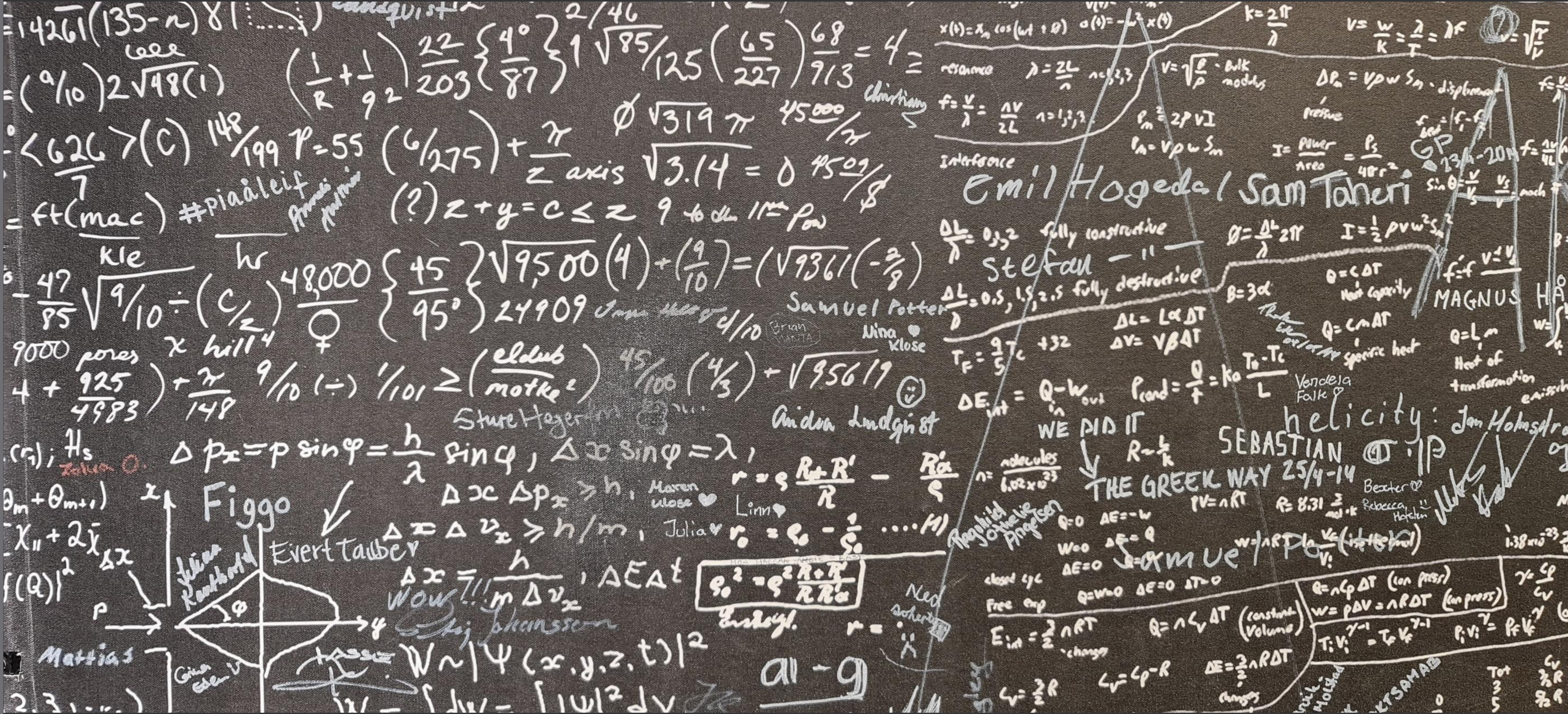


[Photo by Thomas T on Unsplash](https://unsplash.com) (retrieved 2023-06-20)

# A few notions from statistics

# Why statistics skills are vital

1. Measure

Icons from Flaticon (retrieved 2023-06-20)

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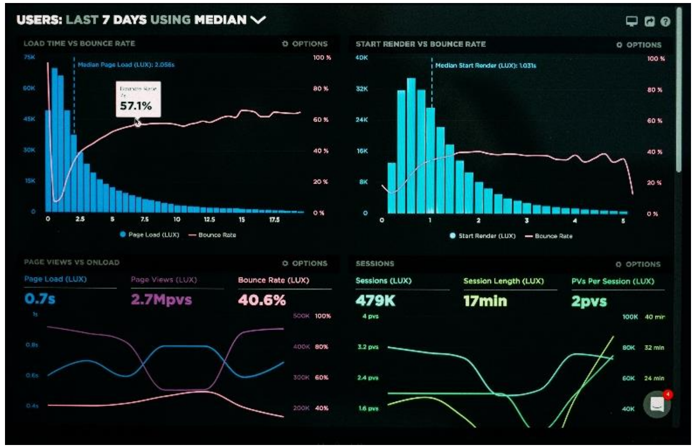


