Combining Tools for Optimization and Analysis of Floating-Point Computations

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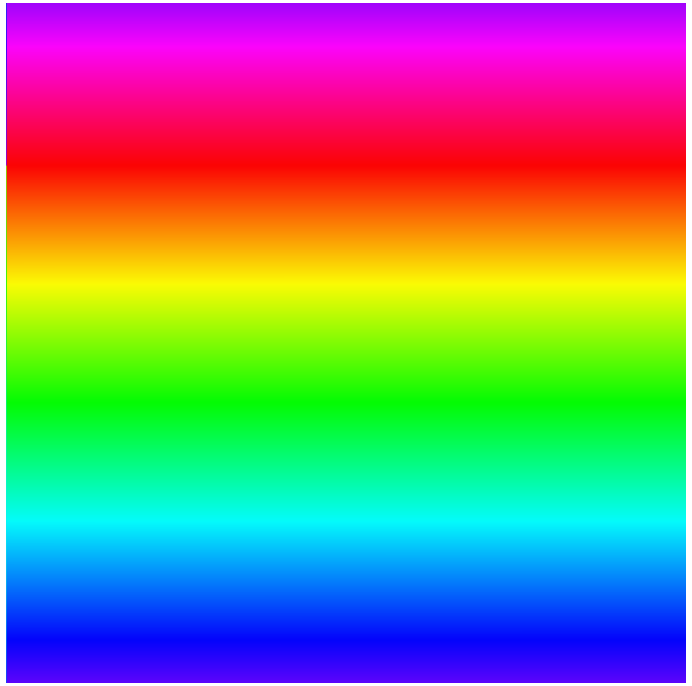
FM 2018, 17.07.2018

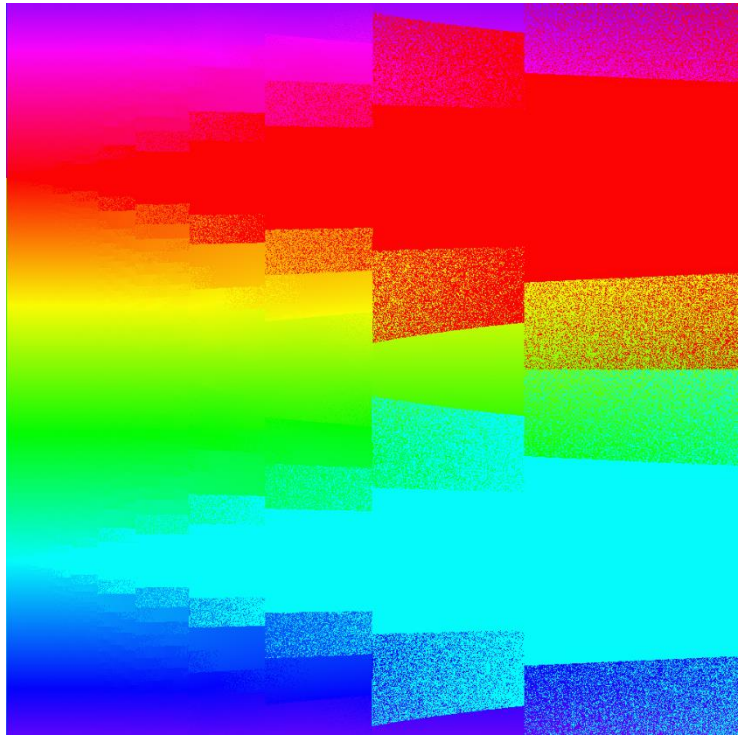
MAX PLANCK INSTITUTE
FOR SOFTWARE SYSTEMS

PLSE

Floating-Point Computations Are Ubiquitous



Floating-Point Computations are Tricky



Rene ols

None of these tools
have been connected together!

Precimonious

Daisy

Precisa

Herbie

VCFloat

FPTuner

???

Gappa

+

- verify Herbies optimizations with Daisy
- compare Herbie's and Daisy's optimizations

$\Longrightarrow$ best run together

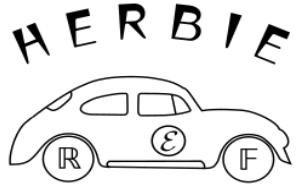
➢ connecting exposes tools to new inputs

➢ found bugs and inaccuracies

$\Longrightarrow$ call to action to connect tools

+

Concretely

- unify input formats
- verify Herbies optimizations with Daisy  $+_{CHECK}$ 
- compare optimization techniques  $+_{OPT}$ 
- fix bugs and inaccuracies

