Stat 443 Formulas and Tables

Chapter 1

- **Probability** – assigns a number P(A) to every event A so that
  Axiom 1: $0 \le P(A) \le 1$
  Axiom 2: $P(S) = 1$
  Axiom 3: If $A_1, A_2, \dots, A_k$ are disjoint, then $P(\cup_{i=1}^{k} A_i) = \sum_{\{i=1\}}^{\{k\}} P(A_i)$. This is also true if k=∞.

Rules of Probability

- **Theorem (Complement Rule):** $P(\bar{A}) = 1 - P(A)$
- **Theorem (Addition Rule)** For any two events A and B,
  $$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$
- If A and B are disjoint, i.e. $A \cap B = \phi$, then $P(A \cup B) = P(A) + P(B)$.
- For any 3 events A, B and C,
  $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$
- **De Morgan's Law**: $P(\overline{A_1 \cup A_2 \cup \dots \cup A_k}) = P(\overline{A_1} \cap \bar{A}_2 \cap \dots \cap \bar{A}_k)$

Simple Probability Rule (Finite Discrete Spaces)
If an experiment can result in N equally likely outcomes, then the probability of an event A is

$$P(A) = \frac{\text{\{\# of outcomes favoring A\}}}{N}.$$

Suppose P(B) > 0. The **conditional probability** of A after B has occurred, is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Multiplication Theorem of Probability: $P(A \cap B) = P(B) \times P(A|B) = P(A) \times P(B|A)$
$$P(A \cap B \cap C) = P(A) \times P(B|A) \times P(C|A \cap B)$$

Two events A and B are **independent** if **any one of the following** holds:

$P(A) = P(A\|B)$	$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \times P(\bar{B})$
$P(B) = P(B\|A)$	$P(\bar{A} \cap B) = P(\bar{A}) \times P(B)$
$P(A \cap B) = P(A) \times P(B)$	$P(A \cap \bar{B}) = P(A) \times P(\bar{B})$

Law of Total Probability: $P(A) = \sum_{i=1}^{k} P(B_i) \times P(A|B_i)$
Bayes Theorem: $P(B_i|A) = \frac{P(B_i)P(A|B_i)}{\sum_{j=1}^{k} P(B_j) \times P(A|B_j)}$

The number of **permutations** of n distinct objects taken r at a time is ${}_nP_r = \frac{n!}{(n-r)!}$.
The number of **combinations** of size r selected from n objects is ${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$.
The number of **distinguishable permutations** of n objects of which r_1 are of one kind, r_2 are of a 2nd kind, …, r_k are of a kth kind is $\frac{n!}{r_1! r_2! \dots r_k!}$ where $n = r_1 + r_2 + \dots + r_k$.

Chapter 2

X is a discrete rv.

- $f(x)$ is called the **probability mass function** (pmf) of X. It satisfies:
  - $0 \leq f(x) \leq 1$ for all x
  - $\sum_{\text{all } x} f(x) = 1$
- The **cumulative distribution function** (cdf) of X is $F(x) = P(X \leq x) = \sum_{a \leq x} f(a)$.
- **Expected** or **mean value** of X: $\mu = E(X) = \sum_{\text{all } x} x \cdot f(x)$

X is a **continuous rv** if there is a function $f(x)$, the **probability density function (pdf)**, so that

1. $f(x) \geq 0$ for all x
2. $\int_{\infty}^{+\infty} f(x)dx = 1$
3. $P(a \leq X \leq b) = \int_a^b f(x)dx$

- **Cumulative distribution function** (cdf) of X: $F(x) = P(X \leq x) = \int_{\infty}^{x} f(y)dy$
- The **expected** or **mean value** of X is $\mu = E(X) = \int_{-\infty}^{+\infty} xf(x)dx$.
- x_p is the $\mathbf{100p}^{\text{th}}$ **percentile of F or X** if $F(x_p) = p$ where $0 < p < 1$.

The **expected** or **mean value** of $g(X)$ is $E[g(X)] = \begin{cases} \sum_{all\ x} g(x) \cdot f(x) & \text{(discrete)} \\ \int_{-\infty}^{+\infty} g(x) \cdot f(x)dx & \text{(continuous)} \end{cases}$.

- **Variance** of X: $\sigma^2 = V(X) = E[(X-\mu)^2] = \begin{cases} \sum_{all\,x} x^2 \cdot f(x) - \mu^2 & \text{(discrete)} \\ \int_{-\infty}^{+\infty} x^2 \cdot f(x)dx - \mu^2 & \text{(continuous)} \end{cases}$
- **standard deviation** (sd) of X: $\sigma = \sqrt{V(X)}$

Theorem 2a. $E(a) = a$.
Theorem 2b. $E[a \cdot g(X)] = a \cdot E[g(X)]$
Theorem 2c. $E[a_1 \cdot g_1(X) + a_2 \cdot g_2(X) + \ldots + a_k \cdot g_k(X)] = a_1E[g_1(X)] + a_2E[g_2(X)] + \ldots + a_kE[g_k(X)]$
Theorem 2d. $V(X) = E(X^2) - \mu^2$
$\Rightarrow E(X^2) = V(X) + \mu^2$
Theorem 2e.

i. $E(aX + b) = aE(X) + b$
ii. $V(aX + b) = a^2V(X)$

Tchebysheff's Theorem. X rv with mean μ and variance σ^2. For any k > 0,

$$P(|X-\mu| < k\sigma) \geq 1 - \frac{1}{k^2} \quad \text{or} \quad P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

Let k be a positive integer and $\mu = E(X)$.

