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e^{At} e^{-At}

$\Phi(t)$ $\Phi(-t)$

$e^{At} e^{-At} = I$

Hoja de trabajo No. 9

1. $\dot{x} = \begin{pmatrix} 0 & 100 \\ -1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, \quad x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

segunda prop:
$x(-t) = \Phi^{-1}(t)\, x(0)$

a) Calcular la matriz de transición de estado.

b) Calcular $x_T(-5)$, $x_T(0)$, y $x_T(5)$. → por la primera propiedad $x_T(0) = x(0)$

→ solo hago e^{A5}

c) Calcular $x_T(t)$ si $x(10) = \begin{pmatrix} 0.86 \\ 0.05 \end{pmatrix}$

3era prop. $\Phi(t_1)\,\Phi(t_2) = \Phi(t_1 + t_2)$

$t_0 > 0$

$x(0) = \Phi(-t_0)\, x(t_0)$

$x(t) = \Phi(t)\, \Phi(-t_0)\, \underline{x}(t_0)$, $t > t_0$

$x(t) = \Phi(t - t_0)\, \underline{x}(t_0)$

a. $e^{At} = \begin{bmatrix} \cos(10t) & 10\,\text{sen}(10t) \\ -\frac{1}{10}\,\text{sen}(10t) & \cos(10t) \end{bmatrix}$

b. $x_T = e^{At}\, \underline{x}(0)$

$x(t) = \Phi(t)\, \underline{x}(0)$

$\begin{bmatrix} \cos(10t) & 10\,\text{sen}(10t) \\ -\frac{1}{10}\,\text{sen}(10t) & \cos(10t) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$x_T = \begin{bmatrix} \cos(10t) \\ -\frac{1}{10}\,\text{sen}(10t) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad \underline{x_T}(0) = \underline{x}(0)$

$x_T(-5) = \Phi^{-1}\, x(0)$ $\quad \Phi^{-1} = e^{-At}$

$= \begin{bmatrix} 0.96 \\ -0.03 \end{bmatrix}$

* no hay que sacar inversa

$\Phi(-t)\, \underline{x}(0)$

$\begin{bmatrix} \cos(10t) & 10\,\text{sen}(10t) \\ -\frac{1}{10}\,\text{sen}(10t) & \cos(10t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0.96 & 2.62 \\ -0.03 & 0.96 \end{bmatrix}}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ✓

c.) Calcular $\boldsymbol{x}_T(t)$ si $\boldsymbol{x}(10) = \begin{pmatrix} 0.86 \\ 0.05 \end{pmatrix}$

$$x_T(t) = \Phi(t)\,\Phi(-t_0)\,\underline{x}(t_0)$$

$$\Phi(t)\,\underline{\Phi^{-1}(10)}\,\underline{x}(10)$$

solo multiplico. $\rightarrow$ $\Phi(-10)\,\underline{x}(10)$

3. Calcular el componente transitorio de la solución de las ecuaciones de estado

$$\dot{\boldsymbol{x}} = \begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix}\boldsymbol{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}u, \quad \boldsymbol{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\text{R: } \boldsymbol{x}_T = \begin{pmatrix} \sqrt{34}e^{-t}\cos(t + 1.0304) \\ \sqrt{68}e^{-t}\cos(t - 1.3258) \end{pmatrix}$$

$$x(t) = \Phi(t)\,\Phi(-t_0)\,\underline{x}(t_0)$$

$$\lambda_1 = -1 + i$$

$$\lambda_2 = -1 - i$$

$$M = \begin{pmatrix} -1+i & -1-i \\ 2 & 2 \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} -\frac{i}{2} & \frac{1-i}{4} \\ \frac{i}{2} & \frac{1+i}{4} \end{pmatrix}$$

$*\ a^m \cdot a^n = a^{m+n}$

$$e^{\lambda t} = \begin{pmatrix} e^{-t}e^{it} & 0 \\ 0 & e^{-t}e^{-it} \end{pmatrix}$$

$$e^{-t}\left(\cos(t) + i\,\text{sen}(t)\right)$$

$$e^{-t}\left(\cos(t) - i\,\text{sen}(t)\right)$$

$$e^{At} = \begin{pmatrix} -1+i & -1-i \\ 2 & 2 \end{pmatrix}\begin{pmatrix} e^{-t}e^{it} & 0 \\ 0 & e^{-t}e^{-it} \end{pmatrix}\begin{pmatrix} -\frac{i}{2} & \frac{1-i}{4} \\ \frac{i}{2} & \frac{1+i}{4} \end{pmatrix}$$

$$e^{At} = \begin{pmatrix} \dfrac{-i(-1+i)e^{(-1+i)t} + i(-1-i)e^{(-1-i)t}}{2} & \dfrac{ie^{(-1+i)t} - ie^{(-1-i)t}}{2} \\ -ie^{(-1+i)t} + ie^{(-1-i)t} & \dfrac{(1-i)e^{(-1+i)t} + (1+i)e^{(-1+i)t}}{2} \end{pmatrix}$$

$\bar{o}$ = usando sust.

$$e^{At} = \begin{pmatrix} -\sin(t) + e^{-t}\cos(t) & -\sin(t) \\ 2\sin(t) & e^{-t}(\cos(t) + \sin(t)) \end{pmatrix}$$

$$X(t) = \Phi(t)\,\Phi(-t_0)\,\underline{x}(t_0)$$

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, \quad x(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\text{R: } x_T = \begin{pmatrix} \sqrt{34}e^{-t}\cos(t + 1.0304) \\ \sqrt{68}e^{-t}\cos(t - 1.3258) \end{pmatrix}$$

$$X_T = e^{At}\,\underline{X}(0)$$

$$e^{At} =$$

$$M = \begin{pmatrix} -1-i & -1+i \\ 2 & 2 \end{pmatrix} \quad (M^{-1})^{-1}$$

$$e^{\Lambda t} = \begin{pmatrix} e^{-t}(\cos(t) + i\,\text{sen}(t)) & 0 \\ 0 & e^{-t}(\cos(t) - i\,\text{sen}(t)) \end{pmatrix}$$

$$e^{At} = \begin{pmatrix} e^{-t}(\sin(t) + \cos(t)) & e^{-t}\sin(t) \\ -2e^{-t}\sin(t) & e^{-t}(\cos(t) - \sin(t)) \end{pmatrix}$$

$$X_T = \begin{pmatrix} e^{-t}(\sin(t) + \cos(t)) & e^{-t}\sin(t) \\ -2e^{-t}\sin(t) & e^{-t}(\cos(t) - \sin(t)) \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} e^{-t}(3\cos(t) + 5\sin(t)) \\ e^{-t}(2\cos(t) - 8\sin(t)) \end{pmatrix}$$

suma de 2 sinusoides

$$A\cos(wt) + B\sin(wt) = \sqrt{A^2+B^2} \cdot \cos(wt + \tan^{-1}(B/A))$$

entonces

$$x_T = \begin{pmatrix} e^{-t}\sqrt{34} \cdot \cos(t + 1.0304) \\ e^{-t}\sqrt{68} \cdot \cos(t - 1.3258) \end{pmatrix}$$