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Hoja de trabajo No. 8

1. calcular $e^{At} = M e^{\lambda t} M^{-1}$

$$A = \begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix}$$

$\lambda_1 = -1$ $\quad$ $\lambda_2 = -3.$

$$\underline{x^1} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad \underline{x^2} \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$M = \begin{pmatrix} -1 & -1 \\ 1 & 3 \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$e^{\lambda t} = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-3t} \end{pmatrix}$$

$$e^{At} = \begin{pmatrix} -1 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-3t} \end{pmatrix} \begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} -e^{-t} & -e^{-3t} \\ e^{-t} & 3e^{-3t} \end{pmatrix} \begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$e^{At} = \frac{1}{2} \begin{pmatrix} 3e^{-t} - e^{-3t} & e^{-t} - e^{-3t} \\ -3e^{-t} + 3e^{-3t} & -e^{-t} + 3e^{-3t} \end{pmatrix}$$

$\lambda_1 = -1 \qquad \lambda_2 = -3$

$e^{-t} = \alpha_0 - \alpha_1$

$e^{-3t} = \alpha_0 - 3\alpha_1$

$\to \quad e^{-t} + \alpha_1 = \alpha_0$

$\cdot \; e^{-3t} = e^{-t} + \alpha_1 - 3\alpha_1$

$e^{-3t} - e^{-t} = -2\alpha_1$

$$\frac{e^{-t} - e^{-3t}}{2} = \alpha_1$$

$e^{At} = \alpha_0 I + \alpha_1 A$

Ejemplo:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 27 & -27 & 9 \end{bmatrix}$$

Solución: $\Delta(\lambda) = |A - \lambda I|$

$$= -\lambda^3 + 9\lambda^2 - 27\lambda + 27$$

$$= (3-\lambda)^3 = 0 \Rightarrow \lambda_1, \lambda_2, \lambda_3 = 3$$

$e^{\lambda_1 t} = \alpha_0 + \alpha_1 \lambda_1 + \alpha_2 (\lambda_1)^2$ → derivar con respecto a λ_1

$$e^{3t} = \alpha_0 + 3\alpha_1 + 9\alpha_2$$

$$\rightarrow \quad t e^{\lambda_1 t} = \alpha_1 + 2\alpha_2 \lambda_1$$

$$t e^{\lambda_1 t} = \alpha_1 + 2\alpha_2 \lambda_1$$

$$t e^{3t} = \alpha_1 + 6\alpha_2$$

$$\rightarrow \quad t^2 e^{3t} = 2\alpha_2$$

$$t^2 e^{3t} = 2\alpha_2$$

$$e^{At} = \alpha_0 I + \alpha_1 A + \alpha_2 A^2$$

$$A = \begin{bmatrix} 0 & 100 \\ -1 & 0 \end{bmatrix}$$

$$\lambda_1 = 10i$$

$$\lambda_2 = -10i$$

$$\left.\begin{aligned} e^{10it} &= \alpha_0 + 10i\,\alpha_1 \\ e^{-10it} &= \alpha_0 - 10i\,\alpha_1 \end{aligned}\right\}$$

$$e^{it} = \cos(t) + i\,\text{sen}(t)$$

$$e^{-it} = \cos(t) - i\,\text{sen}(t)$$

$$A = \begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix}$$

$\lambda_1 = -1 \qquad \lambda_2 = -3.$

$$e^{\lambda_1 t} = \alpha_0 + \lambda_1 \alpha_1$$
$$e^{-t} = \alpha_0 - \alpha_1$$
$$e^{\lambda_2 t} = \alpha_0 + \lambda_2 \alpha_1$$
$$e^{-3t} = \alpha_0 - 3\alpha_1$$

$$e^{At} = \alpha_0 I + \alpha_1 \cdot A + \alpha_2 \cdot A^2$$

$$e^{\lambda_1 t} = \alpha_0 + \alpha_1 \lambda_1 + \alpha_2 (\lambda_1)^2$$

$$t\, e^{\lambda_1 t} = \alpha_1 + 2\alpha_2 (\lambda_1)$$

$$t^2 e^{\lambda t} = 2\alpha_2$$

α_n se encuentran a partir de $e^{\lambda t} = \alpha_0 + \alpha_1 \lambda + \dots$

$$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda + \alpha_2 (\lambda)^2$$

$$e^{-2t} = \alpha_0 - 2\alpha_1 + 4\alpha_2$$

$$t e^{-2t} = \alpha_1 + (-2)\alpha_2$$