

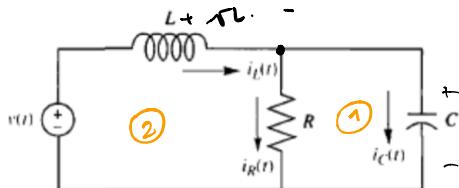
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Hoja de trabajo No. 13

voltajes capacitores y definen cuanta energía tiene el sistema.
corriente inductores

Problema 1: entrada: $v_i(t)$, salida: $i_R(t)$



variables de estado:

$$C \frac{dv_c}{dt} = i_C$$

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1v(t)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_2v(t)$$

$$L \frac{di_L}{dt} = v_L$$

$$y = c_1x_1 + c_2x_2 + d_1v(t)$$

$$1. i_C = -i_R + i_L$$

$$= -\frac{1}{C}v_C + i_L$$

$$2. v_L = -v_C + v(t)$$

$$x_1 : C \frac{dv_c}{dt} = -\frac{1}{L}v_C + i_L$$

$$\frac{dv_c}{dt} = -\frac{1}{LC}v_C + \frac{i_L}{C}$$

$$x_2 : L \frac{di_L}{dt} = -v_C + v(t)$$

$$\frac{di_L}{dt} = -\frac{1}{L}v_C + \frac{1}{L}v(t)$$

$$y = i_R(t)$$

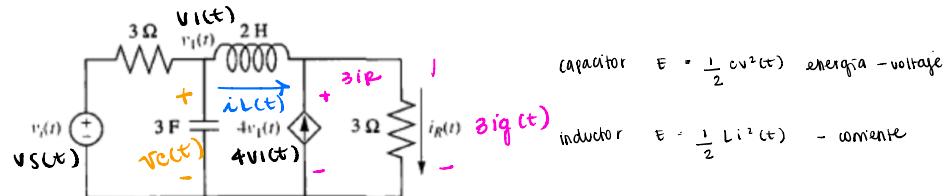
$$i_R = \frac{1}{R} v_C$$

entonces:

$$\dot{x} = \begin{bmatrix} v_C' \\ i_L' \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v(t)$$

$$y = \begin{bmatrix} \frac{1}{R} & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

Problema 2: entrada: $v_i(t)$, salida: $i_R(t)$. La fuente de voltaje dependiente tiene un valor de $4v_1(t)$.



$$\text{capacitor } E = \frac{1}{2} C v^2(t) \quad \text{energía - voltaje}$$

$$\text{inductor } E = \frac{1}{2} L i^2(t) \quad \text{- corriente}$$

definir variables de estado:

$$\underline{x}(t) = \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}$$

$$\text{capacitor } \dot{v} = C \frac{d}{dt} v$$

$$\text{inductor } \dot{i} = L \frac{d}{dt} i$$

$$3 \frac{d}{dt} v_C + i_L = \frac{v_S - v_C}{3}$$

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$\frac{d}{dt} v_C = \frac{v_S - v_C}{9} - \frac{i_L}{3}$$

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1 v_S(t)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_2 v_S(t)$$

$$-v_C + 2 \frac{d}{dt} i_L + 3(i_L + 4v_C) = 0$$

$$\frac{d}{dt} i_L = -\frac{v_C}{2} - \frac{3}{2} i_L - v_C$$

en notación matricial:

$$\dot{x} = \begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{9} & -\frac{1}{3} \\ -\frac{11}{2} & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{9} \\ 0 \end{bmatrix} v_S$$

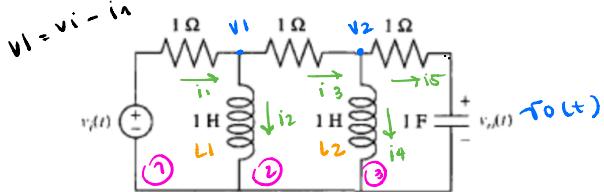
ecuaciones de estado:

$$y = \begin{bmatrix} 12 & 3 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

$$v_D = 3[i_L + 4v_C]$$

$$y_1 = 3i_L + 12v_C$$

Problemas 3: entrada: $v_i(t)$, salida: $v_o(t)$



variables de estado:

$$C \frac{dv_c}{dt} = i_0$$

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1v(t)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_2v(t)$$

