I Ç's

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Problema 1
Calcular

$$\int_C (z+3)dz$$

donde C está dada por $z(t) = x + iy$, donde $x(t) = 2t$, $y(t) = 4t - 1$ con $1 \le t \le 3$.

$x(t) = 2t \qquad 1 \le t \le 3$
$y(t) = 4t - 1$

$z(t) = 2t + i(4t-1)$

$z'(t) = 2 + 4i$

$$\int_C (z+3)dz = \int_1^3 [2t + i(4t-1) + 3][2+4i]\,dt$$

$$= [2+4i]\int_1^3 2t + i(4t-1) + 3\,dt = (2+4i)\left[t^2 + i2t^2 - it + 3t\right]_1^3$$

$$= (2+4i)(8 + i16 - i2 + 6) = (2+4i)(14+14i) = 28 + 28i + 56i - 56$$

$$= -28 + 84i \; \#$$

Problema 2
Calcular

$$\int_{|z|=1} Re(z)dz.$$

$|z| = 1$
$x(t) = \cos(t)$
$y(t) = \text{sen}(t)$

$z(t) = \cos(t) + i\,\text{sen}(t)$

$z(t) = e^{it}, \quad t: 0 \to 2\pi$

$z'(t) = ie^{it}$

$$\int_{|z|=1} Re(z)\,dz = \int_0^{2\pi} (\cos(t))\, i e^{it}\,dt = i\int_0^{2\pi} \cos^2(t) + i\,\text{sen}(t)\cos(t)\,dt$$

! $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$

2 2

① $\int_0^{2\pi} \frac{1}{2} + \frac{1}{2}\cos(2t)\, dt = \frac{1}{2} t \Big|_0^{2\pi} + \frac{1}{4}\int_0^{2\pi} \cos(u)du = \pi + \frac{1}{4}\cancel{\operatorname{sen}(2t)}^{0}\Big|_0^{2\pi} = \pi$

$u = 2t$
$du = 2dt$
$dt = \frac{1}{2}du$

② $\int_0^{2\pi} \operatorname{sen}(t)\cos(t)\, dt = \int_0^{2\pi} u\, du = \frac{1}{2}u^2 \Big|_0^{2\pi} = \frac{1}{2}\cancel{\operatorname{sen}^2(t)}^{0}\Big|_0^{2\pi} = 0$

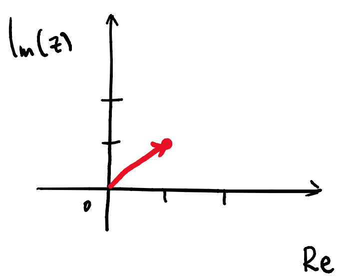
$u = \operatorname{sen}(t)$
$du = \cos(t)dt$

$= i\,[\pi + i(0)] = \pi i$ #

Problema 3
Calcular

$$\int_C e^z dz,$$

donde C es el segmento de recta que va de 0 a $1+i$.



$x(t) = t$
$y(t) = t, \quad t: 0 \to 1$

$\int_0^1 e^{t+it}(1+i)\, dt$

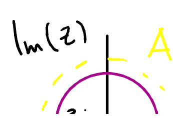
$z(t) = t + it$
$z'(t) = 1 + i$

$= (1+i)\int_0^1 e^{t(1+i)}\, dt = (1+i)\frac{1}{(1+i)}\int_0^1 e^u du = e^u\Big|_0^1$

$u = t(1+i)$
$du = (1+i)dt$
$dt = \frac{du}{(1+i)}$

$= e^{t(1+i)}\Big|_0^1 = e^{1+i} - 1$ #

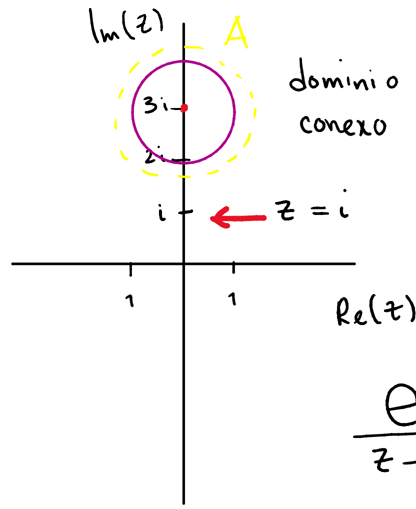
Problema 4
Calcular



dominio simplemente

Problema 4
Calcular

$$\int_{|z-3i|=1} \frac{e^z}{z-i}dz.$$



dominio simplemente conexo

$|z-3i| = 1$

Circunferencia $r=1$ y centro en $3i$

$\frac{e^z}{z-i}$ Es análitica en $\mathbb{C}$ excepto en $z=i$

Por Teorema de Cauchy.

La curva esta contenida en un dominio simplemente conexo A, donde $\frac{e^z}{z-i}$ es analitica en A

entonces $$\int_{|z-3i|=1} \frac{e^z}{z-i}\, dz = 0$$