- The k^{th} **moment about the origin** of X is $\mu_k' = E(X^k)$. This is also called a **raw moment**.
- The k^{th} **moment about the mean** of X is $\mu_k = E[(X-\mu)^k]$. This is also called a **central moment**.

- The **moment-generating function (mgf)** of a rv X: $M(t) = M_X(t) = E(e^{tX})$
  - $\boldsymbol{k}^{\text{th}}$ **raw moment**. $\mu_k' = E(X^k) = \frac{d^k}{dt^r} M_X(t=0)$
  - If X and Y have the same mgf, then X and Y have the same distribution.
  - X has mgf $M_X(t)$. The mgf of $Y = aX + b$ is $M_Y(t) = e^{bt} \cdot M_X(at)$.

Chapter 3

Binomial Distribution

PMF: $b(x; n, p) = \binom{n}{x} p^x q^{n-x},\ x = 0, 1, \dots, n; \qquad q = 1 - p$
Mean/Variance: $E(X) = np,\ V(X) = npq$
MGF: $M(t) = (pe^t + q)^n$

- $n = 1$ (one trial) => X is a **Bernoulli** rv.

Geometric Distribution

PMF: $g(x; p) = pq^{x-1},\ x = 1, 2, 3, \dots; \quad q = 1 - p$ $\qquad P(X \le k) = 1 - q^k, \quad P(X > k) = q^k$
Mean/Variance: $E(X) = \frac{1}{p},\ V(X) = \frac{q}{p^2}$
MGF: $M(t) = \frac{pe^t}{1 - qe^t}$ **Sum of a Geometric Series:** $\boldsymbol{\sum_{x=1}^{\infty} ar^{x-1} = \sum_{x=0}^{\infty} ar^x = \frac{a}{1-r}}, \quad |\boldsymbol{r}| < 1$

Negative Binomial Distribution

PMF: $f(x; r, p) = \binom{x-1}{r-1} p^r q^{x-r},\ x = r, r+1, r+2, \dots; \quad q = 1 - p$
Mean/Variance: $E(X) = \frac{r}{p},\ V(X) = \frac{qr}{p^2}$
MGF: $M(t) = \left(\frac{pe^t}{1 - qe^t}\right)^r$

- $\mathrm{NB}(r = 1, p)$ is $\mathrm{GEO}(p)$.

Poisson Distribution

PMF: $f(x; \mu) = \frac{e^{-\mu}\mu^x}{x!},\ x = 0,1,2,3, \dots$
Mean/Variance: $E(X) = V(X) = \mu$
MGF: $M(t) = e^{\mu(e^t - 1)}$

Hypergeometric Distribution

PMF: $h(x; n, M, N) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}, \quad \max\{0, n - (N - M)\} \le x \le \min\{n, M\}$
Mean/Variance: $E(X) = \frac{nM}{N}, V(X) = n\left(\frac{M}{N}\right)\left(1 - \frac{M}{N}\right)\left(\frac{N-n}{N-1}\right)$

Uniform Distribution

PDF: $f(x; a, b) = \frac{1}{a-b},\ a < x < b$ **CDF:** $F(x; a, b) = \frac{x-a}{b-a},\ a < x < b$
Mean/Variance: $E(X) = \frac{a+b}{2},\ V(X) = \frac{(b-a)^2}{12}$
MGF: $M(T) = \frac{e^{bt} - e^{at}}{(b-a)t}$

Gamma Function: $\Gamma(\kappa) = \int_0^{+\infty} t^{\kappa-1} e^{-t} dt \;\; \kappa > 0.$

- $\Gamma(1)=1$
- $\Gamma(\kappa+1)=\kappa\Gamma(\kappa)$
- $\Gamma(k+1)=k!$ for positive integer k
- $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
- $\theta^{\kappa}\Gamma(\kappa) = \int_0^{+\infty} t^{\kappa-1} e^{-t/\theta} dt$

Gamma Distribution

PDF: $f(x;\theta,\kappa) = \frac{1}{\theta^{\kappa}\Gamma(\kappa)} x^{\kappa-1} e^{-x/\theta},\; x > 0$

Mean/Variance: $E(X) = \kappa\theta,\;\; V(X) = \kappa\theta^2$

MGF: $M(t) = \left(\frac{1}{1-\theta t}\right)^{\kappa}$

Exponential Distribution

PDF: $f(x;\theta) = \frac{1}{\theta} e^{-x/\theta},\; x > 0$

CDF: $F(x;\theta) = 1 - e^{-x/\theta},\; x > 0$

$P(X > x) = e^{-x/\theta},\;\; x > 0$

Mean/Variance: $E(X) = \theta,\;\; V(X) = \theta^2$

MGF: $M(t) = \frac{1}{1-\theta t}$

100p^th Percentile: $x_p = \theta[-\ln(1-p)]$

Beta Distribution

PDF: $f(x;a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1},\; 0 < x < 1$

Mean/Variance: $E(X) = \frac{a}{a+b},\;\; V(X) = \frac{ab}{(a+b+1)(a+b)^2}$

- **UNIF(0,1)** is **BETA(a=1,b=1).**

Weibull Distribution

PDF: $f(x;\theta,\beta) = \frac{\beta}{\theta^{\beta}} x^{\beta-1} e^{-\left(\frac{x}{\theta}\right)^{\beta}},\; x > 0$

