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Hoja de trabajo No. 3

Problema 1

Representar a los sistemas utilizando ecuaciones de estado (en forma canónica observable, en forma canónica controlable, en forma diagonal, y en forma canónica de Jordan) y ecuaciones de salida.

1. $G(s) = \frac{6}{s^3+6s^2+11s+6}$

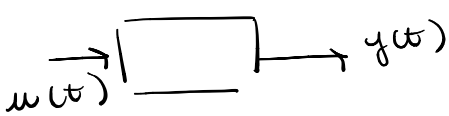
2. $G(s) = \frac{s^2+7s+5}{s^3+9s^2+26s+24}$

3. $\ddot{y} + a\dot{y} + by = u + c\dot{u}$

canonica observable y controlable →

3. $\ddot{y} + a\dot{y} + by = u + c\dot{u}$

$s^2 Y(s) + asY(s) + bY(s) = U(s) = csU(s)$



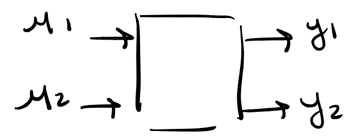
$$\frac{Y(s)}{U(s)} = \frac{cs + 1}{s^2 + as + b}$$ función de transferencia.

Problema 2

Un sistema con dos entradas y dos salidas se describe con las ecuaciones diferenciales $\ddot{y}_1 + 3\dot{y}_1 + 2y_2 = u_1 + 2u_2 + 2\dot{u}_2$ y $\ddot{y}_2 + 4\dot{y}_1 + 3y_2 = \ddot{u}_2 + 3\dot{u}_2 + u_1$. Representar al sistema utilizando ecuaciones de estado en forma canónica observable.

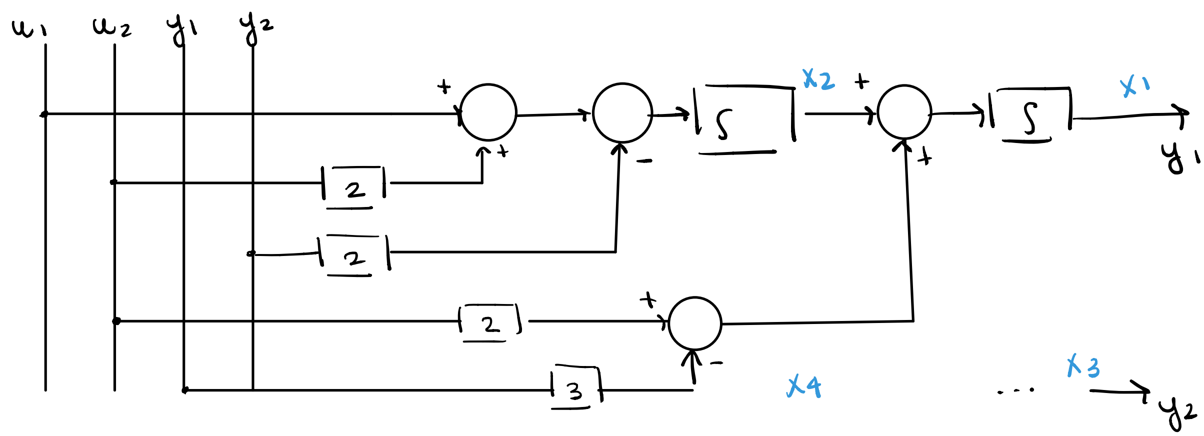
$\ddot{y}_1 = u_1 + 2u_2 + 2\dot{u}_2 - 3\dot{y}_1 - 2y_2$

