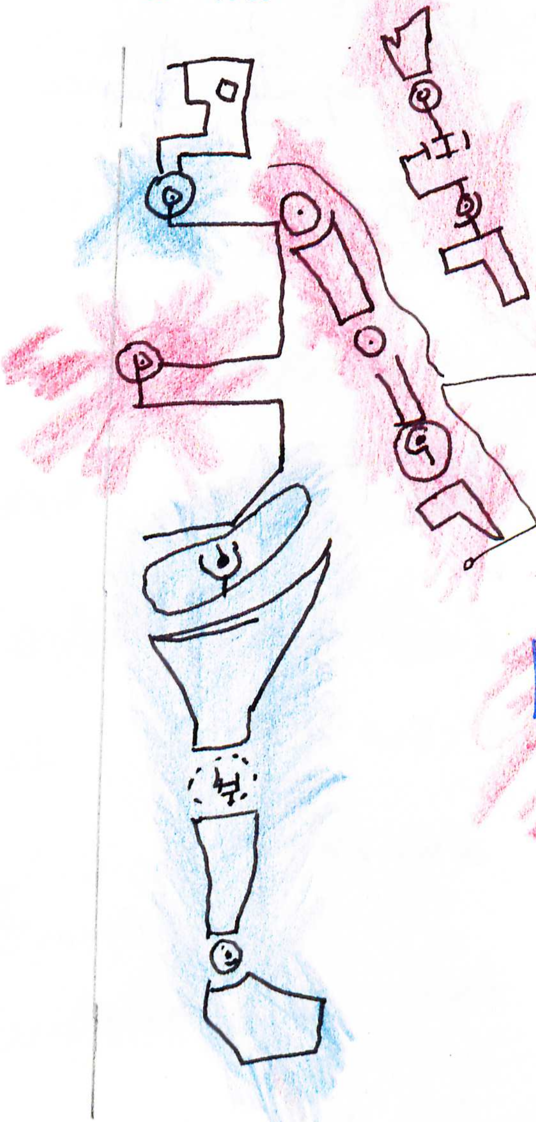
Inverse Kinematics

Inverse kinematics [5]

Human vs Robot

Jacobian forn

$$\begin{pmatrix} x_{11} & x_{12} \cdots \\ x_{21} & \\ \vdots & \\ x_{i1} & \end{pmatrix}$$



A model of human skeleton as a kinematics chain allow positioning using inverse kinematics The Jacobian inverse tech.

Jacobian form.

he

ARM & Degree of freedom

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STATistical Methods

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Statistical Methods
7. Oct. 2016 Prof. Leonardo Ricci

Experement → outcome

samples

sample space Ω = {outcomes}

S

exp: coin toss

outcoms "T" "H"

$\Omega = \{T, H\}$

outcomes: (1, 2, 3, 4, 5, 6)

Ω = (1, 2, 3, 4, 5, 6)

discrete
contable . sample spaces

continuos sample spaces

event: a set of outcomes
i.e. is a subset of Ω

2

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discrete
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continuos sample spaces

event: a set of outcomes

i.e. is a subset of Ω

$A \cap B = \emptyset$

$A \cup B = \Omega$

$A \cap C = (1, 2, \cancel{3} \cancel{3}$

$A \cup C = (1, 2, 3, \cancel{4}, 5,$

$A \setminus C = 5 \quad C \setminus A = 2$

(venn diagram)

A 1 3 5

B 2 4 6

C 1 2 3

D 4 5 6

$P : \text{event} \rightarrow [0, 1)$

1 $P(\text{event}) \geq 0$

2 $P(\Omega) = 1$

3 $A \cap B = \emptyset$

$P(A \cup B) = P(A) + P(B)$

$P(\emptyset) = ?$ 0

$A = \Omega$

$B = \emptyset \quad A \cap B = \emptyset$

$A \cup B = \Omega \quad P(A \cup B) = P(A) + P(B)$

$1 = 1 + P(\emptyset) \quad P(\Omega) = P(\Omega) + P(\emptyset)$

$\Omega - A$

$P(\Omega \setminus A) = ?\quad 1 - P(A)$

$A \quad B \quad = \Omega \setminus A$

$A \cap B = \emptyset$

$A \cup B = \Omega$

$1 = P(A) + P(\Omega \setminus A)$

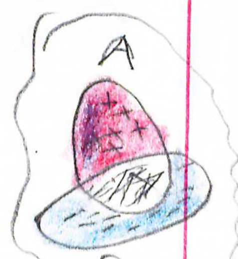
$(\quad P[A \cup (\Omega \setminus A)])$

subjected expression :-

Law of addition

$P(A \cup B) = P(A) + P(\cancel{B} \cap B)$

$A \cup B \quad = B \cup (A \setminus B)$



$B \cap (A \setminus \cancel{B})$

$\emptyset$

$P(A \cup B) = P(B) + P(A \setminus B)$

$(\cancel{A} \setminus B)$

$A = (A \setminus B) \cup (A \cap B)$

$(A \setminus B) \cap (A \cap B) = \emptyset$

$P(A) = P(A \setminus B) + P(A \cap B)$

$P(A \setminus B) = P(A) - P(A \cap B)$

Ex 500 shoppers

308 buy X

266 buy y

103 buy both (x & y)

59 buy neither (X nor y)

$308 + 266 - 103 + 59 = 500$

$530 \neq 500$

C.P.

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$P(B) \neq 0$ * Conditional probability

law of multiplication

$$P(A\cap B) = P(A|B) \cdot P(B)$$

Independent:

A and B are independent iff

$$P(A\cap B) = P(A)\,P(B)$$

$\Longleftrightarrow P(A|B) = P(A)$ $\quad P(B|A) = P(B)$

EX: $A = \{1,2,3\}$ $\quad P(A) = 1/2$

$B = \{1,2,6\}$ $\quad P(B) = 1/2$

$A\cap B = \{2\}$ $\quad P(A\cap B) = \frac{1}{6}$

EX:

$A = \{\heartsuit\}$ $\quad P(\heartsuit) = \frac{1}{4}$

$B = \{K\}$ $\quad P(K) = \frac{1}{13}$

$P(K \cap \heartsuit) = \frac{1}{52}$

(independent)

EX:

$P(\heartsuit) = \frac{13}{53}$

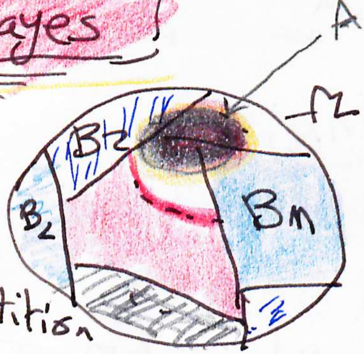
$P(K) = \frac{4}{53}$

$P(K \cap \heartsuit) = \frac{1}{53}$

$\frac{1}{53} \neq \frac{13}{53} \cdot \frac{4}{53} = \frac{52}{53} \frac{1}{53}$

(dependent.)

Bayes



Partition

Assuming that:

$P(B_i)$ $\forall n$

$P(A \backslash B_i)$

known

B_1 --- B_n

prior Prob.

$P(A) = ?$

$$\begin{cases} B_I \cap B_J = \emptyset \\ \bigcup_{n=1}^{n} B_i = \Omega \end{cases}$$

$P(B_i \backslash A) = ?$

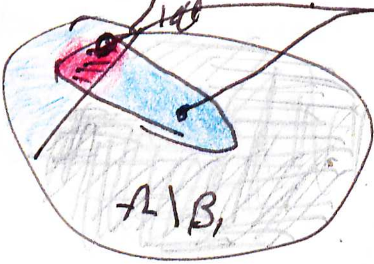
conditional Prob.

conditional Prop.