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# Why statistics skills are vital

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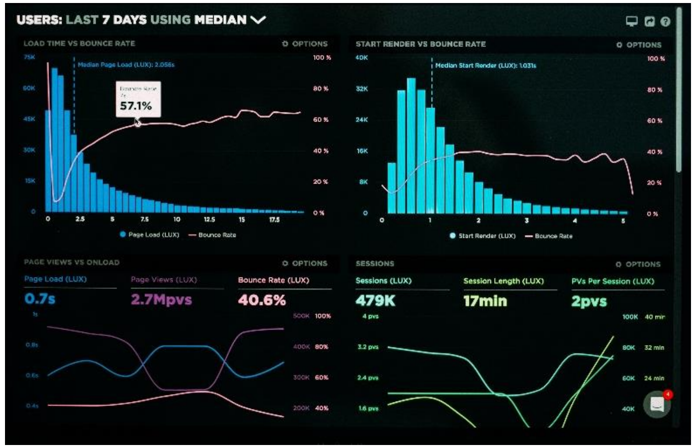


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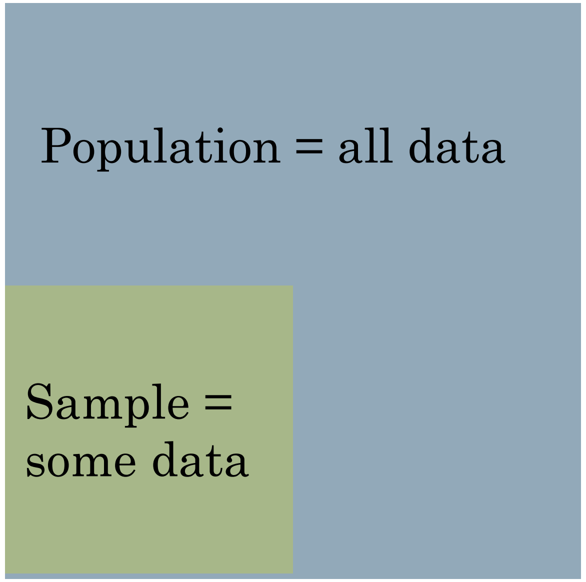
3. Occam's razor

*If two models are equally good, use the simpler one.*

# Statistics - basics

Population vs sample



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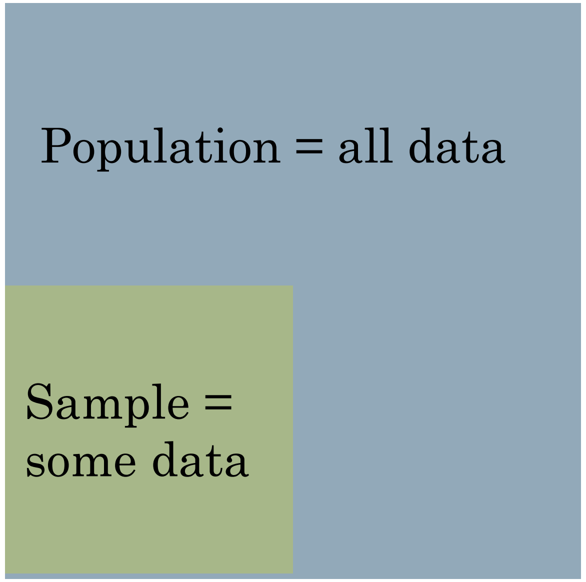
Population vs sample




In Data Science, sampling can be a challenge:

- **iid** for ML → <u>***i**ndependent, **i**dentically **d**istributed*</u>
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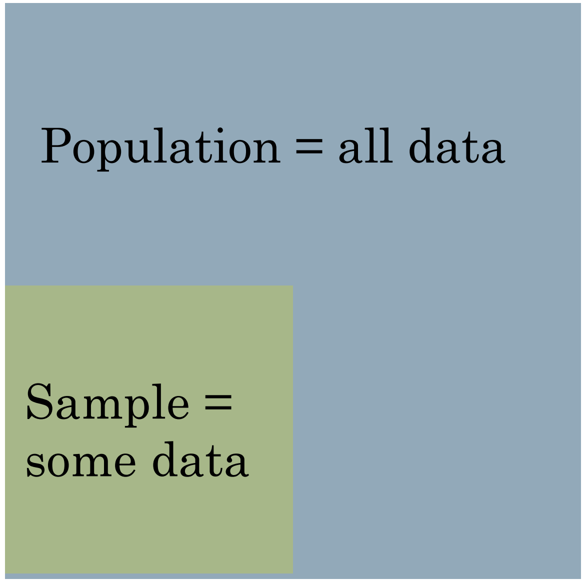


