



[Photo by Lady Escabia from Pexels](#) (retrieved 2023-04-02)

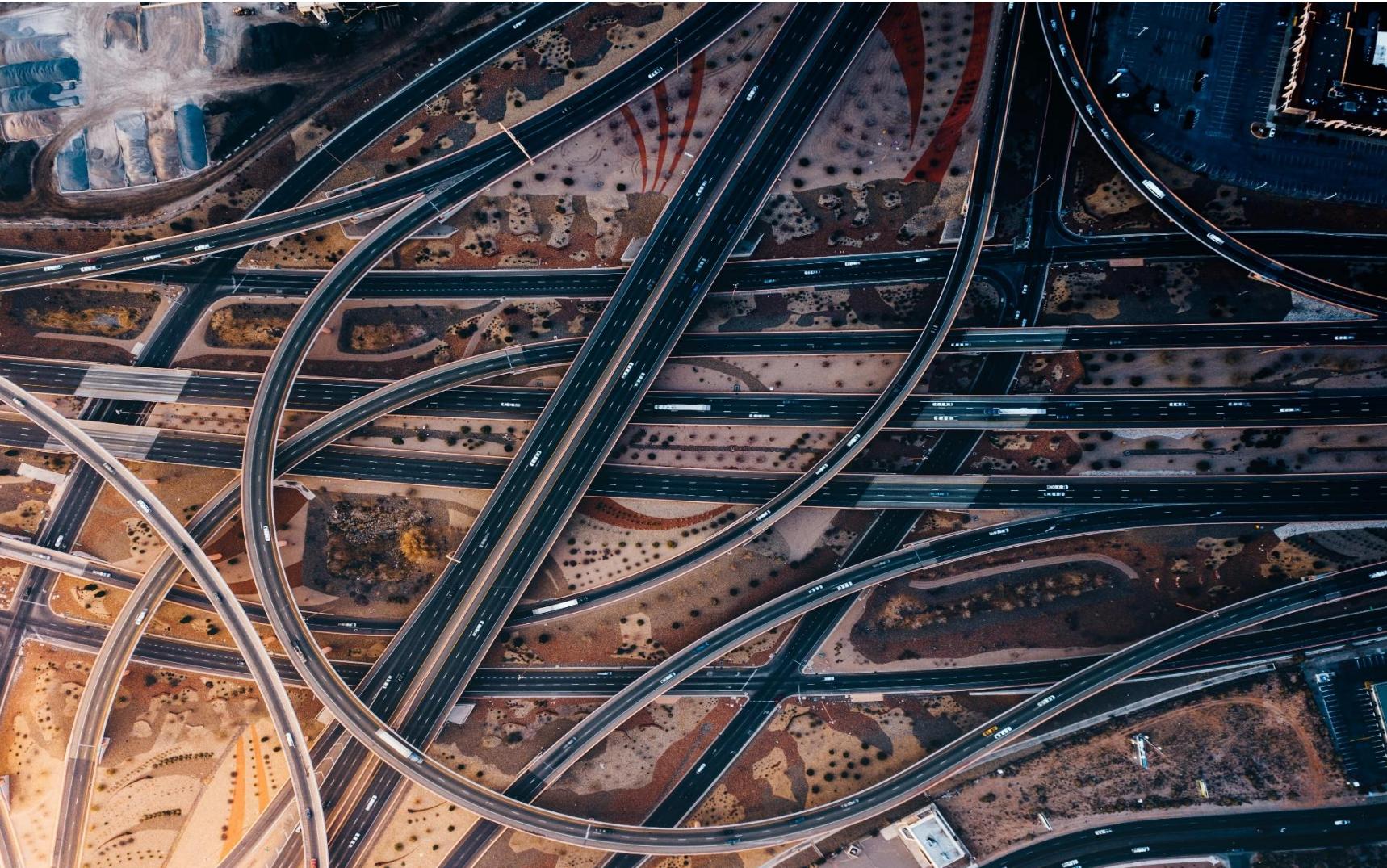
Networks

Networks are everywhere



