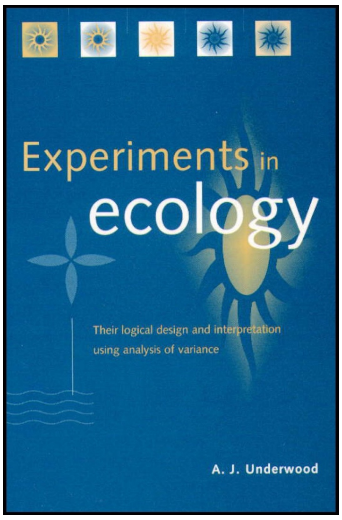
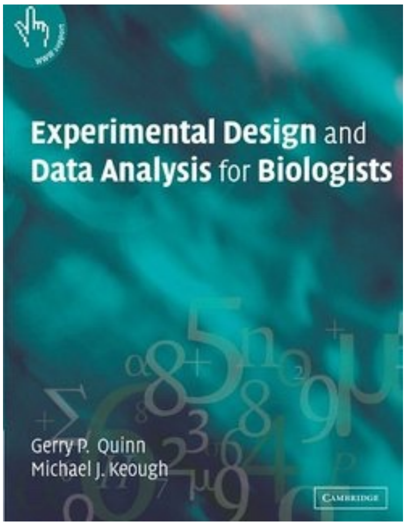
# Some statistical considerations in status- and environmental impact assessment




- Precision of mean estimates
- Cost-benefit optimisation
- Statistical power in hypothesis-testing

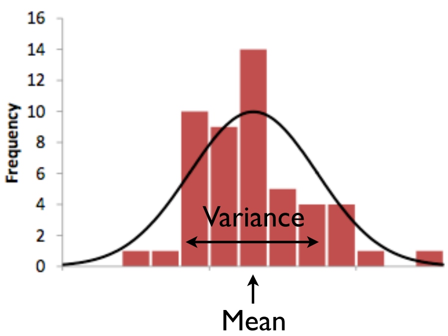
# From samples to populations

Population ("true" parameter) ⟵ Sample (estimated parameter)

$$\mu = \frac{\sum x_i}{n}$$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{n}$$

$$\sigma = \sqrt{\sigma^2}$$

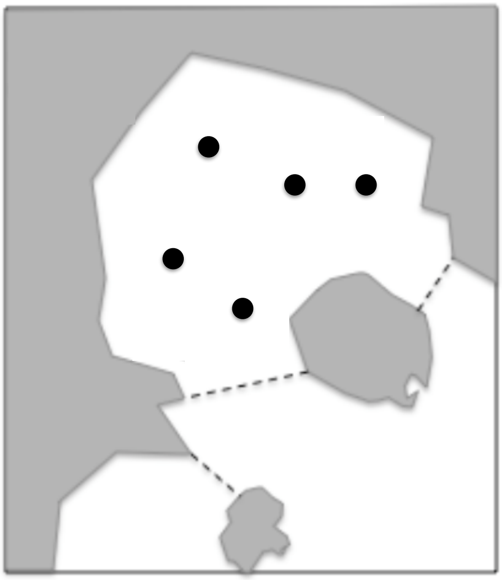



$$\bar{x} = \frac{\sum x_i}{n}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$s = \sqrt{s^2}$$

# A very simple case



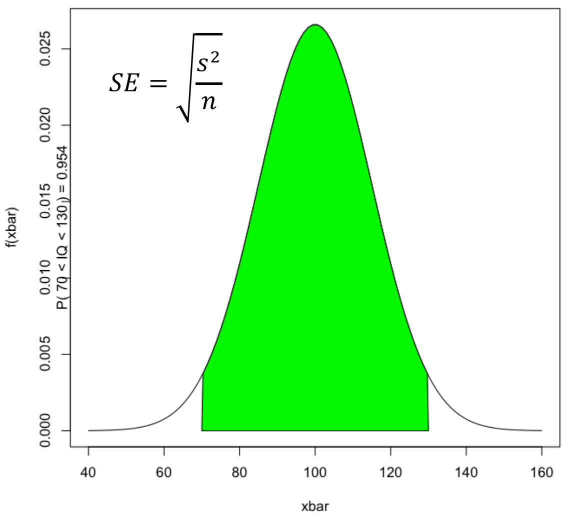
$$\bar{x} = \frac{\sum x_i}{n}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$SE = \sqrt{\frac{s^2}{n}}$$

# Precision

- SE is a measure of uncertainty. How much does an estimated mean on average deviate from the true mean?
- But its simplest formulation holds only under the assumption that samples are independent.
- This is usually not the case because samples are structured in time and space.