Floating-Point Arithmetic

$0.2 + 0.1$ `0.2 + 0.1;`

0.3 $\neq$ `0.30000000000000004`

roundoff error

in Herbie — error of $\mathbf{0}$

in Daisy — error of $\boldsymbol{4e^{-15}}$

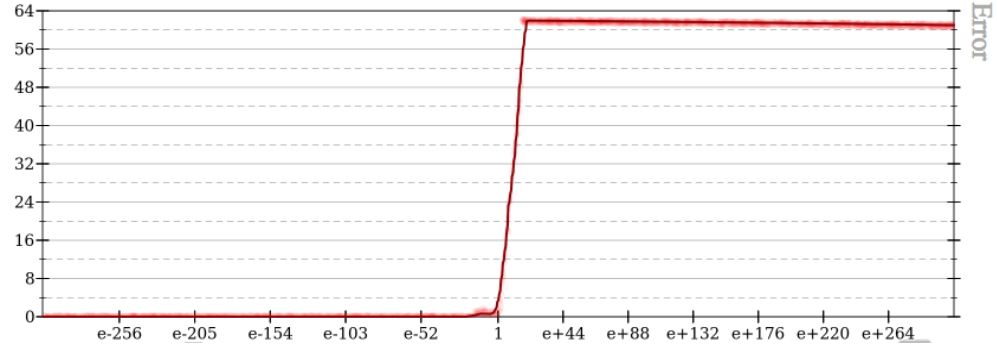
Heuristic Optimization of Floating-Point Programs

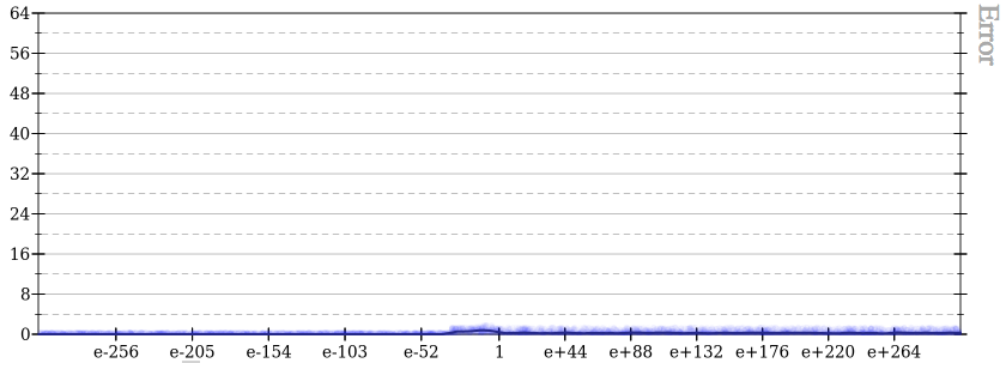
$\sqrt{x+1} - \sqrt{x}$ ⇨ Optimizer ⇨ $\dfrac{1}{\sqrt{x+1}+\sqrt{x}}$

⟹ dynamic analysis

⟹ (possibly) unsound optimizations

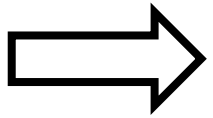
⟹ heuristic hill-climbing algorithm



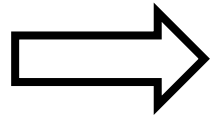


Sound Analysis of Floating-Point Programs

$$f(x) = \sqrt{x+1} - \sqrt{x}$$
$$x \in [10, 100]$$



Static Analyzer



error is 2.34e^{-15}

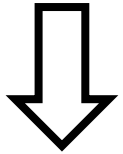
$\Longrightarrow$ sound dataflow analysis

$\Longrightarrow$ 2.34e^{-15} is a sound but possibly pessimistic upper bound

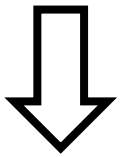
Sound Optimization of Floating-Point Programs

$$-x_1 * x_2 - 2 * x_2 * x_3 - x_1 - x_3$$

$$x_i \in [-15, 15]$$



Static Analyzer + Rewriter

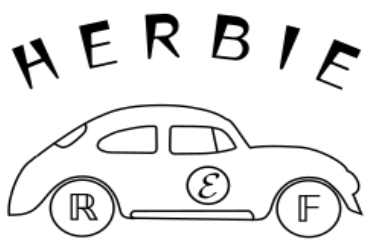


$$(-x_1 * x_2 - (x_1 + x_3)) - (2 * x_2) * x_3$$

$\Longrightarrow$ rewriting optimizes accuracy

$\Longrightarrow$ improved error bound

from 2.95e^{-13} to 1.98e^{-13}



$+_{CHECK}$


```
// portfolio error for Herbies input function
err_src = min{ Daisy(A, FPCore2Scala(f_src ) | A <- AnalysisTypes }

//optimize input with Herbie
f_res = Herbie(f_src)

//portfolio error for Herbies result function
err_res = min{ Daisy(A, FPCore2Scala (f_res ) | A <- AnalysisTypes }
```