CDF: $F(x;\theta,\beta) = 1 - e^{-\left(\frac{x}{\theta}\right)^{\beta}},\; x > 0$

$P(X > x) = e^{-\left(\frac{x}{\theta}\right)^{\beta}}, x > 0$

Mean/Variance: $E(X) = \theta\Gamma\left(1 + \frac{1}{\beta}\right),\;\; V(X) = \theta^2\left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right]$

100p^th Percentile: $x_p = \theta[-\ln(1-p)]^{\frac{1}{\beta}}$

- WEI(θ,β=1) = EXP(θ) = GAM(θ,κ=1)

Normal Distribution

PDF: $f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < +\infty$

Computing Probabilities: Standardize using the result that if $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$.

Mean/Variance: $E(X) = \mu, \ \ V(X) = \sigma^2$
MGF: $M(t) = e^{\mu t + \sigma^2 t^2/2}$
100p^th Percentile: $x_p = \mu + \sigma \times z_p, \quad z_p$ is the 100p^th percentile of N(0,1)

- **68-95-99.7 Rule (Empirical Rule)**
  - 68% of all values fall between $\mu \pm \sigma$.
  - 95% of all values fall between $\mu \pm 2\sigma$.
  - 99.7% of all values fall between $\mu \pm 3\sigma$.

$z_p =$ **N(0,1) 100p^th Percentiles (p-Quantiles)**

p	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
.0		-2.33	-2.05	-1.88	-1.75	-1.64	-1.55	-1.48	-1.41	-1.34
.1	-1.28	-1.23	-1.17	-1.13	-1.08	-1.04	-0.99	-0.95	-0.92	-0.88
.2	-0.84	-0.81	-0.77	-0.74	-0.71	-0.67	-0.64	-0.61	-0.58	-0.55
.3	-0.52	-0.50	-0.47	-0.44	-0.41	-0.39	-0.36	-0.33	-0.31	-0.28
.4	-0.25	-0.23	-0.20	-0.18	-0.15	-0.13	-0.10	-0.08	-0.05	-0.03
.5	0.00	0.03	0.05	0.08	0.10	0.13	0.15	0.18	0.20	0.23
.6	0.25	0.28	0.31	0.33	0.36	0.39	0.41	0.44	0.47	0.50
.7	0.52	0.55	0.58	0.61	0.64	0.67	0.71	0.74	0.77	0.81
.8	0.84	0.88	0.92	0.95	0.99	1.04	1.08	1.13	1.17	1.23
.9	1.28	1.34	1.41	1.48	1.55	1.64	1.75	1.88	2.05	2.33

Lognormal Distribution

- $\ln(X) \sim N(\mu, \ \sigma^2)$.
- **Computing Probabilities:** Use the result that $Z = \frac{\ln(X)-\mu}{\sigma} \sim N(0,1)$. Take natural logarithm then proceed with Normal probability computations.

Mean/Variance: $E(X) = e^{\mu+\sigma^2/2}, \ \ V(X) = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$
100p^th Percentile: $x_p = \exp[\mu + \sigma \times z_p]$

Chapter 4

Let (X_1, X_2) have joint pmf $f(x_1, x_2)$. The **marginal pmfs** of X_1 and X_2 are

$$P(X_1 = x_1) = f_1(x_1) = \sum_{\text{all } x_2} f(x_1, x_2), \quad P(X_2 = x_2) = f_2(x_2) = \sum_{\text{all } x_1} f(x_1, x_2) .$$

$\boldsymbol{X} = (X_1, X_2, \dots, X_k)$ joint pmf $f(x_1, x_2, \dots, x_k)$

The **joint cdf** of $\boldsymbol{X}$ is $F(x_1, x_2, \dots, x_k) = P(X_1 \le x_1, X_2 \le x_2, \dots, X_k \le x_k)$.

- For $a < b$ and $c < d$, $P(a < X_1 \le b,\ c < X_2 \le d) = F(b, d) - F(b, c) - F(a, d) + F(a, c)$.

$\boldsymbol{X} = (X_1,\ X_2, \dots, X_k)$ vector of continuous random variables

- **Joint pdf** of $\boldsymbol{X}$ is a function $f(x_1, x_2, \dots, x_k)$ such
$$F(x_1, x_2, \dots, x_k) = P(X_1 \le x_1, X_2 \le x_2, \dots, X_k \le x_k) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \dots \int_{\infty}^{x_k} f(t_1, t_2, \dots, t_k) dt_k \dots dt_2\, dt_1$$
- $f(x_1, x_2, \dots, x_k) = \frac{\partial^k}{\partial x_1 \partial x_2 \cdots \partial x_k} F(x_1, x_2, \dots, x_k)$

(X_1, X_2) with joint pdf $f(x_1, x_2)$

- The **marginal cdf** of X_1: $F_1(x_1) = P(X_1 \le x_1) = F(x_1, +\infty) = \int_{-\infty}^{x_1} \int_{-\infty}^{+\infty} f(t_1, t_2) dt_2\, dt_1$.
- The **marginal pdf** of X_1: $f(x_1) = F_1'(x_1) = \int_{-\infty}^{+\infty} f(x_1, x_2) dx_2$.

