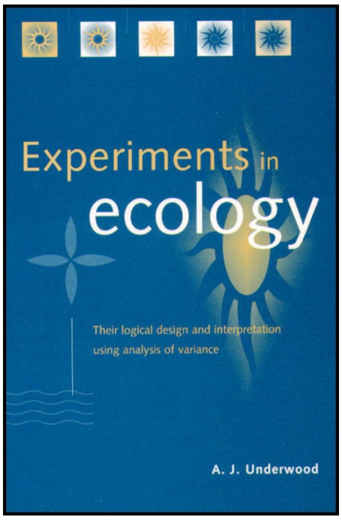
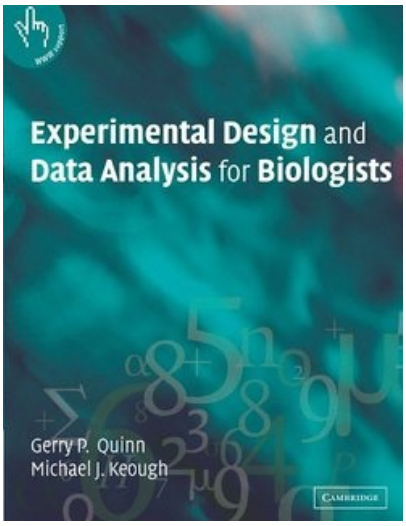
Some statistical considerations in status- and environmental impact assessment





- Precision of mean estimates
- Cost-benefit optimisation
- Statistical power in hypothesis-testing

From samples to populations

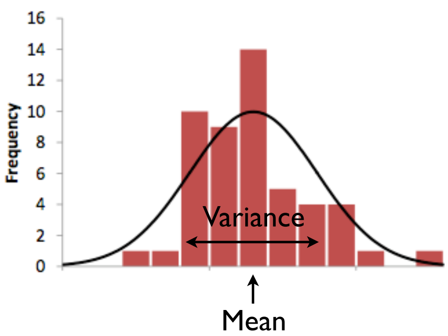
Population
("true" parameter)

Sample
(estimated parameter)

$$\mu = \frac{\sum x_i}{n}$$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{n}$$

$$\sigma = \sqrt{\sigma^2}$$

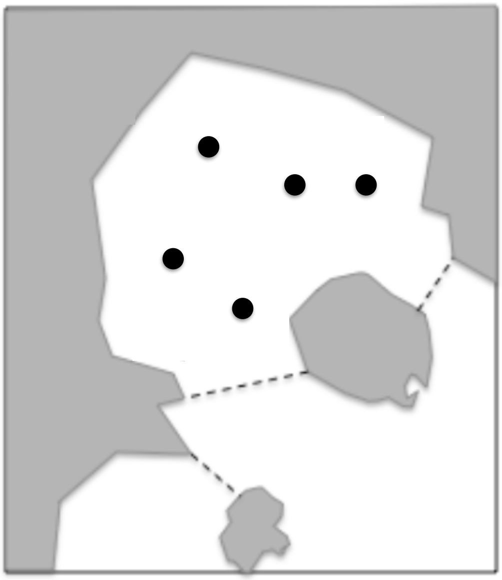


$$\bar{x} = \frac{\sum x_i}{n}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$$

$$s = \sqrt{s^2}$$

A very simple case



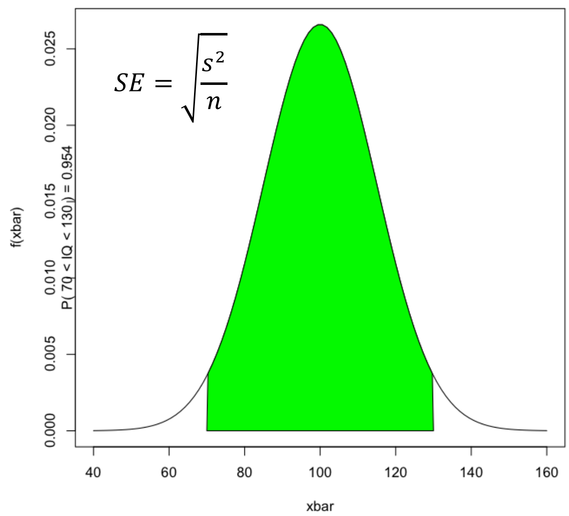
$$\bar{x} = \frac{\sum x_i}{n}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$SE = \sqrt{\frac{s^2}{n}}$$

Precision

- SE is a measure of uncertainty. How much does an estimated mean on average deviate from the true mean?
- But its simplest formulation holds only under the assumption that samples are independent.
- This is usually not the case because samples are structured in time and space.



