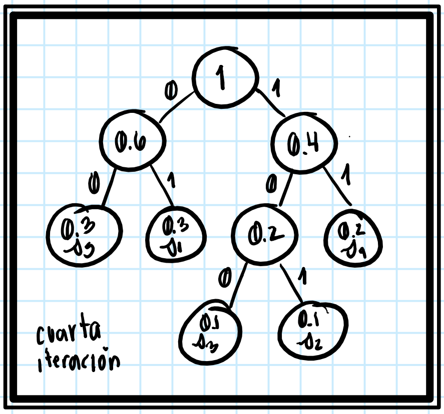











1
0
1
0.6
0.4
0.3
0.3
0.2
0.2
0.1
0.1
cuarta
iteración

Hoja de Trabajo 7

Sunday, September 30, 2018 7:40 PM

1.

a.

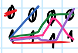
El código A es unívocamente decodificable porque todas las palabras de código tienen la misma longitud y es no singular.

El código B es unívocamente decodificable porque es un código coma.

El código C es unívocamente decodificable porque es un código coma.

El código D no es unívocamente decodificable porque 10110 puede decodificarse como S_2S_3 o S_5S_1 ✗

El código E es unívocamente decodificable porque es sintetizable.



El código F no es unívocamente decodificable porque 00100 puede decodificarse como $S_6S_1S_1$ o $S_1S_1S_2$. ✗

b.

$$\sum_{i=1}^{6} 2^{-l_i} = \frac{6}{8} = 0.75$$ ∴ se cumple la desigualdad de Kraft.

El código A es instantáneo porque se cumple la desigualdad de Kraft y ninguna palabra de código es prefijo de otra.

El código B no es instantáneo porque S_1 es prefijo de todas las palabras de código. ✗

$$\sum_{i=1}^{6} 2^{-l_i} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = 0.9844$$ ∴ se cumple la desigualdad de Kraft.

El código C es instantáneo porque se cumple la desigualdad de Kraft y ninguna palabra de código es prefijo de otra.

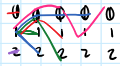
$$\sum_{i=1}^{6} 2^{-l_i} = \frac{1}{2} + \frac{1}{4} + \frac{4}{16} = 1$$ ∴ se cumple la desigualdad de Kraft.

El código E es instantáneo porque se cumple la desigualdad de Kraft y ninguna palabra de código es prefijo de otra.

2.

s_i	l_i	código A
s_1	1	0
s_2	3	100
s_3	4	1010
s_4	5	10110
s_5	1	2
s_6	4	1012
s_7	4	1020
s_8	2	11
s_9	4	1021
s_{10}	3	120

s_i	l_i	código B
s_1	1	0
s_2	4	1000
s_3	2	11
s_4	5	10010
s_5	1	2
s_6	3	101
s_7	2	12
s_8	3	102

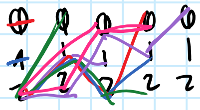


Hoja de Trabajo 8

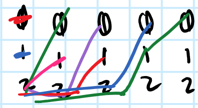
Sunday, September 30, 2018 9:15 PM

1.

s_i	l_i	código A
s_1	1	0
s_2	3	211
s_3	4	2120
s_4	5	22010
s_5	1	1
s_6	4	2121
s_7	4	2122
s_8	2	20
s_9	4	2200
s_{10}	3	210



s_i	l_i	código B
s_1	1	0
s_2	4	2220
s_3	2	20
s_4	5	22210
s_5	1	1
s_6	3	220
s_7	2	21
s_8	3	221

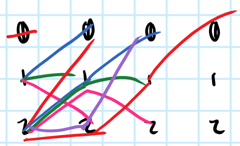


2.

a. $\sum_{i=1}^{10} 3^{-l_i} = \frac{1}{3} + \frac{5}{9} + \frac{4}{27} = 1.037$ ∴ no se cumple la desigualdad de Kraft, no es posible sintetizar un código con esas longitudes.

b. $\sum_{i=1}^{10} 3^{-l_i} = \frac{1}{3} + \frac{4}{9} + \frac{4}{27} + \frac{1}{81} = 0.9383$ ∴ se cumple la desigualdad de Kraft.

s_i	l_i	código
s_1	1	0
s_2	2	10
s_3	2	11
s_4	2	12
s_5	2	20
s_6	3	210
s_7	3	211
s_8	3	212
s_9	3	220
s_{10}	4	2210

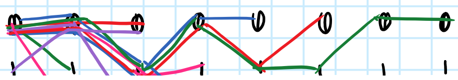


3.

