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Hoja de trabajo No. 3

Problema 1

Representar a los sistemas utilizando ecuaciones de estado (en forma canónica observable, en forma canónica controlable, en forma diagonal, y en forma canónica de Jordan) y ecuaciones de salida.

$$1. G(s) = \frac{6}{s^3 + 6s^2 + 11s + 6}$$

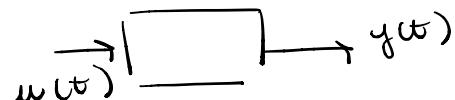
$$2. G(s) = \frac{s^2 + 7s + 5}{s^3 + 9s^2 + 26s + 24}$$

$$3. \ddot{y} + a\dot{y} + by = u + cu$$

canónica observable y controlable \rightarrow

$$3. \ddot{y} + a\dot{y} + by = u + cu$$

$$s^2 Y(s) + asY(s) + bY(s) = U(s) = CSU(s)$$

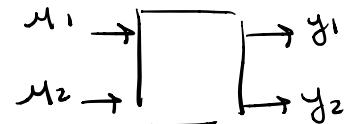


$$\frac{Y(s)}{U(s)} = \frac{CS + 1}{S^2 + AS + B} \quad \text{función de transferencia.}$$

Problema 2

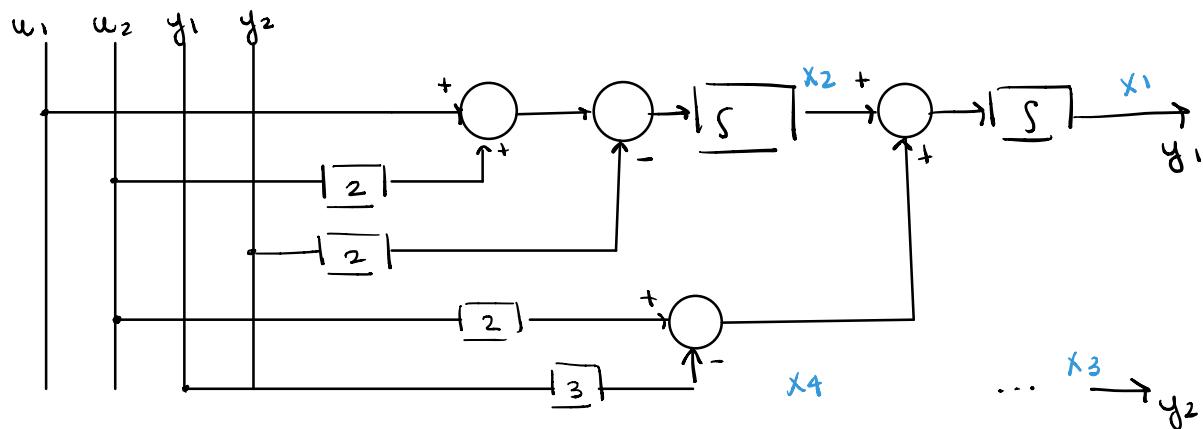
Un sistema con dos entradas y dos salidas se describe con las ecuaciones diferenciales $\ddot{y}_1 + 3\dot{y}_1 + 2y_2 = u_1 + 2u_2 + 2\dot{u}_2$ y $\ddot{y}_2 + 4\dot{y}_1 + 3y_2 = \ddot{u}_2 + 3\dot{u}_2 + u_1$. Representar al sistema utilizando ecuaciones de estado en forma canónica observable.

$$\begin{aligned}\ddot{y}_1 &= u_1 + 2u_2 + 2\dot{u}_2 - 3\dot{y}_1 - 2y_2 \\ &= (2\dot{u}_2 - 3\dot{y}_1) + (u_1 + 2u_2 - 2y_2)\end{aligned}$$