$P(A) = ?$ Rule of elimination
Rule of total probability

$P(B_i \backslash A) =$ Bayes' theorem.

$$A = (A \cap B_1) \cup [A \cap (\Omega \setminus B_1)$$

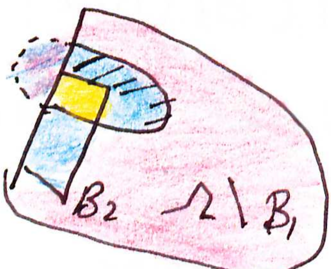


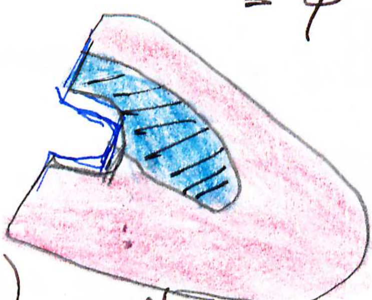
$$(A \cap B_1) \cap [A \cap (\Omega \setminus B_1)] = \emptyset$$

$$P(A) = P(A \cap B_1) + P[A \cap (\Omega \setminus B_1)]$$

$$A \cap (\Omega \setminus B_1) = (A \cap B_2) \cup \{A \cap [\Omega \setminus (B_1 \cup B_2)]\}$$

$$\cap = \emptyset$$





$$P(A) = \sum_{i=1}^{n} P(A \cap B_i) = \sum_{i=1}^{N} P(A \mid B_i) P(B_i)$$

$$P(B_i \mid A) = \frac{P(B_i \cap A)}{P(A)}$$

$$P(A \mid B_i) = \frac{P(A \cap B_i)}{P(B_i)}$$

$$\boxed{P(B_i \mid A) = \frac{P(A \mid B_i)\, P(B_i)}{P(A)}}$$

* A and B are independent

IFF $P(A \cap B) = P(A)\, P(B)$

$\Leftrightarrow P(A \mid B) = P(A) \Leftrightarrow \left(P(B \mid A) = P(B)\right)$

$P(A)\, \cancel{P(B)} \left[= P(A \cap B) =\right] P(A \mid B)\, \cancel{P(B)}$

$\cancel{P(A)}\, P(B) \left[= P(A \cap B) =\right] P(B \mid A)\, \cancel{P(A)}$

$P(A \mid B) = P(A)$

$$P(B \mid A) = \frac{P(A \mid B)\, P(B)}{P(A)}$$

$$= \frac{\cancel{P(A)}\, P(B)}{\cancel{P(A)}} = P(B)$$

EX:

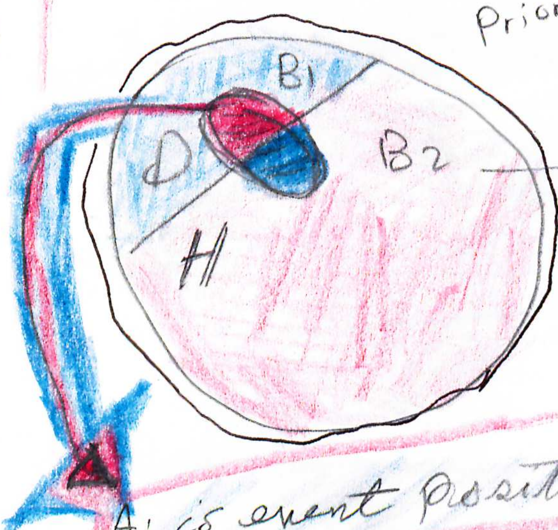
8%	D	over 50S prior prob.	
95%	+	HITS	
5%	−	MISSES	
H	2%	+	FAlse Alarms
	98%	−	Correct Rejections

8% D over 50 years old (prior probability)

	95%	+	Hits
	5%	−	Misses
H	2%	+	False Alarmes
	98%	−	correct Rejection

Find Prob. (having D | Positive Report)

$$= \frac{P(\text{Positive Report} \mid \text{having D}) \cdot P(\text{Having D})}{P(\text{Positive Report})}$$



Prior $P(D) = 0.08$

$P(H) = 0.92$

$P(A|D) = 0.95$

$P(A|H) = 0.02$

conditional Prob.

$\boxed{A: \text{ is event positive test.}}$

$$P(D|A) =? \frac{P(A|D)\,P(D)}{P(A)}$$

$$P(A) = P(A|D) \cdot P(D) + P(A|H)\,P(H)$$

$$= 0.95 * 0.08 + 0.02 * 0.95$$

$$\boxed{P(A) = \frac{760 + 184}{10\,000} = 0.0944}$$

$$= \frac{0.95 \cdot 0.08}{0.0944} = \frac{760}{944}$$

$$\boxed{P(D|A) \simeq 0.805}$$

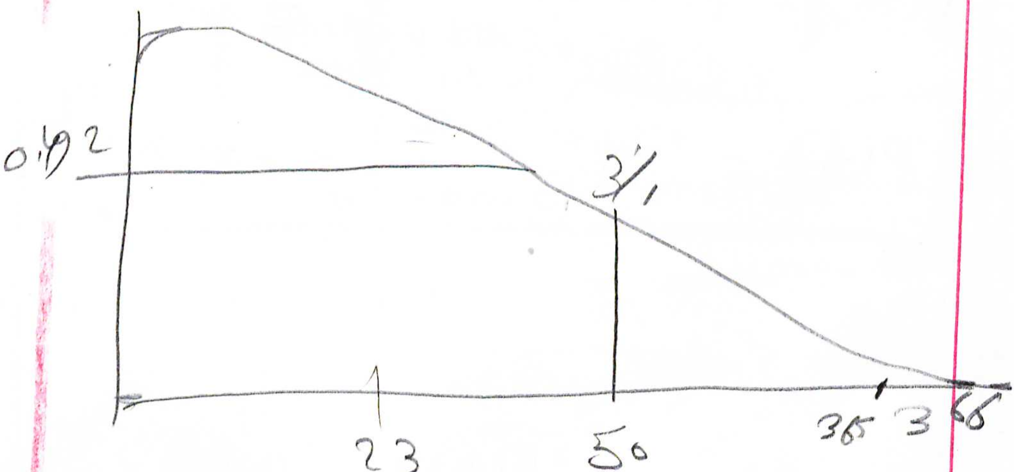
7

$$p = \frac{365!}{365^n (365-n)!}$$

$$P_0(n) = \frac{365}{365^n \; (365-n)!}$$

$$\frac{365}{365^{23} \; 342!}$$

$$\frac{365!}{365^{21} \; 344!}$$



* distinguish between event and probability

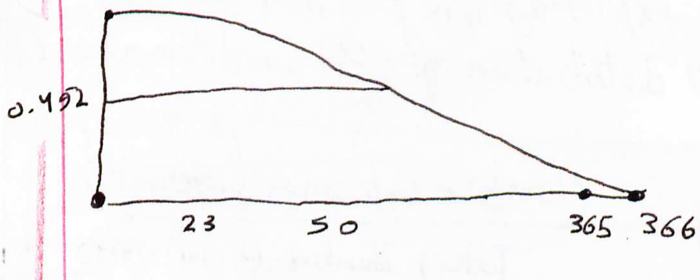
* dependent and independent probability

0.76	0.09	0.02	0.01	0.02	0.10
0	1	2	3	4	5

$$P_0 = \frac{365!}{365^n \; (365-n)!}$$

$$P_{23} = \frac{365!}{365^{23} \; (365-n)!}$$

$$P_{21} = \frac{365!}{365^{21} \; 344!}$$



or Prob

Likelihood

Prior Prob

$$B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

Conditional Prob. Density function

Marginal Likelihood Integrated =

Likelihood: Find β & θ that max

$\mid$ data)

ikelihood: If we can write the function of β

can replace (θ) with the corresponding

say $\theta = g(\beta)$ then we MAX:

$g(\beta) \mid$ data)

Likelihood: Integrat out θ from the likelihood

by exploting the fact that we can identify

bility distribution of (θ) conditional on (β)

$$\text{ility} = \frac{\text{Number outcomes favorable}}{\text{Total Number of Outcomes}}$$

H.M 2.III.; Heider Jeffer, 13 OCT 2016, Bozen

Let F is Fake paint, $\forall\, i = 1, 2 \dots 5$ there is an (i) Fakes out of five paints, therefore:

$$P(F|0) = \frac{0}{5} = 0$$
$$P(F|1) = \frac{1}{5} = 0.2$$
$$P(F|2) = \frac{2}{5} = 0.4$$
$$P(F|3) = \frac{3}{5} = 0.6$$
$$P(F|4) = \frac{4}{5} = 0.8$$
$$P(F|5) = \frac{5}{5} = 1.0$$

← The Density function (Conditional Prob.)

