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Hoja de trabajo No. 5

1. Calcular los valores característicos y los vectores característicos de las siguientes matrices.

a) $A = \begin{pmatrix} 2 & 2 \\ -1 & 5 \end{pmatrix}$ R: $\lambda_1 = 3,\ \lambda_2 = 4$

b) $A = \begin{pmatrix} 1 & -1 \\ \frac{4}{9} & -\frac{1}{3} \end{pmatrix}$ R: $\lambda_{1,2} = \frac{1}{3}$

c) $A = \begin{pmatrix} -4 & -17 \\ 2 & 2 \end{pmatrix}$ R: $\lambda_{1,2} = -1 \pm j5$

d) $A = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 0 & 1 \\ -1 & -1 & 2 \end{pmatrix}$ R: $\lambda_1 = 2,\ \lambda_{2,3} = 1$

e) $A = \begin{pmatrix} 4 & -2 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ R: $\lambda_{1,2} = 3 \pm j,\ \lambda_3 = 6$

a) $A = \begin{pmatrix} 2 & 2 \\ -1 & 5 \end{pmatrix}$

$$\det(A - \lambda I) = \begin{pmatrix} 2 & 2 \\ -1 & 5 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 2-\lambda & 2 \\ -1 & 5-\lambda \end{pmatrix}$$

$$\det = [(2-\lambda)(5-\lambda)] - (-1 \cdot 2)$$

$$10 - 2\lambda - 5\lambda + \lambda^2 + 2$$

$$\lambda^2 - 7\lambda + 12$$

$$(\lambda - 4)(\lambda - 3)$$

$$\lambda_1 = 4 \quad \lambda_2 = 3$$

b) $A = \begin{pmatrix} 1 & -1 \\ \frac{4}{9} & -\frac{1}{3} \end{pmatrix}$ R: $\lambda_{1,2} = \frac{1}{3}$

$$\det(A - \lambda I) = \begin{pmatrix} 1-\lambda & -1 \\ \frac{4}{9} & -\frac{1}{3}-\lambda \end{pmatrix} = [(1-\lambda)(-1/3 - \lambda)] + 4/9$$

$= \frac{-1}{3} - \lambda + \frac{1}{3}\lambda + \lambda^2 + \frac{4}{9}.$ $\qquad \frac{4-3}{9}$

$= \lambda^2 - \frac{2}{3}\lambda + \frac{1}{9}$

$\Rightarrow \lambda^2 - \frac{2}{3}\lambda + \frac{1}{9} = 0$

$(\lambda - \frac{1}{3})^2 = 0 \ /\!/ \sqrt{\ }$

$\lambda - \frac{1}{3} = 0$

$\lambda = \frac{1}{3}$

c) $A = \begin{pmatrix} -4 & -17 \\ 2 & 2 \end{pmatrix}$ R: $\lambda_{1,2} = -1 \pm j5$

$\det(A - \lambda I) = \begin{pmatrix} -4-\lambda & -17 \\ 2 & 2-\lambda \end{pmatrix} = [(-4-\lambda)(2-\lambda)] + 34$

$= -8 + 4\lambda - 2\lambda + \lambda^2 + 34$

$= \lambda^2 + 2\lambda + 26$

$\Rightarrow \lambda^2 + 2\lambda + 26 = 0$

$\lambda_1 = -1 - 5i$

$\lambda_2 = -1 + 5i$

d) $A = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 0 & 1 \\ -1 & -1 & 2 \end{pmatrix}$ R: $\lambda_1 = 2,\ \lambda_{2,3} = 1$

$\det(A - \lambda I) = \begin{pmatrix} 2-\lambda & 1 & -1 \\ -1 & -\lambda & 1 \\ -1 & -1 & 2-\lambda \end{pmatrix} =$

$(-\lambda+2)(-\lambda)(-\lambda+2) + (1)(1)(-1) + (-1)(-1)(-1) - (-1)(-\lambda)(-1) -$
$(-1)(1)(-\lambda+2) - (-\lambda+2)(1)(1) = -\lambda^3 + 4\lambda^2 - 5\lambda + 2$

$\Rightarrow -\lambda^3 + 4\lambda^2 - 5\lambda + 2 = 0$

$$\lambda_1 = 1$$

$$\lambda_2 = 2$$

$$\lambda_3 = 1$$

2. Definir una matriz modal, M, para cada una de las matrices del problema 1 y calcular $J = M^{-1}AM$. ¿Qué se puede concluir?

* Diagonalizar en symbolab !