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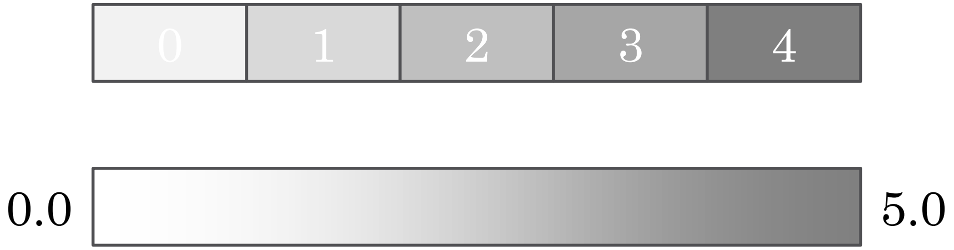
Discrete vs. continuous



# Statistics - basics

(Probability) distributions

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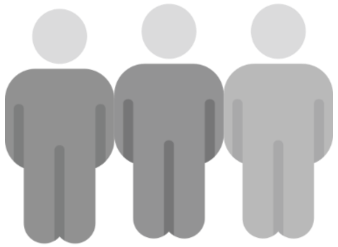
# Statistics - basics



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(Probability) distributions

# Statistics - basics

- Mode
- Median
- Mean

- Most frequent number
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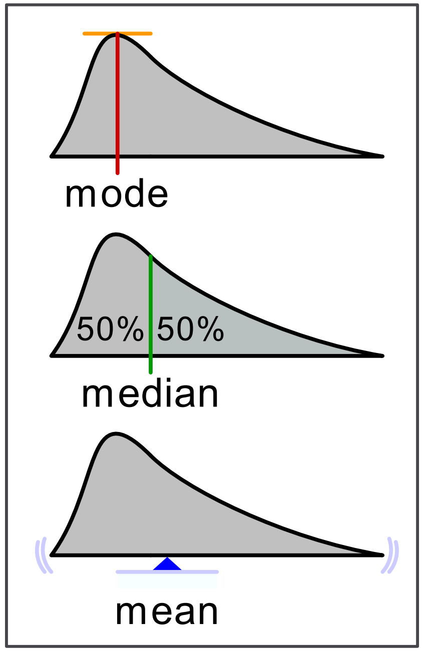

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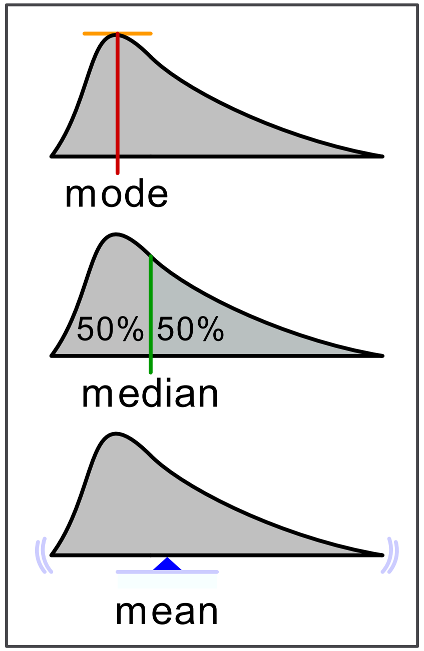


[1, 2, 4, 5, 7, 11, 17, 29, 29, 29, 30, 32, 65, 107, 125]

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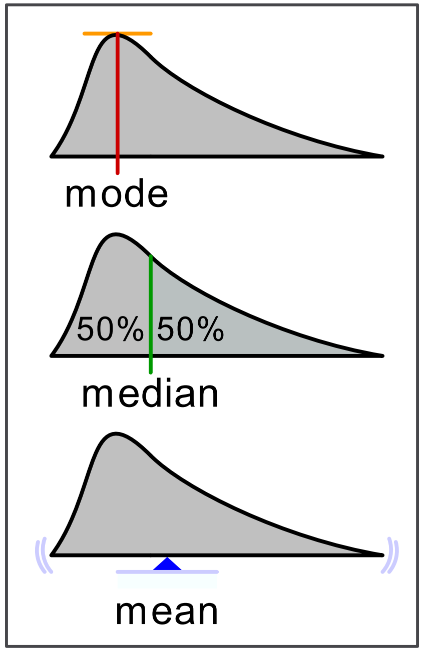


