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Tarea Demostración

Demostrar que $e^{At} = Me^{\lambda t}M^{-1} = \begin{bmatrix} \cos t + \operatorname{sen} t & -2\operatorname{sen} t & 0 \\ \operatorname{sen} t & \cos t - \operatorname{sen} t & 0 \\ 0 & 0 & e^{3t} \end{bmatrix}$

$$M^{-1} = \left[\begin{array}{ccc|ccc} 0 & 1+i & 1-i & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} F1 \leftrightarrow F3 \\ F2 \leftrightarrow F3 \\ F3 \leftarrow F3 - \left(\frac{1-i}{2}\right) \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1+i & 1-i & 1 & 0 & 0 \\ 0 & 0 & 1+i & \frac{1-i}{2} & 1 & 0 \end{array}\right]$$

$$\xrightarrow{F_3 \leftarrow \left(\frac{1-i}{2}\right)F_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1+i & 1-i & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{i}{2} & \frac{1-i}{2} & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\frac{i}{2} & \frac{1+i}{2} & 0 \\ 0 & 0 & 1 & \frac{i}{2} & \frac{1-i}{2} & 0 \end{array}\right]$$

$$e^{At} = Me^{\lambda t}M^{-1} =$$

$$\begin{bmatrix} 0 & 1+i & 1-i \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e^{6t} & 0 & 0 \\ 0 & e^{3t}\cos t + ie^{3t}\operatorname{sen} t & 0 \\ 0 & 0 & e^{3t}\cos t - ie^{3t}\operatorname{sen} t \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ -\frac{i}{2} & \frac{1+i}{2} & 0 \\ \frac{i}{2} & \frac{1-i}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & (1+i)\, e^{3t}\cos t + ie^{3t}\operatorname{sen} t & (1-i)\, e^{3t}\cos t - ie^{3t}\operatorname{sen} t \\ 0 & e^{3t}\cos t + ie^{3t}\operatorname{sen} t & e^{3t}\cos t - ie^{3t}\operatorname{sen} t \\ e^{6t} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ -\frac{i}{2} & \frac{1+i}{2} & 0 \\ \frac{i}{2} & \frac{1-i}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} e^{3t}\cos t + e^{3t}\operatorname{sen} t & -2e^{3t}\operatorname{sen} t & 0 \\ e^{3t}\operatorname{sen} t & e^{3t}\cos t - e^{3t}\operatorname{sen} t & 0 \\ 0 & 0 & e^{6t} \end{bmatrix} \; // \; * \; \frac{1}{e^{3t}}$$

$$= \begin{bmatrix} \cos t + \operatorname{sen} t & -2 \operatorname{sen} t & 0 \\ \operatorname{sen} t & \cos t - \operatorname{sen} t & 0 \\ 0 & 0 & e^{3t} \end{bmatrix}$$