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Hoja de trabajo N°. 10.

Problema 1

Calcular espectro y vectores propios de $A = \begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$.

$$\det(A - \lambda I) = \det \begin{pmatrix} -1-\lambda & 1 & 2 \\ 0 & -2-\lambda & 0 \\ 0 & 1 & 3-\lambda \end{pmatrix} = (-1-\lambda)(\lambda^2 - \lambda - 6) = 0$$
$$= -\lambda^3 + 7\lambda + 6$$

$$-\lambda^3 + 7\lambda + 6 = 0$$

$$-(\lambda+1)(\lambda+2)(\lambda-3) = 0$$

$$\lambda_1 = -1, \lambda_2 = -2, \lambda_3 = 3$$

$$A = \begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

vectores propios

$$\lambda_1 = -1 \Rightarrow \underline{x}^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_2 = -2 \Rightarrow \underline{x}^2 = \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$$

$$\lambda_3 = 3 \Rightarrow \underline{x}^3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Problema 2

Para $A = \begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ y $\mathbf{q}_T(0) = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ calcular $\mathbf{q}_T(t)$.

Matriz modal

$$M = \begin{pmatrix} 1 & 3 & 1 \\ 0 & -5 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{5} & 0 \\ 0 & \frac{1}{10} & \frac{1}{2} \end{pmatrix}$$

$$e^{At} = \begin{pmatrix} e^{-t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & e^{3t} \end{pmatrix}$$

$$\mathbf{x}^{At} = M e^{At} M^{-1}$$

$$= \begin{pmatrix} 1 & 3 & 1 \\ 0 & -5 & 0 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} e^{-t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & e^{3t} \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{5} & 0 \\ 0 & \frac{1}{10} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-t} & 3e^{-2t} & e^{3t} \\ 0 & -5e^{-2t} & 0 \\ 0 & e^{-2t} & 2e^{3t} \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{5} & 0 \\ 0 & \frac{1}{10} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-t} & \frac{5e^{-t} - 4e^{-2t} + e^{3t}}{10} & \frac{e^{3t} - e^{-t}}{2} \\ 0 & e^{-2t} & 0 \\ 0 & \frac{-e^{-2t} + e^{3t}}{5} & e^{3t} \end{pmatrix}$$

$$\mathbf{x}_T = e^{At} \mathbf{x}(0)$$

$$= \begin{pmatrix} e^{-t} & \frac{5e^{-t} - 4e^{-2t} + e^{3t}}{10} & \frac{e^{3t} - e^{-t}}{2} \\ 0 & e^{-2t} & 0 \\ 0 & \frac{-e^{-2t} + e^{3t}}{5} & e^{3t} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5e^{-t} - 4e^{-2t} + e^{3t}}{10} + 2 \cdot \frac{e^{3t} - e^{-t}}{2} \\ \frac{e^{-2t}}{5} + 2e^{3t} \end{pmatrix}$$

$$\mathbf{x}_T = \begin{pmatrix} \frac{1}{10} (11e^{4t} - 4e^{-t} + 5t) \\ \frac{e^{-2t}}{5} \\ \frac{1}{5} (-e^{-2t} + 11e^{3t}) \end{pmatrix}$$

Problema 3

Dar ecuaciones de estado y de lectura en la forma \mathbf{A}_* diagonal para los sistemas

$$3.1 \quad \dot{\mathbf{q}}(t) = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \mathbf{q} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} y, \quad x = (1 \ 0) \mathbf{q}$$

$$\det(\mathbf{A} - \lambda \mathbf{I}) = \det \begin{pmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{pmatrix} = \lambda(3+\lambda) + 2 = \lambda^2 + 3\lambda + 2$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$(\lambda + 1)(\lambda + 2) = 0$$

$$\lambda_1 = -1, \quad \lambda_2 = -2$$

vectores propios

• λ_1

$$(\mathbf{A} - \lambda_1 \mathbf{I}) = \begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix} \xrightarrow{F_2 \leftarrow F_2 + 2F_1} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 + x_2 &= 0 \\ x_1 &= -x_2 \end{aligned} \quad \underline{x} = \begin{pmatrix} -x_2 \\ x_2 \end{pmatrix}; \quad x_2 = 1 \quad \underline{x}^2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