$P(F|5)$ is the density function $= 1$ and $P(5)$ is prior prob. $= 0.1$, While the Marginal likelihood is:

$$\sum_{i=0}^{5} P(i)\, P(F|i)$$

Therefore

$$\frac{\text{likelihood} \times \text{prior prob.}}{\text{Marginal likelihood}} = \text{Posterior prob} = P(5|F)$$

$$\frac{P(F|5) \times P(5)}{\sum_{i=0}^{5} P(i)\, P(F|i)} = P(5|F)$$

9

$= 1 \times 0.1 / (0 \times 0.76 + 0.2 \times 0.09 + 0.4 \times 0.02 + 0.6 \times 0.01 + 0.8 \times 0.02 + 0.10 \times 1)$

$= 0.10 / 148 = 0.675675675$

Posterior Prob

Likelihood

Prior Prob

$$P(A \backslash B) = \frac{P(B \backslash A) \cdot P(A)}{P(B)}$$

Conditional Prob. Density function

Marginal Likelihood Integrated =

Maximum Likelihood: Find β & θ that max $L(\beta, \theta \backslash data)$

Partial Likelihood: If we can write the function of β then we can replace (θ) with the corresponding function say $\theta (= g(\beta)$ then we MAX:

$L(\beta, g(\beta) \backslash data)$

Marginal Likelihood: Integrat out θ from the likelihood equation by exploting the fact that we can identify the probability distribution of (θ) conditional on (β)

$$\text{Probability} = \frac{\text{Number outcomes favorable}}{\text{Total Number of Outcomes}}$$

H.M 2.III. ; Heider Jeffer, 13 OCT 2016, Bozen

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$p(F|5) = \frac{5}{5} = 1.0$ ← The Density function (Conditional Prob.)

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$= 1 \times 0.1 / (0 \times 0.76 + 0.2 \times 0.09 + 0.4 \times 0.02 + 0.6 \times 0.01 + 0.8 \times 0.02 + 0.10 \times 1)$

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14 . OCT . 2016

- Outcomes
- Events
- Sample space Ω of contable outcome

Let X Outcomes $\rightarrow \mathbb{R} \longleftrightarrow X$ a discrete random variable

= stochastic =

= variate

Prob : {events} $\rightarrow [0,1]$

$f(x) = P(X = x)$, $x \in \left\{ \begin{array}{l} \text{set of possible} \\ \text{values of } X \end{array} \right\}$

Distribution fn.

1) $0 \leq f(x) \leq 1$

2) $\sum_{\forall x} f(x) = 1$

$P(G\,G)$ $6/56$

$P(G\,B)$ $15/56$

$P(B\,G)$ $15/56$

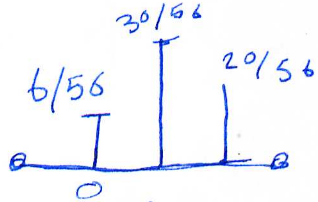
$P(B\,B)$ $20/56$

shadow
$33+44+55$

$F(x)$ Comulative Dist Function.

$$F(x) \equiv P(X \leq x)$$

$$X \in \mathbb{R}$$



$F(-\infty) = 0$ $\quad F(\infty) = 1$

$a < b$ $\quad F(a) \leq F(b)$

$X : \{ \text{outcome} \} \longrightarrow \mathbb{R} \Rightarrow X$

1. Continious random variable

$$f(x_i; \Delta x) \equiv P(x \leq X \leq x + \Delta)$$

$$g(x) = \frac{1}{\partial x} \partial P(x \leq X \leq x + \partial x)$$

$$! \quad x \in \left\{ \begin{array}{c} \text{set of possible} \\ \text{values of } X \end{array} \right\} \quad !$$

Probability Density fn. ---

1)

M = median

Permutation is an ordered Combination

Permulations with Replacement (n^k)

n distinc objects

k position

10	10	10

$k = 3$

$n = 10$

$n^k = 10^3 = 1000$

$n \geq k$

Permutatio without Replacement

$n(n-1)(n-2) \cdots$

$n-(k-1) = n-k+1$

8	9	10

$n(n-1)(n-2)\cdots(n-k+1)$

720

000000 $= \dfrac{n!}{(n-k)!}$!!

$k = 5$

$\dfrac{11!}{(11-5)!} = \dfrac{11!}{6!}$

$n = 11$

$= 55440$

$X : \{ \text{outcome} \} \longrightarrow \mathbb{R} \Leftrightarrow X$

1. Continious random variable

$$f(x_i ; \Delta x) \equiv P(x \leq X \leq x + \Delta)$$

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$$! \; X \in \left\{ \begin{array}{c} \text{set of possible} \\ \text{values of } X \end{array} \right\} !$$

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$g(x) = \frac{1}{\partial x} \partial P(X \leq X \leq X + \partial x)$

$\left| \, X \in \left\{ \text{set of possible values of } X \right\} \right|$

Probability Density fn. - - -

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720

00000 $= \frac{n!}{(n-k)!}$

$k = 5$

$\frac{11!}{(11-5)!} = \frac{11!}{6!}$

$n = 11$

$= 5544$

Permutation — k distinic obj

w. o. Repl. — n position

order is metter — $n \geq k$

$n!/(n-k)!$

$$\frac{8!}{(8-3)!}$$

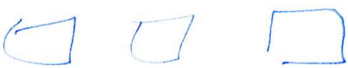
$$\frac{8!}{5!} = \cancel{8}\cdot\cancel{7}\cdot 6\cdot 5$$

permutation — n dist obj

w. o. Rep — n position

order matter

$n!$



3 Chairs — 3 choice

$3! = 6$

Combination

w.o Replacement

n distic obj

k position

k distinct obj

n position

order do not matter

$$\boxed{C_n^k = \frac{n!}{k!\,(n-k)!}}$$

$n \geq k$

How many possible way to choose 5 player from 11 where the order Do not matter:-

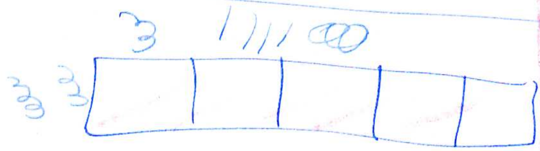
$$\frac{11!}{(11-5)!\,5!} = 462$$

Permutation — k distinic obj

W. O. Repl. — n position

order is matter — $n \geq k$

$n! / (n-k)!$



$$\frac{8!}{(8-3)!}$$

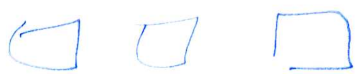
$$\frac{8!}{5!} = 8\cdot 7\cdot 6\cdot 5$$

permutation — n dist obj

W. O. Rep — n position

order matter

$n!$



3 chairs 3 choice

$3! = 6$

Combination
w.o. Replacement

n distic obj
k position

k distinct obj
n position

order do not matter

$$\boxed{C_n^K = \frac{n!}{K!\,(n-K)!}}$$

$n \geq k$

How many possible way to choose 5 player from 11 where the order Do not matter;

$$\frac{11!}{(11-5)!\,5!} = 462$$

Basketball Choss 5 players from 12 players order dosnot matter

$$\frac{12!}{5!\,(12-5)!}$$

$$\frac{12!}{5!\;7!} = 792$$

$$\binom{12}{5} = \frac{12!}{5!\,(12-5)!}$$

Combination | |
+ Replacement
+ order dosent matter

k question

$\forall$ each question n answers where there is only one question is correct.

