

Funciones de transferencia

1. Un sistema se describe con la ecuación diferencial

* problema 11 cap. 2

$$\frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 3x = 1; x(0) = 1 \text{ y } \frac{dx(0)}{dt} = -1$$

Describa al sistema utilizando una función de transferencia (escriba la función de transferencia, la señal de entrada y la señal de salida).

Si asumimos $x(0) = 0$ y $\frac{dx}{dt}(0) = 0$

condiciones iniciales = 0

$$s^2 x(s) + 2s x(s) + 3x(s) = Y(s)$$

$$(s^2 + 2s + 3)x(s) = Y(s)$$

$$H(s) = \frac{x(s)}{Y(s)} = \frac{1}{s^2 + 2s + 3}$$

$$y(t) \rightarrow \left| \frac{1}{s^2 + 2s + 3} \right| \rightarrow x(t)$$

$$\begin{aligned} & y(t) \rightarrow \boxed{x(t)} \\ & \frac{dx}{dt} + 2x = y(t); x(0) = x_0 \\ & y(t) = H(t) \rightarrow \boxed{x(t)} \\ & x(t) = \dots, t > 0 \\ & \frac{d^2x}{dt^2} x + 2 \frac{dx}{dt} x + 3x = H(t) \end{aligned}$$

describimos el sistema por medio de una función de transferencia.

Ahora lo usaremos cuando las condiciones iniciales $\neq 0$.

entonces:

$$s^2 x(s) + 2s x(s) + 3x(s) = Y(s)$$

$$(s^2 x(s) - s + 1) + 2[s x(s) - 1] + 3x(s) = Y(s)$$

$$(s^2 + 2s + 3)x(s) = Y(s) + s + 1$$

$$x(s) = \frac{Y(s)}{s^2 + 2s + 3} + \frac{s+1}{s^2 + 2s + 3}$$

$$\begin{aligned} & \text{(calcular } x(s) \text{ para } y(t) = H(t)) \\ & X(s) = \left[\frac{1}{s^2 + 2s + 3} \right] \frac{1}{s} = K(s) - U(s) \approx Y(s) \\ & y(t) = \mathcal{L}^{-1}[X(s)] \quad \text{condiciones iniciales iguales a 0} \end{aligned}$$

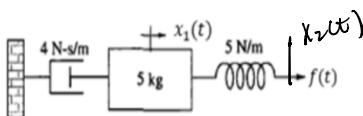
$$\begin{aligned} \frac{df}{dt} & \rightarrow sF(s) - f(0) \\ \frac{d^2f}{dt^2} & \rightarrow s^2 F(s) - sf(0) - f'(0) \end{aligned}$$

$$(s^2 x(s) - s + 1)$$

$$\begin{aligned} & s^2 x(s) - s x(0) - x'(0) \\ & \text{condiciones iniciales } x(0) = 1 \\ & s^2 x(s) - s - (-1) \quad \text{ver def. prob.} \end{aligned}$$

$$Y(s) \rightarrow + \rightarrow \boxed{\frac{1}{s^2 + 2s + 3}} \rightarrow x(s)$$

2. Encuentre la función de transferencia $G(s) = X_1(s)/F(s)$.



$s^2 \cdot$ inercia

$s \cdot$ viscoso

$K \cdot$ resorte

$$x_2(s) \leftarrow \begin{bmatrix} M_1 \\ \end{bmatrix} \rightarrow x_1(s) \\ \rightarrow s^2 x_1(s) \\ \rightarrow s x_1(s)$$

$$x_1(s) \leftarrow \begin{bmatrix} M_2 \\ \end{bmatrix} \rightarrow F(s) \\ \rightarrow x_2(s)$$

$$1) \quad x_1(s) + s^2 x_1(s) + s x_1(s) = x_2(s)$$

$$(s^2 + s + 1)x_1(s) - x_2(s) = 0 \quad \therefore (s^2 + s + 1)x_1(s) = x_2(s)$$

$$2) \quad F(s) + x_2(s) = x_1(s)$$

$$-x_1(s) + x_2(s) = F(s)$$

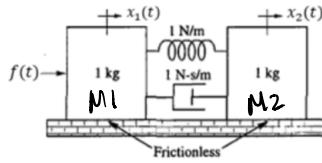
$$\Rightarrow -x_1(s) + (s^2 + s + 1)x_1(s) = F(s)$$

$$(s^2 + s + 1)x_1(s) = F(s)$$

$$(s^2 + s) x_1(s) = F(s)$$

$$\frac{x_1(s)}{F(s)} = \frac{1}{s^2 + s} \quad \square$$

3. Encuentre la función de transferencia $G(s) = X_2(s)/F(s)$.



$$\begin{array}{c} x_1(s) \\ s x_1(s) \\ s^2 x_1(s) \end{array} \left[\begin{array}{c} M_1 \\ \downarrow \\ \end{array} \right] \begin{array}{l} \xrightarrow{\quad} F(s) \\ \xrightarrow{\quad} s x_2(s) \\ \xrightarrow{\quad} x_2(s) \end{array}$$

$$\begin{array}{c} x_2(s) \\ s x_2(s) \\ s^2 x_2(s) \end{array} \left[\begin{array}{c} M_2 \\ \downarrow \\ \end{array} \right] \begin{array}{l} \xrightarrow{\quad} s x_1(s) \\ \xrightarrow{\quad} x_1(s) \\ \xrightarrow{\quad} \end{array}$$

$$1) (s^2 + s + 1) x_1(s) - (s+1) x_2(s) = F(s)$$

$$2) - (s+1) x_1(s) + (s^2 + s + 1) x_2(s) = 0$$

⇒ resolviendo:

