Algebra

ax^2

z

x

π

y

$2ab$

$\sqrt{x}$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Formula

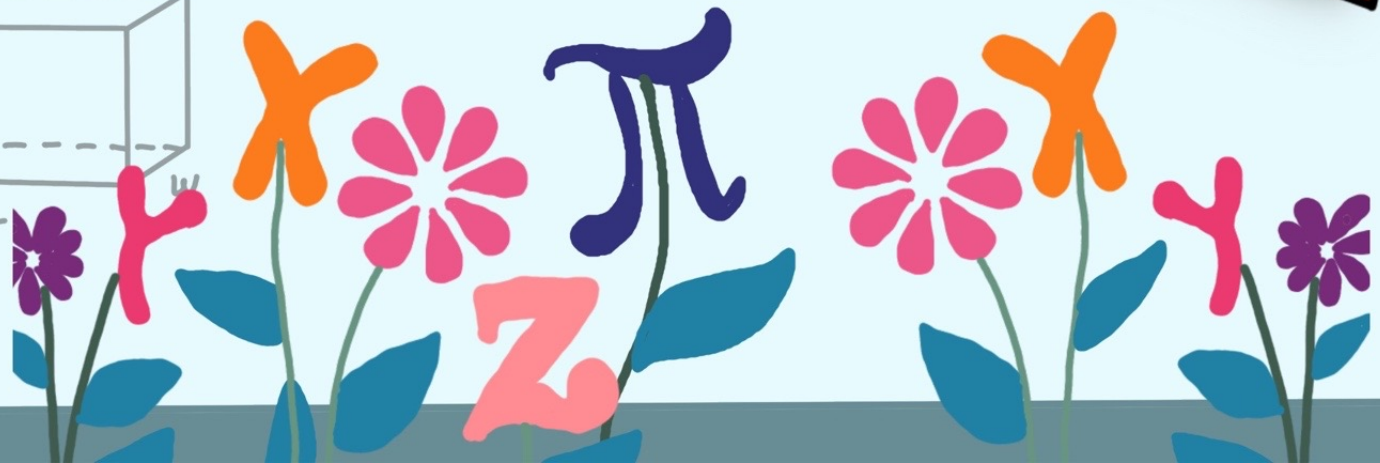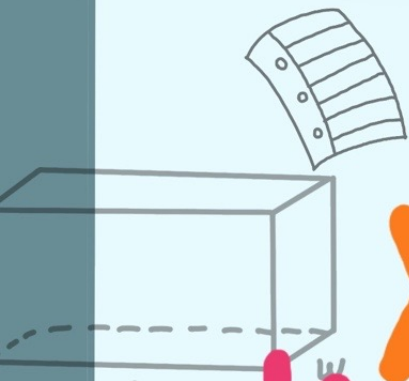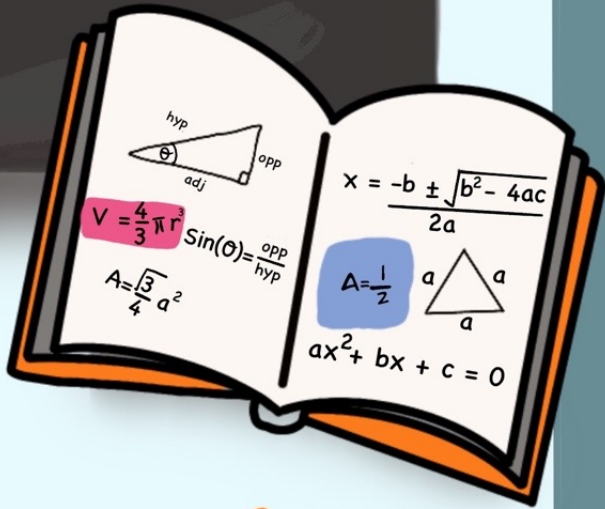$(x+a)(x+b) =$
$x^2 + (a+b)x + ab$

$a^2 - b^2 = (a-b)(a+b)$

Formula

1435

$V = \frac{4}{3}\pi r^3$

$\sin(\theta) = \frac{opp}{hyp}$

$A = \frac{\sqrt{3}}{4}a^2$

$A = \frac{1}{2}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$ax^2 + bx + c = 0$