$$\dot{y}_1 = (2u_2 - 3y_1) + \int (u_1 + 2u_2 - 2y_2)$$

$$y_1 = \int [2u_2 - 3y_1 + \int (u_1 + 2u_2 - 2y_2) dt] dt'$$



$$\dot{x}_1 = x_2 + 2u_2 - 3x_1$$

$$\dot{x}_2 = u_1 + 2u_2 - 2x_2$$

$$\dot{x}_3 = u_1 + 2u_2 - 2x_3$$

$$\dot{x}_4 = u_2 + 3x_4 - 2(x_3 + x_4)$$

$$y_1 = x_1$$

$$y_2 = x_3$$

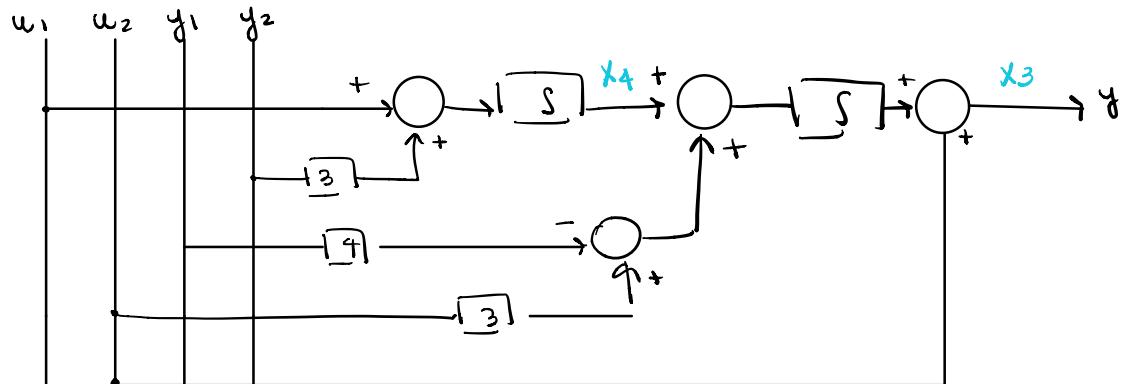
$$y_2 = x_4$$

$$\ddot{y}_2 + 4\dot{y}_1 + 3y_2 = \ddot{w}_2 + 3\dot{w}_2 + w_1$$

$$\ddot{y}_2 = \ddot{w}_2 + [-4\dot{y}_1 + 3\dot{w}_2] + [w_1 - 3y_2]$$

$$\ddot{y}_2 = \ddot{w}_2 + [-4y_1 + 3w_2] + \int [w_1 - 3y_2] dt$$

$$y_2 = w_2 + \int [-4y_1 + 3w_2] + \int [w_1 - 3y_2] dt'$$



$$\dot{x}_3 = x_4 - 4y_1 + 3w_2 = x_4 + 4x_1 + 3w_2 \quad y_1 = x_1$$

$$\dot{x}_4 = u_1 - 3y_2 \quad y_2 = x_3 + u_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ -4 & 0 & 0 & 1 \\ 0 & 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & 0 \\ 0 & 3 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\dot{x}_1 = x_2 + 2u_2 - 3x_1$$

$$\dot{x}_4 = u_1 - 3x_2$$

$$\dot{x}_2 = u_1 - 2x_3$$

$$= u_1 - 3(x_3 + u_2)$$

$$\dot{x}_3 = x_4 + 4x_1 + 3w_2$$

$$= u_1 - 3x_3 - 3u_2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Problema 3

Un sistema se describe con la función de transferencia $G(s) = \frac{s+1}{s^2+7s+6}$.

Representar al sistema utilizando ecuaciones de estado en forma canónica controlable. (El factor común $s + 1$ no debe cancelarse. Si se cancela el sistema sería erróneamente confundido por un sistema de primer orden.)

$$b(s) = \frac{s+1}{s^2+7s+6} = \frac{c(s)}{u(s)}$$

$$(s^2+7s+6)c(s) = u$$

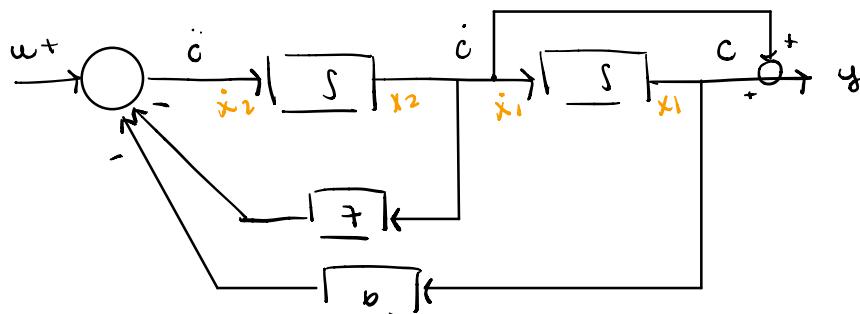
$$\ddot{c} + 7\dot{c} + 6c = u$$

$$\ddot{c} = u - 7\dot{c} - 6c$$

$$y(s) = (s+1)c$$

$$y = cs + c$$

$$y = \dot{c} + c$$



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = u - 7x_2 - 6x_1$$

$$y = x_1 + x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad y = [1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] u.$$