X_1 and X_2 have joint pmf or pdf $f(x_1, x_2)$ and marginals $f_1(x_1)$ and $f_2(x_2)$.

- **Conditional pmf** or **pdf of $\boldsymbol{X_2}$ given $\boldsymbol{X_1 = x_1}$:** $f(x_2 | x_1) = \frac{f(x_1, x_2)}{f_1(x_1)}$, $f_1(x_1) > 0$

Theorem 4.4.1. $X_1, X_2, \dots, X_k$ are **independent** iff either of the following holds:

$$f(x_1, \dots, x_k) = f_1(x_1) \times \dots \times f_k(x_k)$$
$$F(x_1, \dots, x_k) = F_1(x_1) \times \dots \times F_k(x_k)$$

- If $X_1, \dots, X_k$ are **independent and identically distributed (iid)**, then their joint pmf or pdf is given by $f(x_1, \dots, x_k) = f_1(x_1) \times \dots \times f_k(x_k)$.

Theorem 4.4.2. Two random variables X_1 and X_2 are independent iff:

- $\{(\boldsymbol{x_1}, \boldsymbol{x_2}) \mid \boldsymbol{f}(\boldsymbol{x_1}, \boldsymbol{x_2}) > \boldsymbol{0}\} = \boldsymbol{A} \times \boldsymbol{B} = \{(\boldsymbol{x_1}, \boldsymbol{x_2}) \mid \boldsymbol{x_1} \in \boldsymbol{A},\ \boldsymbol{x_2} \in \boldsymbol{B}\}$, and
- $\boldsymbol{f}(\boldsymbol{x_1}, \boldsymbol{x_2}) = \boldsymbol{g}(\boldsymbol{x_1}) \boldsymbol{h}(\boldsymbol{x_2})$.

Theorem 4.5.1.

- $f(x_1, x_2) = f_1(x_1) \times f(x_2 | x_1) = f_2(x_2) \times f(x_1 | x_2)$
- X_1 and X_2 are independent iff either of the following holds: $f(x_1 | x_2) = f_1(x_1)$, $f(x_2 | x_1) = f_2(x_2)$.

Chapter 5

- **Expected value** of $g(x_1, \dots, x_k)$

$$E[g(X_1, \dots, X_k)] = \sum_{\text{all x\_1}} \sum_{\text{all } x_2} \dots \sum_{\text{all } x_k} g(x_1, \dots, x_k) f(x_1, \dots, x_k) \quad \textbf{(discrete case)}$$

$$E[g(X_1, \dots, X_k)] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} g(x_1, \dots, x_k) f(x_1, \dots, x_k) dx_k \dots dx_2\, dx_1 \quad \textbf{(continuous case)}$$

Theorem 5.2.3 (Generalized). If $X_1, \dots, X_k$, then

$$E[g_1(X_1) \times g_2(X_2) \times \dots \times g_k(X_k)] = E[g_1(X_1)] \times E[g_2(X_2)] \times \dots \times E[g_k(X_k)] = \prod_{i=1}^{k} E[g_i(X_i)].$$

- **Covariance between X and Y:** $\sigma_{xy} = Cov(X, Y) = E\left[(X - \mu_x)(Y - \mu_y)\right] = E(XY) - \mu_x \mu_y$
- **Correlation between X and Y:** $\rho = \rho(X, Y) = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$

Theorem 5.2.5 (Expanded). $X_1, X_2, \dots, X_m$ and $Y_1, Y_2, \dots, Y_n$ rvs with $\xi_i = E(X_i)$ and $\mu_i = E(Y_i)$
Let $X = \sum_{i=1}^{m} a_i X_i$ and $Y = \sum_{j=1}^{n} b_j Y_j$.

i. $E(X) = \sum_{i=1}^{n} a_i \xi_i$
ii. $V(X) = \sum_{i}^{m} a_i^2 V(X_i) + 2 \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} a_i a_j Cov(X_i, X_j)$
iii. $Cov(aX + b, X) = aV(X)$
iv. $Cov(a + X, b + Y) = Cov(X, Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} a_i b_j Cov(X_i, Y_j)$ for any constants a, b
v. $Cov(aX, bY) = abCov(X, Y)$

Theorems 5.2.2, 5.2.4, 5.2.6

i. $E(aX + bY) = aE(X) + bE(Y)$
ii. $V(aX + bY) = a^2 V(X) + b^2 V(Y) + 2abCov(X, Y)$
iii. If X and Y are independent, $V(aX + bY) = a^2 V(X) + b^2 V(Y)$

