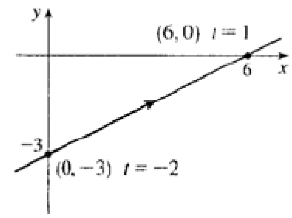
HOJA DE TRABAJO No. 6 - SOLUCION
Curvas Definidas por Ecuaciones Paramétricas

1)

a.

1. (a) $x = 2t + 4$, $y = t - 1$

t	-3	-2	-1	0	1	2
x	-2	0	2	4	6	8
y	-4	-3	-2	-1	0	1

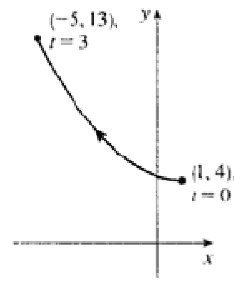


(b) $x = 2t + 4$, $y = t - 1 \quad \Rightarrow \quad x = 2(y + 1) + 4 = 2y + 6$ or
$y = \frac{1}{2}x - 3$

b.

3. (a) $x = 1 - 2t$, $y = t^2 + 4$, $0 \le t \le 3$

t	0	1	2	3
x	1	-1	-3	-5
y	4	5	8	13



(b) $x = 1 - 2t \quad \Rightarrow \quad 2t = 1 - x \quad \Rightarrow \quad t = \dfrac{1 - x}{2} \quad \Rightarrow$

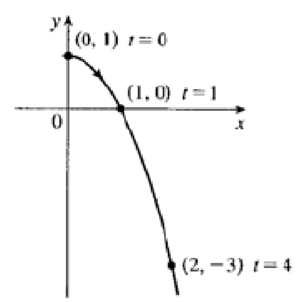
$y = t^2 + 4 = \left(\dfrac{1 - x}{2}\right)^2 + 4 = \frac{1}{4}(x - 1)^2 + 4$ or

$y = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{17}{4}$

c.

(a) $x = \sqrt{t}$, $y = 1 - t$

t	0	1	2	3	4
x	0	1	1.414	1.732	2
y	1	0	-1	-2	-3

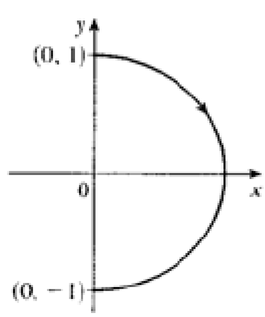


(b) $x = \sqrt{t} \quad \Rightarrow \quad t = x^2$. $y = 1 - t = 1 - x^2$. Since $t \ge 0, x \ge 0$.

2.a.

7. (a) $x = \sin\theta$, $y = \cos\theta$, $0 \le \theta \le \pi$.
$x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1, 0 \le x \le 1.$

(b)

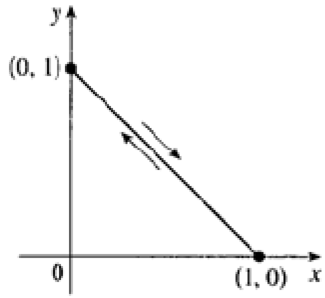


b.

9. (a) $x = \sin^2\theta$, $y = \cos^2\theta$.

$x + y = \sin^2\theta + \cos^2\theta = 1, 0 \le x \le 1.$
Note that the curve is at (0, 1) whenever $\theta = \pi n$ and is at (1, 0) whenever $\theta = \frac{\pi}{2}n$ for every integer n.

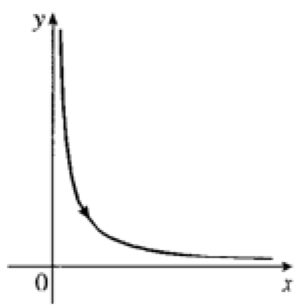
(b)



c.

11. (a) $x = e^t$, $y = e^{-t}$, $y = 1/x$, $x > 0$

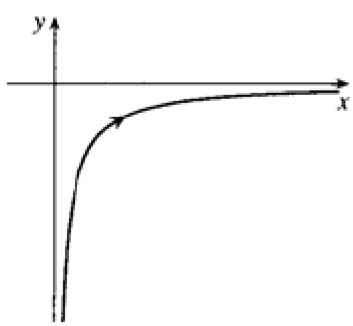
(b)



d.

13. (a) $x = \tan\theta + \sec\theta$, $y = \tan\theta - \sec\theta$,
$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. $xy = \tan^2\theta - \sec^2\theta = -1$
$\Rightarrow \quad y = -1/x, x > 0.$

(b)



3.

a.

. $x = t^4 - 1, y = t - t^2 \Rightarrow \frac{dy}{dt} = 1 - 2t, \frac{dx}{dt} = 4t^3, \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-2t}{4t^3} = \frac{1}{4}t^{-3} - \frac{1}{2}t^{-2};$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = -\frac{3}{4}t^{-4} + t^{-3}, \frac{d^2y}{dx^2} = \frac{d\,(dy/dx)\,/dt}{dx/dt} = \frac{-\frac{3}{4}t^{-4} + t^{-3}}{4t^3} \cdot \frac{4t^4}{4t^4} = \frac{-3+4t}{16t^7}.$$

b.

15. $x = \sin \pi t,\ y = \cos \pi t.\ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\pi \sin \pi t}{\pi \cos \pi t} = -\tan \pi t;$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d\,(dy/dx)\,/dt}{dx/dt} = \frac{-\pi \sec^2 \pi t}{\pi \cos \pi t} = -\sec^3 \pi t.$$

c.

16. $x = 1 + \tan t,\ y = \cos 2t \Rightarrow \frac{dy}{dt} = -2\sin 2t,\ \frac{dx}{dt} = \sec^2 t,$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin 2t}{\sec^2 t} = -4\sin t \cos t \cdot \cos^2 t = -4 \sin t \cos^3 t;$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = -4\sin t\,(3\cos^2 t)\,(-\sin t) - 4\cos^4 t = 12\sin^2 t \cos^2 t - 4\cos^4 t,$$

$$\frac{d^2y}{dx^2} = \frac{d\,(dy/dx)\,/dt}{dx/dt} = \frac{4\cos^2 t\,(3\sin^2 t - \cos^2 t)}{\sec^2 t} = 4\cos^4 t\,(3\sin^2 t - \cos^2 t).$$

d.

18. $x = 1 + t^2,\ y = t \ln t.\ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 + \ln t}{2t};\ \frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{2t\,(1/t) - (1 + \ln t)\,2}{(2t)^2} = -\frac{\ln t}{2t^2};$

$$\frac{d^2y}{dx^2} = \frac{d\,(dy/dx)\,/dt}{dx/dt} = -\frac{\ln t}{4t^3}.$$