problema extra:

$$b(s) = \frac{s+3}{s^3 + 9s^2 + 4s + 20} = \frac{Y(s)}{W(s)}$$

• canónica observable:

$$(s^3 + 9s^2 + 4s + 20)Y(s) = (s+3)W(s)$$

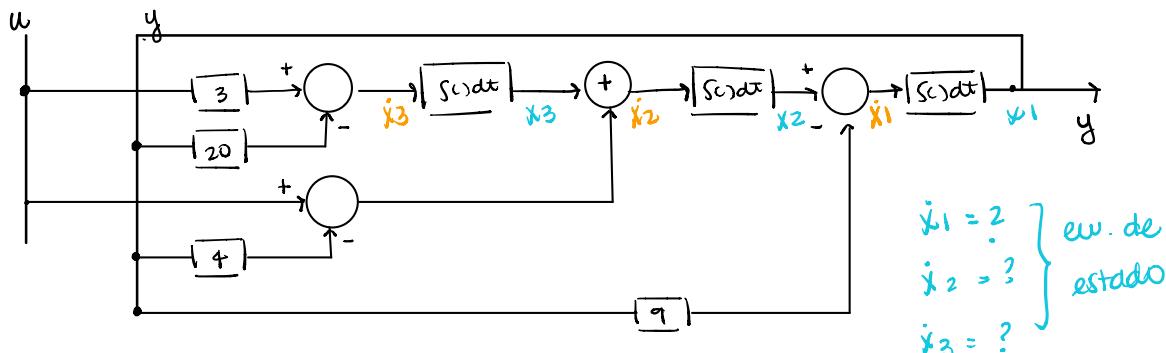
$$\ddot{y}(t) + 9\dot{y}(t) + 4y(t) + 20y(t) = u(t) + 3w(t)$$

$$\begin{aligned}\ddot{y}(t) &= u(t) + 3w(t) - 9\dot{y}(t) - 4y(t) + 20y(t) \\ &= -9\dot{y}(t) + [u(t) - 4y(t)] + [3w(t) - 20y(t)] // s(t)\end{aligned}$$

$$\dot{y}(t) = -9y(t) + [u(t) - 4y(t)] + \int (3w(t) - 20y(t)) dt // s(t)$$

$$\dot{y}(t) = -9y(t) + \int [u - 4y] dt + \int (3w - 20y) dt' \neq dt'' // s(t)$$

$$y(t) = \underbrace{\int [-9y + \int [u - 4y] + \int (3w - 20y) dt' \neq dt'']}_{dt'''}$$



$$\dot{x}_1 = x_2 - 9y = x_2 - 9x_1$$

$$\dot{x}_2 = x_3 + w - 4y = x_3 + w - 4x_1$$

$$\dot{x}_3 = 3w - 20x_1$$

$$y = x_1$$

$$\left. \begin{array}{l} \dot{x}_1 = ? \\ \dot{x}_2 = ? \\ \dot{x}_3 = ? \end{array} \right\} \begin{array}{l} \text{ew. de} \\ \text{estado} \end{array}$$

$$y = ? \quad \begin{array}{l} \text{er de} \\ \text{salida} \end{array}$$

→ usando notación matricial. (FCO: forma canónica observable)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -9 & 1 & 0 \\ -4 & 0 & 1 \\ -20 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} u$$