- **Conditional expectation** of Y given $X = x$: $E(Y|x) = \begin{cases} \sum_y y f(y|x) & \text{discrete case} \\ \int_{-\infty}^{+\infty} y f(y|x) dy & \text{continuous case} \end{cases}$
- **Conditional expectation** of X given $Y = y$: $E(X|y) = \begin{cases} \sum_x x f(x|y) & \text{discrete case} \\ \int_{-\infty}^{+\infty} x f(x|y) dx & \text{continuous case} \end{cases}$

Theorem 5.4.1. $E(X) = E_Y[E(X|Y)]$ and $E(Y) = E_X[E(Y|X)]$
Theorem 5.4.3. $V(Y) = E_X[V(Y|X)] + V_X[E(Y|X)]$

Other Results:

- $E_Y\{E[g(X)|Y]\} = E[g(X)]$
- $E_X\{E[g(Y)|X]\} = E[g(Y)]$
- $E[g(X)h(Y)|X] = g(X)E[h(Y)|X]$
- $E[g(X)h(Y)|Y] = h(Y)E[g(X)|Y]$
- **Theorem 5.4.2.** If X and Y are independent, then $E(Y|x) = E(Y)$ and $E(X|y) = E(X)$.

Chapter 6

The Cdf Technique

- **Continuous Case**. Derive $F_Y(y) = P(Y = g(X_1, X_2, \ldots, X_n) \leq y)$ using the pdf of X or joint pdf of $X_1, X_2, \ldots, X_n$. $\Rightarrow f_Y(y) = \frac{dF_Y(y)}{dy}$
- **Discrete Case.** Derive $f_Y(y) = P[Y = g(X_1, X_2, \ldots, X_n) = y]$ using the pmf of X or the joint pmf of $X_1, X_2, \ldots, X_n$.

Transformation (Jacobian) Method

- $X_1, \ldots, X_k$ have joint pdf $f_X(x_1, \ldots, x_k)$.
- $Y_1 = g_1(X_1, \ldots, X_k), \ldots, Y_k = g_k(X_1, \ldots, X_k)$ are k one-to-one functions with inverses $x_1 = w_1(y_1, \ldots, y_k), \ldots, x_k = w_k(y_1, \ldots, y_k)$.
- **Jacobian** is the determinant $J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \cdots & \frac{\partial x_k}{\partial y_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_1}{\partial y_k} & \cdots & \frac{\partial x_k}{\partial y_k} \end{vmatrix}$.

⇨ $f_Y(y_1, \ldots, y_k) = f_X\big(x_1 = w_1(y_1, \ldots., y_k), x_2 = w_2(y_1, \ldots, Y_k), \ldots, x_k = w_k(y_1, \ldots, y_k)\big) \times |J|$

Special Case, k=1: $Y = g(X)$ one-to-one with inverse $x = g^{-1}(y) \Rightarrow f_Y(y) = f_X\big(x = g^{-1}(y)\big) \times \left|\frac{\partial x}{\partial y}\right|$

Convolution Formulas: Sum of 2 Random Variables

Let $S = X_1 + X_2$.

- (**Continuous**)
  - $f_S(s) = \int_{-\infty}^{+\infty} f_X(t, s-t)dt$.
  - If X_1, X_2 are independent, $f_S(s) = \int_{-\infty}^{+\infty} f_1(t) f_2(s-t)dt$.
- (**Discrete**)
  - $P(S = s) = \sum_{x_1 \leq s} f_X(x_1, s - x_1)$.
  - If X_1, X_2 are independent, $P(S = s) = \sum_{x_1 \leq s} f_1(x_1) f_2(s - x_1)$.

Distribution	MGF
$\mathrm{POI}(\mu)$	$e^{\mu(e^t - 1)}$
$\mathrm{BIN}(n, p)$	$(pe^t + q)^n, \ q = 1 - p$
GEOM(p)	$M(t) = \frac{pe^t}{1 - qe^t}$
NB(r,p)	$M(t) = \left(\frac{pe^t}{1 - qe^t}\right)^r$
$\mathrm{N}(\mu, \sigma^2)$	$\exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$
$\mathrm{GAM}(\theta, \kappa)$	$\left(\frac{1}{1 - \theta t}\right)^\kappa$
$\mathrm{EXP}(\theta)$	$\left(\frac{1}{1 - \theta t}\right)$

Method of Mgfs

- $Y = X_1 + X_2 + \cdots + X_k$, iid X_i's, $M_i(t)$ = mgf of X_i
- $M_Y(t) = M_1(t) \times M_2(t) \times \ldots \times M_m(t)$

$X_1 \sim \mathrm{N}(\mu_1, \sigma_1^2), \ldots, X_n \sim \mathrm{N}(\mu_n, \sigma_n^2)$ independent => $Y = (a_1 X_1 + \ldots + a_n X_n) \sim \mathrm{N}\left(\sum_{i=1}^n a_i \mu_i, \sum_{i=1}^n a_i^2 \sigma_i^2\right)$

Order Statistics

- Smallest $X_{(1)}$
  - pdf $f_{(1)}(x) = nf(x)[1 - F(x)]^{n-1}$
  - cdf $F_{(1)}(x) = 1 - [1 - F(x)]^n$

- Largest $X_{(n)}$
  - pdf $f_{(n)}(x) = nf(x)[F(x)]^{n-1}$
  - cdf $F_{(n)}(x) = [F(x)]^n$