$$\frac{1}{n} \cdots\cdots \frac{1}{n} = \frac{1}{n^k}$$

$$\frac{\# \text{ of ``bbelled'' possibilities}}{\text{total } \# \text{ of possibilities}}$$

13

Permutations
- with~~out~~ Replacement
- order matters

n distinct objects

k position

$$n^k$$

Permutations
without Replacement
order matter

n distinct

k position $n \geq k$

$$\frac{n!}{(n-k)!}$$

Combination
W. O. Replacement
order doesn't matter

n distinct obj

k posit ob

$$n \geq K$$

$$\frac{n!}{(n+k)!\, K!}$$

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Combination
with Replacement
order doesn't matter

$$\frac{(n+k-1)!}{k!\,(n-1)!}$$

* Permutation: is the choice of (k) things from a set of (n), and where the ORDER IS MATTERS:

$$\boxed{{}_nP_k = \frac{n!}{(n-k)!}}$$

n: Total
k: want

${}_nP_0 = 1$, ${}_nP_1 = n$, ${}_nP_n = n!$

* Combination: is the choice of (k) things from a set of (n) and where the ORDER DOES NOT MATTER

$$\boxed{{}_nC_k = \frac{n!}{k!\,(n-k)!}}$$

${}_nC_0 = {}_nC_n = 1$

${}_nC_1 = {}_nC_{n-1} = n$

${}_nC_k = {}_nC_{n-k}$

24

* $${}_nC_k = \frac{{}_nP_k}{k!} = \frac{n!}{k!\,(n-k)!}$$

Discrete Random Variables

is a variable which can only take a countable number of values.

Ex: If coin is tossed "three" times the number of heads obtained can be 0, 1, 2, or 3, and the probability of each of these possibilities will be

Number of Heads	0	1	2	3
Prob.	1/8	3/8	3/8	1/8

$2 \times 2 \times 2 = 8$ elements in the ~~sample~~ space

C^n_x is

~~is Probability~~ is the number of ways to get X heads out of n coin tosses

$$P(A) = \frac{\text{The number of ways Event (A) can occur}}{\text{The Total number of possible outcomes}}$$

$$C^3_2 = \frac{3!}{(3-2)!\,2!} = 3$$

$$C^3_1 = \frac{3!}{(3-1)!\,1!} = 3$$

$$C^3_3 = \frac{3!}{\cancel{3!}\,\cancel{(3-3)!}} = 1$$

in general:

$$\boxed{P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}}$$

$$0 = \binom{3}{0} \frac{1}{2}^0 \left(1-\frac{1}{2}\right)^{3-0}$$

* The combination for 1 head and 3 tails is {h, t, t, t} and gives $\frac{4!}{3!1!}$ permutation

Thus the probability for 1 head and 3 tails

is $P(X=1) = \binom{4}{1} p^x (1-p)^{n-x}$

$$= \frac{4!}{3!\,1!} \left(\frac{1}{2}\right)^3 \left(1-\frac{1}{2}\right)^{4-3}$$

$$= \frac{4!}{3!\,1!} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1$$

$$= \frac{4!}{3!\,1!} \left(\frac{1}{2}\right)^4$$

~~Hence that Prob of get head or~~

~~Probability of get~~

The combination of geting 3 head and 1 tail is $\binom{4}{3}$ permutation

$$\binom{4}{3} p^x (1-p)^{n-x}$$

$$= \frac{4!}{(4-3)!\,3!} \left(\frac{1}{2}\right)^3 \left(1-\frac{1}{2}\right)^{4-3}$$

The combination of ~~geti~~ for 2heads 2 tails

is $\binom{4}{2}$ *

$$P(X=2) = \binom{4}{2} p^2 (1-p)^{4-2}$$

$$= \frac{4!}{(4-2)!\,2!} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$$

$$= \frac{4!}{2!\,2!} \quad \frac{1}{16}$$

$$= \frac{\overset{2}{\cancel{4}} \cdot 3 \cdot \cancel{2}}{\underset{1}{\cancel{2}} \cdot \cancel{2}} \cdot \frac{1}{16} = \frac{6}{16}$$

In General:

The number of ways to get x heads out of n coin tosses is

$$\binom{n}{x}$$

and the probability "p" of getin "head" is:

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

for a fair coin $p = \frac{1}{2}$.

~~Uniform Dist~~

In General: huge it's huge tasks

~~The number~~

~~numeber of~~

Computing Definite Integrals

let $f(x)$ is a continuous function on $[a,b]$ and $F(x)$ is any anti-Derivative for $f(x)$ then;

$$\int_a^b f(x)\,\partial x = F(x)\Big|_a^b = F(b) - F(a)$$

Ex
$$\int_0^2 x^2 + 1\,\partial x = \left[\frac{1}{3}x^3 + x^2\right]\Big|_0^2$$
$$= \left(\frac{1}{3}2^3 + 2^2\right) - \left(\frac{1}{3}0^3 + 0^2\right)$$
$$= \frac{14}{3}$$

Ex solve
$$\int y^2 + y^{-2}\,dy = \frac{1}{3}y + y^{-1} + c$$

Ex solve

17

$$\int_1^2 y^2 + y^{-2}\,\partial y$$
$$= \left[\frac{1}{3}y^3 + y^{-1}\right]\Big|_1^2 = \left(\frac{1}{3}2^3 + 2^{-1}\right) - \left(\frac{1}{3}1^3 + 1^{-1}\right)$$
$$= 17/6$$

Ex $\int_{-3}^{1} 6x^2 + 5x + 2 \, \partial x$?

$$\int_{-3}^{1} 6x^2 + 5x + 2x = \left[\frac{6}{3}x^3 + \frac{5}{2}x^2 + \frac{2}{1}x \right]_{-3}^{1}$$

$$= \left(\frac{6}{3}1^3 + \frac{5}{2}1^2 + 2 \times 1 \right) - \left(\frac{6}{3}(-3)^3 + \frac{5}{2}(-3)^2 + 2(-3) \right)$$

$$= 84$$

EX: $\int_{1}^{2} \frac{2w^5 - w + 3}{w^2} \partial w$

sol $\int_{1}^{2} 2w^3 - \frac{1}{w} + \frac{3}{w^2} \partial w = \int_{1}^{2} 2w^3 - \frac{1}{w} + 3w^{-2}$

$= \frac{2}{4}w^4 - \ln|w| + -3(w)^{-1} \Big|_{1}^{2}$

$= \frac{1}{2}w^4 - \ln|w| - \frac{3}{w} \Big|_{1}^{2}$

$= \left\{\frac{1}{2}(2)^4 - \ln|2| - \frac{3}{2}\right\} - \left\{\frac{1}{2}(1)^4 - \ln(1) - \frac{3}{1}\right\}$

$= 9 - \ln 2 = 8.3068528194$

Common fn.	fn.	Integral.
Constant	$\int a \, \partial x$	$ax + C$
Variable	$\int x \, \partial x$	$\frac{1}{2}x^2 + C$
square	$\int x^2 \, \partial x$	$\frac{1}{3}x^3 + C$
Reciprocal	$\int \frac{1}{x} \, \partial x$	$\ln\lvert x\rvert + C$
Exponential	$\int e^x \, \partial x$	$e^x + c$
	$\int a^x \, \partial x$	$\frac{a^x}{\ln(a)} + C$
	$\int \ln(x) \, \partial x$	$x\ln(x) - x + C$
Trigonometry	$\int \cos(x) \, dx$	$\sin(x) + C$
	$\int \sin(x) \, dx$	$-\cos(x) + c$
	$\int \sec^2(x) \, dx$	$\tan(x) + c$