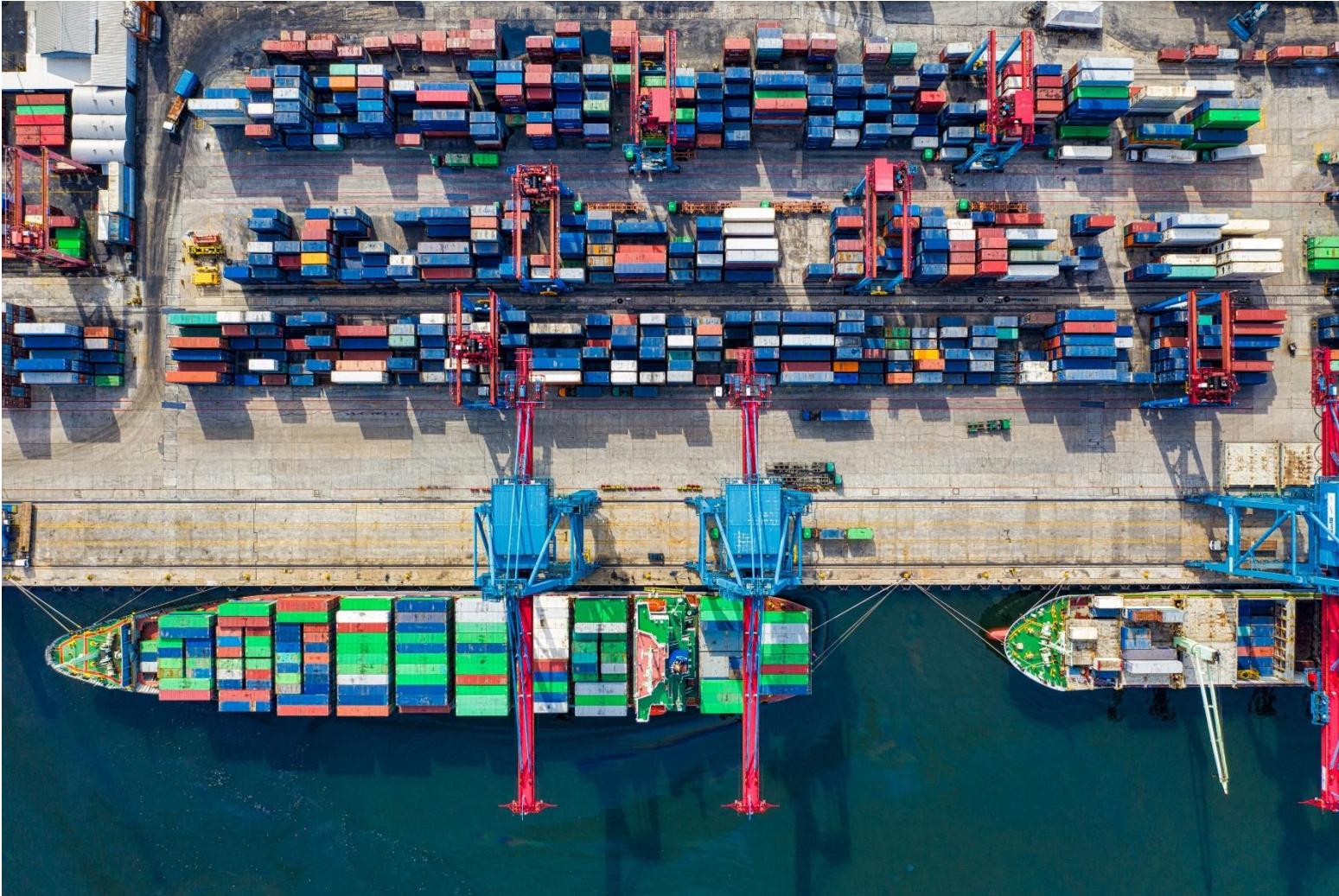
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Networks are everywhere