- $k^{th}$ Quantile
  - The number covering $k^{th}$ part of the values

[1, 2, 4, 5, 7, 11, 17, 29, 29, 29, 30, 32, 65, 107, 125]

- Standard deviation
  - $\sigma = \sqrt{\frac{1}{N}\sum_i^N (x_i - \mu)^2}$

Variance = $\sigma^2$

- Z-score
  - $z_i = \frac{x_i - \mu}{\sigma}$ , standardized: $z_{\bar{x}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

# Statistics - basics

**Hypothesis testing**

Null hypothesis $\boldsymbol{H_0}$: the ‘base case’ / ‘status quo’ - what if it’s just randomness?

Alternative hypothesis $\boldsymbol{H_1}$: the ‘test case’ - what if it’s a thing?

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Question: Does drinking *Red Einstein* impact your $ spending?

1. Define $H_0$ and $H_1$

- $H_0$: *Red Einstein* consumption $k$ does not change your spending results $r$
- Collect results $r_A$ from group $A$ with $k > 0$
- Collect results $r_B$ from group $B$ with $k = 0$
- $H_0$: $\mu_A = \mu_B$
- $H_1$: $\mu_A \neq \mu_B$

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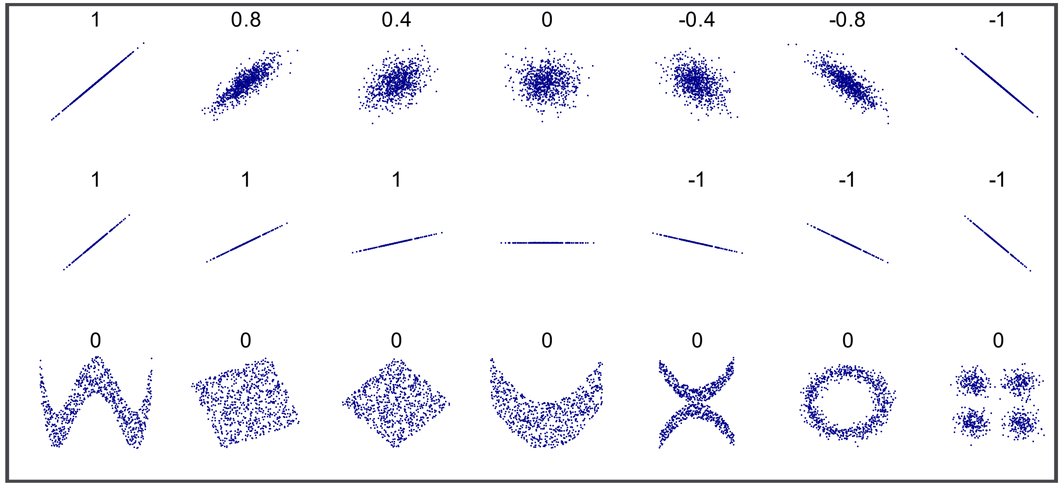
3. Look up the probability related to the z-score in the Z table (e.g. in ztable.net)
4. If $p <$ p-value, reject $H_0$

# Statistics - basics

## Correlation coefficients

Pearson correlation

$$\rho_{x,y} = \frac{1}{N\ \sigma_x \sigma_y} \sum_{i}^{N} (x_i - \mu_x)^2 \left(y_i - \mu_y\right)^2 = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

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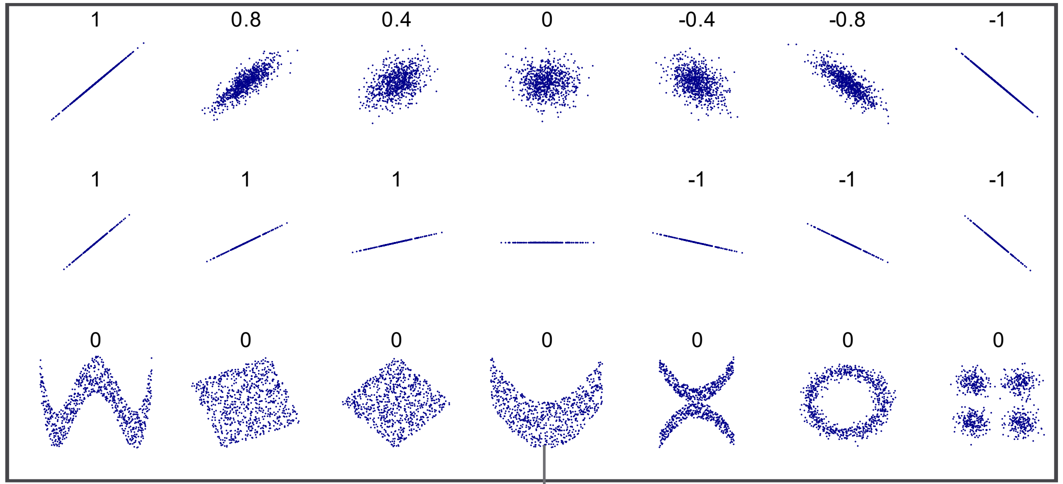


