Hoja 4

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Harim Palma 18001882

Problema 1

Encuentre la serie de Fourier compleja de:

$$f(x) = \begin{cases} 0 & -1 < x < 0 \\ e^{-x} & 0 < x < 1 \end{cases}$$

$L = 1 \qquad w_n = n\pi$

$$C_n = \frac{1}{2L}\int_{-L}^{L} f(x)\, e^{-iw_n x}\, dx$$

$$\Rightarrow C_n = \frac{1}{2}\left(\cancel{\int_{-1}^{1} 0\, e^{-in\pi x}\, dx} + \int_{-1}^{1} e^{-x} \cdot e^{-in\pi x}\, dx\right)$$

$$\Rightarrow C_n = \frac{1}{2}\int_{-1}^{1} e^{-x-in\pi x}\, dx = \frac{1}{2}\int_{-1}^{1} e^{x(-1-in\pi)}\, dx$$

$$= \frac{1}{2}\left(\frac{1}{-1-in\pi}\, e^{x(-1-in\pi)}\Big|_{-1}^{1}\right)$$

$$= \frac{1}{2(-1-in\pi)}\left(e^{(-1-in\pi)} - e^{(1+in\pi)}\right)$$

$$\hat{f}(x) = \sum_{n=-\infty}^{\infty} \frac{1}{2(-1-in\pi)}\left(e^{(-1-in\pi)} - e^{(1+in\pi)}\right)\left(e^{in\pi x}\right)$$

Problema 2

Calcule la serie de Fourier compleja de:

$$f(x) = \sin x \quad 0 < x < \frac{\pi}{2}$$

$L = \frac{\pi}{4} \qquad w_n = \frac{\pi n}{L} = \frac{\cancel{\pi} n 4}{\cancel{\pi}} = 4n$

$$C_n = \frac{1}{2L}\int_{-L}^{L} f(x)\, e^{-iw_n x}\, dx$$

$$\Rightarrow \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin x\, e^{-i4nx}\, dx$$

$U = \sin(x) \qquad V = \frac{-1}{i4n}\, e^{-i4nx}$

$du = \cos(x) \qquad dv = e^{-i4nx}$

$$= \frac{2}{\pi}\left(-\frac{\sin(x)}{i4n}\, e^{-i4nx} + \frac{1}{i4n}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-i4nx} \cos(x)\, dx\right)$$

$U = \cos(x) \qquad V = \frac{-1}{i4n}\, e^{-i4nx}$

$du = -\sin(x) \qquad dv = e^{-i4nx}$

Integral cíclica:

$$\int \sin x \, e^{-i4nx} dx = \frac{-\sin(x)}{i4n} e^{-i4nx} + \frac{1}{i4n}\left(\frac{-\cos(x)\, e^{-i4nx}}{i4n} - \frac{1}{i4n}\int \sin(x)\, e^{-i4nx} dx\right)$$

$$\int \sin x \, e^{-i4nx} dx + \frac{1}{(i4n)^2}\int \sin(x)\, e^{-i4nx} dx = \frac{-\sin(x)}{i4n} e^{-i4nx} - \frac{\cos(x)\, e^{-i4nx}}{(i4n)^2}$$

$$\int \sin x \, e^{-i4nx}\left(1 + \frac{1}{(i4n)^2}\right) = \frac{-\sin(x)}{i4n} e^{-i4nx} - \frac{\cos(x)\, e^{-i4nx}}{(i4n)^2}$$

$$\int \sin x \, e^{-i4nx} dx = \frac{\frac{-\sin(x)}{i4n} e^{-i4nx} - \frac{\cos(x)\, e^{-i4nx}}{(i4n)^2}}{\left(1 + \frac{1}{(i4n)^2}\right)}$$

evaluamos limites y multiplicamos el $\frac{2}{\pi}$

$$\frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin x \, e^{-i4nx} dx = \frac{2}{\pi}\left(\frac{\frac{-\sin(x)}{i4n} e^{-i4nx} - \frac{\cos(x)\, e^{-i4nx}}{(i4n)^2}}{\left(1 + \frac{1}{(i4n)^2}\right)}\Bigg|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\right)$$

