Fórmulas de producto a suma:

$$\operatorname{sen}\theta\cos\beta = \tfrac{1}{2}\left[\operatorname{sen}(\theta+\beta)+\operatorname{sen}(\theta-\beta)\right]$$

$$\cos\theta\operatorname{sen}\beta = \tfrac{1}{2}\left[\operatorname{sen}(\theta+\beta)-\operatorname{sen}(\theta-\beta)\right]$$

$$\cos\theta\cos\beta = \tfrac{1}{2}\left[\cos(\theta+\beta)+\cos(\theta-\beta)\right]$$

$$\operatorname{sen}\theta\operatorname{sen}\beta = \tfrac{1}{2}\left[\cos(\theta-\beta)-\cos(\theta+\beta)\right]$$

Fórmulas de suma a producto

$$\operatorname{sen}\theta+\operatorname{sen}\beta = 2\operatorname{sen}\frac{\theta+\beta}{2}\cos\frac{\theta-\beta}{2}$$

$$\operatorname{sen}\theta-\operatorname{sen}\beta = 2\cos\frac{\theta+\beta}{2}\operatorname{sen}\frac{\theta-\beta}{2}$$

$$\cos\theta+\cos\beta = 2\cos\frac{\theta+\beta}{2}\cos\frac{\theta-\beta}{2}$$

$$\cos\theta-\cos\beta = -2\operatorname{sen}\frac{\theta+\beta}{2}\operatorname{sen}\frac{\theta-\beta}{2}$$

$$\int\cos(ax)dx = \frac{1}{a}\sin(ax)+C$$

$$\int\sin(ax)dx = -\frac{1}{a}\cos(ax)+C$$

In general,

$$\sin\left(\tfrac{n\pi}{2}\right) = \begin{cases} 0 & \text{if } n=2k \\ 1 & \text{if } n=4k+1 \\ -1 & \text{if } n=4k-1 \end{cases}$$

$$\sin(-t) = -\sin(t)$$

$$\cos(-t) = \cos(t)$$

$$\tan(-t) = -\tan(t)$$

$$\|\alpha f\| = |\alpha|\ \|f\|.$$

$$\|f+g\| \leq \|f\|+\|g\|$$

1. $(z^*)^* = z.$
2. $(z+w)^* = z^*+w^*.$
3. $(z\,w)^* = z^*\,w^*.$
4. $z\,z^* = |z|^2.$

$$cos\theta = \tfrac{1}{2}\left(e^{i\theta}+e^{-i\theta}\right) = \operatorname{Re} e^{i\theta}.$$

$$sin\theta = \tfrac{1}{2i}\left(e^{i\theta}-e^{-i\theta}\right) = \operatorname{Im} e^{i\theta}.$$

Dirichlet

$$\frac{1}{2}\left[\lim_{x\to a^+} f(x) + \lim_{x\to a^-} f(x)\right]$$

$$\alpha_k = \frac{1}{\|f_k\|^2}\,\langle f_k|\,f\rangle$$

$$\hat{f}(x) = a_0 + \sum_{n=1}^{\infty}\{a_n\cos nx + b_n\sin nx\}$$

la serie de Fourier de una función periódica con periodo 2π, entonces para todo $n\in\mathbb{Z}^+$:

- $a_0 = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\,dx.$
- $a_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\cos nx\,dx.$
- $b_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\sin nx\,dx.$

Sea $f(x)$ una función periódica con periodo $T = 2L$ entonces su serie de Fourier respecto a la familia ortogonal $\{1, \cos\omega_n x, \sin\omega_n x\}$, donde $\omega_n = \frac{n\pi}{L}$, está dada por:

$$\hat{f}(x) = a_0 + \sum_{k=1}^{\infty}\{a_n \cos\omega_n x + b_n \sin \omega_n x\},$$

con:

1. $a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx.$
2. $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\omega_n x\,dx.$
3. $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\omega_n x\,dx.$

1. f se llama impar si $f(-x) = -f(x)$ para todo $x \in I$.
2. f se llama par si $f(-x) = f(x)$ para todo $x \in I$.

Sea $f(x)$ una función par periódica con semiperiodo L entonces:

$$\hat{f}(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\omega_n x.$$

Con:

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\omega_n x\,dx.$$

Donde $\omega_n = \frac{\pi n}{L}$ para $n \in \mathbb{Z}^+$.

Si $f(x)$ es una función impar periódica con semiperiodo L entonces:

$$\hat{f}(x) = \sum_{n=1}^{\infty} b_n \sin\omega_n x.$$

Con:

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\omega_n x\,dx.$$

Donde $\omega_n = \frac{\pi n}{L}$ para $n \in \mathbb{Z}^+$.

2. Utilice el teorema de Dirichlet para probar que:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$

Sabemos que la función es continua en $x = \frac{1}{2}$, entonces

$$\frac{1}{2} = f\left(\frac{1}{2}\right) = \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n\pi}\sin\left(\frac{n\pi}{2}\right) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi}{2}\right)$$

$$\frac{1}{2} = \frac{2}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$