# A crossed monitoring design

$$y = \mu + YEAR + STATION + YEAR * STATION + PATCHINESS$$

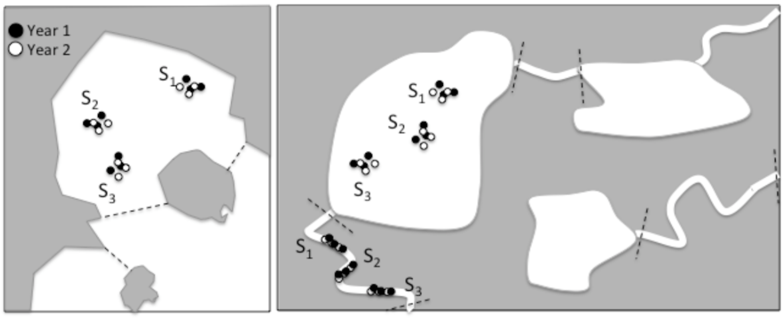



**FIGURE 3.1**
Illustration of crossed monitoring designs in a coastal water body (left) and in a lake and stream (right). In the examples, *a* = 2 years, *b* = 3 stations, and *n* = 3 replicates.

# A crossed monitoring design

- The variance around a mean (and thus the SE) is determined by several components of variability and the replication at various levels.
- It is scale-dependent!

Variance of a mean within a 6-yr period

$$V[\bar{y}] = \frac{s_Y^2 * (1 - \frac{a}{Y})}{a} + \frac{s_S^2}{b} + \frac{s_{Y*S}^2}{ab} + \frac{s_e^2}{abn}$$

Variance of a mean within a year

$$V[\bar{y}_{WB\_YEAR}] = \frac{s_S^2}{b} + \frac{s_e^2}{bn}$$

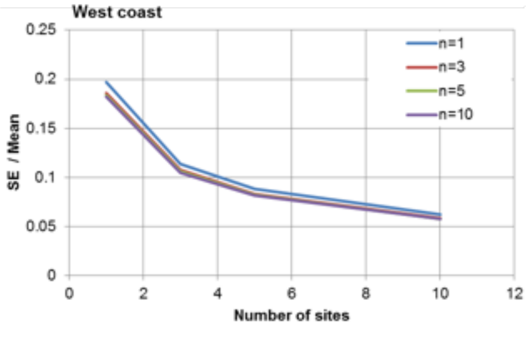
a=number of sampled years
b=number of sites per year
n=number of samples per site and year

# A crossed monitoring design

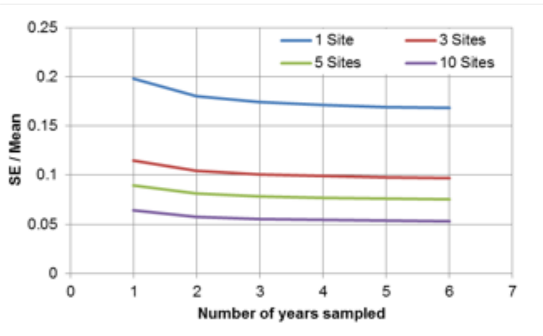
**TABLE 4.1**

The variance components used in calculating the overall uncertainty of the status of benthic invertebrates in a coastal water body within a single year and over a 6-year assessment period.

| Source | West Coast | Baltic Proper | Gulf of Bothnia |
|---|---|---|---|
| $s_Y^2$ | 0.03 | 0.13 | 0.16 |
| $s_S^2$ | 2.59 | 2.15 | 1.71 |
| $s_{Y*S}^2$ | 0.63 | 0.59 | 0.19 |
| $s_e^2$ | 0.64 | 1.06 | 1.24 |



