e.v. V (vectores) $\mathbb{C}/\mathbb{R}$ (escalares) $\leftarrow$ Campo

$|x|$ (valor absoluto)

$|z|$ (módulo) $\mathbb{R} \subseteq \mathbb{C}$

$$\langle \cdot | \cdot \rangle : V \times V \longrightarrow \mathbb{C}/\mathbb{R} \qquad \alpha, \beta \in \mathbb{C}; \quad f, g, h \in V$$

Lineal conjugada por izq. $\langle \alpha f + \beta g | h \rangle = \alpha^* \langle f | h \rangle + \beta^* \langle g | h \rangle$

Simetria Conjugada Hermite $\langle f | g \rangle = \langle g | f \rangle^*$

$z = a + ib \Rightarrow z^* = a - ib$. Conjugado.

Positivamente definida: $\langle f | f \rangle \geq 0$ vector nulo.

NO degenerada: $\langle f | f \rangle = 0$ ssi $f = \bar{0}$.

Ejemplo: $\mathcal{E}_n$ $\mathbb{C}^n = \{ (z_1, \ldots, z_n) \,|\, z_i \in \mathbb{C} \}$

$$\langle (z_1, \ldots, z_n) | (w_1, \ldots, w_n) \rangle = \sum_{j=1}^{n} z_j^* \cdot w_j$$

$$\langle (1+i, 2+3i, 2i) | (1, 3, 1+i) \rangle =$$

$$(1+i)^* \cdot 1 + (2+3i)^* \cdot 3 + 2i^* \cdot (1+i) = 9 - 12i \quad \#$$

Teorema: Sean $f_1, \dots, f_n, g \in V$. $\alpha_1, \dots, \alpha_n \in \mathbb{C}$

$$\left\langle g \middle| \sum_{k=1}^{n} \alpha_k f_k \right\rangle = \sum_{k=1}^{n} \alpha_k \langle g | f_k \rangle$$

dem:

Por inducción en n.

Base $n=1$: $\left\langle g \middle| \sum_{k=1}^{1} \alpha_k f_k \right\rangle = \langle g | \alpha_1 f_1 \rangle = \langle \alpha_1 f_1 | g \rangle^*$ // sim conj.

$= \langle \alpha_1 f_1 + 0 \cdot \bar{0} | g \rangle^*$ // neutro $= [\alpha_1^* \langle f_1 | g \rangle + 0^* \langle \bar{0} | g \rangle]^*$

// lineal conj. izq. $= [\alpha_1^* \langle f_1 | g \rangle]^* = \alpha_1^{**} \langle f_1 | g \rangle^*$ / sim. conj.

$\alpha_1 \langle g | f_1 \rangle$.

Inducción: Sup. $\left\langle g \middle| \sum_{k=1}^{m} \alpha_k f_k \right\rangle = \sum_{k=1}^{m} \alpha_k \langle g | f_k \rangle$ H.I.

A probar: $\left\langle g \middle| \sum_{k=1}^{m+1} \alpha_k f_k \right\rangle = \sum_{k=1}^{m+1} \alpha_k \langle g | f_k \rangle$

$$\left\langle g \middle| \sum_{k=1}^{m+1} \alpha_k f_k \right\rangle = \left\langle g \middle| \sum_{k=1}^{m} \alpha_k f_k + \alpha_{m+1} f_{m+1} \right\rangle \text{ // sim. conj.}$$

$$= \left\langle 1 \cdot \sum_{k=1}^{m} \alpha_k f_k + \alpha_{m+1} f_{m+1} \middle| g \right\rangle^* = \left[1^* \left\langle \sum_{k=1}^{m} \alpha_k f_k \middle| g \right\rangle + \alpha_{m+1}^* \langle f_{m+1} | g \rangle \right]^*$$

$$= \left\langle \sum_{k=1}^{m} \alpha_k f_k \middle| g \right\rangle^* + \alpha_{m+1} \langle f_{m+1} | g \rangle^* =$$

$$\left\langle g \middle| \sum_{k=1}^{m} \alpha_k f_k \right\rangle + \alpha_{m+1} \langle g | f_{m+1} \rangle = \sum_{k=1}^{m} \alpha_k \langle g | f_k \rangle + \alpha_{m+1} \langle g | f_{m+1} \rangle$$

$$= \sum_{k=1}^{m+1} \alpha_k \langle g | f_k \rangle \quad \square$$