RULES	FUNCTION	INTEGRAL
Multiplication by Constant	$\int c f(x)\, \partial x$	$c \int f(x)\, \partial x$
Power Rule $(n \neq -1)$	$\int x^n$	$\frac{x^{n+1}}{n+1} + c$
Sum Rule	$\int (f+g)\, \partial x$	$\int f\, \partial x + \int g\, \partial x$
Difference Rule	$\int (f-g)\, \partial x$	$\int f\, \partial x - \int g\, \partial x$

INTEGRATION By SUBSITUTION

$\int f(g(x))\, g'(x)\, \partial x$ { g(x) and its derivative g'(x)

$\int f(u)\, \partial u$

$\int \cos(x^2)\, 2x\, \partial x$

$\int \cos(u)\, \partial u$

INTEGRATION By PARTS

$$\int u v\, \partial x = u \int v\, \partial x - \int u' \left(\int v\, \partial x\right) \partial x$$

where u is the fn $u(x)$

v is the fn $v(x)$

19

$$\int u\, v\, \partial x$$

$$u \int v\, \partial x - \int u' \left(\int v\, \partial x\right) \partial x$$

INTEGRATION BY PARTS

$$\int u\,v\,dx = u\int v\,dx - \int u'\left(\int v\,dx\right)dx$$

$$\int u\ v\ dx$$

u' $\int v\,dx$

$$u\int v\,dx - \int u'(v\,dx)\,dx$$

EX: $\int x\cos(x)\,dx$?

sol

$u = x$, $V = \cos(x)$

$\underline{u'} = \cancel{x}' = 1$

Differentiate u

Integrate v: $\int v\,dx = \int \cos(x)\,dx = \sin(x)$

Now we can put it together:

$$\int x\cos(x)\,dx$$

$$x\sin(x) - \int 1\,(\sin(x))\,dx$$

$$x\sin(x) - \int \sin(x)\,dx = x\sin(x) + \cos(x) + C$$

EX: $\int \frac{\ln(x)}{x^2}\,dx$?

Sol:

$u = \ln(x)$

$v = \frac{1}{x^2}$

Defferentiate u: $\ln(x)' = \frac{1}{x}$

Integrate v: $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = -x^{-1}$

$= \frac{-1}{x}$

Now put it together:-

$$\int \ln x \, \frac{1}{x^2}\,dx$$

$$\ln x \, \frac{-1}{x} - \int \frac{1}{x}\left(\frac{-1}{x}\right) dx$$

$$-\frac{\ln(x)}{x} - \int -\frac{1}{x^2}\,dx = -\frac{\ln(x)}{x} - \frac{1}{x} + c$$

$$\frac{-(\ln(x)+1)}{x} + c$$

20

EX: $\int \ln(x)\, dX$?

sol $u = \ln(x) \quad v = 1$

$u : \ln(x)' = \frac{1}{x}$

$v : \int 1\, dx = x$

$u \int v\, dx - \int u' \left(\int v\, dx\right) dx$

$\int \ln(x) \cdot 1\, dx$

$\ln x \cdot x - \int \frac{1}{x}(x)\, dx$

$= x\ln(x) - \int 1\, dx = x\ln(x) - x + c$

AGAIN

$$\int U V dx$$

$$u\int v dx - \int u'\left(\int v dx\right) dx$$

Ex: $\int \underline{e^x}_{u} \underline{x}_{v} dx$

$u = e^x$

$V = X$

Defferentiate u : $(e^x)' = e^x$

Integreate V : $\int x dx = \frac{x^2}{2}$

Now put it togethere ;

$$\int e^x x dx$$

21

$$e^x \frac{x^2}{2} - \int e^x \left(\frac{x^2}{2}\right) dx =$$

It's disaster so we can choose differen u and v.

$\{uvu'\}$

$u = x$

$v = e^x$

$u' = 1$

$\int v\,dx = \int e^x\,dx = e^x$

$\int u v\,dx$

$= u \int v\,dx - \int u' (\int v\,dx)\,dx$

$= x\, e^x - \int 1(e^x)\,dx$

$= x e^x - e^x + c$

$= (x-1)\,e^x + c$

Ex: $\int e^x \sin(x)\, dx$

$\int uv\, dx = u \int v\, dx - \int u' (\int v\, dx)\, dx$

$u = \sin x$

$v = e^x$

$\int v\, dx = \int e^x dx = e^x$

$u' = (\sin(x))' = \cos(x)$

$\sin x \cdot e^x - \int \cos(x)\, e^x\, dx$

looks BAD ~~switch~~ v, u again!

$u = \cancel{e^x}\cos(x)$

$v = \cancel{\sin x}\; e^x$

$u' = \sin(x)' = \cos(x)$

22

$\int v\, dx = \int e^x dx = e^x$

$\int \sin(x)\, e^x\, dx = \sin(x) e^x - (\cos(x)) e^x -$

$\int -\sin(x)\, e^x\, dx$

$$\int u v\,dx = \boxed{u\int v\,dx} - \int u'\left(\int v\,dx\right)dx$$

many

many by few

Product ROLE FOR DERIVATIVES

$(uv)' = uv' + u'v$

Inegrate both sides and rearange:

$$\int (uv)'\,dx = \int uv'\,dx + \int u'v\,dx$$

$$uv = \int uv'\,dx + \int u'v\,dx$$

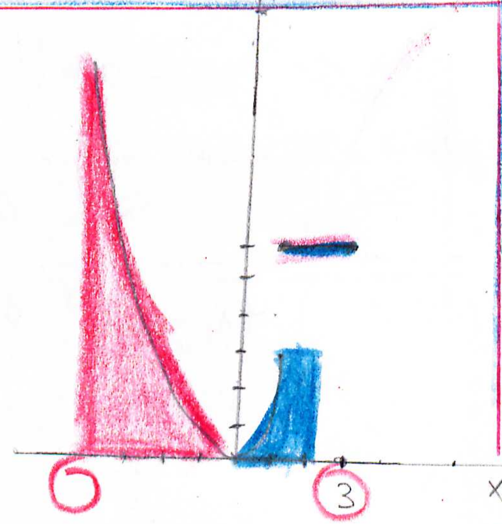
$$\int uv'\,dx = uv - \int u'v\,dx$$

$$\rightarrow \int uv\,dx = u\int v\,dx - \int u'\left(\int v\,dx\right)dx$$

EX

a) $\int_{10}^{20} f(x)\,dx$

b) $\int_{-2}^{3} f(x)\,dx$



$$f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$$

Sol;

a) $\int_{10}^{22} f(x)\,dx = \int_{10}^{22} 6\,dx = 6x \Big|_{10}^{22}$

$= 132 - 60$

$= 72$

b) $\int_{-2}^{3} f(x)\,dx$

$= \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$

$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$

23

$= \frac{3}{3}x^3 \Big|_{-2}^{1} + 6x \Big|_{1}^{3} = (1)^3 - (-2)^3 + (6(3) - 6(1))$

$= 1 + 8 + (18 - 6)$

$= 21$

Ex : $\int_0^1 4x - 6\sqrt[3]{x^2}\, dx$

Sol:

$$= \int_0^1 4x - 6x^{\frac{2}{3}}\, dx$$

$$= \frac{4}{2}x^2 - \frac{6}{\frac{3}{3}} x^{\frac{2}{3}+\frac{3}{3}} \Big|_0^1$$

$$= 2x^2 - \frac{18}{3} x^{\frac{5}{3}} \Big|_0^1$$

$$= \left(2(1)^2 - \frac{18}{3} 1^{\frac{5}{3}}\right) - \left(2(0)^2 - \frac{18}{3} (0)^{\frac{5}{3}}\right)$$

$$= 2 - \frac{18}{3} - (0) = -\frac{8}{5}$$

Notes

we have to compute it well and never forget if there's space between X

$$P(A) = \frac{\text{The Numbers of Ways Event (A) Occur}}{\text{The Total Number of Possible outcome}}$$