Question 1

Find the value of $5a - 3b$ when $a = 7$ and $b = -2$. [2]

$$5(7) - 3(-2) = 41$$

Question 2

Make q the subject of the formula $p = 2q^2$. [2]

$$q^2 = \frac{p}{2}$$

$$q = \sqrt{p/2}$$

Question 3

Make a the subject of the formula.

$$x = y + \sqrt{a}$$ [2]

$$\sqrt{a} = x - y$$

$$a = (x - y)^2$$

The Maths Society

Question 4

$$s = ut + 16t^2$$

[2]

Find the value of s when $u = 2$ and $t = 3$.

$$s = 2(3) + 16(3)^2$$
$$= 150$$

Question 5

$$y = \frac{qx}{p}$$

Write x in terms of p, q and y.

[2]

$$qx = yp$$
$$x = \frac{yp}{q}$$

The Maths Society

Question 6

Make p the subject of the formula. [3]

$$rp + 5 = 3p + 8r$$

$$rp - 3p = 8r - 5$$

$$p(r - 3) = 8r - 5$$

$$p = \frac{8r - 5}{r - 3}$$

Question 7

Solve the equation. [2]

$$6(y + 1) = 9$$

$$y + 1 = \frac{9}{6}$$

$$y = \frac{3}{2} - 1$$

$$y = \frac{1}{2}$$

The Maths Society

Question 8

Make x the subject of the formula.

$$y = ax^2 + b$$

[3]

$$y - b = ax^2$$

$$x^2 = \frac{y-b}{a}$$

$$x = \sqrt{\frac{y-b}{a}}$$
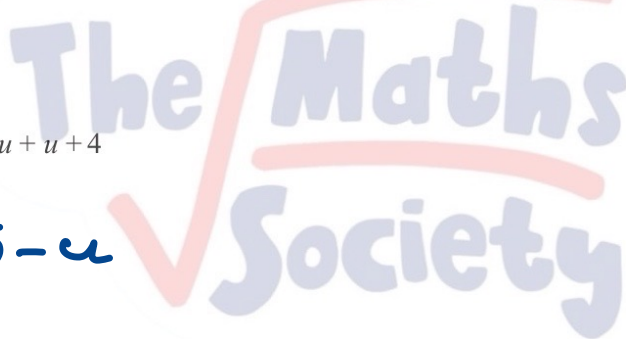
Question 9

Simplify.

$$1 - 2u + u + 4$$

[2]

$$5 - u$$

The Maths Society

Question 10

Make r the subject of this formula.

$$v = \sqrt[3]{p + r}$$

[2]

$$v^3 = p + r$$

$$r = v^3 - p$$

Question 11

Make x the subject of the formula.

$$y = 2 + \sqrt{x - 8}$$

[3]

$$y - 2 = \sqrt{x - 8}$$

$$(y-2)^2 = x - 8$$

$$x = (y-2)^2 + 8$$

The Maths Society

Question 12

$$y = \frac{2}{x^2} + \frac{x^2}{2}$$

Find the value of y when $x = 6$.
Give your answer as a mixed number in its simplest form.   [2]

$$y = \frac{2}{6^2} + \frac{6^2}{2}$$

$$= \frac{2}{36} + \frac{36}{2}$$

$$= \frac{1}{18} + 18$$

$$= 18 \frac{1}{18}$$

Question 13

Make x the subject of the formula.

$$y = (x - 4)^2 + 6$$ [3]

$$y - 6 = (x - 4)^2$$

$$\sqrt{y - 6} = x - 4$$

$$x = \sqrt{y - 6} + 4$$

The Maths Society

Question 1

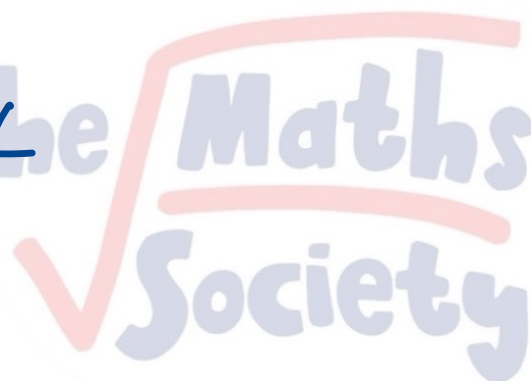$$V = \frac{1}{3}Ah$$

(a) Find V when $A = 15$ and $h = 7$.   [1]

