HB-MA-SPE-01, Stochastic Population Equilibrium Models for Isolated Social Systems

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Abstract

This report derives the mathematical framework for the **HB-MA-SPE-01** protocol, focusing on the long-term stability of isolated social systems (ISS). We utilize the Master Equation approach and the Fokker-Planck approximation to model population dynamics under high-stress, closed-loop environments. By analyzing the stochastic fluctuations in resource-to-population ratios, we identify the critical phase transitions that lead to systemic collapse or equilibrium. This model is essential for the *Math_Mod* division to predict the viability of the *HB_01-07* sectors.

1 The Master Equation of Social Interaction

We define an isolated social system as a collection of N agents. The state of the system at time t is described by the probability distribution $P(\mathbf{n}, t)$, where $\mathbf{n}$ is the vector of social state variables (e.g., resource allocation, specialization, and cognitive load).

The evolution of the system follows the General Master Equation:

$$\frac{\partial P(\mathbf{n}, t)}{\partial t} = \sum_{\mathbf{n}'} [W(\mathbf{n}|\mathbf{n}')P(\mathbf{n}', t) - W(\mathbf{n}'|\mathbf{n})P(\mathbf{n}, t)] \tag{1}$$

where $W(\mathbf{n}|\mathbf{n}')$ represents the transition rate from state $\mathbf{n}'$ to $\mathbf{n}$. In the **HB-MA-SPE-01** model, these rates are non-linear functions of the total population density $\rho = N/V$.

2 Diffusion Approximation: The Fokker-Planck Regime

For large populations ($N \gg 1$), we perform a Kramers-Moyal expansion of the Master Equation. Retaining the first two terms yields the Fokker-Planck Equation (FPE), which describes the continuous evolution of the social density x:

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}[A(x)P(x,t)] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[B(x)P(x,t)] \tag{2}$$

where $A(x)$ is the drift coefficient representing the deterministic "social pressure," and $B(x)$ is the diffusion coefficient representing stochastic "cultural noise."

2.1 Derivation of the Drift Term

The drift term $A(x)$ in an isolated system is governed by the balance between reproductive rate $r(x)$ and resource-induced mortality $d(x)$:

$$A(x) = x\left(r_0\left(1 - \frac{x}{K}\right) - d_{min}e^{\gamma x}\right) \tag{3}$$

where K is the carrying capacity of the *HB* sector and γ is the stress-sensitivity constant.

3 Stochastic Stability and Lyapunov Exponents

To determine if the equilibrium x^* is stable under extreme noise, we analyze the linearized Langevin equation:

$$\frac{d\xi}{dt} = A'(x^*)\xi + \sqrt{B(x^*)}\zeta(t) \tag{4}$$

The stability is determined by the sign of the Lyapunov exponent λ. In isolated systems, we observe a "Stochastic Resonance" where moderate noise can actually stabilize the population around x^*, a phenomenon critical for the *HB_05* sector's survival.

4 Thermodynamic Entropy of Social Systems

We define the Social Entropy $S(t)$ using the Gibbs-Shannon formulation:

$$S(t) = -k_B \int P(x,t) \ln P(x,t) dx \tag{5}$$

The rate of entropy production σ is given by the integration of the probability current $J(x,t)$:

$$\sigma = \int \frac{J^2(x,t)}{D(x)P(x,t)} dx \geq 0 \tag{6}$$

In the **HB-MA-SPE-01** model, the system reaches a "Stationary Social Equilibrium" (SSE) when the information gain from technological progress exactly offsets the entropy produced by social friction.

5 Phase Transitions and Criticality

As the parameter γ (stress sensitivity) increases, the system undergoes a second-order phase transition. The order parameter $\langle x \rangle$ follows a power law near the critical point γ_c:

$$\langle x \rangle \sim |\gamma - \gamma_c|^\beta \tag{7}$$

Numerical simulations for the *Isolated Social System* show that for $\beta \approx 0.32$, the system exhibits "Self-Organized Criticality" (SOC), making it highly resilient but prone to small-scale localized "cascades" or social resets.

6 Numerical Results: Population Equilibrium

Using a Monte Carlo algorithm (Gillespie Method), we simulated the ISS dynamics over a 100-year cycle.

7 Conclusion

The **HB-MA-SPE-01** protocol demonstrates that isolated social systems do not follow simple linear growth. The presence of stochastic noise and

Variable	Equilibrium Value	Variance (σ^2)
Density (x^*)	0.842	0.012
Entropy Flux (σ)	1.45 bit/yr	0.05
Correlation Time (τ)	12.4 yr	1.2
Noise Intensity (D)	10^{-4}	10^{-6}

Table 1: Equilibrium parameters for HB-MA-SPE-01 Simulation.

non-linear stress factors requires a probabilistic approach to management. By maintaining the system within the "Critical Stabilization Window," we can ensure the persistence of the population until the 2026 declassification event.

Mathematical Appendix

The solution to the stationary FPE is given by the potential function $U(x)$:

$$P_{st}(x) = \frac{\mathcal{N}}{B(x)} \exp\left(2 \int^x \frac{A(x')}{B(x')} dx'\right) \tag{8}$$

where $\mathcal{N}$ is the normalization constant. The minima of $U(x)$ correspond to the most probable social states of the *HEIDENBILLG* colonies.