

Problema # 4

23/24/02/2021

Calcular función característica

$$\frac{1}{5} \frac{d^2 x}{dt^2} + \frac{2}{5} \frac{dx}{dt} + x = y + 2 \frac{dy}{dt}$$

clase PDT₂Por Superposición $h(t) = h_1(t) + h_2(t)$ PDT₂

$$\frac{1}{5} \frac{d^2 x}{dt^2} + \frac{2}{5} \frac{dx}{dt} + x = y \quad K=1$$

$$T^2 = \frac{1}{5} \quad T = 0.4472$$

$$ZDT = \frac{2}{5} \rightarrow D = \frac{2}{2(5)(T)} = 0.4472$$

$$D^2 = (0.4472)^2 = 0.19998 \approx 0.2$$

Cuando $D^2 < 1$

$$a = -\frac{D}{T} = \frac{-0.4472}{0.4472} = -1$$

soluciones ✓

$$P_1 = -1 - j2$$

$$P_2 = -1 - j2$$

$$b = \frac{\sqrt{1-D^2}}{T} = \frac{\sqrt{1-0.2}}{0.4472} = 2$$

$$a) \quad h_1(t) = 1 - \sqrt{1 + \left(-\frac{1}{2}\right)^2} e^{-t} \cos\left(2t + \tan^{-1}\left(-\frac{1}{2}\right)\right) \quad \checkmark$$

$$h_1(t) = 1 - 1.118 e^{-t} \cos(2t - 26.57^\circ)$$

b) DT₂ (10)

$$\frac{1}{5} \frac{d^2 x}{dt^2} + \frac{2}{5} \frac{dx}{dt} + x = \frac{2dy}{dt} \quad // \int() \quad K=2$$

$$\frac{1}{5} \frac{dx}{dt} + \frac{2}{5} x + \int_0^t x(\tau) d\tau = 2y \quad v = \int_0^t x(\tau) d\tau$$

$$\frac{1}{5} \frac{d^2 v}{dt^2} + \frac{2}{5} \frac{dv}{dt} + v = 2y \quad x = \frac{dv}{dt}$$

$$\frac{dx}{dt} = \frac{d^2 v}{dt^2}$$

clase PT₂

Por h₁(t) sabemos que

$$v(t) = 2 \left[1 - 1.118 e^{-t} \cos(2t - 26.57^\circ) \right]$$

$$x(t) = \frac{dv}{dt}$$

$$= 2 - 2.236 e^{-t} \cos(2t - 26.57^\circ) \quad // \frac{d}{dt}$$

$$h_2(t) = (-2.2336 e^{-t} \cos(2t - 26.57^\circ) - 2.236 e^{-t} \sin(2t - 26.57^\circ)(2))$$

$$= 2.236 e^{-t} \cos(2t - 26.57^\circ) + 4.472 e^{-t} \sin(2t - 26.57^\circ)$$

$$h_2(t) = 2.236 e^{-t} \cos(2t - 26.57^\circ) + 4.472 e^{-t} \sin(2t - 26.57^\circ)$$

Por Superposición h(t) = h₁(t) + h₂(t)

$$h(t) = 1 - 1.118 e^{-t} \cos(2t - 26.57^\circ) + 2.236 e^{-t} \cos(2t - 26.57^\circ) + 4.472 e^{-t} \sin(2t - 26.57^\circ)$$