$= (2\dot{u}_2 - 3\dot{y}_1) + (u_1 + 2u_2 - 2y_2)$



$\dot{y}_1 = (2u_2 - 3y_1) + \int(u_1 + 2u_2 - 2y_2$

$y_1 = \int[2u_2 - 3y_1 + \int(u_1 + 2u_2 - 2y^2)\,d\tau'\,]\,d\tau''$



$\dot{x}_1 = x_2 + 2u_2 - 3\dot{x}_1$

$\dot{x}_2 = u_1 + 2u_2 - 2x_2$

$y_1 = x_1$

$y_2 = x_3.$

$\dot{x}_2 = u_1 + 2u_2 - 2y_2$

$\dot{x}_2 = u_1 \cancel{+2u_2} - 2(x_3 \cancel{+u_2})$

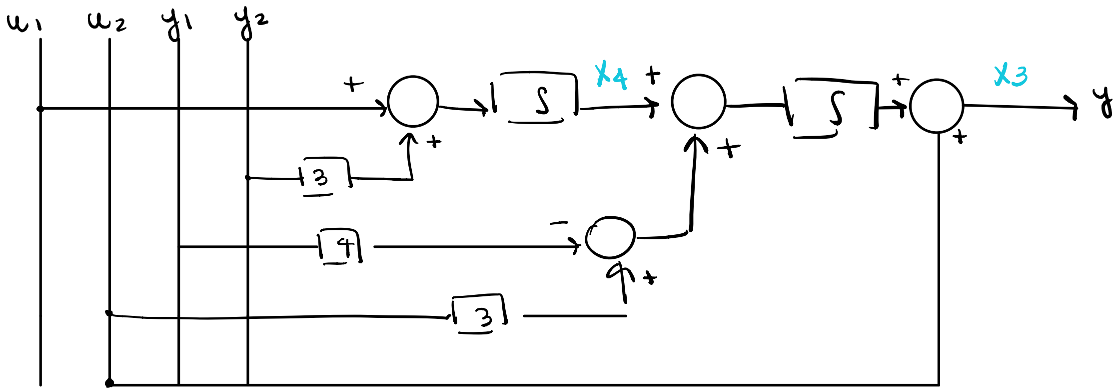
$\dot{x}_2 = u_1 - 2x_3$

$$\ddot{y}_2 + 4\dot{y}_1 + 3y_2 = \ddot{u}_2 + 3\dot{u}_2 + u_1$$

$$\ddot{y}_2 = \ddot{u}_2 + [-4\dot{y}_1 + 3\dot{u}_2] + [u_1 - 3y_2]$$

$$\dot{y}_2 = \dot{u}_2 + [-4y_1 + 3u_2] + \int [u_1 - 3y_2]\,dt$$

$$y_2 = u_2 + \int \Big[-4y_1 + 3u_2 + \int [u_1 - 3y_2]\,dt'\Big]\,dt''$$



$$\dot{x}_3 = x_4 - 4y_1 + 3u_2 = x_4 + 4x_1 + 3u_2 \qquad y_1 = x_1$$

$$\cancel{\dot{x}_4 = v_1 - 3y_2} \qquad y_2 = x_3 + v_2$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ x_4\end{bmatrix} = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} -3 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ -4 & 0 & 0 & 1 \\ 0 & 0 & -3 & 0 \end{bmatrix} \end{array} \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_0\end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & 0 \\ 0 & 3 \\ 1 & -3 \end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}$$

$$\dot{x}_1 = x_2 + 2v_2 - 3\dot{x}_1 \qquad \dot{x}_4 = v_1 - 3x_2$$

$$\dot{x}_2 = v_1 - 2x_3 \qquad = v_1 - 3(x_3 + v_2)$$

$$\dot{x}_3 = x_4 + 4x_1 + 3u_2 \qquad = v_1 - 3x_3 - 3v_2$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}$$

