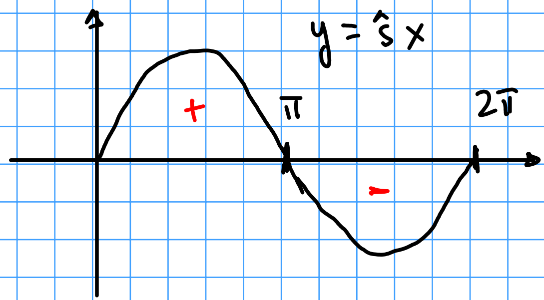
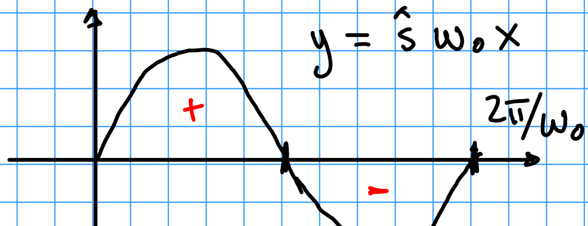


$$\int_0^{2\pi} \hat{s}\, x \, dx = 0$$



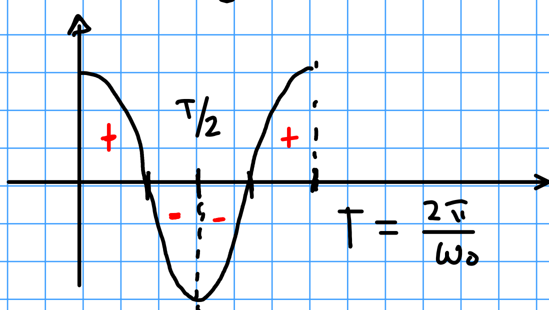
$$\int_0^{2\pi/\omega_0} \hat{s}\, \omega_0 x \, dx = 0$$

ω_0 frecuencia angular

$$f = \frac{\omega_0}{2\pi} \quad Hz$$

$$f = \frac{1}{T} \quad T \text{ periodo}$$

$$T = \frac{2\pi}{\omega_0}$$



$$T = \frac{2\pi}{\omega_0}$$

$$\int_0^{T/2} \cos \omega_0 x \, dx = 0$$

$$e^{i\phi} = \cos\phi + i \sin\phi \qquad e^{-i\phi} = \cos\phi - i\sin\phi$$

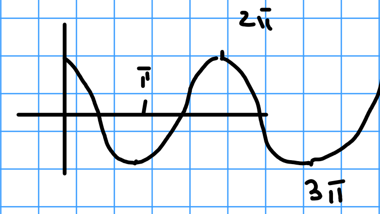
$$(e^{i\phi})^* = e^{-i\phi}$$

$$e^{f+i\omega} = e^{f}\cos\omega + i e^{f}\sin\omega = e^{f} \operatorname{cis}\omega$$

$$\left.\begin{aligned} z &= e^{i\phi} = \cos\phi + i\sin\phi \\ z^* &= e^{-i\phi} = \cos\phi - i\sin\phi \end{aligned}\right\} \Rightarrow \begin{aligned} 2\,\mathrm{Re}\, e^{i\phi} &= \cos\phi \\ 2i\,\mathrm{Im}\, e^{i\phi} &= \sin\phi \end{aligned}$$

$$\int e^{i\omega t}\, dt = \frac{1}{i\omega} e^{i\omega t}$$

$$\int_{-\pi}^{\pi} \cos mx \sin nx \, dx =$$



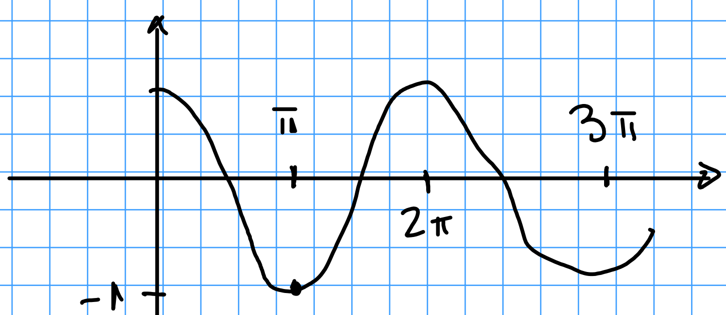
$m, n \in \mathbb{Z} - \{0\}$

$m \neq n$

$$\frac{1}{2}\int_{-\pi}^{\pi} \left(\sin(m+n)x - \sin(m-n)x\right) dx = \frac{1}{2}\left[-\frac{\cos(m+n)x}{m+n} + \frac{\cos(m-n)x}{m-n} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{2}\left(\frac{-(-1)^{m+n}}{m+n} + \frac{(-1)^{m-n}}{m-n} \right) - \frac{1}{2}\left(-\frac{(-1)^{m+n}}{m+n} + \frac{(-1)^{m-n}}{m-n} \right) = 0$$