A crossed monitoring design

$$y = \mu + YEAR + STATION + YEAR * STATION + PATCHINESS$$

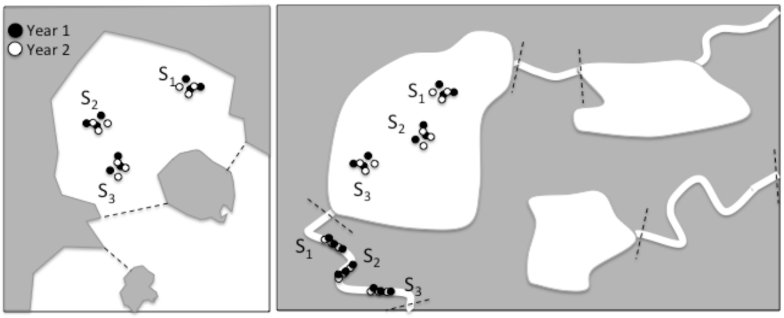


FIGURE 3.1
Illustration of crossed monitoring designs in a coastal water body (left) and in a lake and stream (right). In the examples, a = 2 years, b = 3 stations, and n = 3 replicates.

A crossed monitoring design

- The variance around a mean (and thus the SE) is determined by several components of variability and the replication at various levels.
- It is scale-dependent!

Variance of a mean within a 6-yr period

$$V[\bar{y}] = \frac{s_Y^2 * (1 - \frac{a}{Y})}{a} + \frac{s_S^2}{b} + \frac{s_{Y*S}^2}{ab} + \frac{s_e^2}{abn}$$

Variance of a mean within a year

$$V[\bar{y}_{WB\_YEAR}] = \frac{s_S^2}{b} + \frac{s_e^2}{bn}$$

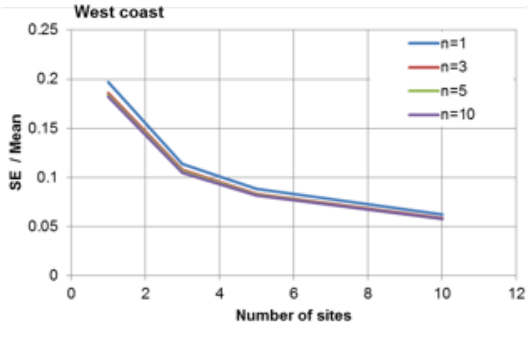
a=number of sampled years
b=number of sites per year
n=number of samples per site and year

A crossed monitoring design

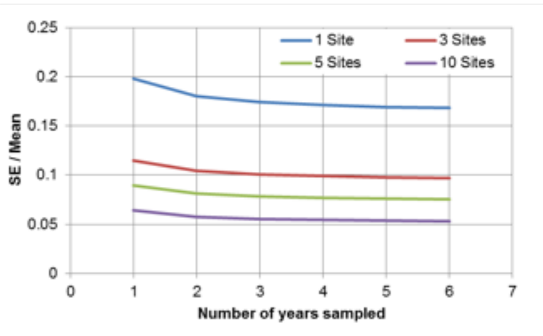
TABLE 4.1

The variance components used in calculating the overall uncertainty of the status of benthic invertebrates in a coastal water body within a single year and over a 6-year assessment period.

Source	West Coast	Baltic Proper	Gulf of Bothnia
s_Y^2	0.03	0.13	0.16
s_S^2	2.59	2.15	1.71
s_{Y*S}^2	0.63	0.59	0.19
s_e^2	0.64	1.06	1.24



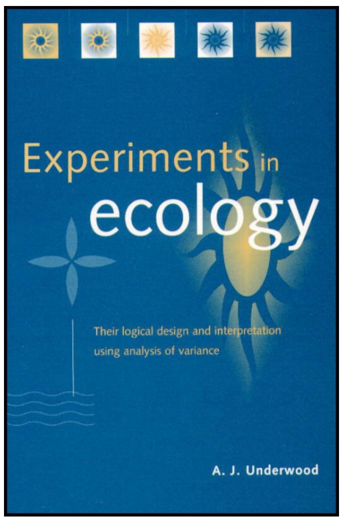
SE within years and WB
as a function of n and b



