Altcoin Options Pricing

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1 Executive Summary

We present here our methodology to compute mark prices for options on altcoins. This methodology is used to value options contracts trading on Aevo OTC (https://otc.aevo.xyz/).

The methodology consists in fitting a SABR model for BTC and ETH to model implied volatility using quotes data from Deribit. Once the model is fitted, we adjust the model parameters to reflect the relationship between the realised volatility of the reference asset (ETH in our case) and the altcoin. We also model the forward of the altcoin using perpetual futures data.

2 Introduction

Context: While options on alternative coins (referred to as "altcoins") have been trading OTC and through Decentralised Option Vaults (DOVs) for almost 2 years, there are no liquid markets for these.

Statement of the problem: How can one build a reliable pricing framework to value options on altcoin without a listed options market?

3 Data

In this section, we first describe the relevant data used to calibrate the model along with the data sources. Then, we go over the pre-processing and data cleaning procedures.

3.1 Market Data

As most of the volume (in notional terms) is traded on Deribit (80% of daily notional traded across ETH and BTC options), our primary data source is quotes data from Deribit. For spot and funding data, we use Binance.

3.2 Data Pre-processing

We extract order book data and convert premium prices into implied volatility numbers. To do so, we respect Deribit's conventions of using an annualising factor for 365 for the time to maturity, 0% interest rate and use the forward price as the underlying asset price. We then apply filters to remove illiquid options and in-the-money (ITM) options.

3.2.1 Filtering OTM options

According to put-call parity, implied volatility should be the same for both put and call options with the same strike and same expiry, assuming no transaction costs. However, in the presence of the latter, this conclusion does not hold. As such, we filter out only the most liquid instruments where the spread is the tightest, which will be out-of-the-money options by looking at strikes greater (resp. lower) than the current forward for calls (resp. for puts).

3.2.2 Filtering illiquid options

In addition to filtering out ITM options, we remove quotes where no volume has been traded in the past:

- 24h for options with more than 2 days to expiry
- 12h for options with less than 2 days to expiry

4 SABR Model Specification

The SABR model, which stands for Stochastic Alpha Beta Rho model, is stochastic volatility model which attempts to capture the volatility skew exhibited in derivatives markets. The model, first published by Hagan, Kumar, Lesniewski, and Woodward in 2002, has the following dynamics:

$$
\begin{aligned}
dF_t &= \sigma_t F_t^{\beta} dW_t^1, \\
d\sigma_t &= \alpha \sigma_t dW_t^2, \\
\rho dt &= dW_t^1 \cdot dW_t^2.
\end{aligned}
$$

where:

- F_t is the forward price,
- α, σ_0, β, and ρ are parameters,
- W_t^1 and W_t^2 are two correlated Wiener processes with correlation ρ,
- $\langle dW_t^1, dW_t^2 \rangle$ denotes the quadratic covariation of W_t^1 and W_t^2.

The parameters have the following interpretations:

- α is the volatility of volatility,
- σ_0 is the initial volatility,
- β governs the elasticity of the forward F (with $\beta = 1$ the model reduces to stochastic log-normal, with $\beta = 0$, it reduces to stochastic normal, and with $\beta = 1/2$, it reduces to the stochastic CIR model),
- ρ is the correlation between the forward price and its volatility.

The implied volatility computation for the SABR model is given by:

$$\sigma_{impl} = \frac{\sigma_0 \left(1 + \left(\frac{(1-\beta)^2\sigma_0^2}{24(F_tK)^{(1-\beta)}} + \frac{\rho\beta\alpha\sigma_0}{4(F_tK)^{\frac{1-\beta}{2}}} + \frac{(2-3\rho^2)\alpha^2}{24}\right)T\right)}{(F_tK)^{\frac{1-\beta}{2}}\left(1 + \frac{(1-\beta)^2(\ln(\frac{F_t}{K}))^2}{24} + \frac{(1-\beta)^4(\ln(\frac{F_t}{K}))^4}{1920}\right)} \cdot \frac{z}{\chi(z)}$$

where

$$z = \frac{\alpha}{\sigma_0}(F_tK)^{\frac{1-\beta}{2}} \ln \frac{F_t}{K},$$
$$\chi(z) = \ln\left(\frac{\sqrt{1 - 2\rho z + z^2} + z - \rho}{1 - \rho}\right).$$

which reduces to the following equation for at-the-money options:

$$\sigma_{atm} = \frac{\sigma_0}{F_t^{(1-\beta)}}\left(1 + T\left(\frac{(1-\beta)^2\sigma_0^2}{24F_t^{2-2\beta}} + \frac{\rho\beta\sigma_0\alpha}{4F_t^{1-\beta}} + \frac{(2-3\rho^2)\alpha^2}{24}\right)\right)$$

Once the parameters α, β, σ_0 and ρ are estimated, the implied volatility is only a function of the forward price F_t and and the strike price K.