$m = n \Rightarrow \displaystyle\int_{-\pi}^{\pi} \cos mx \sin mx \, dx = \frac{1}{2}\int_{-\pi}^{\pi} \sin 2mx \, dx = 0$ por simetría.



$$\cos k\pi = \begin{cases} +1 & \text{si } k \text{ par} \\ -1 & \text{si } k \text{ impar} \end{cases}$$

f periodica con periodo 2π

$\{1, \cos x, \cos 2x, \cos 3x, \ldots, \sin x, \sin 2x, \ldots\}$ son ortogonales

con

$$\langle f | g \rangle = \int_{-\pi}^{\pi} f(x) g(x)\, dx$$

- $\langle 1 | \cos mx \rangle = \int_{-\pi}^{\pi} \cos mx\, dx = 0$ — $\langle 1 | \sin mx \rangle = \ldots$
- $\langle \cos mx | \sin nx \rangle = \begin{cases} m = n \\ m \neq n & \text{ver arriba} \end{cases} = 0.$
- $\langle \cos mx | \cos nx \rangle = 0 \quad m \neq n$ — $\langle \sin mx | \sin nx \rangle = 0$

$$f = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \ldots$$

Sabemos que $a_k = \dfrac{1}{\|\cos kx\|^2} \langle \cos kx | f \rangle.$

por simetria.

$$\|\cos kx\|^2 = \langle \cos kx | \cos kx \rangle = \int_{-\pi}^{\pi} \cos^2 kx\, dx = \frac{1}{2} \int_{-\pi}^{\pi} 1 + \underbrace{\cancel{\cos 2kx}}_{0}\, dx$$

$$= \frac{1}{2}\, 2\pi = \pi.$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx\, dx \qquad b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx\, dx$$

$\boxed{\text{Euler - Fourier}}$

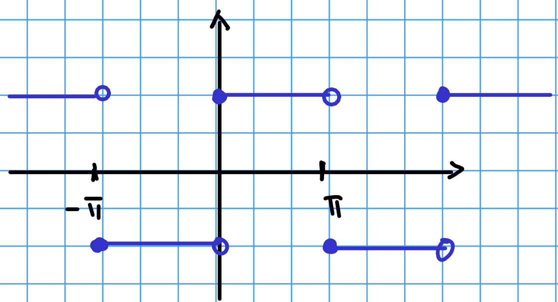
$k = 0: \quad \|1\|^2 = 2\pi. \qquad a_0 = \dfrac{1}{2\pi} \displaystyle\int_{-\pi}^{\pi} f(x)\, dx$

Ej:

$$f(x) = \begin{cases} -k & -\pi \leq x < 0 \\ k & 0 \leq x < \pi \end{cases} \qquad f(x+2\pi) = f(x)$$

$$\hat{f}(x) = a_0 + \sum_{n=1}^{\infty} \{ a_n \cos nx + b_n \sin nx \}$$ serie de Fourier de f.

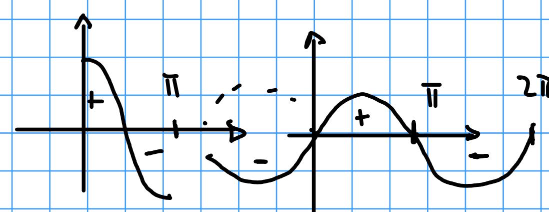
↑ mejor opción $\min \| \hat{f} - f \|$



$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx = 0 \quad \text{por simetría.}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\,dx =$$

$$\frac{1}{\pi} \int_{-\pi}^{0} -k \cos nx\,dx + \frac{1}{\pi} \int_{0}^{\pi} k \cos nx\,dx = 0.$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\,dx = \frac{1}{\pi} \int_{-\pi}^{0} -k \sin nx\,dx + \frac{1}{\pi} \int_{0}^{\pi} k \sin nx\,dx =$$

$$2\frac{k}{\pi} \int_{0}^{\pi} \sin nx\,dx = -\frac{2k}{\pi} \int_{-\pi}^{0} \sin nx\,dx = \frac{2k}{\pi} \cdot \left. -\frac{\cos nx}{n} \right|_{0}^{\pi}$$

$$= \frac{2k}{\pi} \left(\frac{-(-1)^n}{n} - \frac{-1}{n} \right) = \frac{2k}{\pi} \left(\frac{1-(-1)^n}{n} \right) = \frac{2k(1-(-1)^n)}{\pi n}.$$

$$\hat{f}(x) = 0 + \sum_{n=1}^{\infty} \left\{ 0 \cos nx + \frac{2k(1-(-1)^n)}{\pi n} \sin nx \right\}$$

$$\hat{f}_N(x) = \sum_{n=1}^{N} \frac{2k(1-(-1)^n)}{\pi n} \sin nx$$

$$= \frac{4k}{\pi}\sin x + \frac{4k}{3\pi}\sin 3x + \frac{4k}{5\pi}\sin 5x + \dots$$

$$\hat{f}(x) = \frac{4k}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1} = \frac{4k}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)x}{2n+1}$$