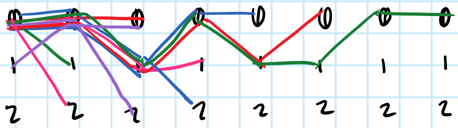
a.

s_i	l_i	código
s_1	3	000
s_2	5	00100
s_3	2	01
s_4	4	0011
s_5	3	002

$\sum_{i=1}^{10} 3^{-l_i} = \frac{2}{9} + \frac{3}{27} + \frac{2}{81} + \frac{1}{243} + \frac{1}{729} + \frac{1}{3^8} = 0.3637$ ∴ se cumple la desigualdad de Kraft.



s_3	2	01
s_4	4	0011
s_5	3	002
s_6	6	001010
s_7	4	0012
s_8	8	00101100
s_9	2	02
s_{10}	3	100



b.

$0 = n_1 \le r = 3$

$2 = n_2 \le r^2 - \cancel{n_1 r} = 9$

$3 = n_3 \le r^3 - \cancel{n_1 r^2} - n_2 r = 21$

$2 = n_4 \le r^4 - \cancel{n_1 r^3} - n_2 r^2 - n_3 r = 54$

Hoja de Trabajo 9

Sunday, September 30, 2018 9:58 PM

1.

a. $$\boxed{L = \sum_{i=1}^{7} p_i l_i = \sum_{i=1}^{q} p_i \log_r\left(\frac{1}{p_i}\right) = H_r(S)}$$

b.

s_i	p_i	l_i	código
s_1	0.5	1	0
s_2	0.125	3	100
s_3	0.125	3	101
s_4	0.125	3	110
s_5	0.125	3	111

$$L = \sum_{i=1}^{5} p_i l_i = 2 \ \frac{\text{dig. bin.}}{\text{símbolo}}$$

$$H(s) = \sum_{i=1}^{5} p_i \log \frac{1}{p_i} = 2 \text{ bits}$$

2.

$$L = \sum_{i=1}^{q} p_i l_i$$

$$H(s) = \sum_{i=1}^{q} p_i \log \frac{1}{p_i}$$

3.

a. $$H(s) = \sum_{i=1}^{5} p_i \log \frac{1}{p_i} = 0.5 \log 2 + 0.25 \log 4 + 0.125 \log 8 + 0.1 \log 10 + 0.025 \log 40$$

$$= 0.5 + 0.5 + 0.375 + 0.3322 + 0.133 = 1.840 \text{ bits}$$

∴ como $H(s) > L$ entonces el código no existe.
no se satisface el primer teorema de Shannon.

b.

$\log 2 = 1$

$\log 4 = 2$

$\log 8 = 3$

$\log 10 = 3.322$

$\log 40 = 5.322$

s_i	l_i	código
s_1	1	0
s_2	2	10
s_3	3	110
s_4	4	1110
s_5	6	111100

$$L = \sum_{i=1}^{5} p_i l_i = 1.925 \ \frac{\text{binits}}{\text{símbolo}}$$

∴ como $L > H(s)$ entonces el código satisface el primer teorema de Shannon.

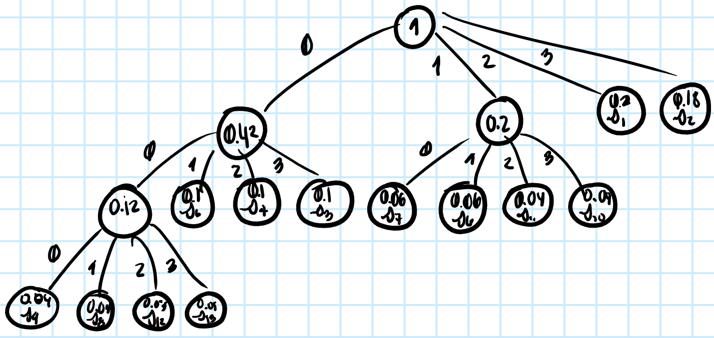
4.