$\Rightarrow$ Symbolab lo simplificara 😅

$$\frac{\frac{-\sin(x)e^{-i\cdot 4nx}}{i\cdot 4n} - \frac{\cos(x)e^{-i\cdot 4nx}}{(i\cdot 4n)^2}}{1+\frac{1}{(i\cdot 4n)^2}} = \frac{-4ine^{-4inx}\sin(x) - e^{-4inx}\cos(x)}{16i^2n^2+1}$$

limites :

$$\Rightarrow \left(\frac{-4in\, e^{-\cancel{4}in\frac{\pi}{\cancel{4}}} \sin\left(\frac{\pi}{4}\right) - e^{-\cancel{4}in\frac{\pi}{\cancel{4}}} \cos\left(\frac{\pi}{4}\right)}{(4in)^2+1} - \frac{-4in\, e^{\cancel{4}in\frac{\pi}{\cancel{4}}} \sin\left(\frac{-\pi}{4}\right) - e^{\cancel{4}in\frac{\pi}{\cancel{4}}} \cos\left(\frac{-\pi}{4}\right)}{(4in)^2+1}\right)$$

$$= \left(\frac{-4in\, e^{-in\pi}\left(\frac{\sqrt{2}}{2}\right) - e^{-in\pi}\left(\frac{\sqrt{2}}{2}\right) + 4in\, e^{in\pi}\left(\frac{\sqrt{2}}{2}\right) + e^{in\pi}\left(\frac{\sqrt{2}}{2}\right)}{(4in)^2+1}\right)$$

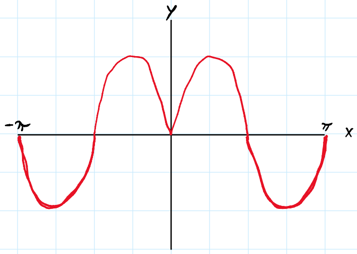
$$\Rightarrow \hat{f}(x) = \sum_{n=-\infty}^{\infty}\left(\frac{-4in\, e^{-in\pi}\left(\frac{\sqrt{2}}{2}\right) - e^{-in\pi}\left(\frac{\sqrt{2}}{2}\right) + 4in\, e^{in\pi}\left(\frac{\sqrt{2}}{2}\right) + e^{in\pi}\left(\frac{\sqrt{2}}{2}\right)}{(4in)^2+1}\right)\left(e^{in\pi x}\right)$$

😖

Problema 3
Para la función $f(x) = \sin x$ para $0 < x < \pi$.

1. Construya una extensión par de la función.
2. Calcule la serie de Fourier compleja de la extensión par.
3. De la serie de Fourier compleja calcule la serie de cosenos de la función original.

1)



2) $w_n = \frac{n\cancel{\pi}}{\cancel{\pi}}$

$$C_n = 2 \cdot \frac{2}{\pi} \int_{-\pi}^{\pi} \sin x \; e^{-inx} \, dx$$

misma integral ciclica del problema 2

$$C_n = \frac{4}{\pi} \left(\frac{-ine^{-inx} \sin(x) - e^{-inx} \cos(x)}{1 + (in)^2} \Bigg|_{-\pi}^{\pi} \right)$$

$$= \frac{4}{\pi} \left(\frac{\cancel{-ine^{-in\pi} \sin(\pi)}^{0} - e^{-in\pi} \cancel{\cos(\pi)}^{-1}}{1 + (in)^2} - \frac{\cancel{-ine^{in\pi} \sin(-\pi)}^{0} - e^{in\pi} \cancel{\cos(-\pi)}^{-1}}{1 + (in)^2} \right)$$

$$= \frac{4}{\pi} \left(\frac{e^{-in\pi} - e^{in\pi}}{1 + (in)^2} \right)$$

$$\hat{f}(x) = \sum_{n=-\infty}^{\infty} \frac{4}{\pi} \left(\frac{e^{-in\pi} - e^{in\pi}}{1 + (in)^2} \right) \left(e^{in\pi x} \right)$$

3) $a_0 = \frac{1}{\pi} \int_0^{\pi} \sin(x) \, dx$

$$= \frac{1}{\pi} \left(-\cos(x) \Big|_0^{\pi} \right)$$

$$= \frac{2}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin(x)\ \cos(nx)\ dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin(x(1+n)) + \cos(x(1-n))\ dx$$

$$= \frac{1}{\pi}\left(\frac{1}{1+n}\left((-1)^n+1\right) + \cancel{\frac{1}{1-n}\sin\left(\pi - n\pi\right)}^{\,0}\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{1+n}\left((-1)^n+1\right)\right)$$

$$f(\hat{x}) = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{1}{\pi}\left(\frac{1}{1+n}\left((-1)^n+1\right)\right) \cos(nx)$$