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Networks are everywhere



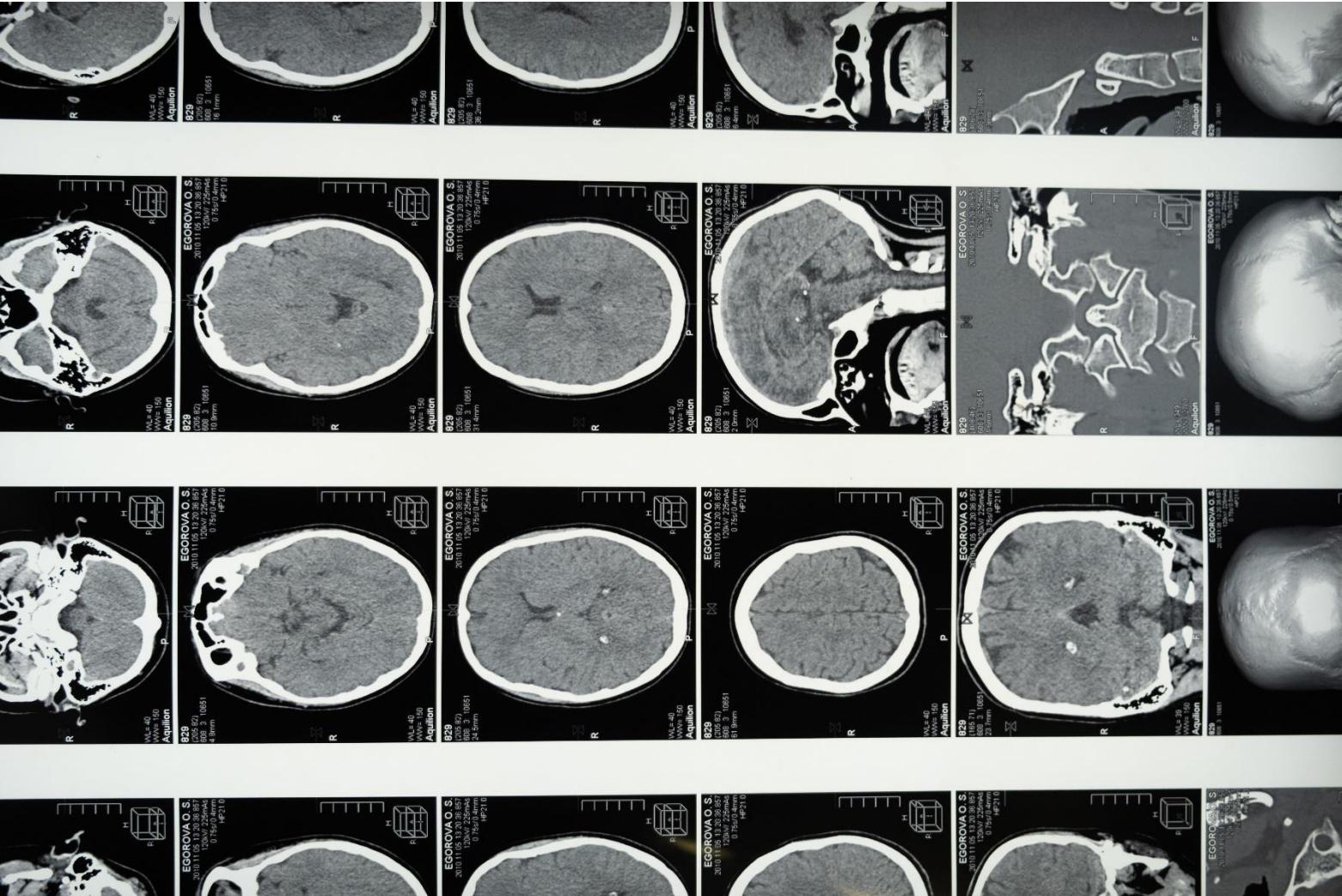
[Photo by Tom Fisk from Pexels](#) (retrieved 2023-04-02)

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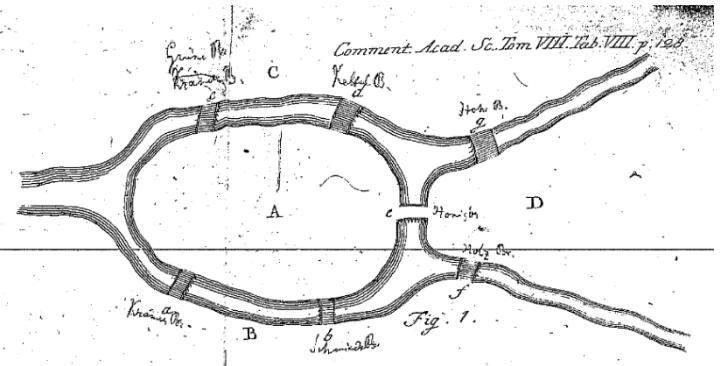