$$V = \frac{1}{3} \times 15 \times 7$$

$$= 35$$

(b) Make h the subject of the formula.   [2]

$$3V = Ah$$

$$h = \frac{3V}{A}$$

Question 2

Rearrange the formula to make x the subject.   [2]

$$y = x^2 + 4$$

$$y - 4 = x^2$$

$$x = \sqrt{y - 4}$$

The Maths Society

Question 3

(a) Expand and simplify $(a + b)^2$. [2]

$$(a+b)(a+b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

(b) Find the value of $a^2 + b^2$ when $a + b = 6$ and $ab = 7$. [1]

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$36 \qquad = a^2 + b^2 + 2 \times 7$$
$$36 - 14 = a^2 + b^2$$
$$22 = a^2 + b^2$$

Question 4

A sphere has a volume of $80\,\text{cm}^3$.

Calculate the radius of the sphere.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [3]

$$\frac{4}{3}\pi r^3 = 80$$
$$\pi r^3 = 80 \times \frac{3}{4}$$
$$\pi r^3 = 60$$
$$r^3 = \frac{60}{\pi}$$
$$r = \sqrt[3]{60/\pi}$$
$$r = 1.2$$

The Maths Society

3

(a)

$$y = \sqrt{8 + \frac{4}{x}}$$

Find y when $x = 2$.
Give your answer correct to 4 decimal places. [2]

$$y = \sqrt{8 + \frac{4}{2}}$$

$$= \sqrt{10}$$

$$= 3.1623$$

(b) Rearrange $y = \sqrt{8 + \frac{4}{x}}$ to make x the subject. [4]

$$y^2 = 8 + \frac{4}{x}$$

$$y^2 - 8 = \frac{4}{x}$$

$$x = \frac{4}{y^2 - 8}$$

The Maths Society

4

Question 6

Expand the brackets.

$$y(3 - y^3)$$

[2]

$$3y - y^4$$

Question 7

Make *y* the subject of the formula.

$$A = \pi x^2 - \pi y^2$$

[3]

$$\pi y^2 = \pi x^2 - A$$

$$y^2 = \frac{\pi x^2 - A}{\pi}$$

$$y = \sqrt{\frac{\pi x^2 - A}{\pi}}$$

$$y = \sqrt{x^2 - \frac{A}{\pi}}$$

The Maths Society

Question 8

Find r when $(5)^{\frac{r}{3}} = 125$. [2]

$$5^{r/3} = 5^3$$

$$r/3 = 3$$

$$r = 9$$

Question 9

Make w the subject of the formula.

$$t = 2 - \frac{3w}{a}$$ [3]

$$ta = 2a - 3w$$

$$3w = 2a - ta$$

$$w = \frac{2a - ta}{3}$$

The Maths Society

Question 10

$$T = 2\pi \sqrt{\frac{l}{g}}$$

(a) Find T when $g = 9.8$ and $\ell = 2$. [2]

$$T = 2\pi \sqrt{\frac{2}{9.8}} = 0.9$$

[3]

(b) Make g the subject of the formula.

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

$$\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g}$$

$$g = l\left(\frac{2\pi}{T}\right)^2$$

The Maths Society

Question 1

Calculate the radius of a sphere with volume 1260 cm^3.

[The volume, V, of a sphere with radius r is $V = \dfrac{4}{3}\pi r^3$.]   [3]

$$\frac{4}{3}\pi r^3 = 1260$$

$$\pi r^3 = 1260 \times \frac{3}{4}$$

$$\pi r^3 = 945$$

$$r^3 = \frac{945}{\pi}$$

$$r = \sqrt[3]{\frac{945}{\pi}} = 3.1$$

Question 2

Rearrange the formula $c = \dfrac{4}{a-b}$ to make a the subject.   [3]

$$a - b = \frac{4}{c}$$

$$a = \frac{4}{c} + b$$

The Maths Society

Question 3

Make x the subject of the formula. $\qquad y = \dfrac{x}{3} + 5$ $\qquad$ [2]

$$y - 5 = \frac{x}{3}$$

$$x = 3(y - 5)$$

Question 4

Expand the brackets and simplify.

