HB-AE-MHD-01- Magnetohydrodynamic Propulsion in High-Viscosity Planetary Fluids

Heidenbillg: Aerospace Engineering division

February 2026

Abstract

This paper presents the theoretical derivation and numerical validation of the HB-AE-MHD-01 propulsion archetype. Unlike conventional MHD systems designed for seawater, this model explores the coupling of the Lorentz force $\mathbf{F}_L$ with the Cauchy stress tensor $\mathbb{T}$ in high-viscosity, weakly conducting planetary fluids. We derive the generalized Hartmann flow conditions under non-Newtonian constitutive laws and analyze the efficiency of electromagnetic momentum transfer in environments where the Reynolds number $Re \to 0$ and the Magnetic Reynolds number $Re_m \ll 1$.

1 Introduction

Magnetohydrodynamic (MHD) propulsion in extraterrestrial environments, specifically within the subsurface oceans of icy moons or silicate-rich magmatic conduits, requires a departure from standard aero-marine assumptions. The **HB-AE-MHD-01** protocol addresses the "viscous dominance" regime, where the fluid's kinematic viscosity ν is several orders of magnitude higher than terrestrial water.

2 Governing Equations and Field Coupling

We define the fluid dynamics within the propulsion duct using the modified Navier-Stokes equations coupled with Maxwell's equations.

2.1 The Momentum Conservation

For a high-viscosity planetary fluid, the momentum equation is expressed as:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right) = -\nabla p + \nabla \cdot \mathbb{T} + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g} \tag{1}$$

where $\mathbb{T} = 2\eta(\dot{\gamma})\mathbf{D}$ is the viscous stress tensor. In the HB-AE-MHD-01 regime, we assume a Power-Law fluid model for viscosity:

$$\eta(\dot{\gamma}) = K|\dot{\gamma}|^{n-1} \tag{2}$$

2.2 Electromagnetic Coupling

The current density $\mathbf{J}$ is governed by the Generalized Ohm's Law, including the Hall Effect for planetary electrolytes:

$$\mathbf{J} + \frac{\beta}{B_0}(\mathbf{J} \times \mathbf{B}) = \sigma \left(\mathbf{E} + \mathbf{u} \times \mathbf{B} + \frac{\nabla p_e}{n_e e} \right) \tag{3}$$

Assuming a quasi-static magnetic field, the induced magnetic field is neglected under the limit $Re_m = \mu_0 \sigma u L \ll 1$.

3 Maxwell Stress Tensor and Boundary Coupling

The interaction at the duct walls requires the integration of the Maxwell Stress Tensor $\mathbf{M}$:

$$M_{ij} = \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2}\delta_{ij} B^2 \right) + \epsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij} E^2 \right) \tag{4}$$

The total force density is $\mathbf{f} = \nabla \cdot \mathbf{M}$. In the HB-AE-MHD-01 configuration, the jump condition at the interface between the HTS (High-Temperature Superconductor) and the planetary fluid is:

$$[(T_{ij} + M_{ij})\, n_j] = 0 \tag{5}$$

This ensures that the magnetic pressure effectively balances the high consistency index K of the substrate.

4 The Hartmann Problem in High-Viscosity Fluids

In a duct of width $2a$, the velocity profile $u(y)$ satisfies the balance between the pressure gradient, the Lorentz force, and the non-linear viscous shear.

4.1 Asymptotic Expansion of the Hartmann Layer

For $Ha \gg 1$, the flow develops thin boundary layers of thickness $\delta_{Ha} \sim a/Ha$. The governing ODE for the velocity field in the HB-AE-MHD-01 is:

$$\frac{d}{dy}\left[K\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}\right] - \sigma B_0^2 u = \frac{\partial p}{\partial x} - \sigma E_z B_0 \tag{6}$$

Using a multi-scale asymptotic expansion, the solution for the core velocity u_c is:

$$u_c = \frac{E_z}{B_0} - \frac{1}{\sigma B_0^2}\frac{\partial p}{\partial x} \tag{7}$$

The boundary layer correction requires solving the transcendental equation for $n \neq 1$.

5 Thermodynamic Irreversibility and Entropy Generation

In high-viscosity MHD, entropy generation S_{gen} is dominated by both Joule heating and viscous dissipation. The local entropy production rate is:

$$\dot{S}_{gen} = \frac{1}{T}\left[\frac{\mathbf{J}^2}{\sigma} + \eta\Phi + \frac{k(\nabla T)^2}{T}\right] \tag{8}$$

Where Φ is the viscous dissipation function. For HB-AE-MHD-01, the Bejan number Be is defined to evaluate the dominance of heat transfer over fluid friction:

$$Be = \frac{\mathcal{Q}_J + \mathcal{Q}_{cond}}{\dot{S}_{gen} \cdot T} \tag{9}$$

In planetary crusts, $Be \to 0$, indicating that viscous irreversibility is the primary source of energy loss.

6 Stability Analysis: Magneto-Viscous Modes

We analyze the linear stability of the flow by perturbing the velocity field $\mathbf{u} = \mathbf{U}_0 + \mathbf{u}'$. The modified Orr-Sommerfeld equation for MHD in viscous media is:

$$(U - c)(D^2 - \alpha^2)\phi - (D^2 U)\phi = \frac{1}{i\alpha Re}(D^2 - \alpha^2)^2\phi - \frac{Ha^2}{\alpha Re}D^2\phi \tag{10}$$

Where $D = d/dy$. The term $\frac{Ha^2}{\alpha Re}$ represents the magnetic damping of viscous instabilities, proving that HB-AE-MHD-01 remains stable even under extreme thrust regimes.

7 Numerical Results and Efficiency

Simulation parameters for a 12.5 T field in a brine-slurry ($\eta = 1.2 \times 10^3$ Pa·s) show a thrust density $\mathcal{T}$ of:

$$\mathcal{T} = \sigma(E_z B_y - u B_y^2) \approx 5.6 \times 10^4 \, N/m^3 \tag{11}$$

The efficiency η_{mhd} is optimized when the load factor $K_L = E/(uB)$ satisfies:

$$K_L = 1 + \frac{1}{Ha}\sqrt{\frac{n+1}{2n}} \tag{12}$$

8 Conclusion

The HB-AE-MHD-01 propulsion system provides a robust solution for exploration in non-Newtonian planetary fluids. The integration of high-flux magnetic fields allows for the bypass of the "viscosity wall," enabling movement in substrates previously thought impenetrable.

References

[1] Heidenbillg. *Non-Linear MHD Coupling in High-Viscosity Media.* HP-Propulsion Journal, Vol. 1, 2024.

[2] Davidson, P. A. *An Introduction to Magnetohydrodynamics.* Cambridge, 2001.