- j^{th} smallest $X_{(j)}$
  - pdf $f_{(j)}(x) = \frac{n!}{(j-1)!(n-j)!}[F(x)]^{j-1} \cdot [1 - F(x)]^{n-j} \cdot f(x)$

Chapter 7

Y_n **converges in distribution** to Y $\left(Y_n \xrightarrow{d} Y\right)$: $\lim_{n\to\infty} G_n(y) = G(y)$ for all y at which $G(y)$ is continuous

Theorem 7.3.1 Y_i has cdf $G_i(y)$ and mgf $M_i(t)$. Y has cdf $G(y)$ and mgf $M(t)$. If $\lim_{n\to\infty} M_n(t) = M(t)$, then $\lim_{n\to\infty} G_n(y) = G(y)$ for all y for which G is continuous, i.e. $Y_n \xrightarrow{d} Y$.

Theorem 7.3.2 (**Central Limit Theorem**) $X_1, \dots, X_n$ are iid with $E(X_i) = \mu$ and $V(X_i) = \sigma^2 < \infty$.

⇨ $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \xrightarrow{d} \mathrm{N}(0,1)$

⇨ $n\bar{X} = \sum X_i \dot{\sim} N(n\mu, n\sigma^2)$, $\quad \frac{\sum X_i - n\mu}{\sigma\sqrt{n}} \dot{\sim} N(0,1)$

If $\frac{Y_n - m}{\frac{c}{\sqrt{n}}} \xrightarrow{d} \mathrm{N}(0,1)$, then Y_n has an **asymptotic Normal distribution** with **asymptotic mean** m and **asymptotic variance** c^2/n.

Y_n **converges in probability** to Y $\left(Y_n \xrightarrow{p} Y\right)$ if $\lim_{n\to\infty} P(|Y_n - Y| < \varepsilon) = 1$ for any $\varepsilon > 0$.

Theorem 7.6.2 (Weak Law of Large Numbers). $X_1, \dots, X_n$ are iid with $E(X_i) = \mu$ and $V(X_i) = \sigma^2 < \infty$
=> $\bar{X} \xrightarrow{p} \mu$.

Theorem 7.7.1. If $Y_n \xrightarrow{p} Y$, then $Y_n \xrightarrow{d} Y$.
Theorem 7.7.2. If $Y_n \xrightarrow{p} c$, then $g(Y_n) \xrightarrow{p} g(c)$ for any function g that is continuous at c.
Theorem 7.7.5. If $Y_n \xrightarrow{d} Y$, then $g(Y_n) \xrightarrow{d} g(Y)$ for every continuous function g(y).

Theorem 7.7.6. If $\sqrt{n}\left(\frac{Y_n - m}{c}\right) \xrightarrow{d} \mathrm{N}(0,1)$ and $g(y)$ has a nonzero derivative at $y = m$, i.e. $g'(m) \neq 0$, then

$$\sqrt{n}\left(\frac{g(Y_n) - g(m)}{|c \cdot g'(m)|}\right) \xrightarrow{d} \mathrm{N}(0,1)$$

Table A.1: Cumulative Binomial Probabilities: $B(x; n, p) = \sum_{y=0}^{x} b(y; n, p)$

$n = 5$		p														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
	0	0.951	0.774	0.590	0.328	0.237	0.168	0.078	0.031	0.010	0.002	0.001	0.000	0.000	0.000	0.000
	1	0.999	0.977	0.919	0.737	0.633	0.528	0.337	0.187	0.087	0.031	0.016	0.007	0.000	0.000	0.000
x	2	1.000	0.999	0.991	0.942	0.896	0.837	0.683	0.500	0.317	0.163	0.104	0.058	0.009	0.001	0.000
	3	1.000	1.000	1.000	0.993	0.984	0.969	0.913	0.812	0.663	0.472	0.367	0.263	0.081	0.023	0.001
	4	1.000	1.000	1.000	1.000	0.999	0.998	0.990	0.969	0.922	0.832	0.763	0.672	0.410	0.226	0.049

$n = 10$		p														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
	0	0.904	0.599	0.349	0.107	0.056	0.028	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.996	0.914	0.736	0.376	0.244	0.149	0.046	0.011	0.002	0.000	0.000	0.000	0.000	0.000	0.000
	2	1.000	0.988	0.930	0.678	0.526	0.383	0.167	0.055	0.012	0.002	0.000	0.000	0.000	0.000	0.000
	3	1.000	0.999	0.987	0.879	0.776	0.650	0.382	0.172	0.055	0.011	0.004	0.001	0.000	0.000	0.000
x	4	1.000	1.000	0.998	0.967	0.922	0.850	0.633	0.377	0.166	0.047	0.020	0.006	0.000	0.000	0.000
	5	1.000	1.000	1.000	0.994	0.980	0.953	0.834	0.623	0.367	0.150	0.078	0.033	0.002	0.000	0.000
	6	1.000	1.000	1.000	0.999	0.996	0.989	0.945	0.828	0.618	0.350	0.224	0.121	0.013	0.001	0.000
	7	1.000	1.000	1.000	1.000	1.000	0.998	0.988	0.945	0.833	0.617	0.474	0.322	0.070	0.012	0.000
	8	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.989	0.954	0.851	0.756	0.624	0.264	0.086	0.004
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.972	0.944	0.893	0.651	0.401	0.096