The FPBench Project

```
(FPCore
 (x)
 :name "Example"
 :precision binary64
 (- (sqrt (+ x 1)) (sqrt x))
 )
```

- collection of benchmarks
- new standardized input format
- converters to input formats
  - FPCore → Gappa
  - FPCore → C
  - **FPCore → Scala**

HERBIE + *CHECK* Daisy

103 FPBench benchmarks in total

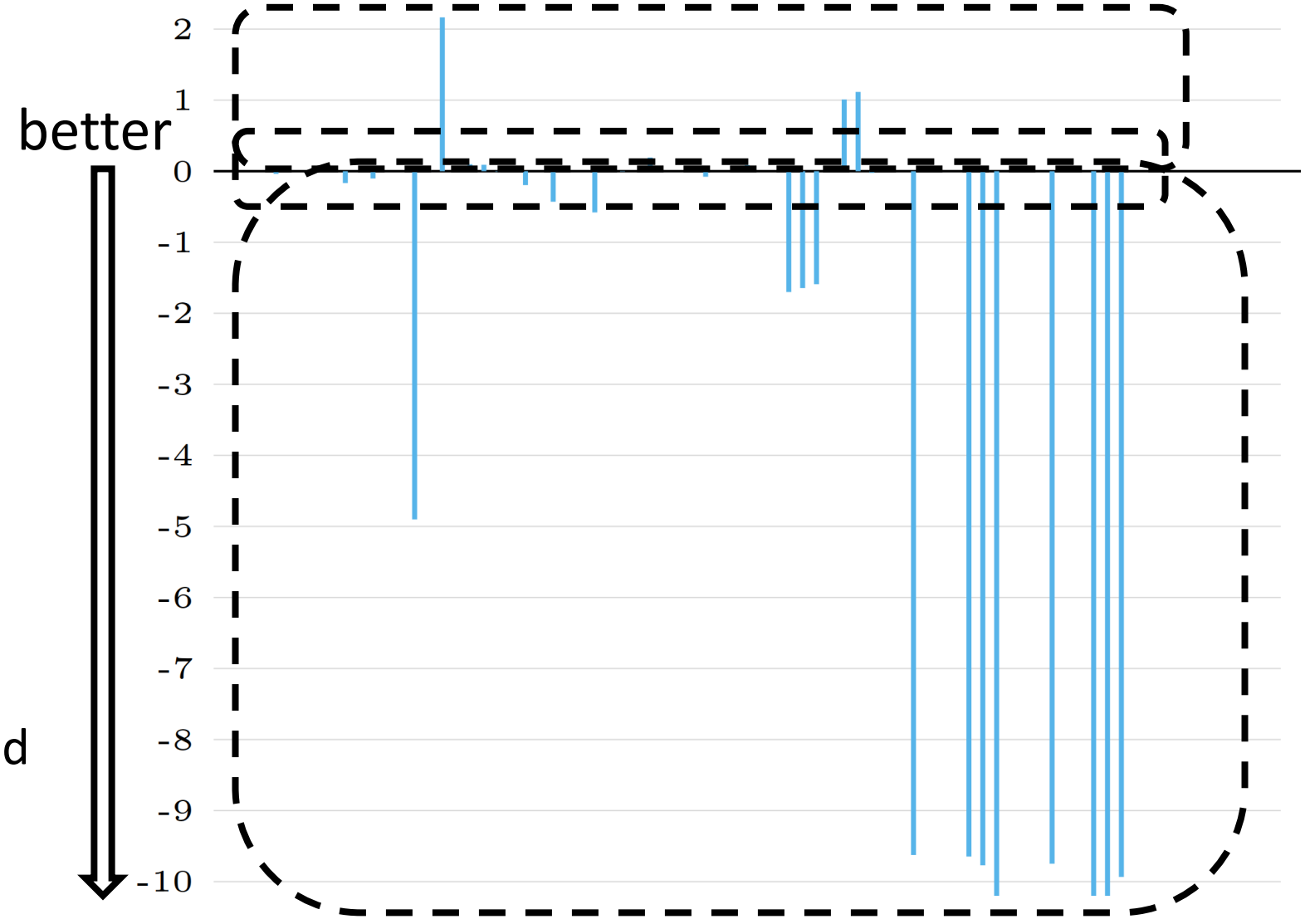
Herbie **times out** on **34**

Daisy raises an **alarm for 22**
13 alarms on **source programs**
9 alarms on **Herbies result**