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Networks - history



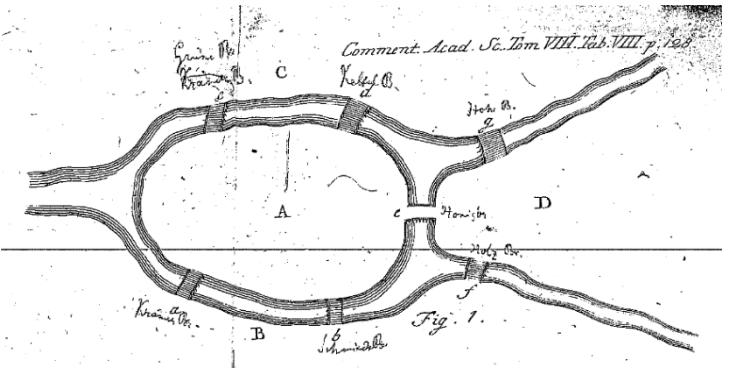
by **Leonard Euler, 1726**

Euler, L. (1741). Solutio problematis ad geometriam
situs pertinentis. *Commentarii academiae scientiarum
Petropolitanae*, 128-140.



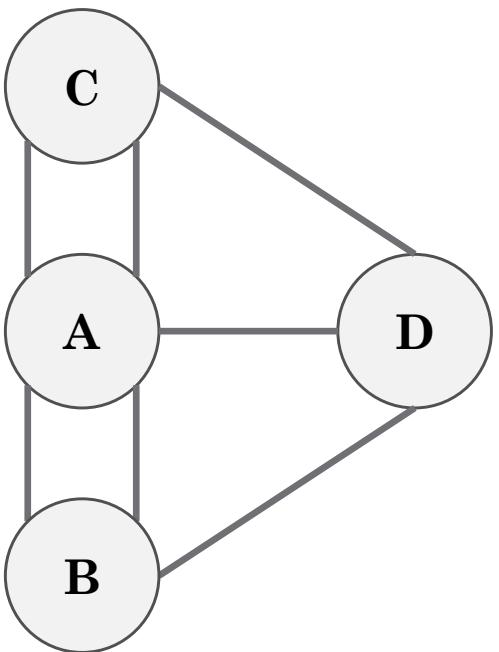
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Networks - history



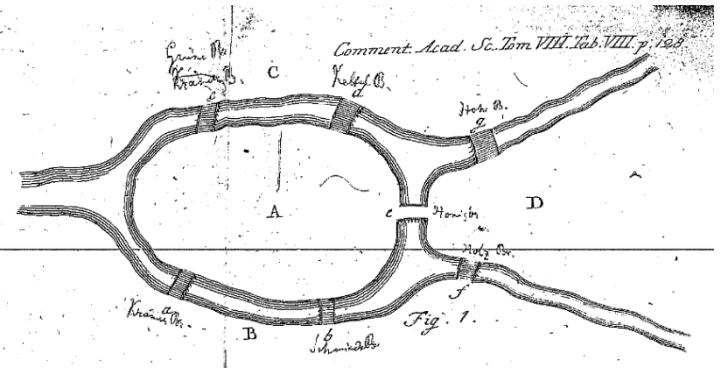
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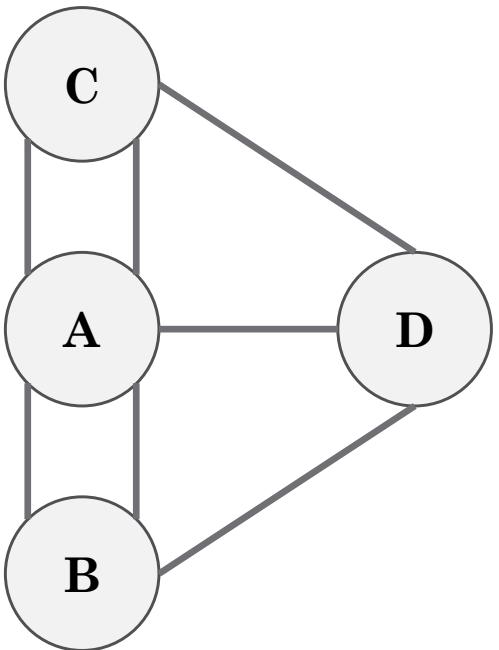


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Networks - history



Euler, L. (1741). Solutio problematis ad geometriam situs pertinentis. *Commentarii academiae scientiarum Petropolitanae*, 128-140.