SE within 6-yr period and WB
as a function of a and b, n=1

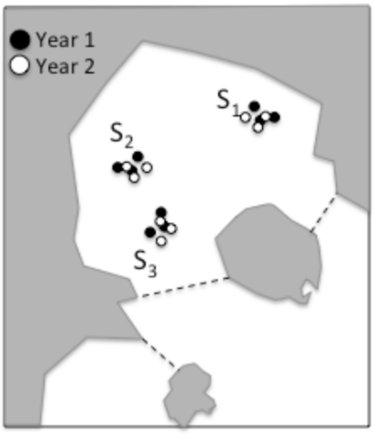
Optimising monitoring

- The sampling design matters!
- How do we achieve highest possible precision for certain resource?
- How do we achieve a targeted precision at the lowest cost?
- Cost-benefit optimisation



Optimisation in a WB in <u>one</u> year

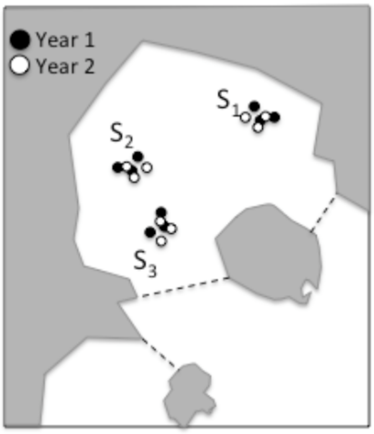
$$y = \mu + S + RES$$



From pilot studies and constrains we get the following constants

s_S^2=variability among sites
s_e^2=variability among samples
C_{site}=cost for sampling one site (sampling and sorting not included)
C_{sample}=cost for one sample (travelling and preparations not included)
$Cost\ per\ WB$=defined by budget

Optimisation in a WB in <u>one</u> year



Central expressions

1. Expression for total variance

$$V[\bar{y}_{WB}] = \frac{s_S^2}{b} + \frac{s_e^2}{bn}$$

2. Expression of total cost

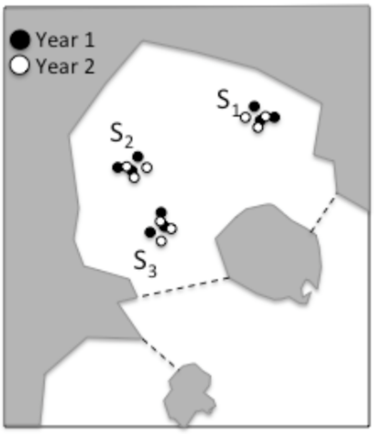
$$Cost\ per\ WB = bnC_{sample} + bC_{site}$$

3. Expression for minimisation of V*C (we want low variance and cost!)

$$VC = \left[\frac{s_S^2}{b} + \frac{s_e^2}{bn}\right] * \left[bnC_{sample} + bC_{site}\right]$$

$$\frac{d(VC)}{dn} = -\frac{C_{sample} * s_e^2}{n^2} + C_{site} * s_S^2 = 0$$

Optimisation in a WB in <u>one</u> year



It can be shown that given constants and total costs the optimal design is:

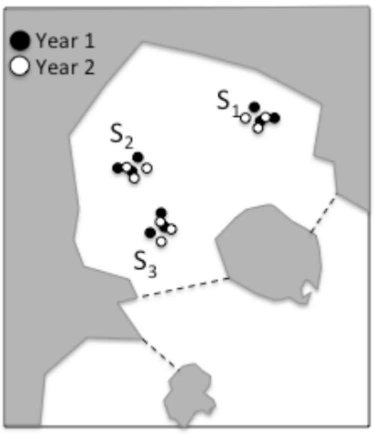
4. Find optimal n at minimum

$$n_{opt} = \sqrt{\frac{C_{site} * s_e^2}{C_{sample} * s_S^2}}$$

5. Find optimal b

$$b_{opt} = \frac{Cost\ per\ WB}{n_{opt} C_{sample} + C_{site}}$$

Optimisation in a WB in <u>one</u> year



It can be shown that given constants and total costs the optimal design is:

