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J6004336   AN   Mate 6.

1. $\dfrac{3t e^{-9|t|}}{t^{4}\ \ f(t)} = 3\left[i\dfrac{d}{dw}\ \hat{f}(w)\right]$

$\mathscr{F}_i\left\{e^{-9|t|}\right\} = \dfrac{z(9)}{(81)+w^2}$

$= 3\left[i\dfrac{d}{dw}\left(\dfrac{18}{81+w^2}\right)\right] = 3\left[i\dfrac{d}{dw}\left(18(81+w^2)^{-1}\right)\right]$

$= 54\,i\dfrac{d}{dw}\left(81+w^2\right)^{-1} = -54\,i\,(81+w^2)^{-2}(2w)$

$= -108\,i\,(81+w^2)^{-2}(w).$

2) $2\pi \mathcal{H}(t)\, t\, e^{-2t}$

$$2\pi\, \mathcal{F}\left\{ t\, H(t)\, e^{-2t} \right\} = 2\pi \left(i \frac{d}{dw} \frac{1}{2+iw} \right)$$

$$= 2\pi \left(i \frac{d}{dw} (2+iw)^{-1} \right) = 2\pi i \left(-1 (2+iw)^{-2} (i) \right)$$

$$= -2\pi i^2 (2+iw)^{-2} = \underline{2\pi (2+iw)^{-2}}$$

3. $\dfrac{d}{dt}\left(H(t)\, e^{-3t} \right)$

$$\mathcal{F}\left\{ \frac{d}{dt}\left(H(t)\, e^{-3t} \right) \right\} = iw\, \frac{1}{3+iw}$$

Serie #3.

$$y'' + 6y' + 5y + \delta(t-3)$$

$$\mathscr{F}\{y'' + 6y' + 5y\} = \mathscr{F}\{\delta(t-3)\}$$

$$= (iw)^2\, \hat{y}(w) + 6(iw)\,\hat{y}(w) + 5\hat{y} = e^{-iw3}$$

$$\hat{y}(w)\left[(iw)^2 + 6(iw) + 5\right] = e^{-iw3}$$

$$\hat{y}(w) = \frac{e^{-iw3}}{\left[(iw)^2 + 6(iw) + 5\right]} = \frac{e^{-iw3}}{(iw+1)(iw+5)}$$

$$y(t) = \mathscr{F}^{-1}\left\{\frac{e^{-iw3}}{(1+iw)} \cdot \frac{1}{(iw+5)}\right\}$$

$$y(t) = H(t-3)\, e^{-(t-3)} \ast H(t)\, e^{-5t}$$

Serie #4.

$$\int_{-\infty}^{\infty} |f(t)|^2 \, dt. \Big/ \qquad f(t) = H(t) e^{-2t}$$

$$\int_{0}^{\infty} e^{-4t} \, dt = \frac{-e^{-4t}}{4} \Big|_{0}^{\infty} = \frac{1}{4}$$

Serie 2.

1) $\dfrac{1}{(1+iw)^2} = \dfrac{1}{(1+iw)} \cdot \dfrac{1}{(1+iw)}$

$\mathcal{F}\{f(t) * g(t)\} = \hat{f}(w)\hat{g}(w)$

$\mathcal{F}^{-1}\left(\dfrac{1}{(1+iw)^2}\right) = H(t)e^{-t} * H(t)e^{+t}$

2. $\dfrac{\sin 3w}{w(2+iw)} = \dfrac{\sin 3w}{w} \cdot \dfrac{1}{2+iw}$

$\mathcal{F}^{-1}\left\{\dfrac{\sin(3w)}{w} \cdot \dfrac{1}{2+iw}\right\} =$

$\mathcal{F}^{-1}\left\{\dfrac{1}{2}\left(\dfrac{2}{w}\sin(3w)\right) \cdot \dfrac{1}{2+iw}\right\} =$

$(H(t+3) - H(t-3)) * H(t)e^{-2t}$