• λ_2

$$(\mathbf{A} - \lambda_2 \mathbf{I}) = \begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix} \xrightarrow{F_2 \leftarrow F_1 + F_2} \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}$$

$$2x_1 + x_2 = 0 \\ x_1 = -\frac{x_2}{2}$$

$$\underline{x} = \begin{pmatrix} -x_2/2 \\ x_2 \end{pmatrix}; x_2 = 2 \quad \underline{x} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Matrix Modal

$$M = \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix} \quad M^{-1} = \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\dot{\underline{x}}(t) = (M^{-1}AM)\underline{x} + (M^{-1}B)\underline{v} \\ = \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} \underline{v}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \underline{x} + \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} \underline{v}$$

$$\dot{\underline{x}}(t) = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \underline{x} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} \underline{v}$$

$$\dot{\underline{y}}(t) = (CM)\underline{x} + 0 \underline{w} \\ = (1 \ 0) \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix} \underline{x} \\ = (-1 \ -1) \underline{x}$$

$$3.2 \quad \dot{\underline{q}}(t) = \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & -3 & 1 \end{pmatrix} \underline{q} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ -1 & 0 \end{pmatrix} \underline{y}, \quad \underline{x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \underline{q}$$

$$\det(A - \lambda I) = \det \begin{pmatrix} -\lambda & -3 & 1 \\ 1 & -4-\lambda & 1 \\ 0 & -3 & 1-\lambda \end{pmatrix} = \begin{array}{l} \lambda(\lambda+1)(\lambda+2) - 3(-4-\lambda) - 3\lambda + 3 - 3\lambda \\ \lambda(\lambda^2 + 3\lambda + 2) - 3(-4-\lambda) - 3\lambda + 3 - 3\lambda \\ \lambda(-\lambda^2 - 3\lambda - 2) - 3(-4-\lambda) - 3\lambda + 3 - 3\lambda \end{array}$$

$$\lambda(-\lambda^2 - 3\lambda - 2) = 0$$

$$\lambda(\lambda+1)(\lambda+2) = 0$$

$$\lambda_1 = 0, \quad \lambda_2 = -1, \quad \lambda_3 = -2$$

$$(A - \lambda_1 I) = \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & -3 & 1 \end{pmatrix} \xrightarrow{F_3 \leftarrow -F_3 + F_1} \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_2 \leftarrow -F_1 + F_2}$$

$$\begin{pmatrix} 0 & -3 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad -3x_2 + x_3 = 0 \\ x_2 = \frac{1}{3}x_3$$

$$\underline{x} = \begin{pmatrix} x_1 \\ \frac{1}{3}x_3 \\ x_3 \end{pmatrix}; \quad x_1 = 1 \\ x_3 = 3 \quad \underline{x}^1 = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$

$$(A - \lambda_2 I) = \begin{pmatrix} 1 & -3 & 1 \\ 1 & -3 & 1 \\ 0 & -3 & 0 \end{pmatrix} \xrightarrow{F_1 \leftarrow -F_2 + F_1} \begin{pmatrix} 0 & 0 & 0 \\ 1 & -3 & 1 \\ 0 & -3 & 0 \end{pmatrix}$$

$$x_1 - 3x_2 + x_3 = 0 \\ -3x_2 = 0 \\ x_3 = -x_1$$

$$\underline{x} = \begin{pmatrix} x_3 \\ \frac{2}{3}x_3 \\ x_3 \end{pmatrix}; \quad x_3 = 3 \quad \underline{x}^2 = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$$

$$(A - \lambda_3 I) = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 0 & -3 & 3 \end{pmatrix} \xrightarrow{\begin{array}{l} F_2 \leftarrow F_2 - \frac{1}{2}F_1 \\ F_2 \leftrightarrow F_3 \end{array}} \begin{pmatrix} 2 & -3 & 1 \\ 0 & -3 & 3 \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \xrightarrow{F_3 \leftarrow F_3 - \frac{1}{6}F_2}$$

