Appendix

Formula

1. Random Fluctuations and Why They Are Used

- **Reason**: Engagement levels in real-world systems (e.g., healthcare participation) are inherently unpredictable and influenced by numerous factors such as personal behavior, external policies, or environmental changes.
- **Purpose in the Simulation**:
    - They introduce variability, making the model more realistic by simulating both increases and decreases in engagement levels over time.
    - Random fluctuations also allow the model to reflect potential short-term disruptions (e.g., staff shortages, new initiatives).
- **Example**: If "Patients" engagement starts at 0.6 and fluctuates randomly within [−0.02,+0.02][-0.02, +0.02][−0.02,+0.02], the variability mimics real-world behaviors such as patients being more engaged during health campaigns and less engaged during holidays.

2. Uptake Rate and Why It Is Used

- **Reason**: The uptake rate summarizes the system's overall engagement by averaging the contributions of all stakeholder groups (patients, doctors, nurses, administrators).
- **Purpose in the Simulation**:
    - It provides a high-level metric to evaluate the program's success.
    - By tracking uptake over time, it identifies patterns and trends that can inform decision-making (e.g., whether engagement is improving or declining).
    - Aggregating individual engagement levels into a single uptake rate is useful for comparing across time or scenarios.
- **Example**: If engagement for doctors is high but low for administrators, the uptake rate highlights this gap. Monitoring it monthly ensures timely adjustments.

3. Feedback Loop and Why It Is Used

- **Reason**: Engagement in one period influences future participation, as people’s behavior often follows momentum (positive or negative). For instance:
    - A stakeholder's higher engagement in one month could lead to better results or satisfaction, encouraging continued involvement.
    - Conversely, disengagement could compound due to frustration, leading to further drops.
- **Purpose in the Simulation**:
    - The feedback loop captures these cascading effects, helping simulate long-term dynamics.
    - It reflects realistic cause-and-effect relationships, showing how policies or external shocks affect sustained participation.
- **Example**: If "Doctors" engagement decreases slightly in Month 2, the new lower level sets the baseline for Month 3, leading to a compounding effect unless counteracted by positive fluctuations.

4. Visualizing Trends and Why It Is Important

- **Reason**: Trends over time help stakeholders understand the overall direction of the system and pinpoint areas for intervention.
- **Purpose in the Simulation**:

    - Identifying whether engagement levels are increasing, stable, or declining over time helps evaluate program success.
    - Visualization simplifies complex data, making it accessible for decision-makers.
- **Example**: A plot showing consistent drops in administrator engagement could prompt targeted measures (e.g., new training or incentives).

Why Combine These Elements?

- **Holistic Analysis**: The combination of random fluctuations, uptake rate, and feedback loop provides a realistic, dynamic view of how engagement evolves over time.
- **Practical Decision-Making**: Decision-makers can:
    - Evaluate overall system health (via uptake rate).
    - Understand variability and uncertainty (via random fluctuations).
    - Plan interventions (based on trend analysis and feedback loops).

In summary, these features ensure that the simulation closely mirrors real-world complexities, allowing for actionable insights and effective planning in dynamic systems like healthcare.

Numerical Example - Simulation Steps

- Step 1: Define Initial Engagement Levels
- Step 2: Simulate Engagement Fluctuations for 12 Months
- Step 3: Calculate Uptake Rate for Month 2
- Step 4: Simulate for Month 3
- Step 5: Calculate Uptake Rate for Month 3
- Step 6: Repeat for Remaining Months

Step 1: Define Initial Engagement Levels

We begin with the following **initial engagement levels** for each stakeholder:

- **Patients**: 0.6 (60% engaged)
- **Doctors**: 0.8 (80% engaged)
- **Nurses**: 0.75 (75% engaged)
- **Administrators**: 0.5 (50% engaged)

Step 2: Simulate Engagement Fluctuations for 12 Months

Next, we apply random fluctuations (within the range $[-0.05, +0.05]$[-0.05, +0.05][−0.05,+0.05]) each month to the initial engagement levels. Let's simulate this process over the first few months, keeping the fluctuations smaller for simplicity.

Month 1:

- **Initial Engagement** (for all stakeholders):
    - **Patients**: 0.6
    - **Doctors**: 0.8
    - **Nurses**: 0.75
    - **Administrators**: 0.5

Month 2 (First Fluctuation):
Assume the random fluctuations for each stakeholder are as follows:

- **Patients**: $\Delta = +0.03$
- **Doctors**: $\Delta = -0.02$
- **Nurses**: $\Delta = +0.01$
- **Administrators**: $\Delta = +0.04$

Now, apply the formula for **New Engagement**:

$$\text{New Engagement}_t = \max(0, \min(1, \text{Previous Engagement}_{t-1} + \Delta))$$

- **Patients**:

$$0.6 + 0.03 = 0.63$$

No need for clipping, so the new engagement for Patients is **0.63**.

