Hoja 2

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Problema 1

Considere $m, n \in \mathbb{Z}$ demuestre las siguientes:

1. $\int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = \begin{cases} \frac{L}{2} & m = n \\ 0 & m \neq n \end{cases}$.

$sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$.

identidad

si $m \neq n$:

$$\frac{1}{2}\int_0^L \cos\left(\frac{n\pi x - m\pi x}{L}\right) - \cos\left(\frac{n\pi x + m\pi x}{L}\right) dx$$

$$\frac{1}{2}\int_0^L \cos\left(x\left(\frac{n\pi - m\pi}{L}\right)\right) - \cos\left(x\left(\frac{n\pi + m\pi}{L}\right)\right) dx$$

$$\frac{1}{2}\left(\frac{L}{n\pi - m\pi} \operatorname{sen}\left(x\left(\frac{n\pi - m\pi}{L}\right)\right) - \frac{L}{n\pi + m\pi} \operatorname{sen}\left(x\left(\frac{n\pi + m\pi}{L}\right)\right)\Big|_0^L\right)$$

$$\frac{1}{2}\left(\frac{L}{n\pi - m\pi} \operatorname{sen}(\pi(n-m)) - \frac{L}{n\pi + m\pi} \operatorname{sen}(\pi(n+m))\right)$$

$$\frac{L}{2}\left(\frac{\overset{0}{\cancel{\operatorname{sen}(\pi(n-m))}}}{n\pi - m\pi} - \frac{\overset{0}{\cancel{\operatorname{sen}(\pi(n+m))}}}{n\pi + m\pi}\right) \quad // \quad \operatorname{sen}(\pi \cdot K) = 0$$

$$= 0$$

si $m = n$

$$\frac{1}{2}\int_0^L \overset{1}{\cancel{\cos\left(\frac{m\pi x - m\pi x}{L}\right)}} - \cos\left(\frac{m\pi x + m\pi x}{L}\right) dx$$

$$= \frac{1}{2}\int_0^L 1 - \cos\left(x\left(\frac{2m\pi}{L}\right)\right) dx$$

$$= \frac{1}{2}\left(x - \frac{L}{2m\pi} \operatorname{sen}\left(x\left(\frac{2m\pi}{L}\right)\right)\Big|_0^L\right)$$

$$= \frac{1}{2}\left(L - \frac{L}{2m\pi} \overset{0}{\cancel{\operatorname{sen}(2m\pi)}}\right)$$

$$= \frac{1}{2}\left(L - \frac{L}{2m\pi}\cancel{\operatorname{sen}(2m\pi)}^{0}\right)$$

$$= \frac{L}{2}$$

2. $\int_0^L \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi x}{L}\right)dx = 0$ si m, n son pares.

Identidad: $\cos\alpha\,\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$.

$$\frac{1}{2}\int_0^L \sin\left(\frac{n\pi x + m\pi x}{L}\right) + \sin\left(\frac{n\pi x - m\pi x}{L}\right) dx$$

$$= \frac{1}{2}\int_0^L \sin\left(\frac{n\pi x + m\pi x}{L}\right) + \sin\left(\frac{n\pi x - m\pi x}{L}\right) dx$$

$$= \frac{1}{2}\int_0^L \sin\left(x\left(\frac{n\pi + m\pi}{L}\right)\right) + \sin\left(x\left(\frac{n\pi - m\pi}{L}\right)\right) dx$$

$$= \frac{1}{2}\left(\frac{-L}{n\pi + m\pi}\cos\left(x\left(\frac{n\pi+m\pi}{L}\right)\right) - \frac{L}{n\pi - m\pi}\cos\left(x\left(\frac{n\pi - m\pi}{L}\right)\right)\Big|_0^L\right)$$

$$= \frac{1}{2}\left(\frac{-L}{n\pi + m\pi}\cos(n\pi + m\pi) - \frac{L}{n\pi - m\pi}\cos(n\pi - m\pi) + \frac{L}{n\pi + m\pi} + \frac{L}{n\pi - m\pi}\right)$$

Para que sea par

$$= \frac{1}{2}\left(\frac{-L}{2\pi(n+m)}(-1)^{2(m+n)} - \frac{L}{2\pi(n-m)}(-1)^{2(m-n)} + \frac{L}{2\pi(n+m)} + \frac{L}{2\pi(n-m)}\right)$$

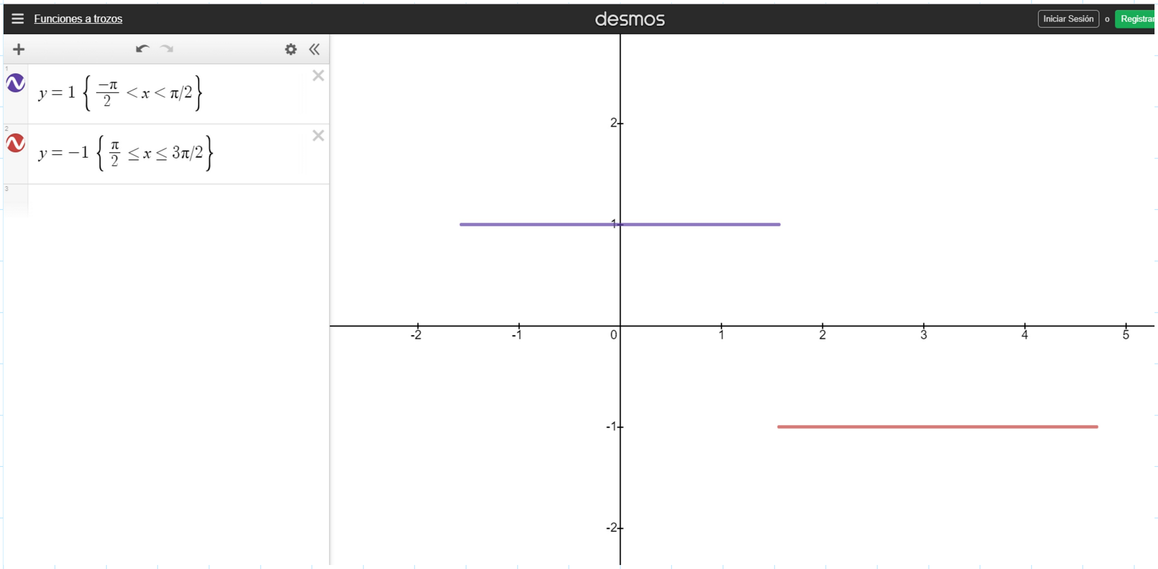
$$= \frac{1}{2}(0)$$

$$= 0$$

Problema 2

Esboce una gráfica de $f(x)$ y calcule su serie de Fourier, dado que:

$$f(x) = \begin{cases} 1 & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ -1 & \frac{\pi}{2} \leq x < \frac{3\pi}{2} \end{cases}, \qquad f(x+2\pi) = f(x).$$



$$a_0 = \frac{1}{2\pi}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \, dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 \, dx\right) = \frac{1}{2\pi}(\pi - \pi)$$

$$a_0 = 0$$

$$a_n = \frac{1}{\pi}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(nx)\, dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos(nx)\, dx\right)$$

$$= \frac{1}{\pi}\Big(\sin(nx)$$