5 Model Calibration for ETH and BTC options

We first set β to 0.5. The choice of β will not have a large impact on the shape of the volatility skew so we set it to a fixed value for simplicity. In theory, we could get a better estimate of β by running a linear regression on a time series of logs of at-the-money volatilities as a function of the forward.

We are then left with 3 parameters to estimate. We are using a sequential calibration where we first estimate σ_0 by solving the below equation. We use the σ_{atm} to obtain σ_0. The reason behind using this method is that it brings more interpretability to the parameters and it will be crucial when adjusting our surface for each altcoin.

We know that for at-the-money option, the implied volatility simplifies to:

$$\sigma_{atm} = \frac{\sigma_0}{F_t^{(1-\beta)}} \left[1 + T \left(\frac{(1-\beta)^2 \sigma_0^2}{24 F_t^{2-2\beta}} + \frac{\rho\beta\sigma_0\alpha}{4F_t^{1-\beta}} + \frac{(2-3\rho^2)\alpha^2}{24} \right) \right]$$

which can be rewritten as:

$$\sigma_0^3 \left[\frac{T(1-\beta)^2}{24F_t^{2-2\beta}} \right] + \sigma_0^2 \left[\frac{\rho\beta\alpha T}{4F_t^{1-\beta}} \right] + \sigma_0 \left[1 + \frac{(2-3\rho^2)\alpha^2 T}{24} \right] - \sigma_{atm} F_t^{(1-\beta)} = 0$$

At every iteration, we find σ_0 in terms of ρ and α by solving the equation above.

Our minimization problem can thus be written as:

$$(\hat{\sigma_0}, \hat{\rho}, \hat{\alpha}) = \arg \min_{\sigma_0, \rho, \alpha} \sum_{i=1}^{n} w_i \left[\sigma_{i,mkt} - \sigma_{impl}(F_{t,i}, K_i, \sigma_0(\rho, \alpha), \rho, \alpha) \right]^2$$

where

$$w_i = \frac{v_i}{\sum_{i=1}^n v_i}$$

with v_i the volume for this specific option over the past 24h.

6 Altcoin Adjustment

Now that we have a volatility skew parametrization for ETH and BTC options, let us adjust these to for non-listed altcoin.

We will explore how to compute the forward price, then look into how we can adjust our volatility surfaces and then investigate how to handle recently listed altcoins which may have very little data.

6.1 Forward Model

While there may not be a futures market with fixed expiries for every altcoin, most cryptocurrencies have a listed perpetual futures market. The funding rate charged on perpetual futures positions will prove very helpful to estimate the forward rate for a fixed expiry.

For a fixed time to expiry T, in days, we look at the historical funding rate on the perpetual futures. After gathering this data, we compute an exponentially weighted moving average over the period from 3T to the current date, using a half life of $\frac{T}{3}$. This provides us the forward rate and can be written as:

$$r_t = \frac{\sum_{i=0}^{3T} \left(\frac{1}{2}\right)^{\frac{i}{T/3}} x_{t-i}}{\sum_{i=0}^{3T} \left(\frac{1}{2}\right)^{\frac{i}{T/3}}} \tag{1}$$

with x_i the daily average perpetual futures funding rate.
We then compute the forward price as:

$$F_t = S_t e^{r_t(T-t)} \tag{2}$$

6.2 At-the-Money Volatility Adjustment

As described in section 4, we have set the parameter σ_0 to match the current at-the-money implied volatility. To adjust the skew parametrization for altcoins, we will make the following assumptions:

- The relationship between the realised volatility of a reference asset (either ETH or BTC) and the realised volatility of an altcoin remains the same in implied volatility space.
- The slope and curvature of the volatility skew as well as the volatility of volatility is the same between the reference asset and the altcoin.

This leaves us with one last parameter to estimate, σ_0, the initial volatility, which, according to our parametrisation, represents the at-the-money volatility.

To estimate this parameter, we follow the procedure described below:

- Compute the realised volatility of the reference asset and the altcoin using two methodologies, the Parkinson volatility and the Close-to-Close volatility, using spot data sampled over 6 different time intervals: every 2 hours, 4 hours, 6 hours, 8 hours, 12 hours and 1 day. If we are looking at the realised volatility of X days, we will use a dataset of 3X days in total.

  The Parkinson realised volatility is defined as follows:

$$\sigma_{\text{Parkinson}} = 100 \times \sqrt{\frac{1}{4\ln(2)} \times \frac{\text{F}}{\text{n}} \times \left(\sum_{i=1}^{\text{n}} \left(\ln \left(\frac{\text{high}_i}{\text{low}_i} \right) \right)^2 \right)}$$

  The Close-to-Close realised volatility is defined as follows:

$$\sigma_{\text{ctc}} = 100 \times \sqrt{\frac{\text{F}}{\text{n}} \times \left(\sum_{i=1}^{\text{n}} \left(\ln \left(\frac{\text{S}_i}{\text{S}_{i-1}} \right) \right)^2 \right)}$$

Where F is the frequency of returns in a year.