$$L \frac{di_0}{dt} = v_2$$

$$y = c_1x_1 + c_2x_2 + d_1v(t)$$

$$1: \frac{di_2}{dt} = v_1$$

$$x = \begin{bmatrix} i_2 \\ i_4 \\ v_o \end{bmatrix}$$

$$2: \frac{di_4}{dt} = v_2$$

$$3: \frac{dv_o}{dt} = i_5$$

entonces

$$v_i - i_1 - i_3 - i_5 - v_o = 0 \quad i_3 = i_1 - i_2$$

$$i_5 = i_3 - i_4$$

$$= v_i - v_1 - (i_1 - i_2) - (i_3 - i_4) - v_o = 0$$

$$= v_i - v_1 - (i_1 - i_2) - ((i_1 - i_2) - i_4) - v_o = 0$$

para $v_1 =$

$$v_i - i_1 - (i_1 - i_2) - ((i_1 - i_2) - i_4) - v_o = 0$$

$$v_i - i_1 - i_1 + i_2 - i_1 + i_2 + i_4 - v_o = 0$$

$$-3i_1 + 2i_2 + i_4 - v_o + v_i = 0$$

$$3i_1 = 2i_2 + i_4 - v_o + v_i$$

$$ii = \frac{2}{3} i_2 + \frac{1}{3} i_4 - \frac{1}{3} v_0 + \frac{1}{3} vi$$

$$\begin{aligned} vi = vi - ii &= -\frac{2}{3} i_2 - \frac{1}{3} i_4 + \frac{1}{3} v_0 + \frac{3}{3} vi - \frac{1}{3} vi \\ &= -\frac{2}{3} i_2 - \frac{1}{3} i_4 + \frac{1}{3} v_0 + \frac{2}{3} vi \end{aligned}$$

$$i_3 = i_1 - i_2 = -\frac{1}{3} i_2 + \frac{1}{3} i_4 - \frac{1}{3} v_0 + \frac{1}{3} vi$$

$$i_5 = i_3 - i_4 = -\frac{1}{3} i_2 - \frac{2}{3} i_4 - \frac{1}{3} v_0 + \frac{1}{3} vi$$

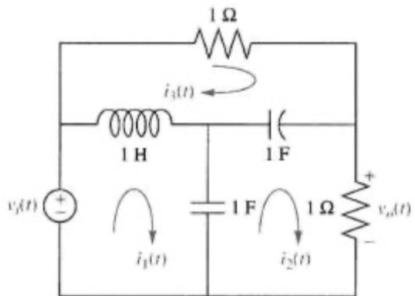
$$v_2 = i_5 + v_0 = -\frac{1}{3} i_2 - \frac{2}{3} i_4 + \frac{2}{3} v_0 + \frac{1}{3} vi$$

usando v_1, v_2 y i_5

$$x = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} i_2 \\ i_4 \\ v_0 \end{bmatrix} + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} vi$$

$$\begin{aligned} y &= v_0(t) \\ &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_2 \\ i_4 \\ v_0 \end{bmatrix} \end{aligned}$$

Problema 4: entrada: $v_i(t)$, salida: $v_o(t)$



variables de estado:

$$C \frac{dv_c}{dt} = i_C$$

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1v(t)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_2v(t)$$

$$L \frac{di_L}{dt} = v_L$$

$$y = c_1x_1 + c_2x_2 + d_1v(t)$$

1. $v_i = \frac{dv_L}{dt} + v_{C1}$

$$x = \begin{bmatrix} i_L \\ v_{C1} \\ v_{C2} \end{bmatrix}$$

$$\frac{dv_L}{dt} = v_i - v_{C1}$$

2. $\frac{dv_{C1}}{dt} = i_1 - i_2 = i_L + v_i - v_{C1} + v_{C2} - (v_{C1} - v_{C2})$
 $i_L + v_i - v_{C1} + v_{C2} - v_{C1} + v_{C2} = i_L - 2v_{C1} + 2v_{C2} + v_i$

3. $\frac{dv_{C2}}{dt} + i_2 - i_3 = v_{C1} - v_{C2} - (v_i - v_{C1} + v_{C2})$
 $= v_{C1} - v_{C2} - v_i + v_{C1} - v_{C2} = 2v_{C1} - 2v_{C2} - v_i$

2. $v_{C1} = v_{C2} + i_2$

∴ $i_2 = v_{C1} - v_{C2}$

outer loop:

$$v_L = i_3 + i_2$$

$$i_3 = v_i - i_2 = v_i - (v_{C1} - v_{C2})$$

$$= v_i - v_{C1} + v_{C2}$$

$$i_1 - i_3 = i_L$$

$$i_1 - i_L + i_3 = i_L + v_i - v_{C1} + v_{C2}$$

ENTRIES:

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_{C1}}{dt} \\ \frac{dv_{C2}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -2 & 2 \\ 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} i_L \\ v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} v_i$$

$$v_0 = i_2$$

$$= v_{C1} - v_{C2}$$

$$y = [0 \ 1 \ -1] \begin{bmatrix} i_L \\ v_{C1} \\ v_{C2} \end{bmatrix}$$