- **Doctors**:

$$0.8 - 0.02 = 0.78$$

No need for clipping, so the new engagement for Doctors is **0.78**.

- **Nurses**:

$$0.75 + 0.01 = 0.76$$

No need for clipping, so the new engagement for Nurses is **0.76**.

- **Administrators**:

$$0.5 + 0.04 = 0.54$$

No need for clipping, so the new engagement for Administrators is **0.54**.

Month 2 Engagement Levels:

- **Patients**: 0.63
- **Doctors**: 0.78
- **Nurses**: 0.76
- **Administrators**: 0.54

Step 3: **Calculate Uptake Rate for Month 2**

The **uptake rate** is the average engagement across all stakeholders:

$$\text{Uptake Rate} = \frac{\sum \text{Engagement Levels}}{N}$$

For Month 2:

$$\text{Uptake Rate} = \frac{0.63 + 0.78 + 0.76 + 0.54}{4} = \frac{2.71}{4} = 0.68$$

So, the uptake rate for **Month 2** is **0.68**.

Step 4: **Simulate for Month 3**

Now, we continue to simulate the engagement levels for Month 3 with new random fluctuations.

Assume the fluctuations for each stakeholder are:

- **Patients**: $\Delta = -0.02$
- **Doctors**: $\Delta = +0.01$
- **Nurses**: $\Delta = -0.03$
- **Administrators**: $\Delta = -0.01$

Apply the formula for **New Engagement**:

- **Patients**:

$$0.63 - 0.02 = 0.61$$

No need for clipping, so the new engagement for Patients is **0.61**.

- **Doctors**:

$$0.78 + 0.01 = 0.79$$

No need for clipping, so the new engagement for Doctors is **0.79**.

- **Nurses**:

$$0.76 - 0.03 = 0.73$$

No need for clipping, so the new engagement for Nurses is **0.73**.

- **Administrators**:

$$0.54 - 0.01 = 0.53$$

No need for clipping, so the new engagement for Administrators is **0.53**.

Month 3 Engagement Levels:

- **Patients**: 0.61
- **Doctors**: 0.79
- **Nurses**: 0.73
- **Administrators**: 0.53

Step 5: **Calculate Uptake Rate for Month 3**

Again, calculate the uptake rate:

$$\text{Uptake Rate} = \frac{0.61 + 0.79 + 0.73 + 0.53}{4} = \frac{2.66}{4} = 0.665$$

So, the uptake rate for **Month 3** is **0.665**.

Summary of the First 3 Months:

Month	Patients	Doctors	Nurses	Administrators	Uptake Rate
1	0.6	0.8	0.75	0.5	-
2	0.63	0.78	0.76	0.54	0.68
3	0.61	0.79	0.73	0.53	0.665

Step 6: Repeat for Remaining Months

The same process is repeated for the remaining months (Month 4 to Month 12), where new fluctuations are applied each month to simulate the evolution of engagement levels and the corresponding uptake rate. The final uptakes are plotted over time to visualize the model's performance.

Final Thoughts:

This numerical example illustrates how:

- Engagement levels fluctuate based on random changes each month.
- The uptake rate is calculated as the average of the engagement levels.
- A feedback loop exists, where each month's engagement influences the next, and this pattern can be visualized to see the overall trend.

By running the entire code, we will get a series of engagement levels and uptake rates, which help simulate and analyze the healthcare participatory model.

Formula for Simulating Engagement:

For each stakeholder, the engagement level fluctuates over 12 months based on a random fluctuation range, where the fluctuation is defined as ±5% (0.05) of the previous month's engagement level. The formula for updating the engagement level each month is:

$$\text{New Engagement}_t = \max(0, \min(1, \text{Previous Engagement}_{t-1} + \Delta))$$

Where:

- New Engagement_t is the engagement level at the current month t,
- $\text{Previous Engagement}_{t-1}$ is the engagement level of the previous month,
- Δ is a random change drawn from a uniform distribution within the range $[-\text{fluctuation}, \text{fluctuation}]$, i.e., $\Delta \sim U(-0.05, 0.05)$,
- The $\max(0, \min(1, \cdot))$ operation ensures the engagement level stays between 0 and 1, as engagement cannot be negative or exceed 1.