- We then compute the ratio of the altcoin realised volatility to the reference asset realised volatility for each sample period.
- We then compute the exponentially weighted moving average of this ratio, with a half life of $\frac{1}{2}$ the realised volatility period.
- We end up with 6 adjustment ratios for the Close-to-Close realised volatility and 6 adjustment factors for the Parkinson volatility. Out of these, in each group, we take the median ratio.
- Subsequently, we take the average of the median ratios which is our adjustment factor.
- Finally, we compute the estimate of the altcoin σ_0 by multiplying the reference asset σ_0 by our adjustment factor.

This procedure gives us a SABR parametrization to compute the implied volatility for the altcoin.

6.3 Adjustment for Recently Listed Underlyers

Some underlyers may not have enough funding or spot data to compute the aforementioned adjustments. In the case where we do not have data points to compute the realised volatility over the period (i.e. 30 days realised volatility but underlyer was listed 25 days ago), we compute the realised volatility and adjustments over the longest possible available time window. We use a similar procedure for funding data.

7 Results

We display below a slice for the reference asset, here ETH, and a slice for an altcoin, here LDO.

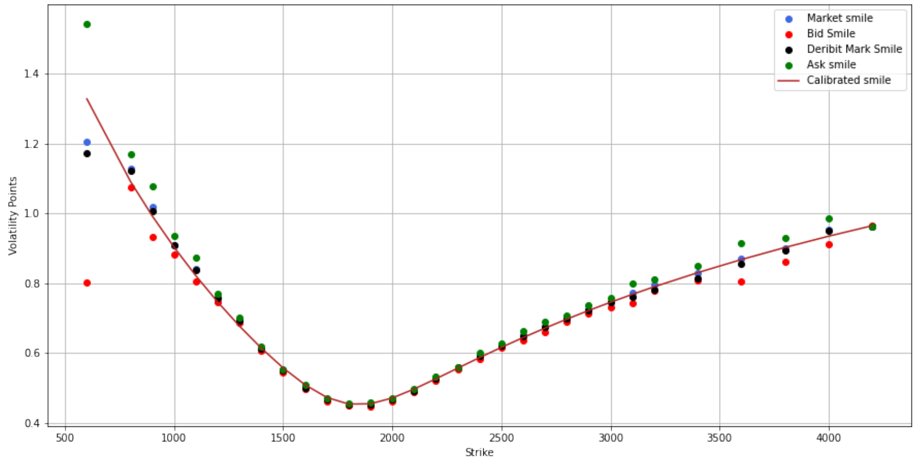

Figure 1: ETH Volatility Smile - 28Jul23 Expiry

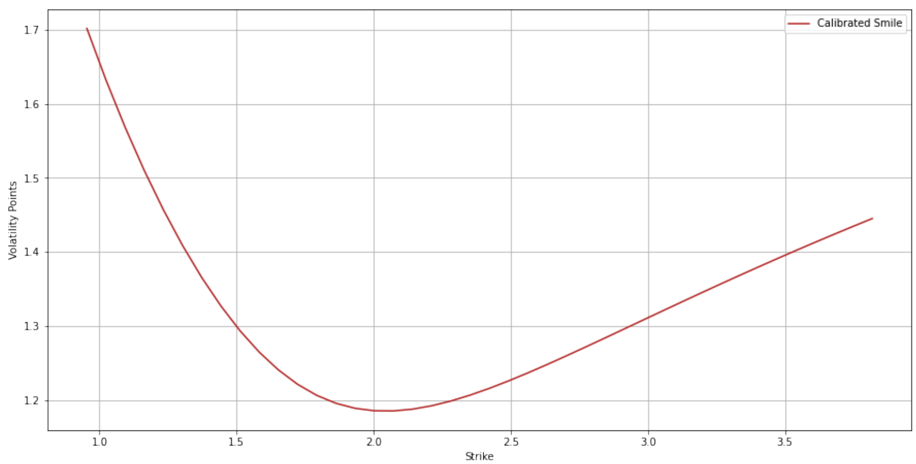

Figure 2: LDO Volatility Smile - 28Jul23 Expiry

Here are the parameters used:

Parameter	ETH	LDO
α	2.8055	2.8055
β	0.5	0.5
ρ	0.0868	0.0868
σ_0	0.4234	1.1130
adj	NA	2.6290

Table 1: Model Parameters for ETH and LDO respectively

8 Limitations

First, volatility does not mean revert in the SABR model. As such, it may not be accurate for longer dated options (empirically, realised volatility mean reverts after 8 months in equities).

Secondly, the model relies a lot on Deribit and Binance data which may become an issue if volume becomes more disitributed across exchanges.

Thirdly, the assumptions made for the altcoin adjustment may not hold, specifically the one regarding the skew having the same curvature as the reference asset, especially in the context of idiosyncratic events affecting the downside/upside skew such as unlocks or other news.

Finally, the model may be inacurrate for recently launched coins due to the lack of data available.