Hoja de Trabajo 10

Friday, September 14, 2018 4:58 PM

5.

s_i	$P(i)$	código cuaternario de Huffman
s_1	0.2	2
s_2	0.18	3
s_3	0.1	03
s_4	0.1	02
s_5	0.1	01
s_6	0.06	11
s_7	0.06	10
s_8	0.04	001
s_9	0.04	000
s_{10}	0.04	13
s_{11}	0.04	12
s_{12}	0.03	002
s_{13}	0.01	003

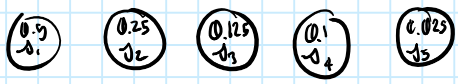


$$L_{HC} = 1.74 \frac{\text{dig cuat.}}{\text{símb}}$$

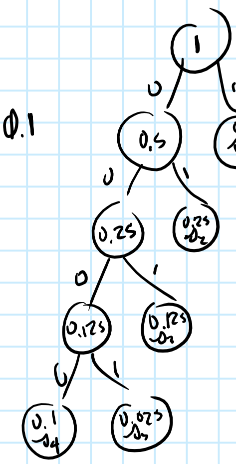
1.

s_i	P_i	código binario de Huffman
s_1	0.5	1
s_2	0.25	01
s_3	0.125	001
s_4	0.1	0000
s_5	0.025	0001

inicialización:



$$L = \sum_{i=1}^{5} P_i l_i = 0.5 + 0.5 + 0.375 + 0.4 + 0.1$$
$$= 1.875$$



Transmisión de Información

Thursday, September 20, 2018 2:09 PM

Teorema de capacidad de un Canal de comunicaciones

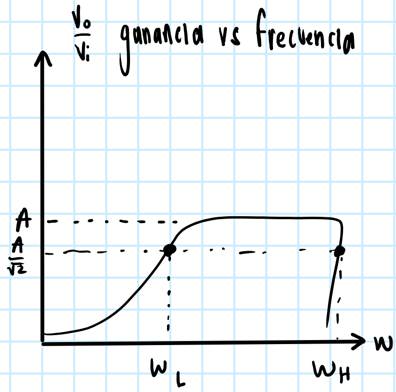
$$\boxed{C = B\log\left(1 + \frac{S}{N}\right)} \left[\frac{bits}{s}\right]$$

B: ancho de banda del canal

S: potencia real de la señal de entrada.

N: potencia del ruido

- distorsión determinística
- distorsión aleatoria (ruido)

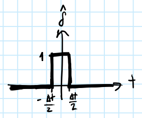


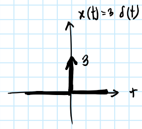
$2\pi B = w_H - w_L$

Muestreo Ideal

Friday, September 21, 2018 4:19 PM

La función impulso $\delta(t)$





Definición:

$$\delta(t) = \begin{cases} \infty & \text{si } t = 0 \\ 0 & \text{si } t \neq 0 \end{cases}$$

$$\text{y} \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

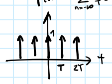
Propiedad de selectividad de la función impulso

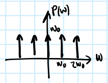
$$\int_a^b f(t)\,\delta(t-t_0)\,dt = \begin{cases} f(t_0) & \text{si } a < t_0 < b \text{ y } f(t) \text{ es continua en } t = t_0 \\ 0 & \text{de otra manera} \end{cases}$$

$x(t) \to \otimes \to x_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$

$$p(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{in\frac{2\pi}{T}t}$$

$$P(w) = w_0 \sum_{n=-\infty}^{\infty} \delta(w - nw_0)$$





Propiedad de multiplicación de la transformada de Fourier.

$$\mathcal{F}\{f_1(t) f_2(t)\} = \frac{1}{2\pi}\left[F_1(w) * F_2(w)\right]$$

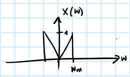
$$\frac{1}{2\pi}\left[X(w) * w_0 \sum_{n=-\infty}^{\infty} \delta(w - nw_0)\right] = \frac{w_0}{2\pi} \int_{-\infty}^{\infty} X(v) \sum_{n=-\infty}^{\infty} \delta(w - v - nw_0)\,dv$$

$$= \frac{w_0}{2\pi} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(w - v - nw_0)\, X(v)\,dv$$

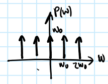
$\delta(w - v - nw_0)$ función par

$$= \frac{w_0}{2\pi} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(v + nw_0 - w)\, X(v)\,dv$$

$$= \frac{w_0}{2\pi} \sum_{n=-\infty}^{\infty} X(w - nw_0)$$



$2\pi f_{máx}$



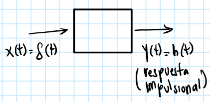
$$w_0 = \frac{2\pi}{T}$$

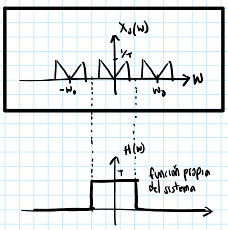
$w_m < w_0 - w_m$

$w_0 > 2w_m \;\|\; * \frac{1}{2\pi}$

$f_0 > 2W_m$

Nyquist





$$C_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-inw_0t}\,dt$$

$$= \frac{1}{T} \int_{-\tau/2}^{\tau/2} e^{-inw_0t}\,dt = -\frac{1}{T}\left(\frac{1}{inw_0}\right) e^{-inw_0t} \Big|_{-\tau/2}^{\tau/2}$$

