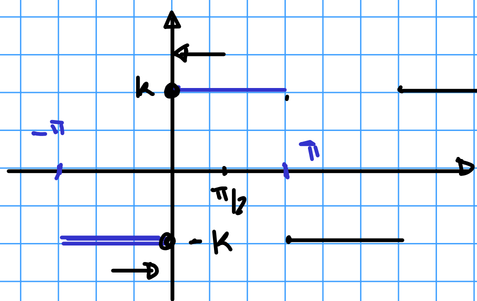
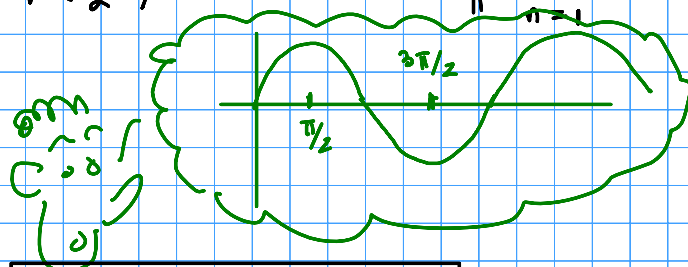
$$f(x) = \begin{cases} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases} \qquad f(x+2\pi) = f(x)$$

$$\hat{f}(x) = \frac{4k}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)x$$

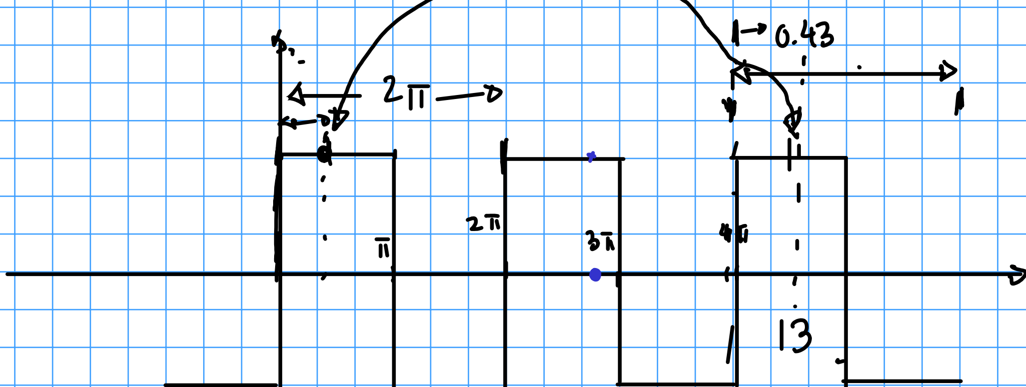


$$\hat{f}\left(\frac{\pi}{2}\right) = k \Rightarrow k = \frac{4k}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\left(\frac{(2n-1)\pi}{2}\right)$$



$$K = \frac{4K}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \Rightarrow \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}}$$

$\hat{f}(0) = 0$ porque $x = 0$ hay discontinuidad $\Rightarrow \hat{f}(0) = \frac{k-k}{2} = 0$.



$f(13)$

$f($

$$g = x^3 - 2x + 1 \qquad f = \cos \pi x$$

$$\int_0^1 (x^3 - 2x + 1)\cos \pi x \, dx$$

$$u = g \Rightarrow du = g' = 3x^2 - 2$$
$$dv = f \Rightarrow v = \int f\,dx = \frac{1}{\pi}\hat{s}\,\pi x$$

$$g^{(0)} f_1 - \int f_1 g^{(1)} dx = (x^3 - 2x + 1)\cdot \frac{1}{\pi}\hat{s}\,\pi x - \int \overbrace{(3x^2 - 2)}^{g^{(1)}} \overbrace{\frac{1}{\pi}\hat{s}\,\pi x}^{f_1} dx$$

$$u = g' \Rightarrow du = g^{(2)} = 6x$$
$$dv = f_1 \Rightarrow v = \int f_1\,dx = f_2 \qquad -\left(g' f_2 - \int g^{(2)} f_2 \, dx\right)$$

$$\int f \cdot g \, dx = g^{(0)} f_1 - g^{(1)} f_2 + g^{(2)} f_3 - \dots$$

$$\int_0^1 (x^3 - 2x + 1)\cos \pi x \, dx$$

g	f
$x^3 - 2x + 1$	$\cos \pi x$
$3x^2 - 2$	$\frac{1}{\pi}\hat{s}\,\pi x$
$6x$	$-\frac{1}{\pi^2}\hat{c}\,\pi x$
6	$-\frac{1}{\pi^3}\hat{s}\,\pi x$
0	$\frac{1}{\pi^4}\hat{c}\,\pi x$

$$(x^3 - 2x + 1)\frac{1}{\pi}\hat{s}\,\pi x - (3x^2 - 2)\frac{-1}{\pi^2}\hat{c}\,\pi x + (6x)\frac{-1}{\pi^3}\hat{s}\,\pi x + 6\frac{1}{\pi^4}\hat{c}\,\pi x \Big|_0^1$$

$$= \frac{-1}{\pi^2} - \frac{6}{\pi^4} - \frac{2}{\pi^2} - \frac{6}{\pi^4} \quad \#$$

Ej: Calcule $\hat{f}(x)$ de $f(x) = x^2 \quad -1 \le x \le 1$ con $f(x+2) = f(x)$.

sol: $L = 1 \qquad \omega_n = \frac{n \cdot \pi}{L} = n\pi.$

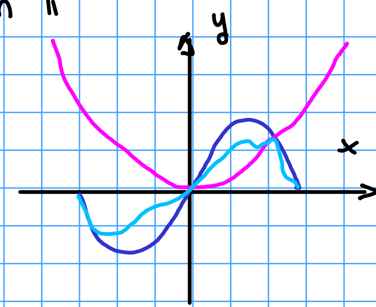
$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2}\cdot\int_{-1}^{1} x^2\,dx = \frac{1}{2}\cdot\frac{1}{3}x^3 \Big|_{-1}^{1} = \frac{1}{6}(1 - (-1)) = \frac{2}{6} = \frac{1}{3}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos \omega_n x \, dx = \int_{-1}^{1} x^2 \cdot \cos(n\pi x)\,dx =$$

$$\begin{array}{ll} x^2 & \cos(n\pi x) \\ 2x & \frac{1}{n\pi}\sin(n\pi x) \\ 2 & \frac{-1}{n^2\pi^2}\hat{c}(n\pi x) \\ 0 & \frac{-1}{n^3\pi^3}\hat{s}(n\pi x) \end{array}$$

$$= \frac{x^2}{n\pi}\hat{s}\, n\pi x + \frac{2x}{n^2\pi^2}\hat{c}\, n\pi x - \frac{2}{n^3\pi^3}\hat{s}(n\pi x)\Big|_{-1}^{1}$$

$$= \frac{2(-1)^n}{n^2\pi^2} - \frac{-2(-1)^n}{n^2\pi^2} = \frac{4(-1)^n}{n^2\pi^2}.$$



$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\omega_n x\,dx = \int_{-1}^{1} \underbrace{x^2 \sin n\pi x}_{\text{impar}}\,dx$$

$$\int_{-1}^{0} x^2 \sin n\pi x\,dx + \int_{0}^{1} x^2 \sin n\pi x\,dx = 0 \text{. por simetria.}$$

$$\boxed{\hat{f}(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2\pi^2}\cos(n\pi x).}$$