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needs axis transformation

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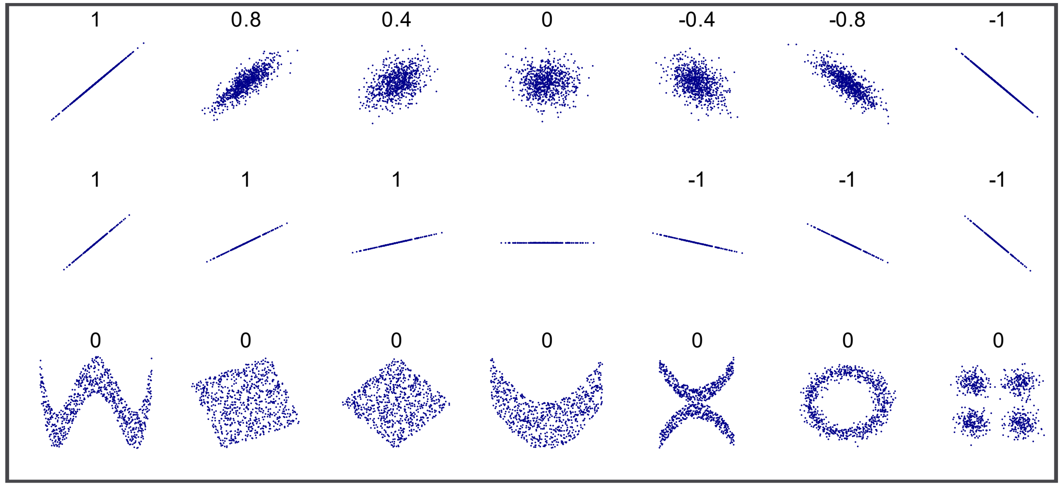


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general problem with means

**Be careful with sample vs population!**

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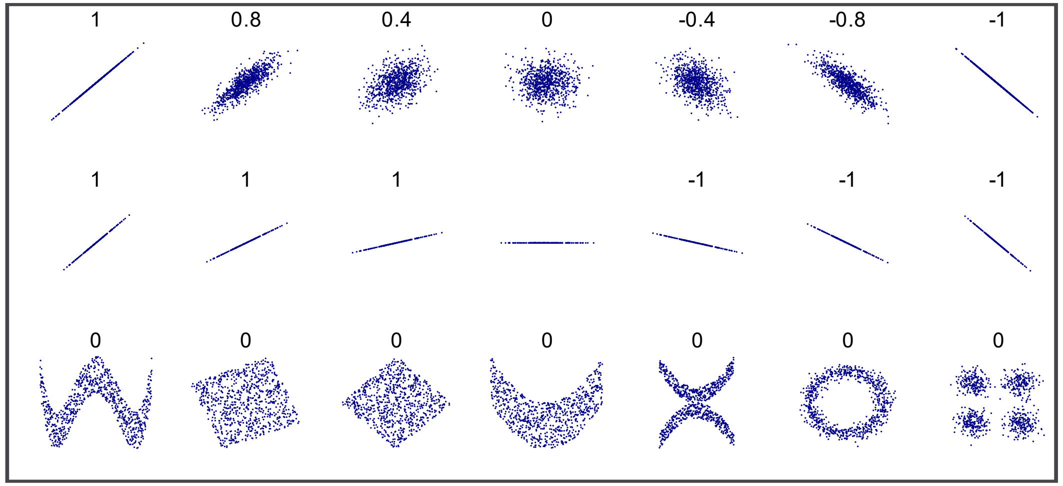


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$\rho_{x,y}$ is susceptible to outliers!

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$R(x_i)$ is the *rank* of $x_i$based on the value of $x_i$

For example:

$x$ = [1, 5, 2, 4, 7, 11, 17, 29, 29, 29, 32, 30, 65]

$R(x)$ = [1, 4, 2, 3, 5, 6, 7, 8, 8, 8, 10, 9, 11]

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Ties? → Use Kendall $\tau$

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**Correlation is not causation!**

- Confounding variables
- Directionality problem
- "*Dumb luck*"