$$\begin{pmatrix} 2 & -3 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\begin{array}{l} F_2 \leftarrow -\frac{1}{3}F_2 \\ F_1 \leftarrow F_1 + 3F_2 \end{array}} \begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_1 \leftarrow \frac{1}{2}F_1}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad x_1 - x_3 = 0 \quad x_1 = x_3 \\ x_2 - x_3 = 0 \quad x_2 = x_3$$

$$\underline{x} = \begin{pmatrix} x_3 \\ x_3 \\ x_3 \end{pmatrix}; x_3=1 \quad x^3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Matrix Modal

$$M = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix}$$

$$M^{-1} = \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 3 & 3 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\substack{F_3 \leftrightarrow F_1 \\ F_2 \leftarrow F_2 - \frac{1}{3}F_1}} \left(\begin{array}{ccc|ccc} 3 & 3 & 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{2}{3} & 0 & 1 & -\frac{1}{3} \\ 1 & 3 & 1 & 1 & 0 & 0 \end{array} \right)$$

$$\xrightarrow{\substack{F_3 \leftarrow F_3 - \frac{1}{3}F_1 \\ F_2 \leftrightarrow F_3}} \left(\begin{array}{ccc|ccc} 3 & 3 & 1 & 0 & 0 & 1 \\ 0 & 2 & \frac{2}{3} & 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{3} & 0 & 1 & -\frac{1}{3} \end{array} \right) \xrightarrow{\substack{F_3 \leftarrow F_3 - \frac{1}{2}F_2 \\ F_1 \leftarrow 3F_3}} \left(\begin{array}{ccc|ccc} 3 & 3 & 1 & 0 & 0 & 1 \\ 0 & 2 & \frac{2}{3} & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{3}{2} & 3 & -\frac{1}{2} \end{array} \right)$$

$$\xrightarrow{\substack{F_2 \leftarrow F_2 - \frac{2}{3}F_3 \\ F_1 \leftarrow F_1 - F_3}} \left(\begin{array}{ccc|ccc} 3 & 3 & 0 & \frac{3}{2} & -3 & \frac{3}{2} \\ 0 & 2 & 0 & 2 & -2 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 3 & -\frac{1}{2} \end{array} \right) \xrightarrow{\substack{F_2 \leftarrow \frac{1}{2}F_2 \\ F_1 + F_1 - 3F_2}} \left(\begin{array}{ccc|ccc} 3 & 0 & 0 & -\frac{3}{2} & 0 & \frac{3}{2} \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 3 & -\frac{1}{2} \end{array} \right)$$

$$\xrightarrow{\substack{F_1 \leftarrow \frac{1}{3}F_1}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 3 & -\frac{1}{2} \end{array} \right)$$

$$M^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ -\frac{3}{2} & 3 & -\frac{1}{2} \end{pmatrix}$$

$$\dot{\underline{x}} = (\mathbf{M}^{-1} \mathbf{A} \mathbf{M}) \underline{x} + (\mathbf{M}^{-1} \mathbf{B}) \underline{w}$$

$$= \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 1 & -1 & 0 \\ -\frac{3}{2} & 3 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 1 & -1 & 0 \\ -\frac{3}{2} & 3 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 1 & -1 & 0 \\ -\frac{3}{2} & 3 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\dot{\underline{x}}(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \underline{x} + \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ 1 & 1 \\ -\frac{5}{2} & -\frac{3}{2} \end{pmatrix} \underline{w}$$

$$y(t) = (\mathbf{C}\mathbf{M}) \underline{x} + 0 \underline{w}$$

3.2. $\dot{q}(t) = \begin{pmatrix} 0 & -3 & 1 \\ 1 & -4 & 1 \\ 0 & -3 & 1 \end{pmatrix} q + \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ -1 & 0 \end{pmatrix} y, \quad x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} q$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 1 \end{pmatrix} \underline{x}$$

$$y(t) = \begin{pmatrix} 1 & 3 & 1 \\ 4 & 5 & 2 \end{pmatrix} \underline{x}$$

Problema 4

Para el sistema dado por la función de transferencia

$$G(p) = \frac{3}{(p+1)^2(p+2)}$$

dar ecuaciones de estado y de lectura con \mathbf{A}_* diagonal.