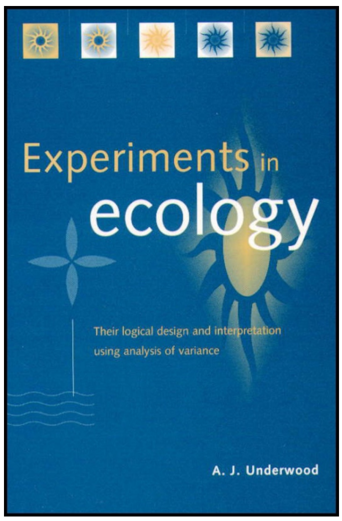

SE within years and WB
as a function of n and b




SE within 6-yr period and WB
as a function of a and b, n=1

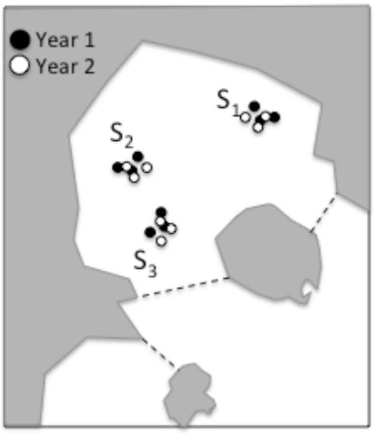
# Optimising monitoring

- The sampling design matters!
- How do we achieve highest possible precision for certain resource?
- How do we achieve a targeted precision at the lowest cost?
- Cost-benefit optimisation



# Optimisation in a WB in <u>one</u> year

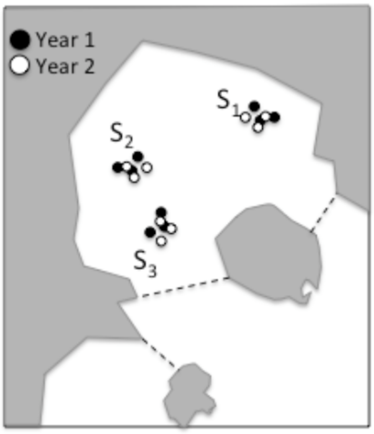
$$y = \mu + S + RES$$




From pilot studies and constrains we get the following constants

$s_S^2$=variability among sites
$s_e^2$=variability among samples
$C_{site}$=cost for sampling one site (sampling and sorting not included)
$C_{sample}$=cost for one sample (travelling and preparations not included)
$Cost\ per\ WB$=defined by budget

# Optimisation in a WB in <u>one</u> year




## Central expressions

1. Expression for total variance

$$V[\bar{y}_{WB}] = \frac{s_S^2}{b} + \frac{s_e^2}{bn}$$

2. Expression of total cost

$$Cost\ per\ WB = bnC_{sample} + bC_{site}$$

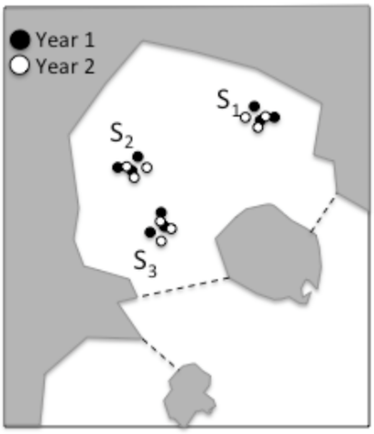
3. Expression for minimisation of V*C (we want low variance and cost!)

$$VC = \left[\frac{s_S^2}{b} + \frac{s_e^2}{bn}\right] * \left[bnC_{sample} + bC_{site}\right]$$

$$\frac{d(VC)}{dn} = -\frac{C_{sample} * s_e^2}{n^2} + C_{site} * s_S^2 = 0$$

# Optimisation in a WB in <u>one</u> year

It can be shown that given constants and total costs the optimal design is:




**4. Find optimal n at minimum**

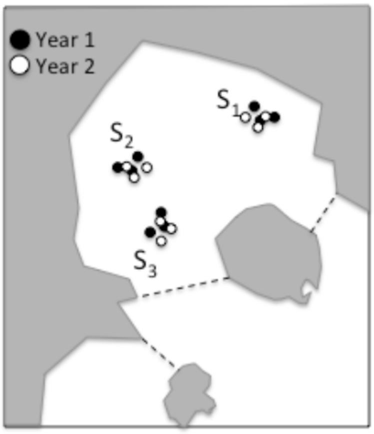
$$n_{opt} = \sqrt{\frac{C_{site} * s_e^2}{C_{sample} * s_S^2}}$$

**5. Find optimal b**

$$b_{opt} = \frac{Cost\ per\ WB}{n_{opt}C_{sample} + C_{site}}$$

# Optimisation in a WB in <u>one</u> year

It can be shown that given constants and total costs the optimal design is:




**6. Calculate variance and SE of optimal solution**

$$V[\bar{y}_{WB}] = \frac{s_S^2}{b_{opt}} + \frac{s_e^2}{b_{opt}n_{opt}}$$

$$\Rightarrow SE = \sqrt{V[\bar{y}_{WB}]}$$

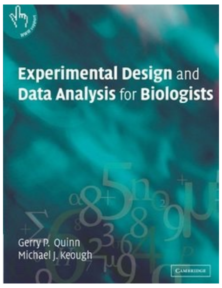
**7. Calculate costs necessary to achieve certain target error, $SE_{target}$.**

$$b_{target} = \left(\frac{1}{SE_{target}^2}\right) * \left(s_S^2 + \frac{s_e^2}{n_{opt}}\right)$$

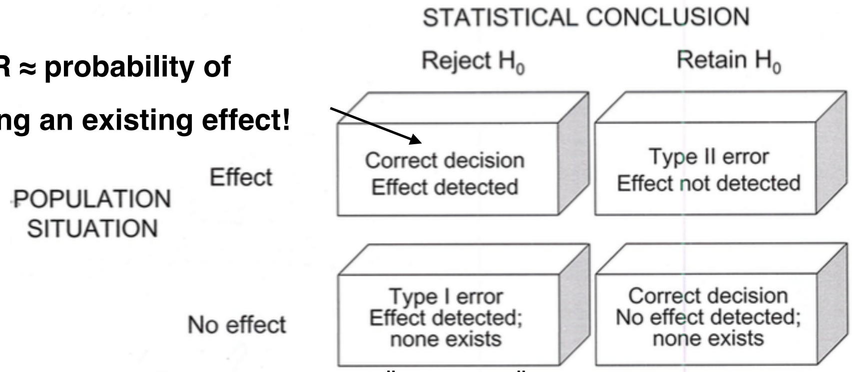
$$Cost\ to\ acheive\ target = b_{target} * n_{opt*}C_{sample} + b_{target} * C_{site}$$

# Statistical power (and errors)

- Statistical tests are used to test hypotheses!
- Decision making!
- Four scenarios




**POWER ≈ probability of detecting an existing effect!**




"β - error"

"α - error"



# Hypothesis-testing

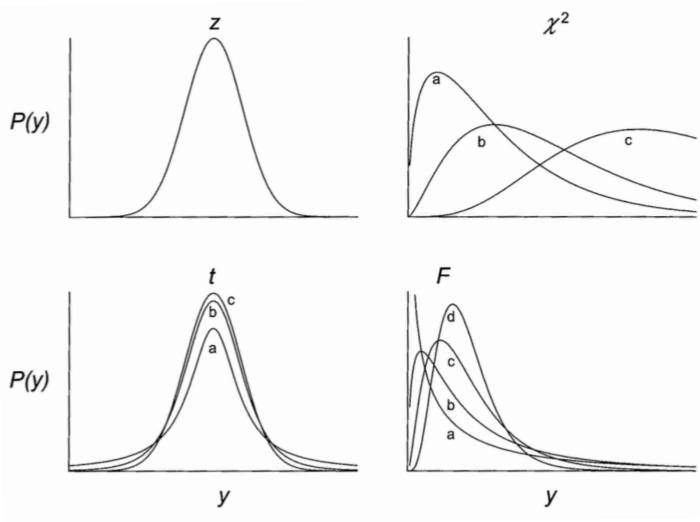
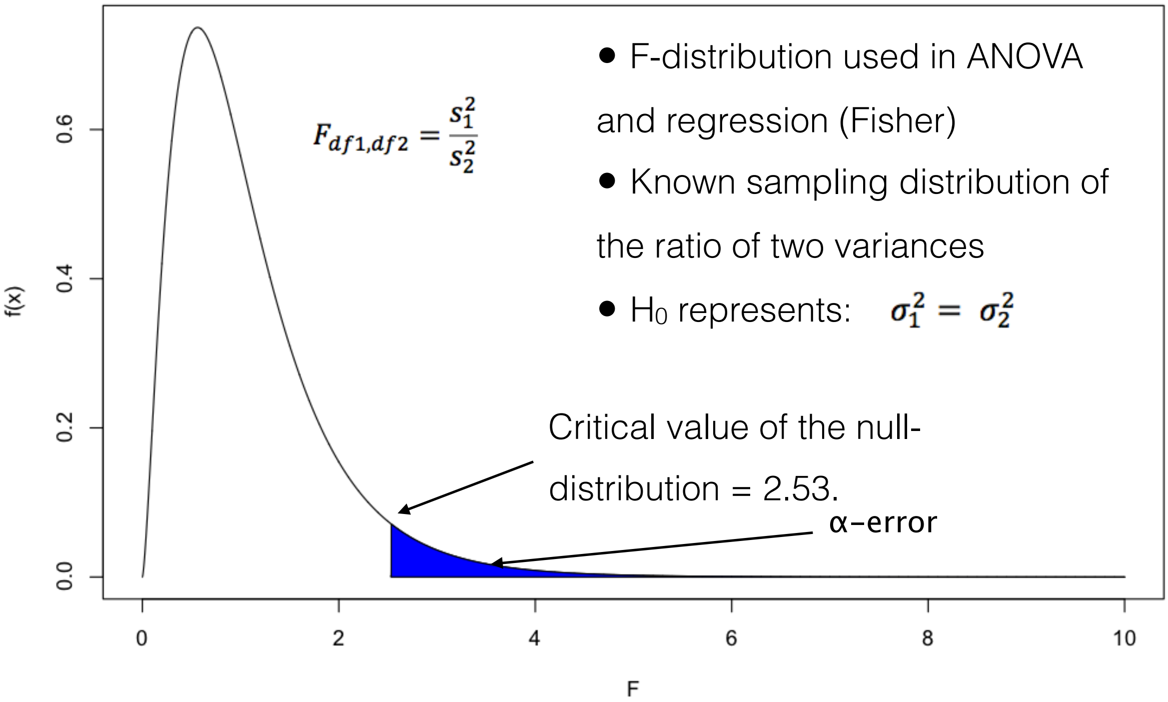