$$\frac{1}{2}(6x - 2) - 3(x - 1) \qquad [2]$$

$$= 3x - 1 - 3x + 3$$

$$= 2$$

The Maths Society

Question 5

Make x the subject of $y = \dfrac{(x+3)^2}{5}$. [3]

$$5y = (x+3)^2$$

$$\sqrt{5y} = x+3$$

$$x = \sqrt{5y} - 3$$

Question 6

Rearrange the formula $J = mv - mu$ to make m the subject. [2]

$$J = m(v-u)$$

$$m = \frac{J}{v-u}$$

The Maths Society

Question 7

$$\frac{g}{2} = \sqrt{\frac{h}{i}}$$

Find i in terms of g and h. [3]

$$\frac{g^2}{4} = \frac{h}{i}$$

$$i = \frac{4h}{g^2}$$

Question 8

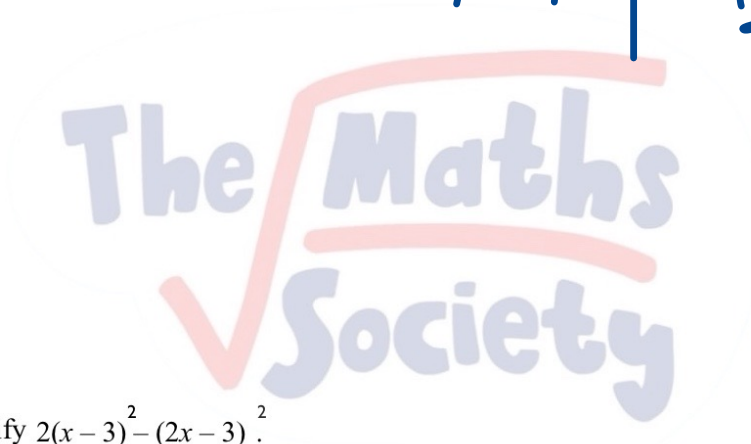Make d the subject of the formula $c = \frac{5d + 4w}{2w}$. [3]

$$2cw = 5d + 4w$$

$$5d = 2cw - 4w$$

$$d = \frac{2cw - 4w}{5}$$

The Maths Society

Question 9

Make x the subject of the formula.

$$P = \frac{x + 3}{x}$$

[4]

$$Px = x + 3$$

$$Px - x = 3$$

$$x(P - 1) = 3$$

$$x = \frac{3}{P - 1}$$

$$P = 1 + \frac{3}{x}$$

$$P - 1 = \frac{3}{x}$$

$$x = \frac{3}{P - 1}$$

Question 10

Expand and simplify $2(x - 3)^2 - (2x - 3)^2$.

[3]

$$2(x - 3)(x - 3) - (2x - 3)(2x - 3)$$

$$= 2(x^2 - 3x - 3x + 9) - (4x^2 - 6x - 6x + 9)$$

$$= 2(x^2 - 6x + 9) - (4x^2 - 12x + 9)$$

$$= 2x^2 - 12x + 18 - 4x^2 + 12x - 9$$

$$= 9 - 2x^2$$

The Maths Society

Question 1

Make y the subject of the formula. $A = \dfrac{r(y + 2)}{5}$ [3]

$$5A = r(y + 2)$$

$$\frac{5A}{r} = y + 2$$

$$y = \frac{5A}{r} - 2$$

Question 2

Simplify $16 - 4(3x - 2)^2$. [3]

$$16 - 4(3x - 2)(3x - 2)$$

$$= 16 - 4(9x^2 - 6x - 6x + 4)$$

$$= 16 - 4(9x^2 - 12x + 4)$$

$$= 16 - 36x^2 + 48x - 16$$

$$= 48x - 36x^2$$

The Maths Society

Question 3

Rearrange the formula to make y the subject.

$$x + \frac{\sqrt{y}}{9} = 1$$

[3]

$$\frac{\sqrt{y}}{9} = 1 - x$$

$$\sqrt{y} = 9(1 - x)$$

$$y = 81(1 - x)^2$$

Question 4

(a) Factorise $ax^2 + bx^2$.