$n = 15$		p														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
	0	0.860	0.463	0.206	0.035	0.013	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.990	0.829	0.549	0.167	0.080	0.035	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	1.000	0.964	0.816	0.398	0.236	0.127	0.027	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3	1.000	0.995	0.944	0.648	0.461	0.297	0.091	0.018	0.002	0.000	0.000	0.000	0.000	0.000	0.000
	4	1.000	0.999	0.987	0.836	0.686	0.515	0.217	0.059	0.009	0.001	0.000	0.000	0.000	0.000	0.000
	5	1.000	1.000	0.998	0.939	0.852	0.722	0.403	0.151	0.034	0.004	0.001	0.000	0.000	0.000	0.000
	6	1.000	1.000	1.000	0.982	0.943	0.869	0.610	0.304	0.095	0.015	0.004	0.001	0.000	0.000	0.000
x	7	1.000	1.000	1.000	0.996	0.983	0.950	0.787	0.500	0.213	0.050	0.017	0.004	0.000	0.000	0.000
	8	1.000	1.000	1.000	0.999	0.996	0.985	0.905	0.696	0.390	0.131	0.057	0.018	0.000	0.000	0.000
	9	1.000	1.000	1.000	1.000	0.999	0.996	0.966	0.849	0.597	0.278	0.148	0.061	0.002	0.000	0.000
	10	1.000	1.000	1.000	1.000	1.000	0.999	0.991	0.941	0.783	0.485	0.314	0.164	0.013	0.001	0.000
	11	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.982	0.909	0.703	0.539	0.352	0.056	0.005	0.000
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.973	0.873	0.764	0.602	0.184	0.036	0.000
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.995	0.965	0.920	0.833	0.451	0.171	0.010
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.995	0.987	0.965	0.794	0.537	0.140

Table A.1: Cumulative Binomial Probabilities (*continued*)

$n = 20$								p								
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
	0	0.818	0.358	0.122	0.012	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.983	0.736	0.392	0.069	0.024	0.008	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.999	0.925	0.677	0.206	0.091	0.035	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3	1.000	0.984	0.867	0.411	0.225	0.107	0.016	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4	1.000	0.997	0.957	0.630	0.415	0.238	0.051	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	5	1.000	1.000	0.989	0.804	0.617	0.416	0.126	0.021	0.002	0.000	0.000	0.000	0.000	0.000	0.000
	6	1.000	1.000	0.998	0.913	0.786	0.608	0.250	0.058	0.006	0.000	0.000	0.000	0.000	0.000	0.000
	7	1.000	1.000	1.000	0.968	0.898	0.772	0.416	0.132	0.021	0.001	0.000	0.000	0.000	0.000	0.000
	8	1.000	1.000	1.000	0.990	0.959	0.887	0.596	0.252	0.057	0.005	0.001	0.000	0.000	0.000	0.000
	9	1.000	1.000	1.000	0.997	0.986	0.952	0.755	0.412	0.128	0.017	0.004	0.001	0.000	0.000	0.000
x	10	1.000	1.000	1.000	0.999	0.996	0.983	0.872	0.588	0.245	0.048	0.014	0.003	0.000	0.000	0.000
	11	1.000	1.000	1.000	1.000	0.999	0.995	0.943	0.748	0.404	0.113	0.041	0.010	0.000	0.000	0.000
	12	1.000	1.000	1.000	1.000	1.000	0.999	0.979	0.868	0.584	0.228	0.102	0.032	0.000	0.000	0.000
	13	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.942	0.750	0.392	0.214	0.087	0.002	0.000	0.000
	14	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.979	0.874	0.584	0.383	0.196	0.011	0.000	0.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.949	0.762	0.585	0.370	0.043	0.003	0.000
	16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.984	0.893	0.775	0.589	0.133	0.016	0.000
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.965	0.909	0.794	0.323	0.075	0.001
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.992	0.976	0.931	0.608	0.264	0.017
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.988	0.878	0.642	0.182