This process is repeated for each stakeholder over the 12 months.

Summary of Key Parameters:

- **Initial Engagement**: This is the base engagement level set for each stakeholder at the start of the simulation (e.g., 0.6 for patients, 0.8 for doctors, etc.).
- **Fluctuation**: This defines the amount by which the engagement level can change from month to month, which is ± 0.05 in the code.
- **Months**: The simulation is carried out over a period of 12 months.

In terms of distribution, this approach simulates fluctuating engagement levels, which is visualized using a histogram with a Kernel Density Estimate (KDE) plot. The KDE provides a smooth estimate of the probability density function of the engagement levels across stakeholders.

In the context of this code, **engagement** refers to the level of involvement, participation, or interaction that different stakeholder groups (such as **patients**, **doctors**, **nurses**, and **administrators**) have with a given project, platform, or system over time. It is typically a measure of how actively these groups are engaging with certain activities, services, or interventions.

Key Points about Engagement:

- **Quantitative Representation**: In the code, engagement is represented as a value between **0 and 1**, where:
  - **0** indicates **no engagement** (complete disengagement or no interaction),
  - **1** indicates **full engagement** (maximum possible participation or interaction).
- **Fluctuations Over Time**: The engagement levels can change over time, which is reflected by the fluctuating values in the simulation. For example, engagement could increase if a stakeholder group becomes more involved or decrease if they lose interest or face barriers to participation.

- **Stakeholder Groups**: The engagement level is simulated for different groups, such as:
  - **Patients**: How involved patients are in a healthcare system, project, or study.
  - **Doctors**: The degree of participation or involvement of doctors in a healthcare setting or initiative.
  - **Nurses**: The level of engagement of nursing staff in activities, events, or initiatives.
  - **Administrators**: The level of involvement of administrative personnel in decision-making, planning, or day-to-day operations.

Examples of Engagement in Different Contexts:

1. **Healthcare Systems**:
   - Patients might be engaged if they are actively participating in their treatment plans, attending check-ups, or using health tracking apps.
   - Doctors could be engaged if they actively contribute to patient care, participate in meetings, or use a digital health platform for communication.
   - Nurses' engagement could be measured by their involvement in patient care, training programs, or digital communication.
   - Administrators' engagement might involve their participation in the planning and execution of policies or strategies within the healthcare system.
2. **Online Platforms**:
   - For social media, **engagement** could mean how often users interact with posts (likes, comments, shares).
   - In a business context, **engagement** might refer to how frequently employees are participating in company programs or contributing to team projects.

Importance of Tracking Engagement:

By tracking engagement, organizations can gain valuable insights into the effectiveness of their strategies, identify which groups are more or less involved, and tailor interventions or improvements accordingly. The fluctuation of engagement levels can indicate areas where additional support or attention is needed to maintain or increase participation.

$$\text{New Engagement}_t = \max(0, \min(1, \text{Previous Engagement}_{t-1} + \Delta))$$

Where:

- $\text{Previous Engagement}_{t-1}$ is the engagement level from the previous month.
- Δ is the random fluctuation (chosen within the range $[-0.05, 0.05]$).
- The $\max(0, \min(1, \cdot))$ ensures that the engagement stays within the bounds of 0 and 1.

Example 1: Starting Engagement = 0.6

Let's assume the **initial engagement** ($\text{Previous Engagement}_{t-1}$) for month 1 is 0.6, and we apply the fluctuation to calculate the new engagement for the next month.

Step 1: Month 1 to Month 2

- **Previous Engagement (Month 1)** = 0.6
- **Random fluctuation (Δ)**: Let's assume $\Delta = +0.03$ (this could be any value in the range $[-0.05, 0.05]$).

$$\text{New Engagement}_2 = \max(0, \min(1, 0.6 + 0.03)) = \max(0, \min(1, 0.63)) = 0.63$$

So, the engagement for **Month 2** is **0.63**.

Step 2: Month 2 to Month 3

- **Previous Engagement (Month 2)** = 0.63
- **Random fluctuation (Δ)**: Let's assume $\Delta = -0.04$ (again, randomly chosen within the fluctuation range).

$$\text{New Engagement}_3 = \max(0, \min(1, 0.63 - 0.04)) = \max(0, \min(1, 0.59)) = 0.59$$

So, the engagement for **Month 3** is **0.59**.

Step 3: Month 3 to Month 4

- **Previous Engagement (Month 3)** = 0.59
- **Random fluctuation (Δ)**: Let's assume $\Delta = +0.07$.

$$\text{New Engagement}_4 = \max(0, \min(1, 0.59 + 0.07)) = \max(0, \min(1, 0.66)) = 0.66$$

So, the engagement for **Month 4** is **0.66**.

