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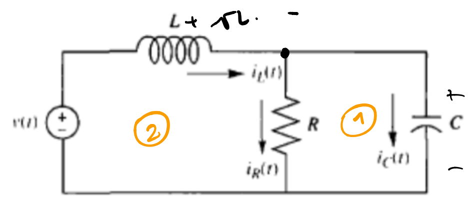
Hoja de trabajo No. 13

voltajes capacitores } definen cuanta energía tiene el sistema.
corriente inductores }

Problema 1: entrada: $v_i(t)$, salida: $i_R(t)$



variables de estado:

$$C\frac{dv_C}{dt} = i_C$$

$$L\frac{di_L}{dt} = v_L$$

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1 v(t)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_2 v(t)$$

$$y = c_1x_1 + c_2x_2 + d_1 v(t)$$

1. $i_C = -i_R + i_L$

$$= -\frac{1}{R}v_C + i_L$$

2. $v_L = -v_C + v(t)$

$\dot{x}_1$: $$C\frac{dv_C}{dt} = -\frac{1}{R}v_C + i_L$$

$$\frac{dv_C}{dt} = \frac{-1}{RC}v_C + \frac{i_L}{C}$$

$\dot{x}_2$: $L \dfrac{di_L}{dt} = -v_C + v(t)$

$$\frac{di_L}{dt} = -\frac{1}{L} v_C + \frac{1}{L} v(t)$$

$y = i_R(t)$

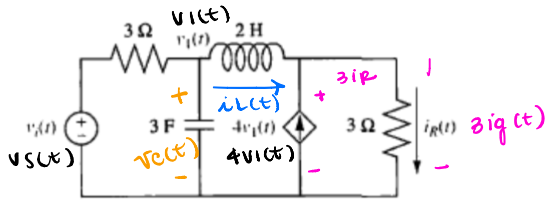
$$i_R = \frac{1}{R} v_C$$

entonces:

$$\dot{x} = \begin{bmatrix} v_C' \\ i_L' \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v(t)$$

$$y = \begin{bmatrix} \frac{1}{R} & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

Problema 2: entrada: $v_i(t)$, salida: $i_R(t)$. La fuente de voltaje dependiente tiene un valor de $4v_1(t)$.



capacitor $E = \frac{1}{2} c v^2(t)$ energía − voltaje

inductor $E = \frac{1}{2} L i^2(t)$ − corriente

definir variables de estado:

$$\underline{x}(t) = \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}$$

capacitor $i = c \frac{d}{dt} v$

inductor $v = L \frac{d}{dt} i$

$$3 \frac{d}{dt} v_c + i_L = \frac{V_s - V_c}{3}$$

$$\frac{d}{dt} v_c = \frac{V_s - V_c}{9} - \frac{i_L}{3}$$

$$-v_c + 2\frac{d}{dt} i_L + 3(i_L + 4v_c) = 0$$

$$\frac{d}{dt} i_L = -11\frac{v_c}{2} - \frac{3}{2} i_L - 6 v_c$$

$$x(t) = A\underline{x}(t) + B\underline{w}(t)$$

$$\dot{x}_1 = a_{11} x_1 + a_{12} x_2 + b_1 V_s(t)$$

$$\dot{x}_2 = a_{21} x_1 + a_{22} x_2 + b_2 V_s(t)$$

en notación matricial:

$$\dot{x} = \begin{bmatrix} v_c' \\ i_L' \end{bmatrix} = \begin{bmatrix} -\frac{1}{9} & -\frac{1}{3} \\ -\frac{11}{2} & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{9} \\ 0 \end{bmatrix} v_s$$

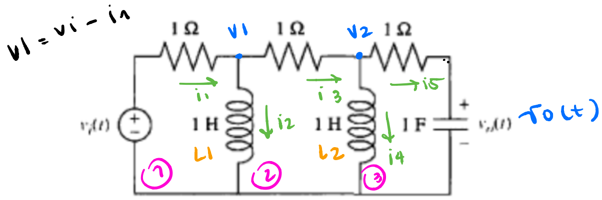
ecuaciones de estado:

$$y = \begin{bmatrix} 12 & 3 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix}$$

$$v_R = 3[i_L + 4v_c]$$

$$y = 3i_L + 12V_c$$

Problemas 3: entrada: $v_i(t)$, salida: $v_o(t)$



variables de estado:

$$C\frac{dv_C}{dt} = i_C \qquad \dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1 v(t)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_2 v(t)$$