$$= -\frac{1}{T}\left(\frac{1}{inw_0}\right)\left(e^{-inw_0\tau/2} - e^{inw_0\tau/2}\right)$$

$$= -\frac{1}{inTw_0}\left[-2i\sin\left(\frac{nw_0\tau}{2}\right)\right]$$

$$= \frac{\sin\left(\frac{nw_0\tau}{2}\right)}{n\pi} = \frac{\sin(\pi d n)}{n\pi}, \quad d = \frac{\tau}{T}, \quad w_0 = \frac{2\pi}{T}$$

ciclo de trabajo

$$p(t) = \sum_{n=-\infty}^{\infty} \frac{d\sin(\pi d n)}{\pi d n} e^{inw_0t}$$

$C_0 = \lim_{n\to 0} d\cos(\pi d n) = d$

$C_1 = \frac{\sin \pi d}{\pi}$

$C_2 = \frac{\sin(2\pi d)}{2\pi}$

$\vdots$

$$P(w) = \mathcal{F}\left\{\sum_{n=-\infty}^{\infty} d\frac{\sin(\pi d n)}{\pi d n} e^{inw_0t}\right\}$$

$$= 2\pi \sum_{n=-\infty}^{\infty} \frac{d\sin(\pi d n)}{\pi d n} e^{inw_0t}$$

$$\mathcal{F}\{x(t)p(t)\} = \frac{1}{2\pi}\left[X(w) * P(w)\right]$$

$$= \frac{1}{2\pi}\left[X(w) * 2\pi \sum_{n=-\infty}^{\infty} \frac{d\sin(\pi d n)}{\pi d n} \delta(w - nw_0)\right]$$

$$= \int_{-\infty}^{\infty} X(v) \sum_{n=-\infty}^{\infty} \frac{d\sin(\pi d n)}{\pi d n} \delta(w - nw_0 - v)\,dv$$

$$= \sum_{n=-\infty}^{\infty} \frac{d\sin(\pi d n)}{\pi d n} \int_{-\infty}^{\infty} X(v)\, \delta(w - nw_0 - v)\,dv$$

$\delta(w - nw_0 - v)$ función par

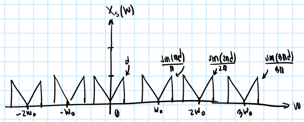
$$= \sum_{n=-\infty}^{\infty} \frac{d\sin(\pi d n)}{\pi d n} \int_{-\infty}^{\infty} X(v)\, \delta(v - w + nw_0)\,dv$$

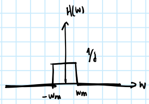
señal periódica

formulita ☺

$$\boxed{C_n = \frac{1}{T} P(w)\Big|_{w = nw_0}}$$

$$= \sum_{n=-\infty}^{\infty} \frac{d\sin(\pi d n)}{\pi d n} X(w - nw_0)$$

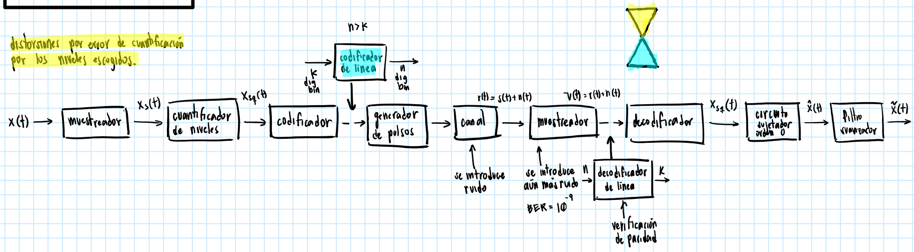




Energía total de la señal

$$E_{total} = \frac{1}{2\pi}\int_{-w_m}^{w_m} |X(w)|^2\,dw, \quad X(w) = \frac{1}{\sqrt{1+w^2}}$$

distorsiones por error de cuantificación por los niveles escogidos.