*Graph theory foundations
by Leonard Euler, 1726*



[Wikimedia](#) (retrieved 2023-04-02)

Networks vs graphs

Graphs: neutral mathematical representations of elements

	a1	a2	a3	a4
a1	0	0	1	1
a2	1	0	1	0
a3	0	1	0	1
a4	1	1	0	0

Adjacency matrix

Networks vs graphs

Graphs: neutral mathematical representations of elements

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Adjacency matrix

Networks: graphs + context
(often real-world-related)

	a1	a2	a3	a4
a1	0	0	1	1
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Networks vs graphs

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Adjacency matrix

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	a1	a2	a3	a4
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a3	0	1	0	1
a4	1	1	0	0

a: actors

Networks vs graphs

“In the scientific literature the terms network and graph are used interchangeably”

A.L. Barabási

(Source: <http://networksciencebook.com/>, retrieved 2023-04-02)

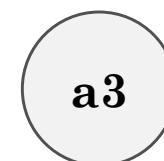
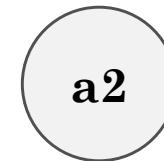
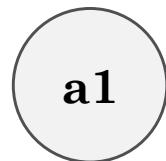


[Wikimedia](#) (retrieved 2023-04-02)

Network components

1. Nodes (vertices)

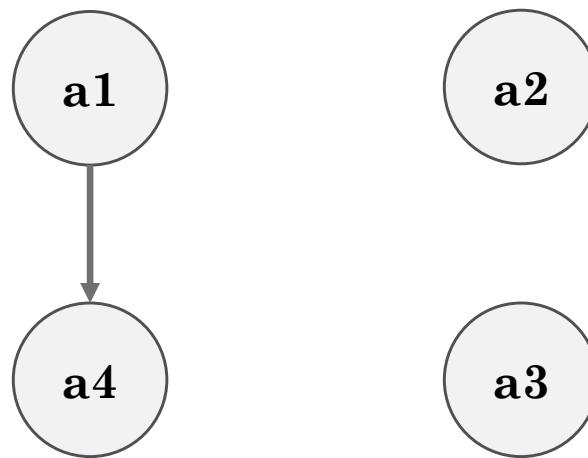
	a1	a2	a3	a4
a1	0	0	1	1
a2	1	0	1	0
a3	0	1	0	1
a4	1	1	0	0



Network components

1. Nodes (vertices)
2. Edges (links)

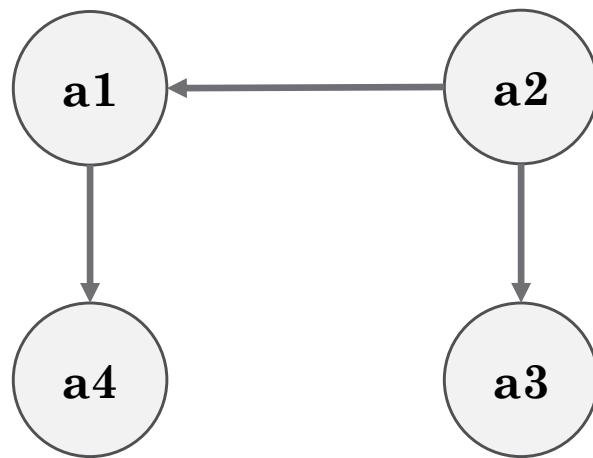
	a1	a2	a3	a4
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a4	1	1	0	0