6. Calculate variance and SE of optimal solution

$$V[\bar{y}_{WB}] = \frac{s_S^2}{b_{opt}} + \frac{s_e^2}{b_{opt} n_{opt}}$$

$$\Rightarrow SE = \sqrt{V[\bar{y}_{WB}]}$$

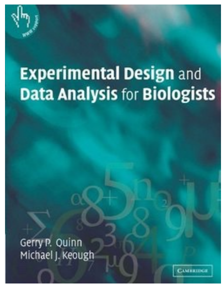
7. Calculate costs necessary to achieve certain target error, SE_{target}.

$$b_{target} = \left(\frac{1}{SE_{target}^2}\right) * \left(s_S^2 + \frac{s_e^2}{n_{opt}}\right)$$

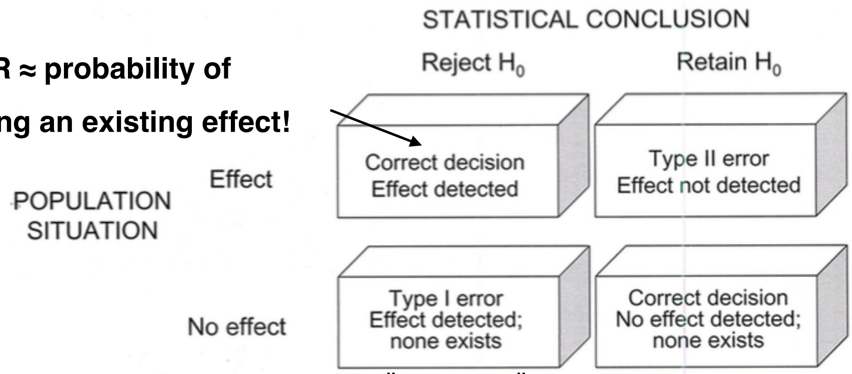
$$Cost\ to\ acheive\ target = b_{target} * n_{opt*} C_{sample} + b_{target} * C_{site}$$

Statistical power (and errors)

- Statistical tests are used to test hypotheses!
- Decision making!
- Four scenarios



POWER ≈ probability of detecting an existing effect!



”β - error”

”α - error”

Hypothesis-testing

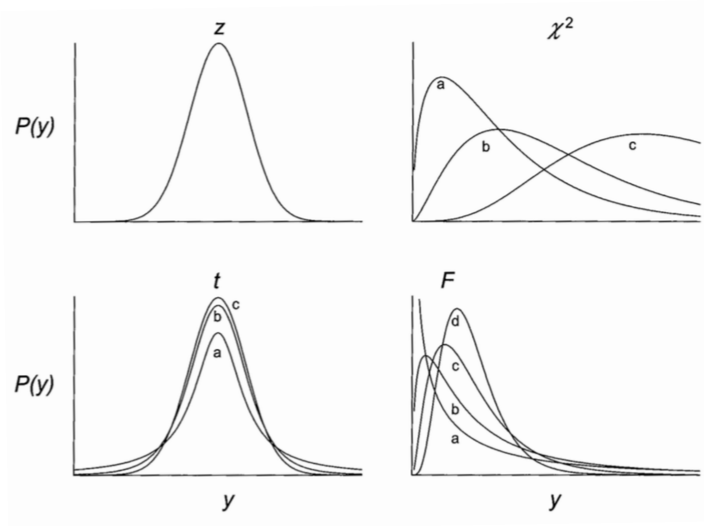
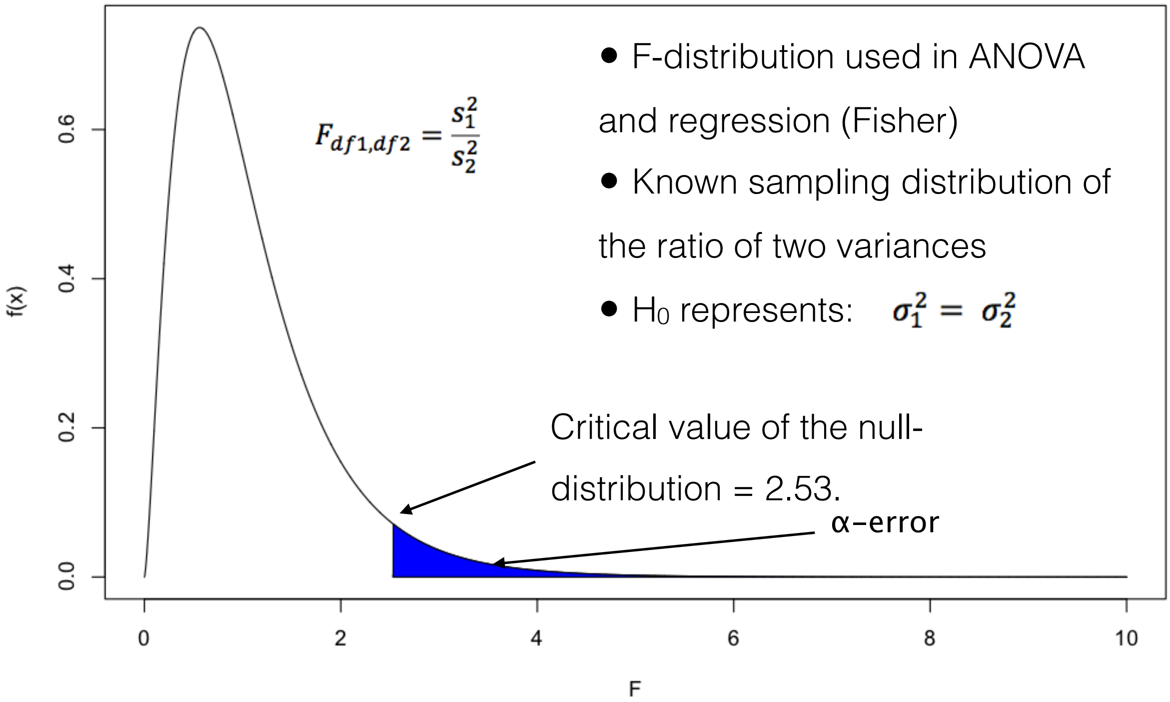