**Figure 1.2** Probability distributions for four common statistics. For the $t$, $\chi^2$, and $F$ distributions, we show distributions for three or four different degrees of freedom (a to d, in increasing order), to show how the shapes of these distributions change.

- Test statistics with known sampling distributions under the null-hypothesis, $H_0$
- An observed value is estimated from sample and compared to the null-distribution.
- The probability of that the observed belongs to the null is calculated (=p-value).

# Hypothesis-testing

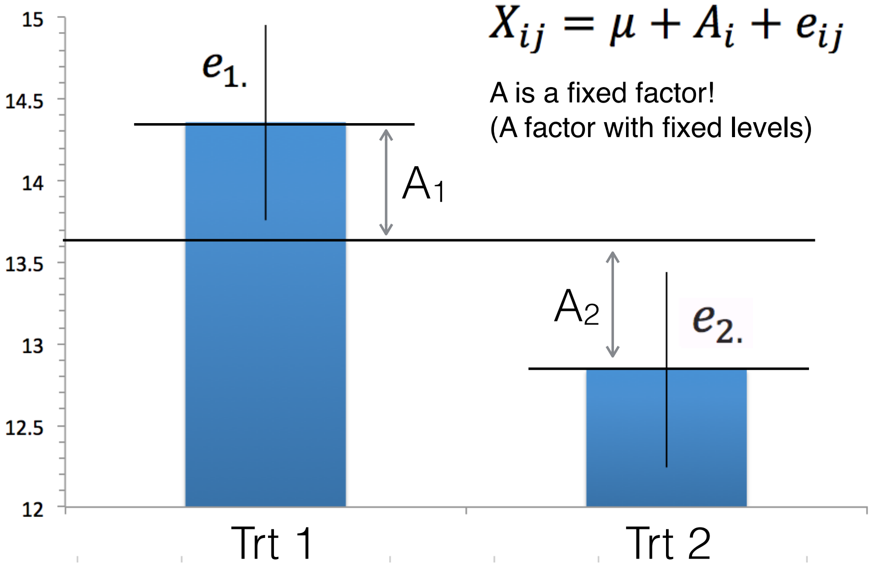



# Hypothesis-testing

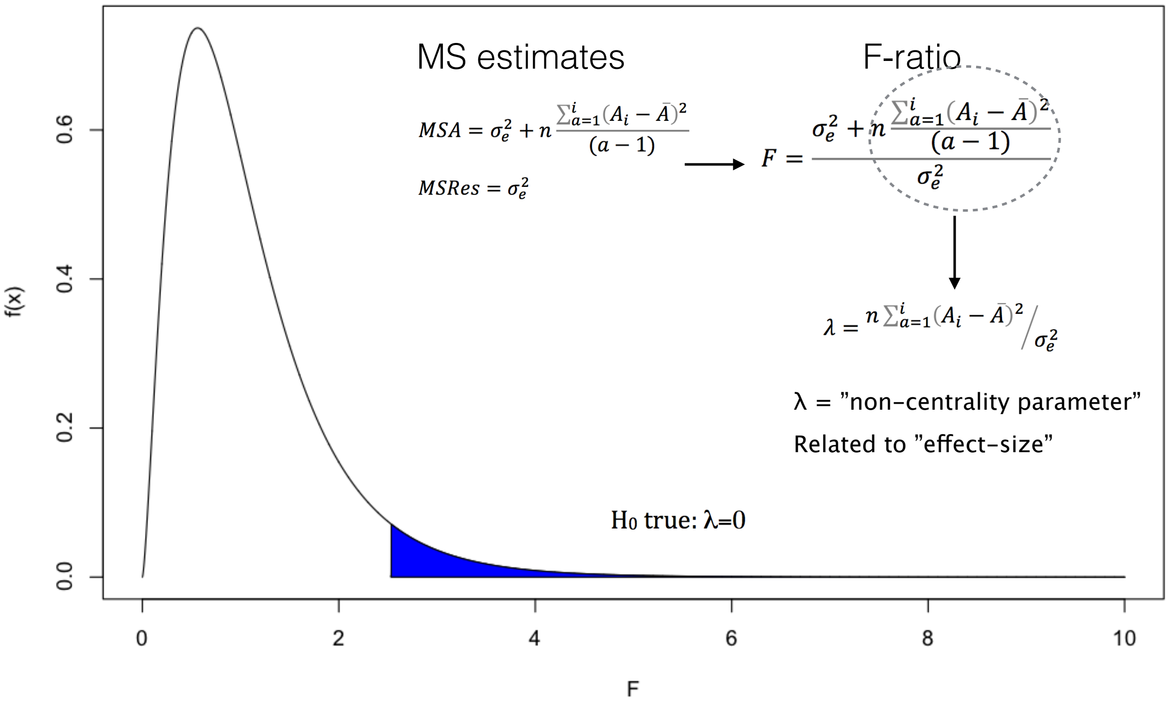
Linear model:

$$X_{ij} = \mu + A_i + e_{ij}$$

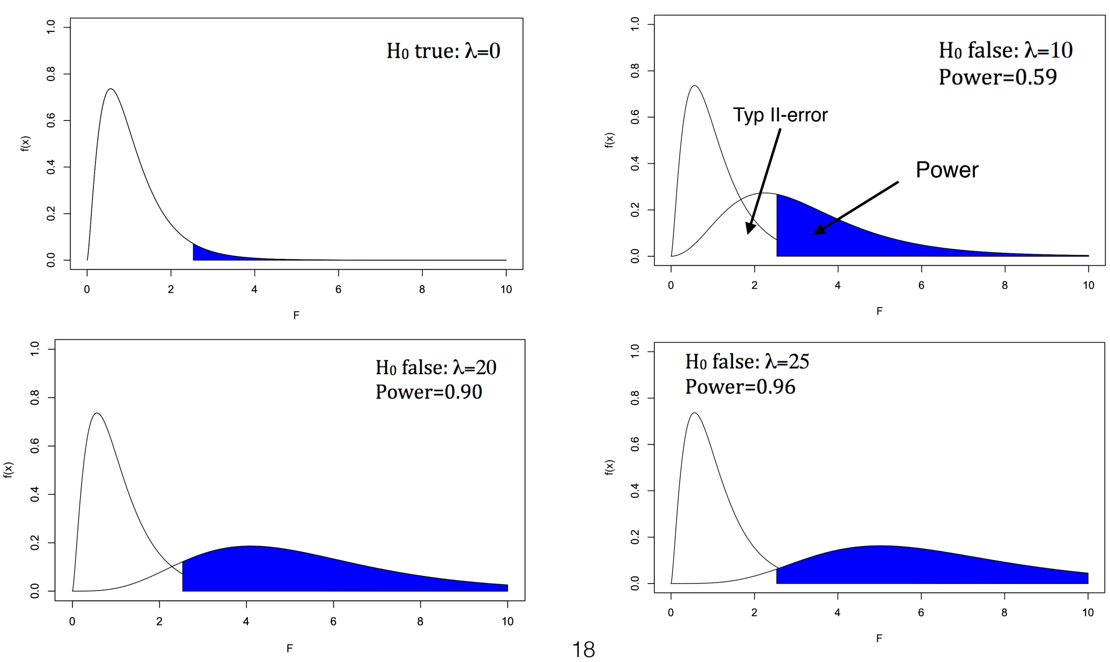
A is a fixed factor!
(A factor with fixed levels)



