SOLUCION HOJA DE TRABAJO No. 7

1)

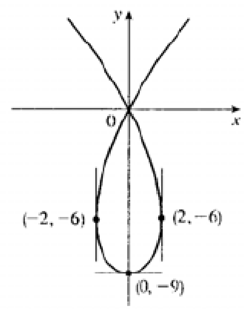
5. $x = t^2 + t,\ y = t^2 - t;\ t = 0.$ $\dfrac{dy}{dt} = 2t - 1,$ $\dfrac{dx}{dt} = 2t + 1,$ so $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t-1}{2t+1}.$ When $t = 0$, $x = y = 0$ and $\dfrac{dy}{dx} = -1$. An equation of the tangent is $y - 0 = (-1)(x - 0)$ or $y = -x$.

8. $x = t\sin t,\ y = t\cos t;\ t = \pi.$ $\dfrac{dy}{dt} = \cos t - t\sin t,$ $\dfrac{dx}{dt} = \sin t + t\cos t,$ and $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\cos t - t\sin t}{\sin t + t\cos t}.$ When $t = \pi$, $(x, y) = (0, -\pi)$ and $\dfrac{dy}{dx} = \dfrac{-1}{-\pi} = \dfrac{1}{\pi}$, so an equation of the tangent is $y + \pi = \frac{1}{\pi}(x - 0)$ or $y = \frac{1}{\pi}x - \pi$.

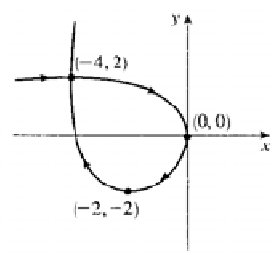
2)

19. $x = t(t^2 - 3) = t^3 - 3t,\ y = 3(t^2 - 3).$ $\dfrac{dx}{dt} = 3t^2 - 3 = 3(t-1)(t+1);$ $\dfrac{dy}{dt} = 6t.$ $\dfrac{dy}{dt} = 0 \Leftrightarrow t = 0 \Leftrightarrow (x, y) = (0, -9).$ $\dfrac{dx}{dt} = 0 \Leftrightarrow t = \pm 1 \Leftrightarrow (x, y) = (-2, -6)$ or $(2, -6)$. So there is a horizontal tangent at $(0, -9)$ and there are vertical tangents at $(-2, -6)$ and $(2, -6)$.

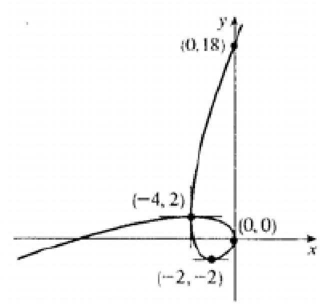
	$t < -1$	$-1 < t < 0$	$0 < t < 1$	$t > 1$
dx/dt	$+$	$-$	$-$	$+$
dy/dt	$-$	$-$	$+$	$+$
x	$\rightarrow$	$\leftarrow$	$\leftarrow$	$\rightarrow$
y	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$
curve	$\searrow$	$\swarrow$	$\nwarrow$	$\nearrow$



20. $x = t^3 - 3t^2,\ y = t^3 - 3t.$ $\dfrac{dx}{dt} = 3t^2 - 6t = 3t(t-2),$ $\dfrac{dy}{dt} = 3t^2 - 3 = 3(t-1)(t+1).$ $\dfrac{dy}{dt} = 0 \Leftrightarrow t = +1$ or $-1 \Leftrightarrow (x, y) = (-2, -2)$ or $(-4, 2)$. $\dfrac{dx}{dt} = 0 \Leftrightarrow t = 0$ or $2 \Leftrightarrow (x, y) = (0, 0)$ or $(-4, 2)$. So the tangent is horizontal at $(-2, -2)$ and vertical at $(0, 0)$. At $(-4, 2)$ the curve crosses itself and there are two tangents, one horizontal and one vertical.