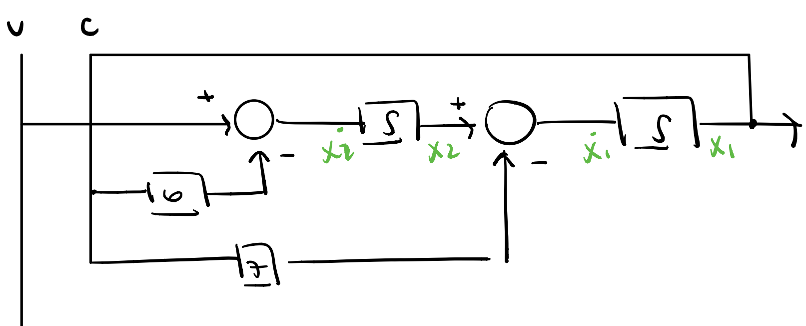
Problema 3

Un sistema se describe con la función de transferencia $G(s) = \frac{s+1}{s^2+7s+6}$. Representar al sistema utilizando ecuaciones de estado en forma canónica controlable. (El factor común $s+1$ no debe cancelarse. Si se cancela el sistema sería erróneamente confundido por un sistema de primer orden.)

$$G(s) = \frac{s+1}{s^2+7s+6} = \frac{C(s)}{U(s)}$$

$$y(s) = (s+1)\,c$$
$$y = cs + c$$
$$y = \dot{c} + c$$

$$(s^2+7s+6)\,C(s) = U$$
$$\ddot{c} + 7\dot{c} + 6c = u$$
$$\ddot{c} = u - 7\dot{c} - 6c$$
$$\dot{c} = -7c + \int (u - 6c)\,dt$$
$$c = \int \left[-7c + \int (u - 6c)\,dt'\right] dt''$$



$$\dot{x}_1 = x_2 - 7x_1$$
$$\dot{x}_2 = u - 6x_1$$
$$y = x_1 + x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -7 & 1 \\ -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]\,u$$

problema extra:

$$G(s) = \frac{s+3}{s^3+9s^2+4s+20} = \frac{Y(s)}{W(s)}$$

- canonica observable:

$$(s^3+9s^2+4s+20)Y(s) = (s+3)W(s)$$

$$\dddot{y}(t)+9\ddot{y}(t)+4\dot{y}(t)+20y(t) = \dot{w}(t)+3w(t)$$

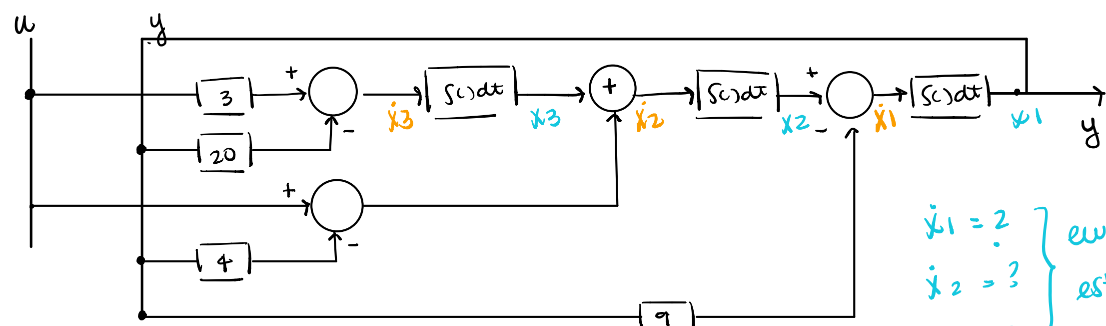
$$\dddot{y}(t) = \dot{w}(t)+3w(t)-9\ddot{y}(t)-4\dot{y}(t)+20y(t)$$

$$= -9\ddot{y}(t) + [\dot{w}(t)-4\dot{y}(t)] + [3w(t)-20y(t)] \quad // \int()$$

$$\ddot{y}(t) = -9\dot{y}(t) + [w(t) - 4y(t))] + \int (3w(t)-20y(t))\,dt \quad // \int()$$

$$\dot{y}(t) = -9y(t) + \int \{ [w-4y] + \int (3w-20y)\,dt' \}\,dt'' \quad // \int()$$

$$y(t) = \int [-9y + \int \{ [w-4y] + \int (3w-20y)\,dt' \}\,dt'']\,dt'''$$



$\dot{x}_1 = ?$
$\dot{x}_2 = ?$
$\dot{x}_3 = ?$
ec. de estado

$y = ?$ ec de salida

$$\dot{x}_1 = x_2 - 9y = x_2 - 9x_1$$

$$\dot{x}_2 = x_3 + w - 4y = x_3 + w - 4x_1$$

$$\dot{x}_3 = 3w - 20x_1$$

$$y = x_1$$

→ usando notación matricial. (FCO: Forma canónica observable)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -9 & 1 & 0 \\ -4 & 0 & 1 \\ -20 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} u$$