Figure 1.2 Probability distributions for four common statistics. For the t, χ^2, and F distributions, we show distributions for three or four different degrees of freedom (a to d, in increasing order), to show how the shapes of these distributions change.

- Test statistics with known sampling distributions under the null-hypothesis, H_0
- An observed value is estimated from sample and compared to the null-distribution.
- The probability of that the observed belongs to the null is calculated (=p-value).

Hypothesis-testing

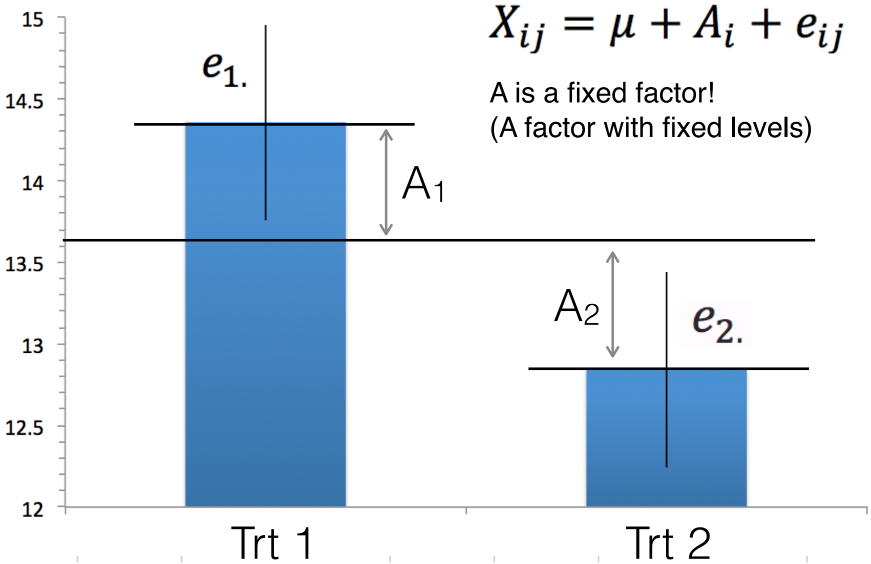


Hypothesis-testing

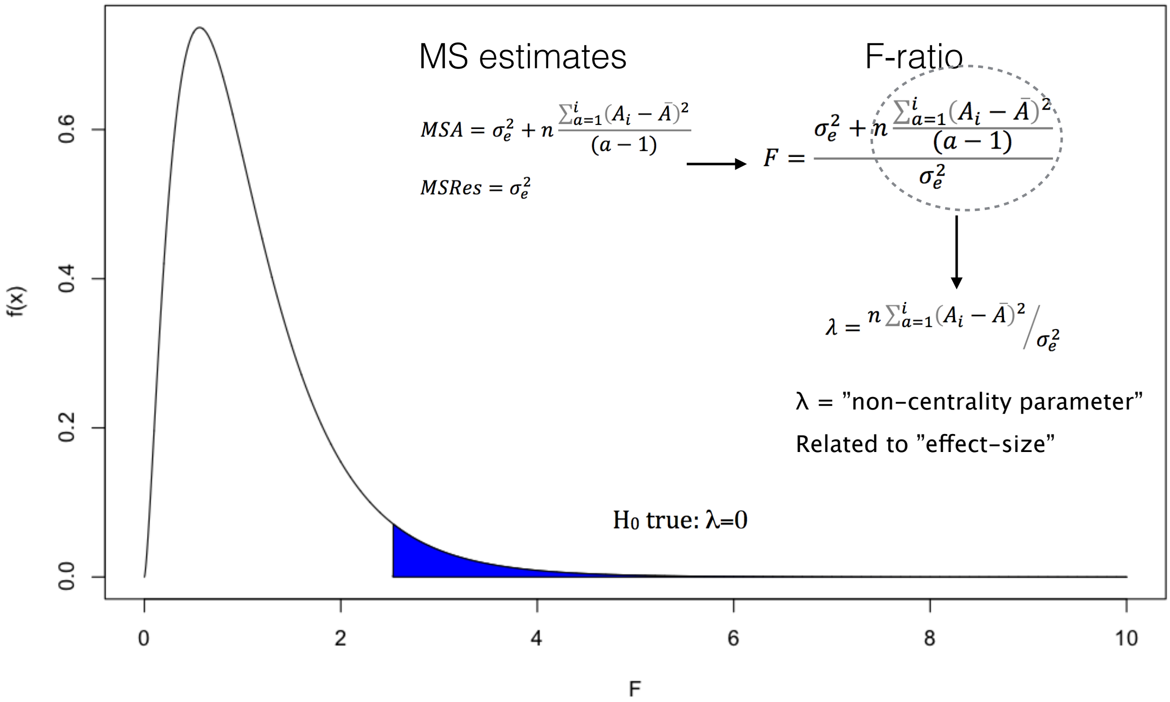
Linear model:

$$X_{ij} = \mu + A_i + e_{ij}$$

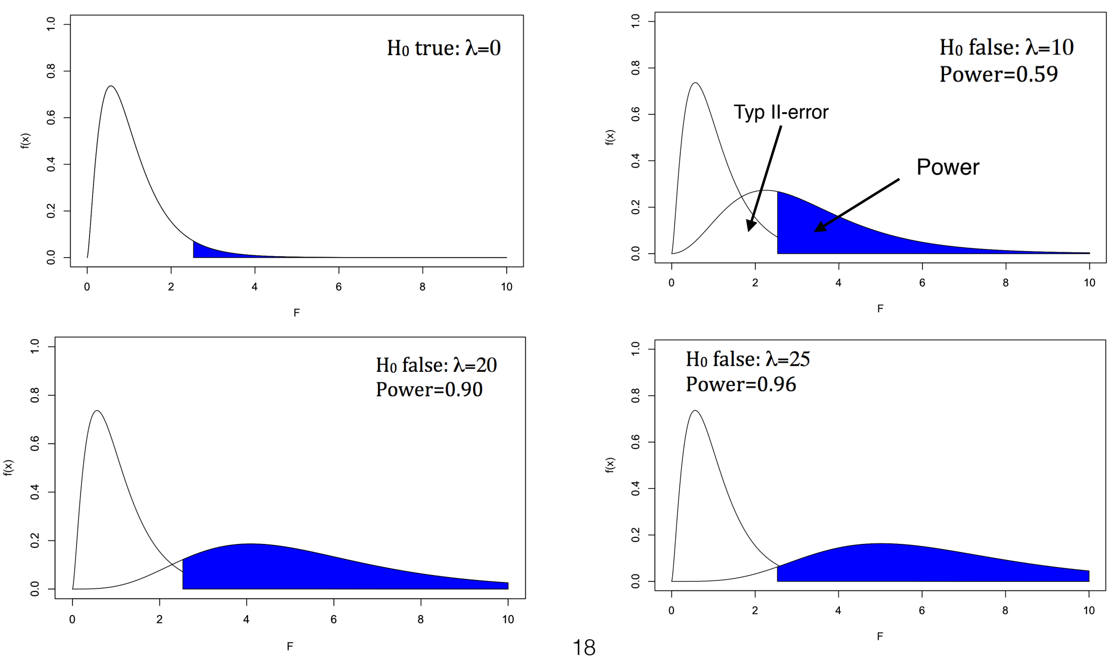
A is a fixed factor!
(A factor with fixed levels)