Normal Distribution $\mathcal{N}(\mu, \sigma^2)$

$$\text{P.D.F} = f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\text{C.D.F} = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

The Quadratic Formula :

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

Normal Distribution

$\mathcal{N}(\mu, \sigma^2)$

$x \in \mathbb{R}$

$$PDF = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$CDF = \frac{1}{2}\left[1+\operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

$$\text{Quantile} = \mu + \sigma\sqrt{2}\,\operatorname{erf}^{-1}(2F-1)$$

μ = Mean = Median = Mode

Variance $= \sigma^2$

Entropy $= \frac{1}{2}\ln(2\sigma^2\pi e)$

Moment Generating function

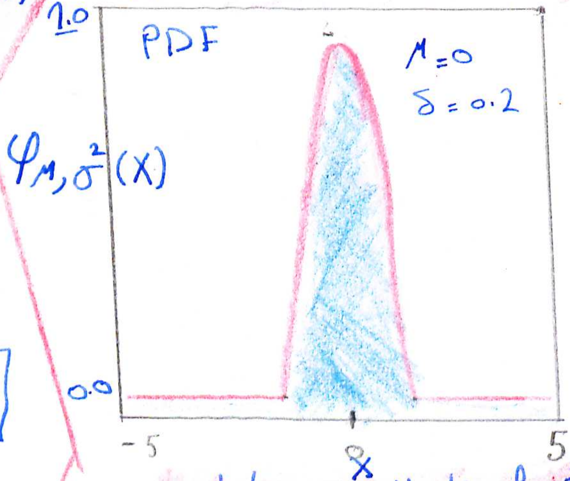
$$MGF = \exp\{\mu t - \frac{1}{2}\sigma^2 t^2\}$$

Characteristic Function

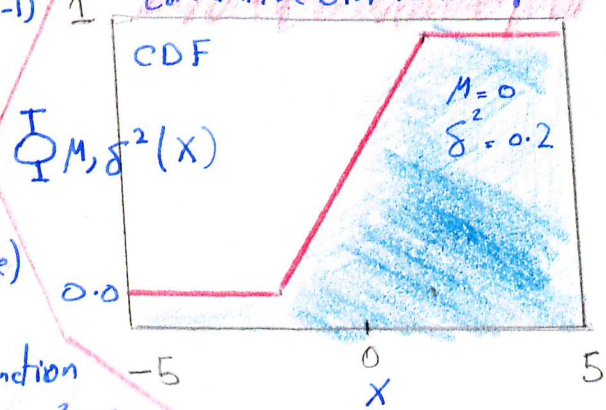
$$CF = \exp\{i\mu t - \frac{1}{2}\sigma^2 t^2\}$$

Fisher Information $= \begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 1/(2\sigma^4) \end{pmatrix}$

Probability Density function



Cumulative Distribution function



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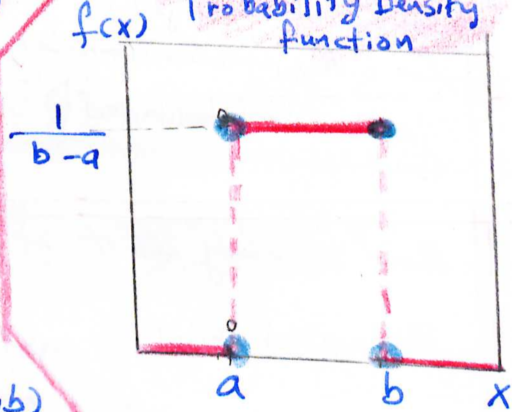
UNIFORM DISTRIBUTION

$\mathcal{U}(a,b)$

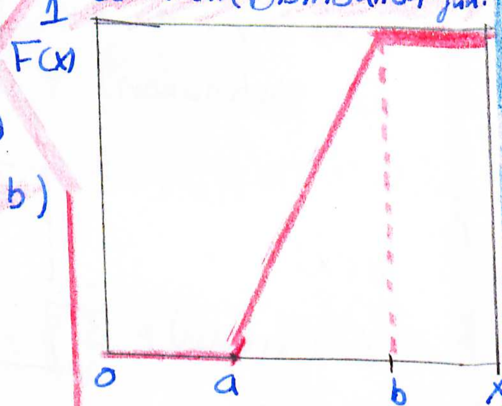
$$\text{PDF} = \begin{cases} \frac{1}{b-a} & \text{if } x \in (a,b) \\ 0 & \text{otherwise} \end{cases}$$

Probability Density function



$$\text{CDF} = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } x \in (a,b) \\ 1 & \text{if } x \geq b \end{cases}$$

Comulative Distribution fun.



$\text{Mean} = \text{Median} = \frac{1}{2}(a+b)$

Mode = any value in (a,b)

$\text{Variance} = \frac{1}{12}(b-a)^2$

$\text{Entropy} = \log(b-a)$

Moment Generating function
MGF

$$\begin{cases} \frac{e^{tb} - e^{ta}}{t(b-a)} & \text{if } t \neq 0 \\ 1 & \text{if } t = 0 \end{cases}$$

Characteristic function
CF

$$\frac{e^{itb} - e^{ita}}{it(b-a)}$$

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MAXIMUM Likelihood Theorem;-
with som highlight:-
for better understanding to these theorem
Maximum Likelihood D
some higlight and short review about MLH apps.

how to play that? 21.OCT.2016 BOZEN

Binomial Dist and Poisson Dist /PDF

Hyp. the Dist. ~~of~~ PDF/
$f(x)$ of a r.v. X. | EXP value is not a r.v.!
is Unknown

$$h(x) \qquad E(h(x)) \begin{cases} \text{discrete } \sum_{\forall x} f(x)h(x) & \sim \text{Weigh average} \\ \text{continuous } \int_{-\infty}^{\infty} f(x)\,d(x) \end{cases}$$

$$E[\gamma h_\gamma(x) + \beta h_b(X)]$$

$$= \gamma\Big[\sum_{\forall x} h_\gamma(x)F(x)\Big] + \beta\Big[\sum_{\forall x} h_b(x)f(x)\Big]$$

$$= \alpha E[h_a(x)] + \beta E[h_b(x)]$$

The Expected operator is linear

$h(x,y) \qquad X, Y \qquad f(x) \qquad f(y)$

$$E(h(X,Y)) = \sum_{\forall (x,y)} h(x,y) \cdot \underbrace{p(X,Y)}_{\text{Joint prob.}}$$

e.g $h(x,y) = \alpha X + \beta y$

$$E(h(x,y)) = \sum_{\forall X} \sum_{\forall y} (\alpha X + \beta y) f(x)\, g(x) =$$

$$\alpha \sum_{\forall x} x f(x) \sum_{\forall y} g(y) + \beta \sum_{\forall X} f(x) \sum_{\forall y} y\, g(y)$$

Moment of R.V. :

$h(x) = X^n \quad n \in \mathbb{N}$

$E(X^n)$: The n-th moment about the origin.

e.g: $E(X^0) = E(1) = \sum_{\forall x} f(x) \cdot 1 = 1$

e.g: $E(X^1) = E(X) = \sum_{\forall x} x f(x)$

e.g: $E(X) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2} = 0.5$

coin expected value

¡ Expected values and its moment are not a r.V. !