End-to-end result for 47:

- **8** have become **worse**
- **18** have the **same** worst-case bound
- **21** have a **provable improvement**

Herbie's error improvement



$+_{OPT}$

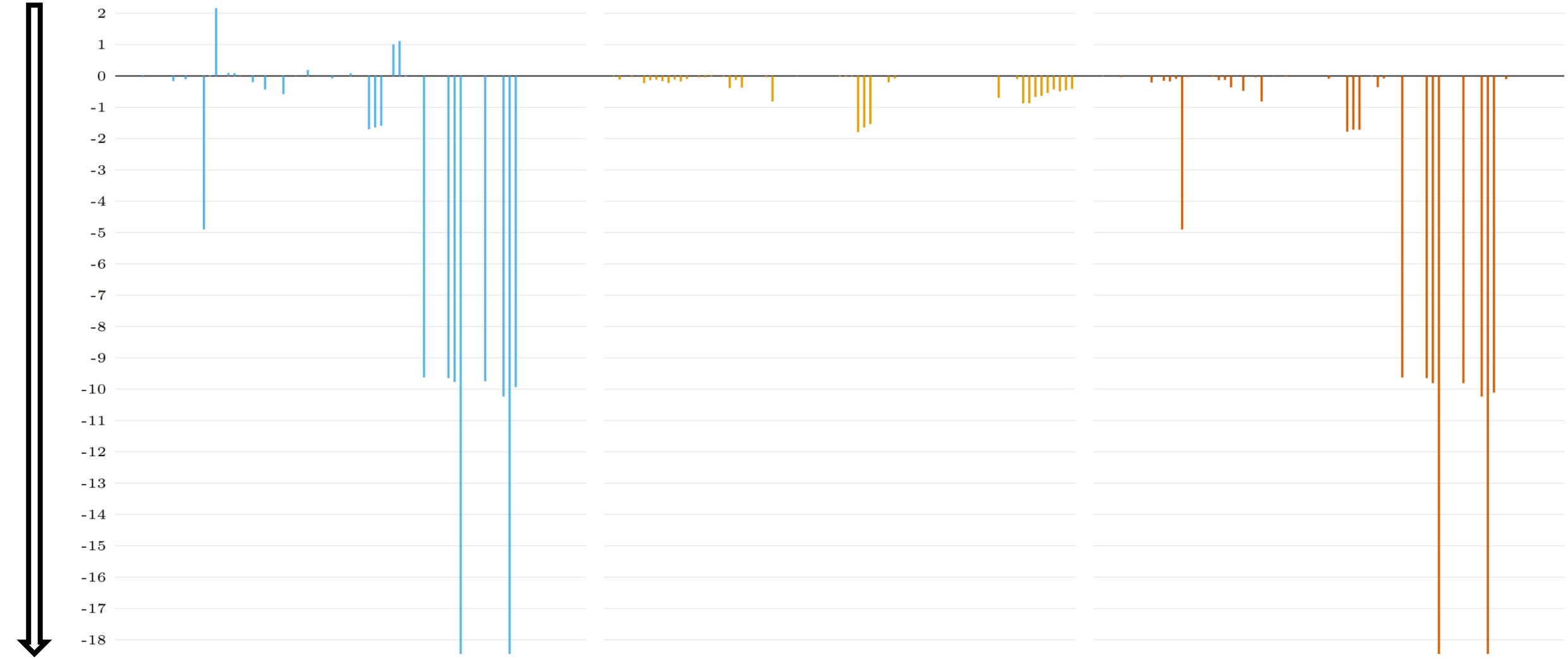

```
//error for Herbies input function
err_src = Daisy(FPCore2Scala(f_src))

//optimize input with Herbie
f_herbie = Herbie(f_src)

//compute worst-case error with Daisy
err_herbie = Daisy(FPCore2Scala(f_herbie))

//optimize input with Daisy only and compute worst-case error
(err_daisy,f_daisy) = Daisy(Rewrite, FPCore2Scala(f_src))

//optimize input with Herbie, then Daisy and compute worst-case error
(err_both,f_both) = Daisy(Rewrite, FPCore2Scala (Herbie (f_src)))
```