Example: one-way ANOVA



Power with increasing effect-size



Power is affected by

Number of replicates

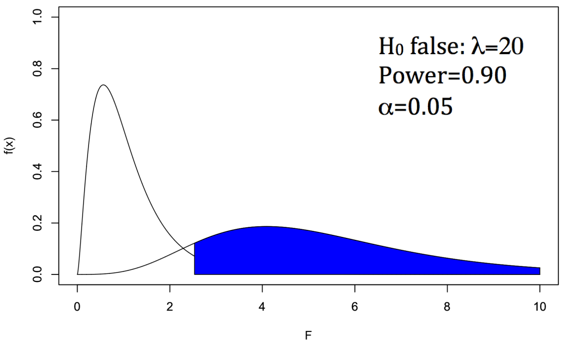
Effect-size

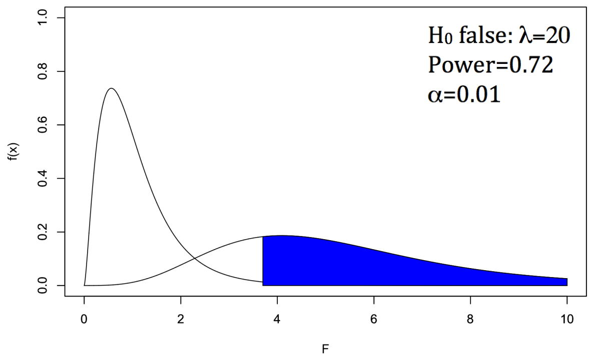
Residual variability

$$\lambda = {n\sum_{a=1}^{i}(A_i - \bar{A})^2}\Big/{\sigma_e^2}$$

α-error

(defined by experimenter)

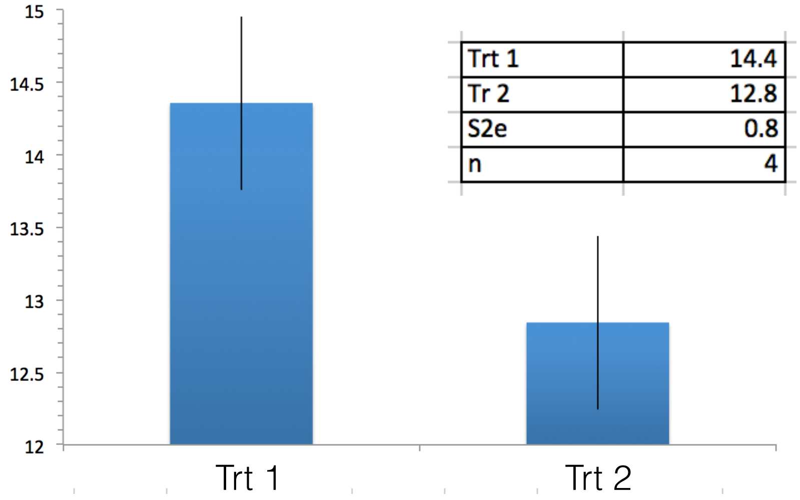




Power calculations

- A priori - Model expected power (and type II-error) <u>before</u> the experiment! Requires definition of alternative hypotheses (H_A)
- A posteriori - Estimate power of an observed, non-significant difference among means, <u>after</u> an experiment (sometimes questioned)
- Applicable to all types of statistical tests (not only F) but complex designs and alternative hypotheses makes it more complicated.
- Analytical or simulation approaches can be used.

Power calculation



Various software and tools available!
_https://www.danielsoper.com/statcalc/default.aspx

To summarise

- The uncertainty of a mean estimate is usually affected by many sources (components) and their joint contributions can be estimated.
- The efficiency of a sampling program can be optimized using information on costs and natural variability.
- The power of a statistical test represents the probability of detecting an existing effect. It is increasing with the effect-size, n and α; decreasing with σ_e^2