síndrome de la palabra recibida

$\delta = Hr$

palabra recibida

matriz de chequeo de paridad

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$H' = \left[\begin{array}{ccc:cccc} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$w = Gd$

palabra que se desea enviar

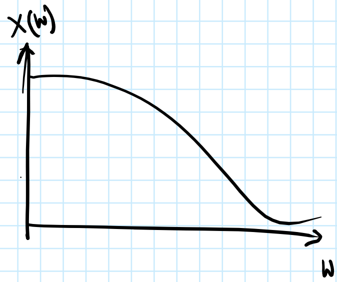
$w \xrightarrow{\text{error}} r$

$w \xrightarrow{\text{sin error}} w$

Hoja de Trabajo 13

Friday, October 12, 2018 5:21 PM

a)



b)

$$\boxed{\begin{array}{l} w = 2(6\pi) = 12\pi \\ w = \frac{2\pi}{T} \quad T = \frac{2\pi}{w} = \frac{1}{6} \end{array}}$$

c)

$x^*(0) = 0$

$x^*(\frac{1}{6}) = 0.1667$

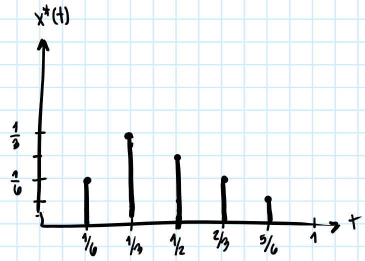
$x^*(\frac{1}{3}) = 0.3333$

$x^*(\frac{1}{2}) = 0.25$

$x^*(\frac{2}{3}) = 0.1667$

$x^*(\frac{5}{6}) = 0.08333$

$x^*(1) = 0$



d)

i	a_i
0	0
1	0.04762
2	0.09524
3	0.1429
4	0.1905
5	0.2381
6	0.2857
7	0.3333

$X_{sq}(0) = 0$

$X_{sq}(1/6) = \frac{3}{21} = 0.1429$

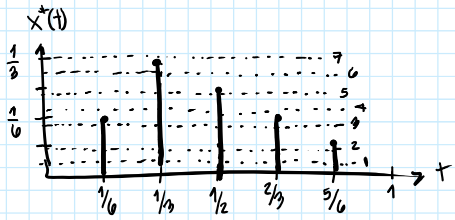
$X_{sq}(1/3) = \frac{7}{21} = 0.3333$

$X_{sq}(1/2) = \frac{5}{21} = 0.2381$

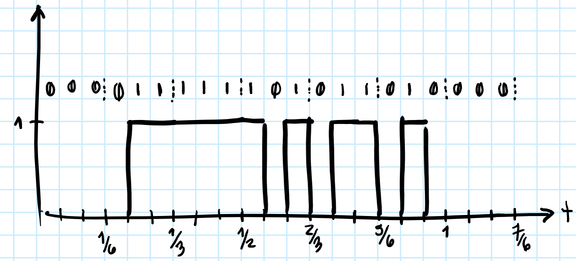
$X_{sq}(2/3) = \frac{3}{21} = 0.1429$

$X_{sq}(5/6) = \frac{2}{21} = 0.09524$

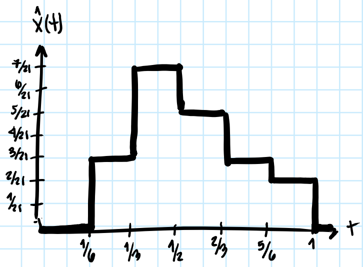
$X_{sq}(1) = 0$



e)



f)



Hoja de Trabajo 14

Monday, November 5, 2018 1:59 AM

1.

a)

p_1	p_2	d_1	p_4	d_2	d_3	d_4
		1		0	1	0
1		1		0		0
	0	1			1	0
			1	0	1	0
1	0	1	1	0	1	0

b)

$$S = Hr = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 4 \end{array}$$

error en posición 6

$$\boxed{1011010}$$

2.

a)

$$S = Hr = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} p_1 \\ p_2 \\ d_1 \\ p_4 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} p_1 + d_1 + d_2 + d_4 \\ p_2 + d_1 + d_3 + d_4 \\ p_4 + d_2 + d_3 + d_4 \end{bmatrix}$$

3.

a)

$$W = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} =$$

$W = Gd$

↑

palabra que se desea enviar

d	w^T
0000	0000000
0001	1101001
0010	0101010
0011	1000011
0100	1001100
0101	0100101
0110	1100110
0111	0001111
1000	1101000
1001	0011001
1010	1011010
1011	0101011
1100	0111100
1101	1010101
1110	0010110
1111	1111111

lunes, 5 de noviembre de 2018 12:50