# Example: one-way ANOVA




# Power with increasing effect-size



# Power is affected by

Number of replicates

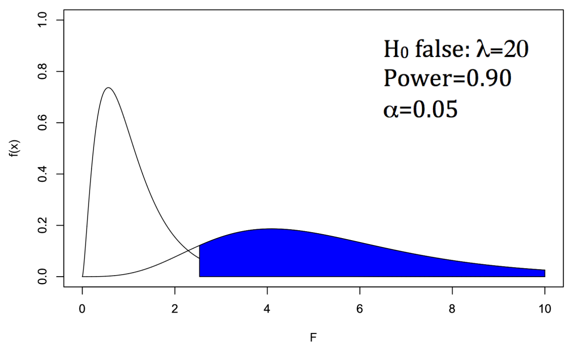
Effect-size

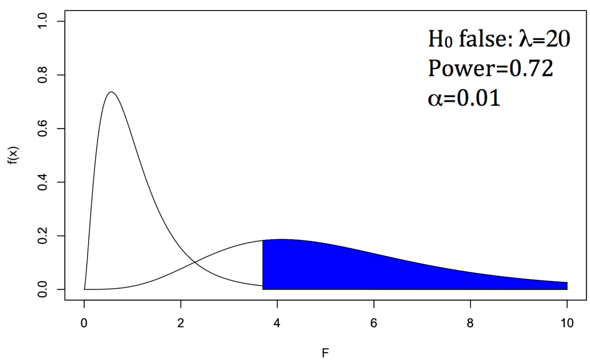
Residual variability

$$\lambda = {n \sum_{a=1}^{i} (A_i - \bar{A})^2} / {\sigma_e^2}$$

α-error

(defined by experimenter)






# Power calculations

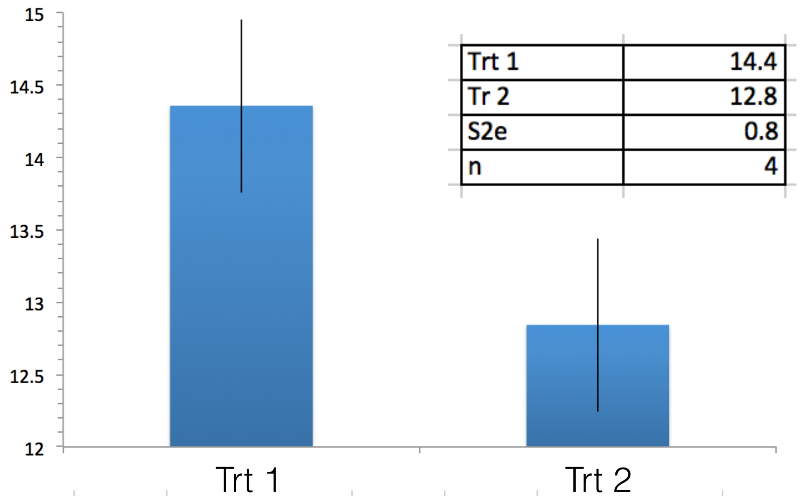
- A priori - Model expected power (and type II-error) <u>before</u> the experiment! Requires definition of alternative hypotheses ($H_A$)

- A posteriori - Estimate power of an observed, non-significant difference among means, <u>after</u> an experiment (sometimes questioned)

- Applicable to all types of statistical tests (not only F) but complex designs and alternative hypotheses makes it more complicated.

- Analytical or simulation approaches can be used.

# Power calculation




Various software and tools available!
_https://www.danielsoper.com/statcalc/default.aspx



# To summarise

- The uncertainty of a mean estimate is usually affected by many sources (components) and their joint contributions can be estimated.
- The efficiency of a sampling program can be optimized using information on costs and natural variability.
- The power of a statistical test represents the probability of detecting an existing effect. It is increasing with the effect-size, n and α; decreasing with $\sigma_e^2$