μ = Population mean

* $h(x) = (X - \mu)^n \quad n \in \mathbb{N}$

$E[(X-\mu)^n] \equiv$ The n-th moment about the mean

$E((X-M)^0) = E(1) = 1$

$E((X-M)^1) = E(X) - E(M) = M - M = 0$

$E((X-M)^2) = \frac{6}{56}(0-\frac{5}{4}) + \frac{30}{56}(1-\frac{5}{4})$

Sucks

$+ \frac{20}{56}(2-\frac{5}{4})$

$= \frac{360}{56 \cdot 16} = \frac{45}{112}$

$= 0.4$

$E((X-M)^2) = \frac{1}{2}(1-\frac{1}{2})^2 + \frac{1}{2}(0-\frac{1}{2})^2$

coin ?

$\delta^2 = $ population variance $= E((X-M)^2)$

$\sqrt{\delta^2} = \delta$ population standered division

$M =$ population mean.

Skewness $= E((X-M)^3)$

$G((X-M)^4) =$ kurtosis.

Population Variance:-

$$\sigma^2 = E((X-\mu)^2) = E(X^2 - 2X\mu + \mu^2)$$
$$= E(X^2) - 2\mu E(X) + E(\mu^2)$$
$$= E(X^2) - 2\mu^2 + \mu^2$$
$$= E(X^2) - \mu^2 = E(X^2) -$$
$$\sigma^2 = E(X^2) - E^2(X)$$

Theorm:-

* The set of The Moments completly ~~Identif~~ characterizes the Distribution $f(x)$:-

Corollary:

[In order to characterize (determine) a dist. $f(x)$, estimate all Moments!

Bernolli Disti

Lablled event

unlablled event

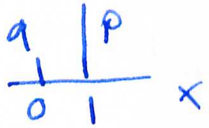
$P(\text{Lablledevent}) \equiv p$

$p(\text{ulablled event}) \equiv 1 - p = q$

X : of lablled event $= X_o$

$f(0) = (P_{x=0}) = q$

$f(1) = (P_{x=1}) = p$



$\mu = E(X) = 0 \cdot q + 1 \cdot p = p$

$$\boxed{\delta^2 = E(x^2) - \mu^2 = p - p^2 = p(1-p) = pq}$$

Coin $p = q = \frac{1}{2}$

Dice : $\mu = \frac{1}{6}$ $\delta^2 = \frac{5}{36}$

Q: a sequence of n Identical and Independent Bernolli-trialls ;

$k \equiv$ # of labelled events

$$\left| \binom{n}{k} p^k (1-p)^{n-k} \right|$$

$$(a+b)^n = \sum_{n=0}^{N} \binom{n}{k} a^k b^{n-k}$$

$$1 = (p+q)^n = \sum_{n=0}^{n} \binom{n}{k} p^k q^{n-k}$$

$$\boxed{\begin{matrix} M_k \\ \delta_k^2 \end{matrix}}$$

$$M = E(K) = \sum_{k=0}^{n} k f(k) =$$

$$= \sum_{k=0}^{n} k \frac{n!}{k!(n-k)!} p^k q^{n-k}$$

$$= \sum \frac{n!}{(k-1)!(n-k)!} p^k q^{n-k}$$

$$= \quad = \sum_{J=0}^{n-1} \frac{n!}{J!\,(n-1-J)!} p^{J+1} q^{n-1-J}$$

$J \equiv k-1$

$k = J+1$

$$np \sum_{J=0}^{n-1} \frac{(n-1)!}{J!\,(n-1-J)!} p^{J} q^{n-1-J}$$

$$= np \underbrace{\sum_{J=0}^{n-1} \binom{n-1}{J} p^{J} q^{n-1-J}}_{1}$$

$$\mu = np$$

$$\delta^2 = \sum_{k=0}^{n} k(k-1) \frac{n!}{k!\,(n-k)!} p^{k} q^{n-k}$$

$$= \sum_{J=0}^{k-2} \frac{n!}{J!\,(n-2-J)!} p^{J+2} q^{n-2-J}$$

$$= n(n-1)\,p^2 \sum_{J=0}^{n-2} \frac{(n-2)!}{J!\,(n-2-J)!} p^{J} q^{n-2-J}$$

$$E(k^2) - \mu_k^2 = E(k + k(k-1)) - n^2 p^2$$

$$= E(k) + E(k(k-1)) - n^2 p^2 = np + n(n-1)p^2 - n^2 p^2$$

$$= npq$$

$$\sigma_k^2 = npk$$

$$r \equiv \frac{k}{n}$$ r is rate

$$\mu_r = p$$

$$\sigma_r^2 = \frac{pq}{n}$$

$$E(r^2) - \mu_r^2$$

$$= E\left(\frac{k^2}{n^2}\right) - p^2$$

$$= \frac{1}{n^2} E(k^2) - p^2$$

$$= \frac{1}{n^2}[E(k^2) - n^2 p^2$$

$$= \frac{1}{n^2} \delta_k^2 = \frac{1}{n^2} npq$$

$$= \frac{pq}{n}$$

e.g: toss coin (Fair) $\Rightarrow p = q = \frac{1}{2}$

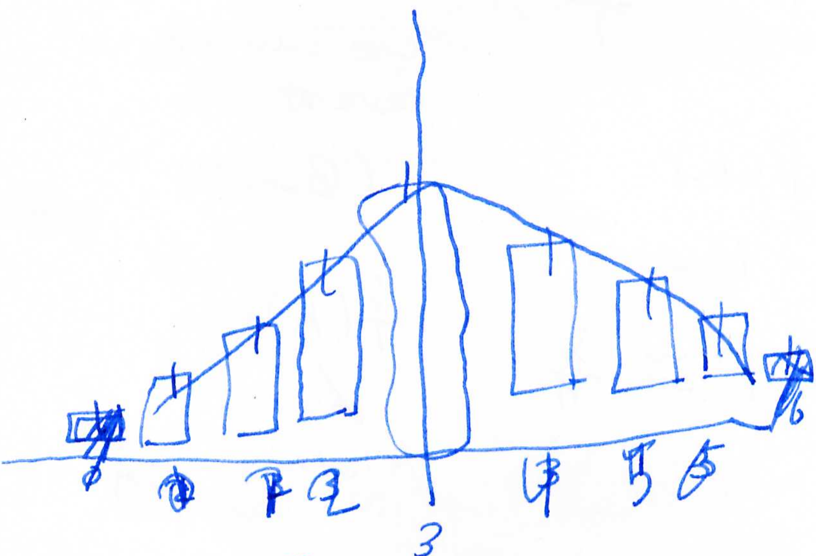
$n = 5$

$$f(0) = \binom{5}{0} \frac{1}{2^0} \frac{1}{2^5} = \frac{1}{52}$$

$n = 6$

$$f(0) = \binom{6}{0} \frac{1}{2^6} \frac{1}{2^0} = \frac{1}{64}$$

$$f(1) = \binom{6}{1} \frac{1}{2^1} \frac{1}{\cancel{2^5}} = \frac{6}{2^6} = \frac{6}{64}$$



30

END

Poisson Dist.