Fraciones parciales:

$$\frac{3}{(p+1)^2(p+2)} = \frac{a_0}{p+1} + \frac{a_1}{(p+1)^2} + \frac{a_2}{p+2}$$

$$\frac{3(p+1)^2(p+2)}{(p+1)^2(p+2)} = \frac{a_0(p+1)^2(p+2)}{(p+1)} + \frac{a_1(p+2)(p+1)^2}{(p+1)^2} + \frac{a_2(p+1)^2(p+2)}{(p+2)}$$

$$3 = a_0(p+1)(p+2) + a_1(p+2) + a_2(p+1)^2$$

$$p = -1$$

$$3 = a_0(-1+1)(-1+2) + a_1(-1+2) + a_2(-1+1)^2$$

$$3 = a_1$$

$$p = -2$$

$$3 = a_0(-2+1)(-2+2) + a_2(-2+1)^2$$

$$3 = a_2$$

$$3 = a_0(p+1)(p+2) + 3(p+2) + 3(p+1)^2$$

$$3 = a_0(p^2 + 3p + 2) + 3p + 6 + 3p^2 + 6p + 3$$

$$3 = p^2 a_0 + 3p a_0 + 2a_0 + 3p^2 + 9p + 9$$

$$3 = p^2(a_0 + 3) + p(3a_0 + 9) + (2a_0 + 9)$$

$$3 = 2a_0 + 9$$

$$a_0 = -3$$

$$\begin{aligned} \frac{3}{(p+1)^2(p+2)} &= \frac{a_0}{p+1} + \frac{a_1}{(p+1)^2} + \frac{a_2}{p+2} \\ &= \frac{3}{(p+1)^2} - \frac{3}{(p+1)} + \frac{3}{(p+2)} \end{aligned}$$

$$x_1(p) = \frac{v(p)}{(p+1)^2}$$

$$\frac{x_1(p)}{x_2(p)} = \frac{3v(p)}{(p+1)^2} * \frac{4(p+1)}{-3v(p)}$$

$$\frac{x_1(p)}{x_2(p)} = \frac{1}{(p+1)}$$

$$x_1(p+1) = x_2$$

$$+ \dot{x}_1 + x_1 = x_2$$

$$x_2(p) = \frac{v(p)}{p+1}$$

$$x_2(p+1) = v$$

$$\dot{x}_2 + x_2 = v$$

$$\dot{x}_2 = -x_2 - v$$

$$x_3(p+2) = v$$

$$\dot{x}_3 + 2x_3 = v$$

$$\dot{x}_3 = -2x_3 + v$$

$$\dot{x}_1 + x_1 = x_2$$

$$\dot{x}_1 = -x_1 + x_2$$

$$\dot{\underline{x}}(t) = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \underline{w}$$

$$\underline{y}(t) = \begin{bmatrix} 3 & -3 & 3 \end{bmatrix} \underline{x}' + [0] \underline{w}$$

Problema 5

¿Son los eigenvalores de una matriz invariantes respecto de una transformación lineal? ¡Demostrar!

$$\det(\lambda I - P^{-1}AP) = 0$$

$$\det(\lambda P^{-1}IP - P^{-1}AP) = 0$$

$$\det(P^{-1}(\lambda IP - AP)) = 0$$

$$\det(P^{-1}(\lambda I - A)P) = 0$$

$$\det(P^{-1}) \cdot \det(\lambda I - A) \cdot \det(P) = 0$$

$$\det(\lambda I - A) = 0$$

Problema 6

Dar A_* diagonal para $A = \begin{pmatrix} 0 & 0 & -4 \\ 1 & 0 & -6 \\ 0 & 1 & -4 \end{pmatrix}$.