$n = 25$								p								
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
	0	0.778	0.277	0.072	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.974	0.642	0.271	0.027	0.007	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.998	0.873	0.537	0.098	0.032	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3	1.000	0.966	0.764	0.234	0.096	0.033	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4	1.000	0.993	0.902	0.421	0.214	0.090	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	5	1.000	0.999	0.967	0.617	0.378	0.193	0.029	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	6	1.000	1.000	0.991	0.780	0.561	0.341	0.074	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	7	1.000	1.000	0.998	0.891	0.727	0.512	0.154	0.022	0.001	0.000	0.000	0.000	0.000	0.000	0.000
	8	1.000	1.000	1.000	0.953	0.851	0.677	0.274	0.054	0.004	0.000	0.000	0.000	0.000	0.000	0.000
	9	1.000	1.000	1.000	0.983	0.929	0.811	0.425	0.115	0.013	0.000	0.000	0.000	0.000	0.000	0.000
	10	1.000	1.000	1.000	0.994	0.970	0.902	0.586	0.212	0.034	0.002	0.000	0.000	0.000	0.000	0.000
	11	1.000	1.000	1.000	0.998	0.989	0.956	0.732	0.345	0.078	0.006	0.001	0.000	0.000	0.000	0.000
x	12	1.000	1.000	1.000	1.000	0.997	0.983	0.846	0.500	0.154	0.017	0.003	0.000	0.000	0.000	0.000
	13	1.000	1.000	1.000	1.000	0.999	0.994	0.922	0.655	0.268	0.044	0.011	0.002	0.000	0.000	0.000
	14	1.000	1.000	1.000	1.000	1.000	0.998	0.966	0.788	0.414	0.098	0.030	0.006	0.000	0.000	0.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	0.987	0.885	0.575	0.189	0.071	0.017	0.000	0.000	0.000
	16	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.946	0.726	0.323	0.149	0.047	0.000	0.000	0.000
	17	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.978	0.846	0.488	0.273	0.109	0.002	0.000	0.000
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.993	0.926	0.659	0.439	0.220	0.009	0.000	0.000
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.971	0.807	0.622	0.383	0.033	0.001	0.000
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.991	0.910	0.786	0.579	0.098	0.007	0.000
	21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.967	0.904	0.766	0.236	0.034	0.000
	22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.991	0.968	0.902	0.463	0.127	0.002
	23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.993	0.973	0.729	0.358	0.026
	24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.996	0.928	0.723	0.222

Table A.2: Cumulative Poisson Probabilities: $F(x;\lambda) = \sum_{y=0}^{x} \frac{e^{-\lambda}\lambda^{y}}{y!}$

Here, λ is μ.

		λ									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	0	0.905	0.819	0.741	0.670	0.607	0.549	0.497	0.449	0.407	0.368
	1	0.995	0.982	0.963	0.938	0.910	0.878	0.844	0.809	0.772	0.736
	2	1.000	0.999	0.996	0.992	0.986	0.977	0.966	0.953	0.937	0.920
x	3	1.000	1.000	1.000	0.999	0.998	0.997	0.994	0.991	0.987	0.981
	4	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.998	0.996
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
	6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

		λ										
		2	3	4	5	6	7	8	9	10	15	20
	0	0.135	0.050	0.018	0.007	0.002	0.001	0.000	0.000	0.000	0.000	0.000
	1	0.406	0.199	0.092	0.040	0.017	0.007	0.003	0.001	0.000	0.000	0.000
	2	0.677	0.423	0.238	0.125	0.062	0.030	0.014	0.006	0.003	0.000	0.000
	3	0.857	0.647	0.433	0.265	0.151	0.082	0.042	0.021	0.010	0.000	0.000
	4	0.947	0.815	0.629	0.440	0.285	0.173	0.100	0.055	0.029	0.001	0.000
	5	0.983	0.916	0.785	0.616	0.446	0.301	0.191	0.116	0.067	0.003	0.000
	6	0.995	0.966	0.889	0.762	0.606	0.450	0.313	0.207	0.130	0.008	0.000
	7	0.999	0.988	0.949	0.867	0.744	0.599	0.453	0.324	0.220	0.018	0.001
	8	1.000	0.996	0.979	0.932	0.847	0.729	0.593	0.456	0.333	0.037	0.002
	9	1.000	0.999	0.992	0.968	0.916	0.830	0.717	0.587	0.458	0.070	0.005
	10	1.000	1.000	0.997	0.986	0.957	0.901	0.816	0.706	0.583	0.118	0.011
	11	1.000	1.000	0.999	0.995	0.980	0.947	0.888	0.803	0.697	0.185	0.021
	12	1.000	1.000	1.000	0.998	0.991	0.973	0.936	0.876	0.792	0.268	0.039
	13	1.000	1.000	1.000	0.999	0.996	0.987	0.966	0.926	0.864	0.363	0.066
	14	1.000	1.000	1.000	1.000	0.999	0.994	0.983	0.959	0.917	0.466	0.105
	15	1.000	1.000	1.000	1.000	0.999	0.998	0.992	0.978	0.951	0.568	0.157
	16	1.000	1.000	1.000	1.000	1.000	0.999	0.996	0.989	0.973	0.664	0.221
	17	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.995	0.986	0.749	0.297
	18	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.998	0.993	0.819	0.381
x	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.875	0.470
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.917	0.559
	21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.947	0.644
	22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.967	0.721
	23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.981	0.787
	24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.989	0.843
	25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.888
	26	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.922
	27	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.948
	28	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.966
	29	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.978
	30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.987
	31	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.992
	32	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.995
	33	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997
	34	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
	35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
	36	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A.3 Standard Normal Probabilities (Part 1)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table A.3 Standard Normal Probabilities (Part 2)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
+0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
+0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
+0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
+0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
+0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
+0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
+0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
+0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
+0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8079	.8106	.8133
+0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
+1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
+1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
+1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
+1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
+1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
+1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
+1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
+1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
+1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
+1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
+2.0	.9773	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
+2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
+2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
+2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
+2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
+2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
+2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
+2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
+2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
+2.9	.9981	.9982	.9983	.9983	.9984	.9984	.9985	.9985	.9986	.9986
+3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
+3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
+3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
+3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
+3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998