$T \Lambda$

T

λ

We take n so large so that

$\cancel{A}$

$\Gamma = \frac{\lambda}{T}$

$\frac{I}{n}$ is so small so that we can only see 0, 1 event

$f(k)$

(Benolli trial)

$k = 0 \cdots n$

$$p \cong \frac{\lambda}{n}$$

$$f(k) = \binom{k}{n} \left(\frac{\lambda}{n}\right)^k \left(1 - \frac{\lambda}{n}\right)^{n-k}$$

$k: 0, \cdots n$ $\quad p^k$ $\quad q^{n-k}$

$$f(k) = \frac{\lambda^k}{k!} \frac{n!}{(n-k)!} \frac{1}{n^k} \left[\left(1 - \frac{\lambda}{n}\right)^n\right]^{\frac{n-k}{n}}$$

$$\lim_{\substack{n \to \infty \\ k \in \mathbb{N}}} f(k) = \frac{\lambda^k}{k!} \frac{\overbrace{n(n-1)(n-2)\cdots(n-k+1)}^{k}}{\underbrace{n \cdot n\, n\, n \cdots n}_{k}}$$

$$\boxed{f(k) = \frac{\lambda^k}{k!} e^{-\lambda}}$$ ... Poisson Distribution

(μ_k) and (σ^2_k)

$\sigma_k^2 =$

$$\boxed{\mu_k = E(k) = \lambda}$$

$$\sigma_k^2 = \lambda$$

$$\sigma_k^2 = \mu_k$$

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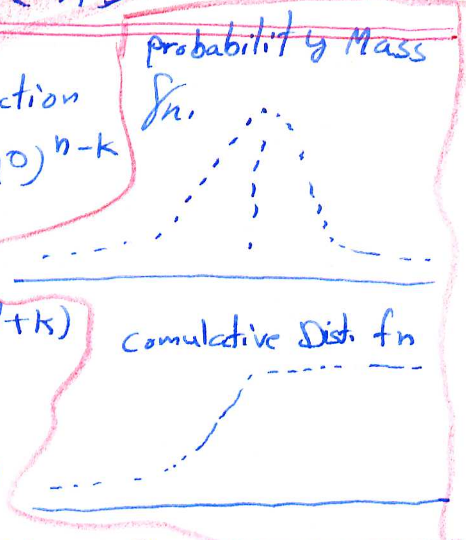
Binomial Dist. $B(n,p)$

probability Mass function

$PMF = \binom{n}{k} p^k (1-p)^{n-k}$



comulative Dist fn

$CDF = I_{1-p}(n-k, 1+k)$

Mean $= np$

median $= [np]$ or $[np]$

Mode $= [(n+1)p]$ or $[(n+1)p] - 1$

Variance $= np(1-p)$

Skeweness $= \dfrac{1-2p}{\sqrt{np(1-p)}}$

Ex. Kurtosis $= \dfrac{1-6p(1-p)}{np(1-p)}$

Entropy $= \frac{1}{2}\log_2(2\pi e np(1-p)) + O\left(\frac{1}{n}\right)$

MGF $= (1-p+pe^t)^n$

CF $= (1-p+pe^{it})^n$

PGF $= G(z)\ [(1-p)+pz]^n$

Fisher Information $= I_n(p) = \dfrac{n}{p(1-p)}$ {for fixed n}

Poisson Dist fn. :-

Probability Mass fn.

$$\text{PMF} = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\text{CDF} = \frac{\Gamma(\lfloor k+1 \rfloor, \lambda)}{\lfloor k \rfloor !} \quad \text{or} \quad e^{-\lambda} \sum_{i=0}^{\lfloor k \rfloor} \frac{\lambda^i}{i!},$$

$$\text{or } Q(\lfloor k+1 \rfloor, \lambda)$$

$k \geqslant 0$ where $\Gamma(x, y)$ is the incomplete gama fn, $\lfloor k \rfloor$ is floor fn and Q is the regularized gamma fn

Mean $= \lambda$

Median $\cong \lfloor \lambda + 1/3 - 0.02/\lambda \rfloor$

Mode $= \lceil \lambda \rceil - 1, \lfloor \lambda \rfloor$

Variance $= \lambda$

32

Skewness $= \lambda^{-1/2}$

Ex. Kurtosis $= \lambda^{-1}$

$$\text{Entropy} = \lambda [1 - \log(\lambda)] + e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k \log(k!)}{k!}$$

For Large λ

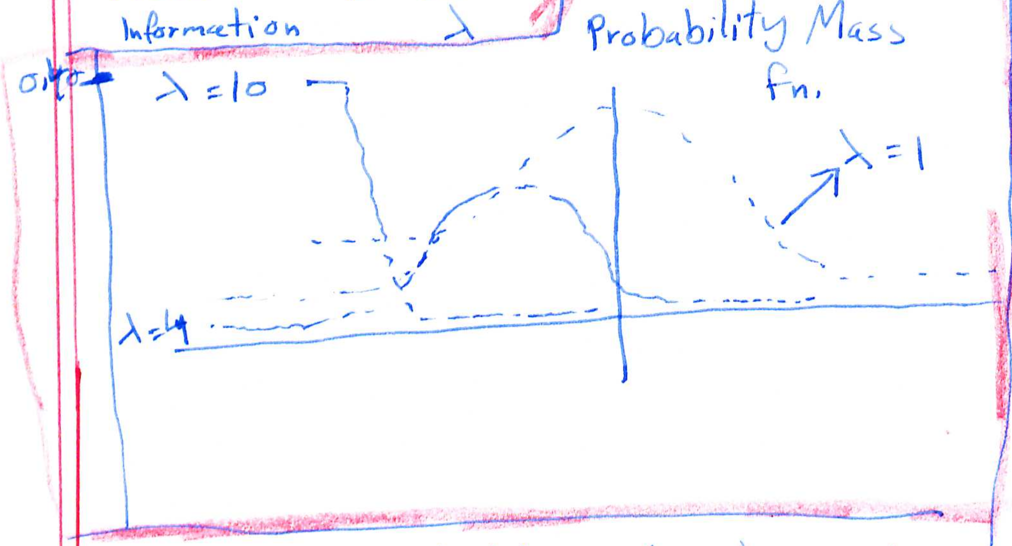
$$\frac{1}{2}\log(2\pi e \lambda) - \frac{1}{12\lambda} - \frac{1}{24\lambda^2} - \frac{19}{360\lambda^3} + O\left(\frac{1}{\lambda^4}\right)$$

$MGF = \exp(\lambda(e^{t}-1))$

$CF = \exp(\lambda(e^{it}-1))$

$PGF = \exp(\lambda(z-1))$

Fisher Information $= \frac{1}{\lambda}$



The exame is about (pass / Fail) according to ~~our~~ the student answer and how he/she answer the exam, The Prof. will decide (Pass / fail)

Bernolli Distribution

$$PMF = \begin{cases} q = (1-p) & \text{if } k = 0 \\ p & \text{if } k = 1 \end{cases}$$

CDF =

$$PDF \begin{cases} 0 & \text{if } k < 0 \\ 1-p & \text{if } 0 \leq k < 1 \\ 1 & \text{if } k \geq 1 \end{cases}$$

$$M = p$$

$$Median = \begin{cases} 0 & \text{if } q > p \\ 0.5 & \text{if } q = p \\ 1 & \text{if } q < p \end{cases}$$

$$Variance = pq$$

$$Skewenss = \frac{1-2p}{\sqrt{qp}}$$

$$Ex.\ kurtosis = \frac{1-6pq}{pq}$$

$$Entropy = -q\ln(q) - p\ln(p)$$

$$MGF = q + pe^{t}$$

$$PGF = q + pz$$

$$\text{Fisher Information} = \frac{1}{p(1-p)}$$

33

The END

Year 1 > SEM 1 {COR 5}

1. Statistical Methods [ϕ]
2. Software Process Management [OLE]
3. Siminars in Human Machine Interaction [SHMI]
4. Requirments and Design of Software Systems [OLE]
5. IT and Service Management.

Year 1 > SEM 2 [COR 4]

1. Technichal And Scientific Communi~~ct~~tion.
2. System security
3. Software Reliability Testing
4. Advanced Internet Technologies

Heider Jeffer Master in Computer Science

Software Engineering and IT Management

Free University of Bozen

Year 2 > SEM 1 {COR 4}

1. Optional
2. Siminars in Software and IT Engineering
3. Research Methods
4. Mobile Systems Engineering

Year 2 > SEM 2 {COR 1}

1. Thesis

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Bolzano

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