$$A^* = M^{-1}AM$$

$$\det(\lambda A - \lambda I) = \det \begin{pmatrix} -\lambda & 0 & -4 \\ 1 & -\lambda & -6 \\ 0 & 1 & -4-\lambda \end{pmatrix} = -\lambda(4\lambda + \lambda^2 + 6) - 0(-\lambda - 4) - 4 = -\lambda^3 - 4\lambda^2 - 6\lambda - 4$$

$$-\lambda^3 - 4\lambda^2 - 6\lambda - 4 = 0$$

$$-(\lambda+2)(\lambda^2 + 2\lambda + 2) = 0$$

$$\lambda_1 = -2$$

$$\lambda_2, \lambda_3 = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1+i, -1-i$$

$$\lambda_1 = -2, \lambda_2 = -1+i, \lambda_3 = -1-i$$

$$(A - \lambda_1 I) = \begin{pmatrix} 2 & 0 & -4 \\ 1 & 2 & -4 \\ 0 & 1 & -2 \end{pmatrix} \xrightarrow{\substack{F_2 \leftarrow F_2 - \frac{1}{2}F_1 \\ F_3 \leftarrow F_3 - \frac{1}{2}F_2}} \begin{pmatrix} 2 & 0 & -4 \\ 0 & 2 & -4 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\substack{F_2 \leftarrow \frac{1}{2}F_2 \\ F_1 \leftarrow \frac{1}{2}F_1}}$$

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} \quad x_1 - 2x_3 = 0 \quad x_1 = 2x_3$$

$$x_2 - 2x_3 = 0$$

$$x_2 = 2x_3$$

$$\underline{x} = \begin{pmatrix} 2x_3 \\ 2x_3 \\ x_3 \end{pmatrix} \quad x_3 = 1 \quad x^1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$(A - \lambda_2 I) = \begin{pmatrix} 1-i & 0 & -4 \\ 1 & 1-i & -4 \\ 0 & 1 & -3-i \end{pmatrix} \xrightarrow{\substack{F_2 \leftarrow F_2 - \frac{1}{2}(1+i)F_1 \\ F_3 \leftarrow F_3 - \frac{1}{2}(1+i)F_2}} \begin{pmatrix} 1-i & 0 & -4 \\ 0 & 1-i & -4+2i \\ 0 & 0 & 0 \end{pmatrix}$$

$$F_2 \leftarrow \frac{1}{2}(1+i)F_2 \longrightarrow \begin{pmatrix} 1-i & 0 & -4 \\ 0 & 1 & -3-i \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_1 \leftarrow \frac{1}{2}(1+i)F_1} \begin{pmatrix} 1 & 0 & -2 - 2i \\ 0 & 1 & -3 - i \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_1 + x_3(-2 - 2i) = 0 \Rightarrow x_1 = -(-2 - 2i)x_3$$

$$x_2 + x_3(-3 - i) = 0 \Rightarrow x_2 = -(-3 - i)x_3$$

$$\underline{x} = \begin{pmatrix} -(-2 - 2i)x_3 \\ -(-3 - i)x_3 \\ x_3 \end{pmatrix} \quad x_3 = 1 \quad x^2 = \begin{pmatrix} 2 + 2i \\ 3 + i \\ 1 \end{pmatrix}$$

$$(A - \lambda_3 I) = \begin{pmatrix} 1+i & 0 & -4 \\ 1 & 1+i & -4 \\ 0 & 1 & -3+i \end{pmatrix} \xrightarrow{\begin{array}{l} F_2 \leftarrow F_2 - \frac{1}{2}(1-i)F_1 \\ F_3 \leftarrow F_3 - \frac{1}{2}(1-i)F_2 \end{array}} \begin{pmatrix} 1+i & 0 & -4 \\ 0 & 1+i & -4+2i \\ 0 & 0 & 0 \end{pmatrix}$$