$$1) A x_1 - B x_2 = F$$

$$2) -C x_1 + D x_2 = 0$$

$$3) -C x_1 + D x_2 = 0$$

$$C x_1 = D x_2$$

$$x_1 = \frac{D}{C} x_2$$

⇒ sustituyendo:

$$A x_1 - B x_2 = F$$

$$A \left(\frac{D}{C} x_2 \right) - B x_2 = F$$

$$\left(\frac{AD}{C} - B \right) x_2 = F$$

$$\left(\frac{AD - BC}{C} \right) x_2 = F$$

$$\frac{x_2}{F} = \frac{C}{AD - BC}$$

$$(s^2 + s + 1) x_1(s) - (s+1) x_2(s) = F(s)$$

$$- (s+1) x_1(s) + (s^2 + s + 1) x_2(s) = 0$$

⇒ reemplazando valores

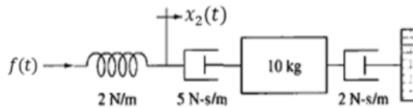
$$\frac{x_2(s)}{F(s)} = \frac{s+1}{(s^2+s+1)(s^2+s+1) - (s+1)(s+1)}$$

$$\frac{x_2(s)}{F(s)} = \frac{s+1}{s^4 + s^3 + s^2 + s^3 + s^2 + s + s + s^2 + s + 1 - s^2 - 2s - 1}$$

$$\frac{x_2(s)}{F(s)} = \frac{s+1}{s^4 + 2s^3 + 3s^2 + 2s + 1 - s^2 - 2s - 1}$$

$$\frac{x_2(s)}{F(s)} = \frac{s+1}{s^4 + 2s^3 + 2s^2} \quad D$$

4. Encuentre la función de transferencia $G(s) = X_2(s)/F(s)$.



25. Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical system shown in Figure P2.11. (Hint: place a zero mass at $x_2(t)$.) [Section: 2.5]

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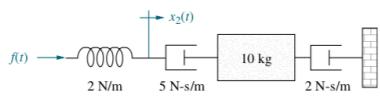


FIGURE P2.11

25.

Let $X_1(s)$ be the displacement of the left member of the spring and $X_3(s)$ be the displacement of the mass.

Writing the equations of motion

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$$\begin{aligned} 2X_1(s) - 2X_2(s) &= F(s) \\ -2X_1(s) + (5s+2)X_2(s) - 5sX_3(s) &= 0 \\ -5sX_2(s) + (10s^2 + 7s)X_3(s) &= 0 \end{aligned}$$

Solving for $X_2(s)$,

$$X_2(s) = \frac{\begin{vmatrix} 5s^2+10 & F(s) \\ -10 & 0 \end{vmatrix}}{\begin{vmatrix} 5s^2+10 & -10 \\ -10 & \frac{1}{5}s+10 \end{vmatrix}} = \frac{10F(s)}{s(s^2+50s+2)}$$

$$\text{Thus, } \frac{X_2(s)}{F(s)} = \frac{1}{10} \frac{(10s+7)}{s(5s+1)}$$

¿Qué hacer? A partir de las ecuaciones de movimiento para el ing. X_3 es \uparrow aquí X_1

$$\begin{aligned} (10s^2 + 7s)X_1(s) - 5sX_2(s) &= 0 \\ -5X_1(s) + (5s+2)X_2(s) - 2X_3(s) &= 0 \\ -2X_2(s) + 2X_3(s) &= F(s) \end{aligned}$$

entonces

$$\begin{bmatrix} 10s^2+7s & -5s & 0 \\ -5 & 5s+2 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F(s) \end{bmatrix}$$

Ahora revisemos que el determinante de la matriz no sea = '0'

paso 1: calcular $\det(A)$ por la fórmula.

seleccionamos primera columna.

$$\det(A) = (-1)^{\begin{matrix} 3+1 \leftarrow \text{columna} \\ \text{fila} \end{matrix}} (0) \det \begin{bmatrix} -5s & 0 \\ ss+2 & -2 \end{bmatrix} = 0$$

$$(-1)^{2+1} (-5s) \det \begin{bmatrix} -5s & 0 \\ -2 & 2 \end{bmatrix} = ss (-10s) = -50s^2$$

$$(-1)^{3+1} (10s^2 + 7s) \det \begin{bmatrix} ss+2 & -2 \\ -2 & 2 \end{bmatrix} = (10s^2 + 7s)(10s + 7 - *) \\ (10s^2 + 7s)(10s) \\ = 100s^3 + 70s^2$$

$$\det(A) = 0 + (-50s^2) + (100s^3 + 70s^2)$$

$$= 100s^3 + 20s^2$$

$$= 20s^2(5s + 1)$$

↓ reemplazo por el resultado.

$$\text{paso 2: } x_2(s) = \frac{\det \begin{bmatrix} 10s^2 + 7s & 0 & 0 \\ -5s & 0 & -2 \\ 0 & F(s) & 2 \end{bmatrix}}{\det(A)}$$

$$\det \rightarrow (-1)^{1+1} (10s^2 + 7s) \det \begin{bmatrix} 0 & -2 \\ F(s) & 2 \end{bmatrix} = (10s^2 + 7s)(2F(s))$$

$$x_2(s) = \frac{(10s^2 + 7s)2F(s)}{100s^3 + 20s^2} = \frac{(20s^2 + 14s)F(s)}{100s^3 + 20s^2}$$

$$\frac{x_2(s)}{F(s)} = \frac{20s^2 + 14s}{100s^3 + 20s^2} = \frac{2s(10s + 7)}{20s^2(5s + 1)} = \frac{10s + 7}{10s(5s + 1)}$$