Step 4: Month 4 to Month 5

- **Previous Engagement (Month 4)** = 0.66
- **Random fluctuation (Δ)**: Let's assume $\Delta = -0.10$ (a larger decrease in engagement).

$$\text{New Engagement}_5 = \max(0, \min(1, 0.66 - 0.10)) = \max(0, \min(1, 0.56)) = 0.56$$

So, the engagement for **Month 5** is **0.56**.

Example Summary of Engagement Levels Over Time:

Month	Previous Engagement	Fluctuation (Δ)	New Engagement
1	0.60	-	0.60
2	0.60	+0.03	0.63
3	0.63	-0.04	0.59
4	0.59	+0.07	0.66
5	0.66	-0.10	0.56

Key Points:

- The engagement fluctuates each month within the range of 0 to 1.
- The randomness in fluctuations introduces variability in the engagement levels over time.
- The formula ensures that engagement never exceeds 1 (maximum engagement) or goes below 0 (no engagement).

This example shows how the engagement level evolves from month to month with random fluctuations based on the initial engagement level.

The code simulates stakeholder engagement levels over time and analyzes the impact of these engagement levels on a model's uptake rate. Let's break down the key formulas and concepts used in this code:

1. Engagement Simulation Formula:

In the function `simulate_stakeholder_engagement`, the **engagement levels** are simulated over a 12-month period for different stakeholders (Patients, Doctors, Nurses, Administrators).

For each stakeholder, the **engagement level** fluctuates monthly, and the formula used to update the engagement for each month is:

$$\text{New Engagement}_t = \max(0, \min(1, \text{Previous Engagement}_{t-1} + \Delta))$$

Where:

- $\text{Previous Engagement}_{t-1}$ is the engagement level from the previous month.
- Δ is the random fluctuation applied to the engagement, drawn from a uniform distribution within the range $[-0.05, +0.05]$, i.e., $\Delta \sim U(-0.05, +0.05)$.
- $\max(0, \cdot)$ ensures that the engagement value cannot go below 0.
- $\min(1, \cdot)$ ensures that the engagement value cannot exceed 1.

This results in an engagement level between 0 and 1 for each month, with fluctuations added due to external factors (like changes in participation or interest).

2. Model Uptake Rate Formula:

The function `model_uptake_rate` calculates the **uptake rate** based on the **engagement levels** of all stakeholders.

The formula for the **uptake rate** is:

$$\text{Uptake Rate} = \frac{\sum \text{Engagement Levels}}{N}$$

Where:

- $\sum \text{Engagement Levels}$ is the total engagement from all stakeholders (sum of engagement levels for all groups).
- N is the number of stakeholders (in this case, 4: Patients, Doctors, Nurses, Administrators).

This formula calculates the average engagement level across all stakeholders, which is used as the **uptake rate**.

3. **Uptake Rate Feedback Loop:**

Each month, the engagement levels of the stakeholders are updated with small random fluctuations, and the **uptake rate** is recalculated based on the updated engagement levels. This is done in the following way:

- **Updated Engagement Levels**: For each stakeholder, a random fluctuation between $[-0.02, +0.02]$ is applied to their current engagement level. The fluctuation is calculated using:

$$\text{Updated Engagement}_t = \text{Previous Engagement}_t + \Delta_{\text{fluctuation}}$$

  where $\Delta_{\text{fluctuation}} \sim U(-0.02, +0.02)$.

- The updated engagement levels are then used to calculate the **new uptake rate** using the formula mentioned earlier.

This process simulates the dynamic change in engagement and its impact on the overall uptake rate over the 12-month period.

Summary of Key Formulas:

1. **Engagement Level Update**:

$$\text{New Engagement}_t = \max(0, \min(1, \text{Previous Engagement}_{t-1} + \Delta))$$

where $\Delta \sim U(-0.05, +0.05)$.

2. **Uptake Rate**:

$$\text{Uptake Rate} = \frac{\sum \text{Engagement Levels}}{N}$$

where N is the number of stakeholders (4 in this case).

3. **Updated Engagement for Feedback Loop**:

$$\text{Updated Engagement}_t = \text{Previous Engagement}_t + \Delta_{\text{fluctuation}}$$

where $\Delta_{\text{fluctuation}} \sim U(-0.02, +0.02)$.

These formulas simulate how engagement fluctuates over time and how that, in turn, impacts the overall uptake of the healthcare model or intervention being studied.