[1]

$$x^2(a + b)$$

(b) Make x the subject of the formula

$$ax^2 + bx^2 - d^2 = p^2.$$

[2]

$$x^2(a + b) - d^2 = p^2$$

$$x^2(a + b) = p^2 + d^2$$

$$x^2 = \frac{p^2 + d^2}{a + b}$$

$$x = \sqrt{\frac{p^2 + d^2}{a + b}}$$

The Maths Society

3

Question 5

Two quantities c and d are connected by the formula $c = 2d + 30$. [1]
Find c when $d = -100$.

$$c = 2(-100) + 30$$
$$= -170$$

Question 6

The number of tennis balls (T) in the diagram is given by the formula

$$T = \frac{1}{2} n (n+1),$$

where n is the number of rows.
The diagram above has 4 rows.
How many tennis balls will there be in a diagram with 20 rows? [1]

$$T = \frac{1}{2} \times 20 (20+1)$$
$$= 10 (21)$$
$$= 210$$

The Maths Society

Question 7

Make *d* the subject of the formula

$$c = \frac{d^3}{2} + 5.$$

[3]

$$c - 5 = \frac{d^3}{2}$$

$$d^3 = 2(c-5)$$

$$d = \sqrt[3]{2c - 10}$$

Question 8

Make *c* the subject of the formula

$$\sqrt{3c - 5} = b.$$

[3]

$$3c - 5 = b^2$$

$$3c = b^2 + 5$$

$$c = \frac{b^2 + 5}{3}$$

The Maths Society

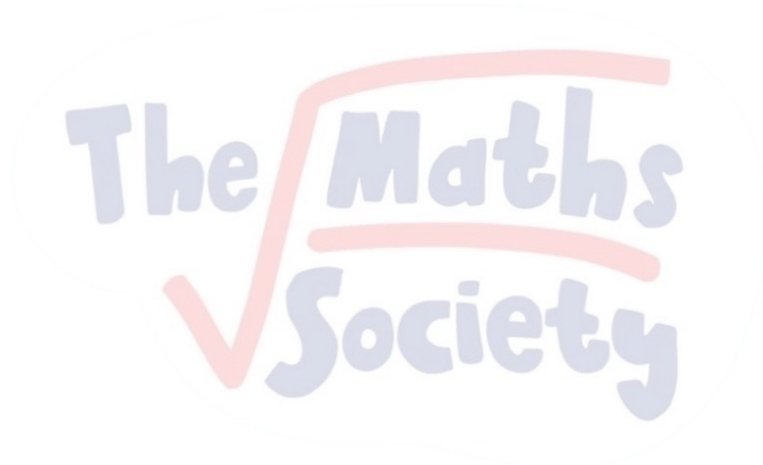Make d the subject of the formula

$$c = kd^2 + e.$$ [3]

$$c - e = kd^2$$

$$d^2 = \frac{c-e}{k}$$

$$d = \sqrt{\frac{c-e}{k}}$$

The Maths Society

Question 1

Make x the subject of the formula.
$$y = \sqrt{x^2 + 1}$$
[3]

$$y^2 = x^2 + 1$$

$$x^2 = y^2 - 1$$

$$x = \sqrt{y^2 - 1}$$

Question 2

$y = p^2 + qr$

(a) Find y when $p = -5$, $q = 3$ and $r = -7$. [2]

$$y = (-5)^2 + 3(-7)$$

$$= 25 - 21$$

$$= 4$$

(b) Write p in terms of q, r and y. [2]

$$y - qr = p^2$$

$$p = \sqrt{y - qr}$$

The Maths Society

Question 3

Make b the subject of the formula.

[3]

$$c = \sqrt{a^2 + b^2}$$

$$c^2 = a^2 + b^2$$

$$b^2 = c^2 - a^2$$

$$b = \sqrt{c^2 - a^2}$$

Question 4

Simplify the expression.