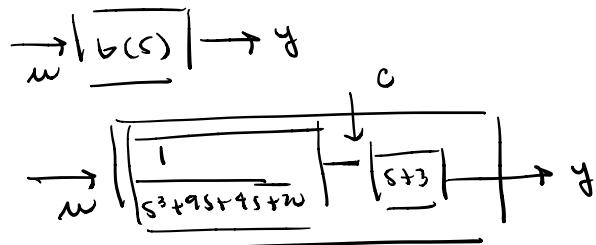
$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

$$y(t) = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0] u$$

$$b(s) = \frac{s+3}{s^3 + 9s^2 + 4s + 20}$$

FCC : Forma canónica controlable



$$b(s) = b_1(s)b_2(s)$$

$$\text{donde } b_1(s) = \frac{1}{s^3 + 9s^2 + 4s + 20}$$

$$b_2(s) = s+3$$

$$\rightarrow \frac{c(s)}{w(s)} = \frac{1}{s^3 + 9s^2 + 4s + 20}$$

$$(s^3 + 9s^2 + 4s + 20)c(s) = w(s)$$

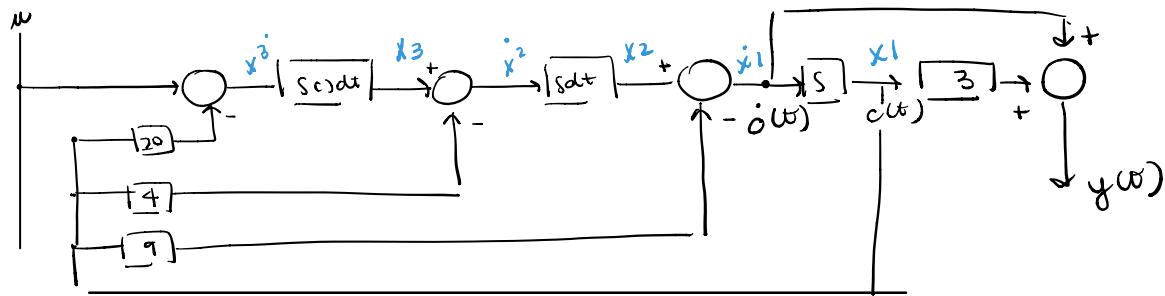
$$\ddot{c}(t) + 9\dot{c}(t) + 4c(t) + 20c(t) = w(t)$$

$$\ddot{c}(t) = -9\dot{c}(t) - 4c(t) + [w(t) - 20c(t)]$$

$$\dot{c}(t) = -9\dot{c}(t) - 4c(t) + \int (w(t) - 20c(t)) dt$$

$$c(t) = -9c(t) + \int \{-4c(t) + \int [w(t) - 20c(t)] dt\} dt$$

$$c(t) = \int (-9c + \int \dot{x}_1 + c + \int [w - 20] dt' \{ dt'' \}) dt'''$$



$$\gamma(s) = (s+3)C(s)$$

$$\dot{x}_1 = x_2 - 9x_1$$

$$y(t) = \dot{o}(t) + 3c(t)$$

$$\dot{x}_2 = x_3 - 4x_1$$

$$\dot{x}_3 = w - 20x_1$$

$$y = 3x_1 + \dot{x}_1$$

$$3x_1 + x_2 - 9x_1$$

$$= x_2 - 6x_1$$

usando mat.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -9 & 1 & 0 \\ -4 & 0 & 1 \\ -20 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w$$

$$y(t) = \begin{bmatrix} -4 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} w$$

Forma normal diagonal \rightarrow "exige factorización"

$$b(s) = \frac{s+3}{s^3 + 9s^2 + 24s + 20}$$

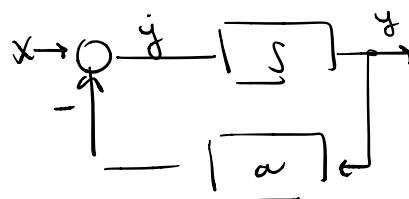
$$= \frac{s+3}{(s+2)^2(s+5)} = \frac{\gamma_1}{\gamma_2} \frac{s+3}{s+2} \rightarrow \frac{1}{s+2} \rightarrow \frac{1}{s+5} \rightarrow q(s)$$

$$x - \left| \frac{1}{s+a} \right| y \quad \frac{y}{x} = \frac{1}{s+a}$$

$$(s+a)y = x$$

$$y + a y = x$$

$$y = x - ay$$



$$\text{jordan} \rightarrow b(s) = \frac{s+3}{s^3 + 9s^2 + 24s + 20}$$

= Fracciones parciales

$$= \frac{2/9}{s+2} + \frac{1/3}{(s+2)^2} - \frac{2/9}{s+5}$$