Error Improvement of

Herbie

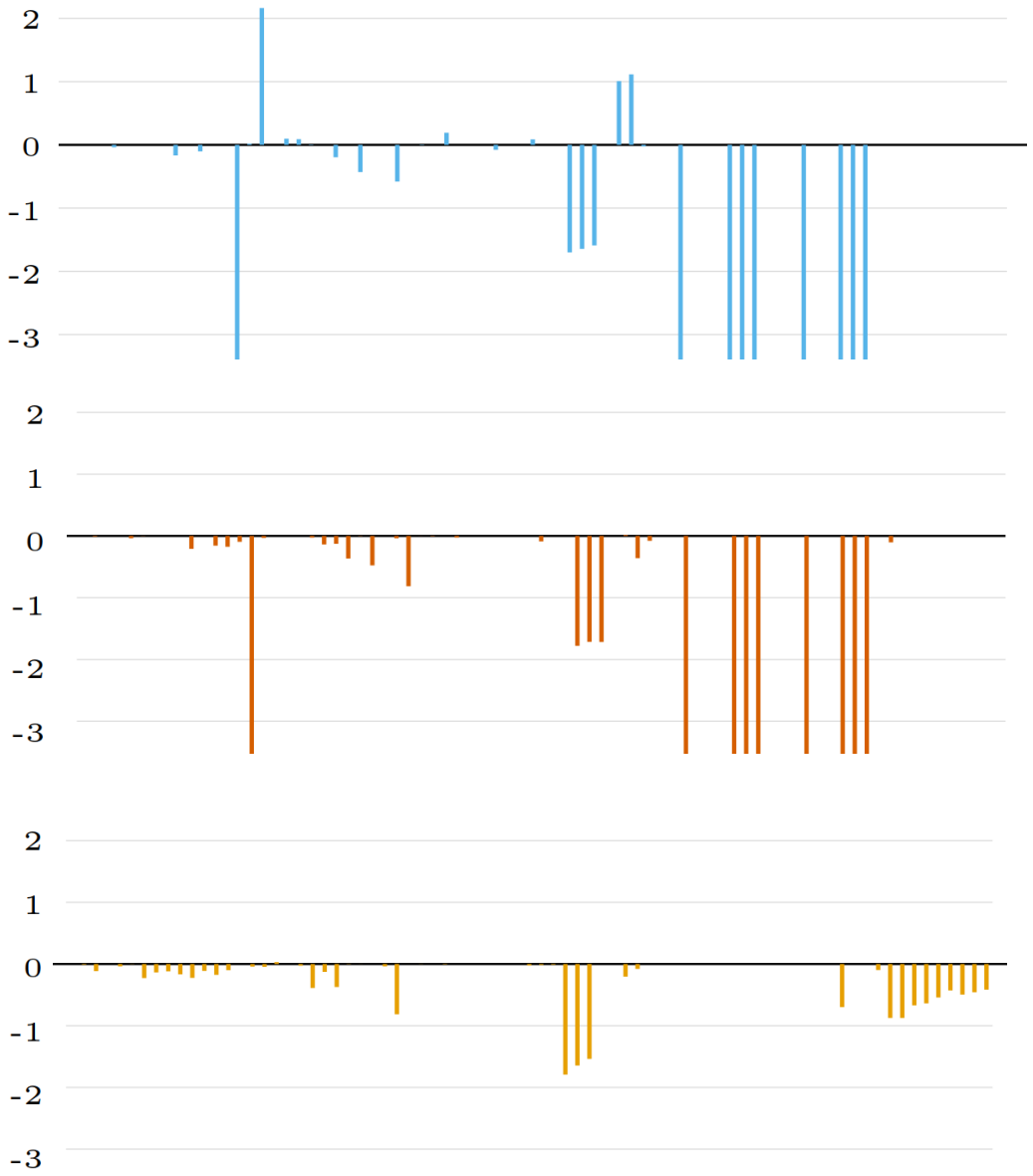
Daisy

Both



better

Evaluation of the optimization algorithms



Herbie

Both

Daisy

Additional Benefit: Improving Tool Robustness

Herbie

- Incorrect Typing rule for let-bindings
- Incorrect handling of duplicate fields
- Infinite loop for some preconditions

Daisy

- Improved analysis of elementary functions
- Improved error reporting

Conclusion

- floating-point tools solve orthogonal problems
- connecting them is easy and exposes bugs
- Daisy is a good verification backend for Herbie
- Herbie's and Daisy's optimizations work best together

- first step on bigger vision of connecting tools

https://fpbench.org

Questions?