[2]

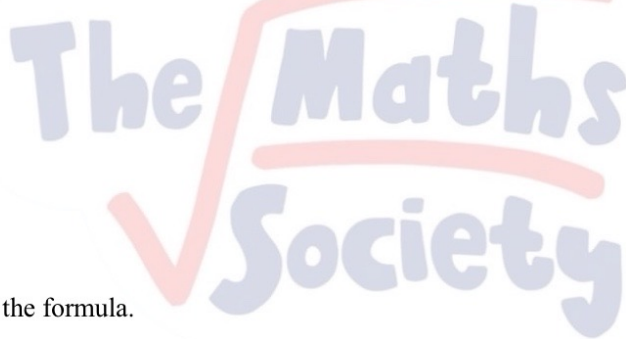$$(a^{\frac{1}{2}} - b^{\frac{1}{2}})(a^{\frac{1}{2}} + b^{\frac{1}{2}})$$

$$a + a^{\frac{1}{2}} b^{\frac{1}{2}} - a^{\frac{1}{2}} b^{\frac{1}{2}} - b$$

$$= a - b$$

The Maths Society

Question 5

Rearrange the formula $y = \dfrac{x+2}{x-4}$ to make x the subject. [4]

$$y(x-4) = x+2$$
$$xy - 4y = x+2$$
$$xy - x = 4y+2$$
$$x(y-1) = 4y+2$$
$$x = \frac{4y+2}{y-1}$$

Question 6

Make w the subject of the formula. [4]

$$c = \frac{4+w}{w+3}$$

$$c(w+3) = 4+w$$
$$cw + 3c = 4+w$$
$$cw - w = 4 - 3c$$
$$w(c-1) = 4 - 3c$$
$$w = \frac{4-3c}{c-1}$$

The Maths Society

Question 7

$$w = \frac{1}{\sqrt{LC}}$$

(a) Find w when $L = 8 \times 10^{-3}$ and $C = 2 \times 10^{-9}$. [3]

Give your answer in standard form.

$$w = \frac{1}{\sqrt{8 \times 10^{-3} \times 2 \times 10^{-9}}}$$

$$= \frac{1}{\sqrt{16 \times 10^{-12}}}$$

$$= \frac{1}{4 \times 10^{-6}} = 0.25 \times 10^{6} = 2.5 \times 10^{5}$$

(b) Rearrange the formula to make C the subject. [3]

$$w^2 = \frac{1}{LC}$$

$$Lw^2 = \frac{1}{C}$$

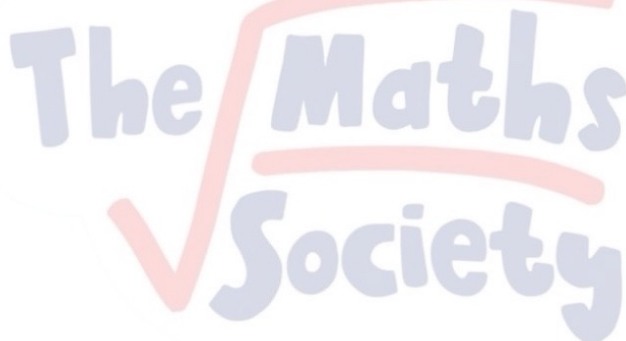$$C = \frac{1}{Lw^2}$$

The Maths Society

Question 8

$$ap = px + c$$

[3]

Write p in terms of a, c and x.

$$ap - px = c$$

$$p(a - x) = c$$

$$p = \frac{c}{a - x}$$

The Maths Society
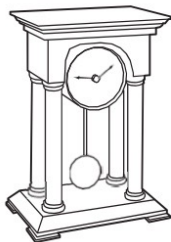
Question 9

The length of time, T seconds, that the pendulum in the clock takes to swing is given by the formula $T = \dfrac{6}{\sqrt{(1+g^2)}}$.

Rearrange the formula to make g the subject. [4]

$$T^2 = \frac{36}{1+g^2}$$

$$1+g^2 = \frac{36}{T^2}$$

$$g^2 = \frac{36}{T^2} - 1$$

$$g = \sqrt{\frac{36}{T^2} - 1}$$

Question 10

(a) $3^x = \frac{1}{3}$.

Write down the value of x. [1]

$$3^x = 3^{-1}$$

$$x = -1$$

(b) $5^y = k$.

Find 5^{y+1}, in terms of k. [1]

$$5^{y+1} = 5^y \cdot 5$$

$$= 5k$$

The Maths Society

7