	$t < -1$	$-1 < t < 0$	$0 < t < 1$	$1 < t < 2$	$t > 2$
dx/dt	$+$	$+$	$-$	$-$	$+$
dy/dt	$+$	$-$	$-$	$+$	$+$
x	$\rightarrow$	$\rightarrow$	$\leftarrow$	$\leftarrow$	$\rightarrow$
y	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$
curve	$\nearrow$	$\searrow$	$\swarrow$	$\nwarrow$	$\nearrow$



2)

36. By symmetry, $A = 4\int_0^a y\,dx = 4\int_{\pi/2}^0 a\sin^3\theta\,(-3a\cos^2\theta\sin\theta)\,d\theta = 12a^2\int_0^{\pi/2}\sin^4\theta\cos^2\theta\,d\theta$. Now

$$\begin{aligned}\int\sin^4\theta\cos^2\theta\,d\theta &= \int\sin^2\theta\left(\tfrac{1}{4}\sin^2 2\theta\right)d\theta = \tfrac{1}{8}\int(1-\cos 2\theta)\sin^2 2\theta\,d\theta\\ &= \tfrac{1}{8}\int\left[\tfrac{1}{2}(1-\cos 4\theta)-\sin^2 2\theta\cos 2\theta\right]d\theta = \tfrac{1}{16}\theta - \tfrac{1}{64}\sin 4\theta - \tfrac{1}{48}\sin^3 2\theta + C\end{aligned}$$

so $\int_0^{\pi/2}\sin^4\theta\cos^2\theta\,d\theta = \left[\frac{1}{16}\theta - \frac{1}{64}\sin 4\theta - \frac{1}{48}\sin^3 2\theta\right]_0^{\pi/2} = \frac{\pi}{32}$. Thus, $A = 12a^2\left(\frac{\pi}{32}\right) = \frac{3}{8}\pi a^2$.

3)

8. $x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \le t \le 3$. $dx/dt = e^t - e^{-t}$ and $dy/dt = -2$, so $(dx/dt)^2 + (dy/dt)^2 = e^{2t} - 2 + e^{-2t} + 4 = e^{2t} + 2 + e^{-2t} = (e^t + e^{-t})^2$ and $L = \int_0^3 (e^t + e^{-t})\,dt = \left[e^t - e^{-t}\right]_0^3 = e^3 - e^{-3} - (1-1) = e^3 - e^{-3}$.

4)

30. $x = e^t - t$, $y = 4e^{t/2}$, $0 \le t \le 1$. $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (e^t - 1)^2 + (2e^{t/2})^2 = e^{2t} + 2e^t + 1 = (e^t + 1)^2$.

$$\begin{aligned}S &= \int_0^1 2\pi\,(e^t - t)\sqrt{(e^t - 1)^2 + (2e^{t/2})^2}\,dt = \int_0^1 2\pi\,(e^t - t)(e^t + 1)\,dt\\ &= 2\pi\left[\tfrac{1}{2}e^{2t} + e^t - (t-1)e^t - \tfrac{1}{2}t^2\right]_0^1 = \pi\left(e^2 + 2e - 6\right)\end{aligned}$$

5)

32. $t+1/t=2.5 \Leftrightarrow t=\frac{1}{2}$ or 2, and for $\frac{1}{2}<t<2$, we have $t+1/t<2.5$. $x=-\frac{3}{2}$ when $t=\frac{1}{2}$ and $x=\frac{3}{2}$ when $t=2$. \

$$A = \int_{-3/2}^{3/2}(2.5-y)\,dx = \int_{1/2}^{2}\left(\frac{5}{2}-t-1/t\right)(1+1/t^2)\,dt\ [x=t-1/t, dx=(1+1/t^2)dt]$$

$$= \int_{1/2}^{2}\left(-t+\frac{5}{2}-2t^{-1}+\frac{5}{2}t^{-2}-t^{-3}\right)dt = \left[\frac{-t^2}{2}+\frac{5t}{2}-2\ln|t|-\frac{5}{2t}+\frac{1}{2t^2}\right]_{1/2}^{2}$$

$$= \left(-2+5-2\ln 2-\frac{5}{4}+\frac{1}{8}\right)-\left(-\frac{1}{8}+\frac{5}{4}+2\ln 2-5+2\right) = \frac{15}{4}-4\ln 2$$