$$L\frac{di_L}{dt} = v_L \qquad y = c_1x_1 + c_2x_2 + d_1 v(t)$$

1: $\frac{di_2}{dt} = v_1$

$$x = \begin{bmatrix} i_2 \\ i_4 \\ v_0 \end{bmatrix}$$

2: $\frac{di_4}{dt} = v_2$

3: $\frac{dv_0}{dt} = i_5$

· entonces

$$v_i - i_1 - i_3 - i_5 - v_0 = 0 \qquad i_3 = i_1 - i_2$$

$$i_5 = i_3 - i_4$$

$$= v_i - i_1 - (i_1 - i_2) - (i_3 - i_4) - v_0 = 0$$

$$= v_i - i_1 - (i_1 - i_2) - ((i_1 - i_2) - i_4) - v_0 = 0$$

para i_1 =

$$v_i - i_1 - (i_1 - i_2) - ((i_1 - i_2) - i_4) - v_0 = 0$$

$$v_i \cancel{-i_1} - i_1 + i_2 - i_1 + i_2 + i_4 - v_0 = 0$$

$$-3i_1 + 2i_2 + i_4 - v_0 + v_i = 0$$

$$3i_1 = 2i_2 + i_4 - v_0 + v_i$$

$$i_1 = \frac{2}{3} i_2 + \frac{1}{3} i_4 - \frac{1}{3} v_0 + \frac{1}{3} v_i$$

$$v_1 = V_i - i_1 = -\frac{2}{3} i_2 - \frac{1}{3} i_4 + \frac{1}{3} v_0 + \frac{3}{3} v_i - \frac{1}{3} v_i$$

$$= \frac{-2}{3} i_2 - \frac{1}{3} i_4 + \frac{1}{3} v_0 + \frac{2}{3} v_i$$

$$i_3 = i_1 - i_2 = -\frac{1}{3} i_2 + \frac{1}{3} i_4 - \frac{1}{3} v_0 + \frac{1}{3} v_i$$

$$i_5 = i_3 - i_4 = -\frac{1}{3} i_2 - \frac{2}{3} i_4 - \frac{1}{3} v_0 + \frac{1}{3} v_i$$

$$v_2 = i_5 + v_0 = -\frac{1}{3} i_2 - \frac{2}{3} i_4 + \frac{2}{3} v_0 + \frac{1}{3} v_i$$

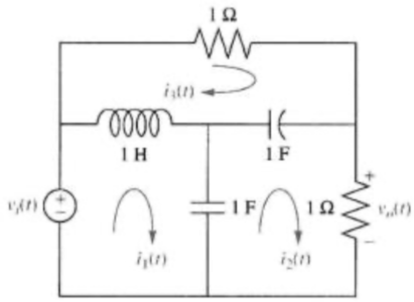
usando v_1, v_2 y i_5

$$\dot{x} = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} i_2 \\ i_4 \\ v_0 \end{bmatrix} + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} v_i$$

$$y = v_0(t)$$

$$= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_2 \\ i_4 \\ v_0 \end{bmatrix}$$

Problema 4: entrada: $v_i(t)$, salida: $v_o(t)$



variables de estado:

$$C\frac{dv_c}{dt} = i_c \qquad \dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1 v(t)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_2 v(t)$$

$$L\frac{di_L}{dt} = v_L \qquad y = c_1x_1 + c_2x_2 + d_1 v(t)$$

1. $v_i = \frac{di_L}{dt} + v_{c1}$

$$\frac{di_L}{dt} = v_i - v_{c1} \qquad x = \begin{bmatrix} i_L \\ v_{c1} \\ v_{c2} \end{bmatrix}$$

2. $\frac{dv_{c1}}{dt} = i_1 - i_2 = i_L + v_i - v_{c1} + v_{c2} - (v_{c1} - v_{c2})$

$$i_L + v_i - v_{c1} + v_{c2} - v_{c1} + v_{c2} = i_L - 2v_{c1} + 2v_{c2} + v_i$$

3. $\frac{dv_{c2}}{dt} = i_2 - i_3 = v_{c1} - v_{c2} - (v_i - v_{c1} + v_{c2})$

$$= v_{c1} - v_{c2} - v_i + v_{c1} - v_{c2} = 2v_{c1} - 2v_2 - v_i$$

2. $v_{c1} = v_{c2} + i_2$

ó $i_2 = v_{c1} - v_{c2}$

outer loop:

$v_i = i_3 + i_2$

$i_3 = v_i - i_2 = v_i - (v_{c1} - v_{c2})$

$= v_i - v_{c1} + v_{c2}$

$i_1 - i_3 = i_L$

$i_1 = i_L + i_3 = i_L + v_i - v_{c1} + v_{c2}$

entonces:

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_{c1}}{dt} \\ \frac{dv_{c2}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -2 & 2 \\ 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} i_L \\ v_{c1} \\ v_{c2} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} v_i$$

$v_0 = i_2$

$= v_{c1} - v_{c2}$

$$y = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_L \\ v_{c1} \\ v_{c2} \end{bmatrix}$$