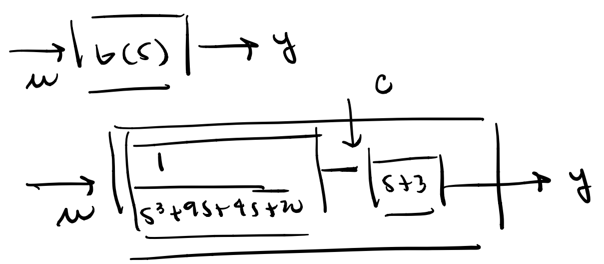
$$\underline{\dot{x}}(t) = A\underline{x}(t) + B\underline{u}(t)$$

$$y(t) = C\underline{x}(t) + Du(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$

$$G(s) = \frac{s+3}{s^3+9s+4s+20}$$

FCC : Forma canónica controlable



$$G(s) = G_1(s)G_2(s)$$

donde $G_1(s) = \dfrac{1}{s^3+9s^2+4s+20}$

$G_2(s) = s+3$

$$\rightarrow \frac{C(s)}{U(s)} = \frac{1}{s^3+9s^2+4s+20}$$

$$(s^3+9s^2+4s+20)C(s) = U(s)$$

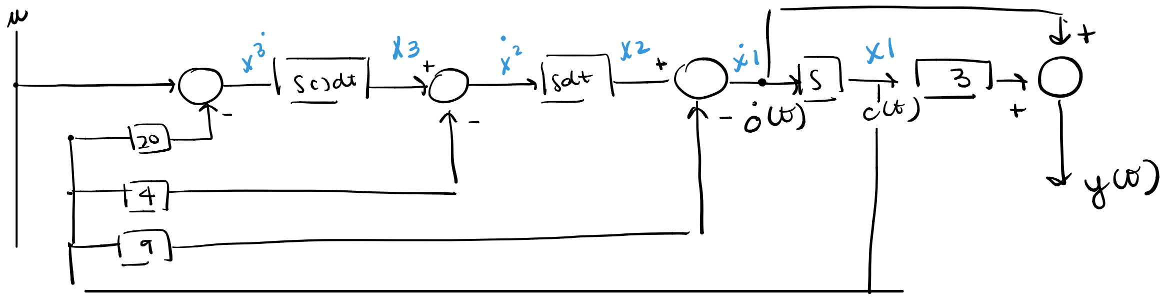
$$\dddot{c}(t) + 9\ddot{c}(t) + 4\dot{c}(t) + 20c(t) = u(t)$$

$$\dddot{c}(t) = -9\ddot{c}(t) - 4\dot{c}(t) + [u(t) - 20c(t)]$$

$$\ddot{c}(t) = -9\dot{c}(t) - 4c(t) + \int (u(t) - 20(t))\,dt$$

$$\dot{c}(t) = -9c(t) + \int \left\{-4c + \int [u - 20]\,dt'\right\} dt''$$

$$c(t) = \int\left(-9c + \int\left\{-4c + \int[u - 20]\,dt'\right\}dt''\right)dt'''$$



$Y(s) = (s+3)\,C(s)$

$y(t) = \dot{c}(t) + 3c(t)$

$\dot{x}_1 = x_2 - 9x_1$

$\dot{x}_2 = x_3 - 4x_1$

$\dot{x}_3 = u - 20x_1$

$y = 3x_1 + \dot{x}_1$

$3x_1 + x_2 - 9x_1$

$= x_2 - 6x_1$

usando not. mat.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -9 & 1 & 0 \\ -4 & 0 & 1 \\ -20 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} -6 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$