$F_2 \leftarrow \frac{1}{2}(1-i)F_2$

$$\longrightarrow \begin{pmatrix} 1+i & 0 & -4 \\ 0 & 1 & -3+i \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_1 \leftarrow \frac{1}{2}(1-i)F_1} \begin{pmatrix} 1 & 0 & -2+2i \\ 0 & 1 & -3+i \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_1 + x_3(-2+2i) = 0 \Rightarrow x_1 = -(-2+2i)x_3$$

$$x_2 + x_3(-3+i) = 0 \Rightarrow x_2 = -(-3+i)x_3$$

$$x = \begin{pmatrix} -(-2+2i)x_3 \\ -(-3+i)x_3 \\ x_3 \end{pmatrix} \quad x_3 = 1 \quad \underline{x}^3 = \begin{pmatrix} 2-2i \\ 3-i \\ 1 \end{pmatrix}$$

Matrⁿ Modul

$$M = \begin{pmatrix} 2 & 2+2i & 2-2i \\ 2 & 3+i & 3-i \\ 1 & 1 & 1 \end{pmatrix}$$

$$M^{-1} = \left(\begin{array}{ccc|ccc} 2 & 2+2i & 2-2i & 1 & 0 & 0 \\ 2 & 3+i & 3-i & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\begin{array}{l} F_2 \leftarrow F_2 - F_1 \\ F_3 \leftarrow F_3 - \frac{1}{2}(1-i)F_1 \end{array}} \left(\begin{array}{ccc|ccc} 2 & 2+2i & 2-2i & 1 & 0 & 0 \\ 0 & 1-i & 1+i & -1 & 1 & 0 \\ 0 & -i & i & -\frac{1}{2} & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} F_2 \leftarrow F_2 \\ F_3 \leftarrow F_3 - (1-i)F_2 \end{array}} \left(\begin{array}{ccc|ccc} 2 & 2+2i & 2-2i & 1 & 0 & 0 \\ 0 & -1 & 1 & \frac{1}{2} & 0 & 1 \\ 0 & 0 & 2 & \frac{1}{2}(-1+i) & 1 & -1-i \end{array} \right) \xrightarrow{\begin{array}{l} F_3 \leftarrow \frac{1}{2}F_3 \\ F_2 \leftarrow F_2 \end{array}} \left(\begin{array}{ccc|ccc} 2 & 2+2i & 2-2i & 1 & 0 & 0 \\ 0 & -i & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4}(-1+i) & \frac{1}{2} & \frac{1}{2}(-1-i) \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} F_1 + F_1 - (2+2i)F_2 \\ F_1 \leftarrow \frac{1}{2}F_1 \end{array}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -1 & 2 \\ 0 & 1 & 0 & \frac{1}{4}(-1-i) & \frac{1}{2} & \frac{1}{2}(-1+i) \\ 0 & 0 & 1 & \frac{1}{4}(-1+i) & \frac{1}{2} & \frac{1}{2}(-1-i) \end{array} \right)$$

$$M^{-1} = \begin{pmatrix} \frac{1}{2} & -1 & 2 \\ \frac{1}{4}(-1-i) & \frac{1}{2} & \frac{1}{2}(-1+i) \\ \frac{1}{4}(-1+i) & \frac{1}{2} & \frac{1}{2}(-1-i) \end{pmatrix}$$

$$A_* = M^{-1} A M$$

$$= \begin{pmatrix} \frac{1}{2} & -1 & 2 \\ \frac{1}{4}(-1-i) & \frac{1}{2} & \frac{1}{2}(-1+i) \\ \frac{1}{4}(-1+i) & \frac{1}{2} & \frac{1}{2}(-1-i) \end{pmatrix} \begin{pmatrix} 0 & 0 & -4 \\ 1 & 0 & -4 \\ 0 & 1 & -4 \end{pmatrix} \begin{pmatrix} 2 & 2+2i & 2-2i \\ 2 & 3+i & 3-i \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 & -4 \\ \frac{1}{2} & \frac{1}{2}(-1+i) & -i \\ \frac{1}{2} & \frac{1}{2}(-1-i) & i \end{pmatrix} \begin{pmatrix} 2 & 2+2i & 2-2i \\ 2 & 3+i & 3-i \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1+i & 0 \\ 0 & 0 & -1-i \end{pmatrix}$$