Network components

1. Nodes (vertices)
2. Edges (links)

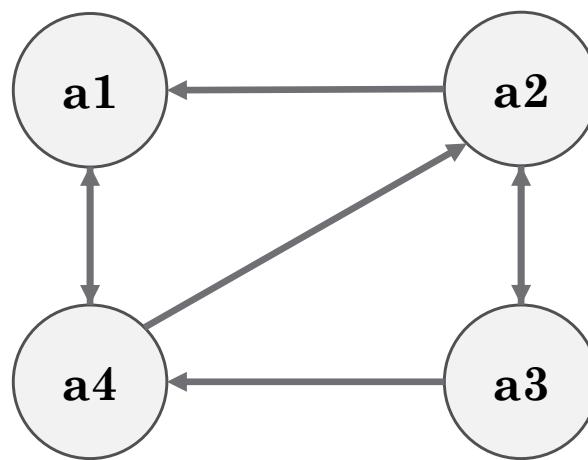
	a1	a2	a3	a4
a1	0	0	0	1
a2	1	0	1	0
a3	0	1	0	1
a4	1	1	0	0



Network components

1. Nodes (vertices)
2. Edges (links)

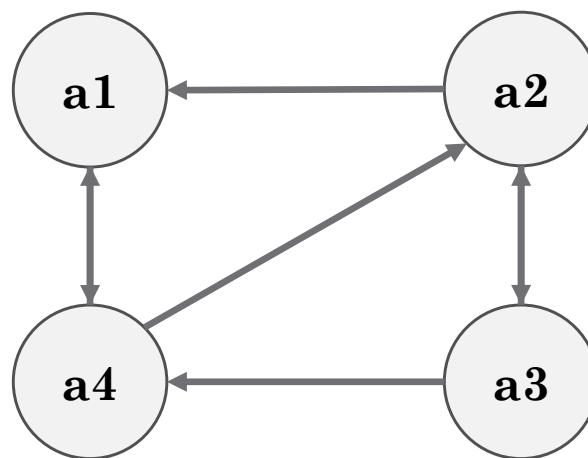
	a1	a2	a3	a4
a1	0	0	0	1
a2	1	0	1	0
a3	0	1	0	1
a4	1	1	0	0



Network components

1. Nodes (vertices)
2. Edges (links) – directed / un-directed

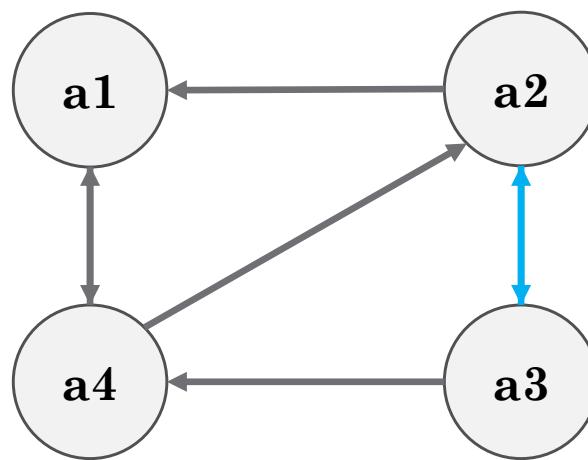
	a1	a2	a3	a4
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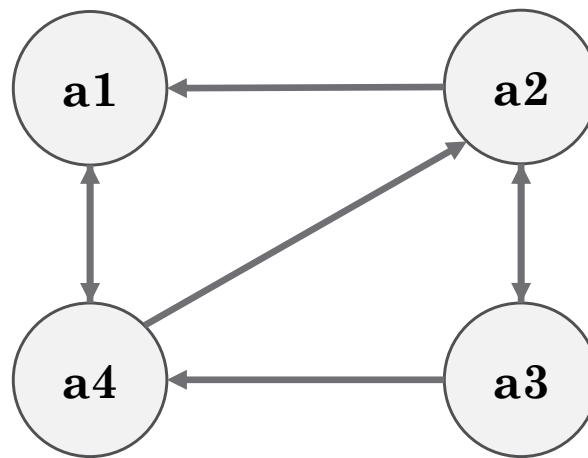


Network components

1. Nodes (vertices)
2. Edges (links) – directed / un-directed

	a1	a2	a3	a4
a1	0	0	0	1
a2	1	0	1	0
a3	0	1	0	1
a4	1	1	0	0

a: actors
l: co-starring

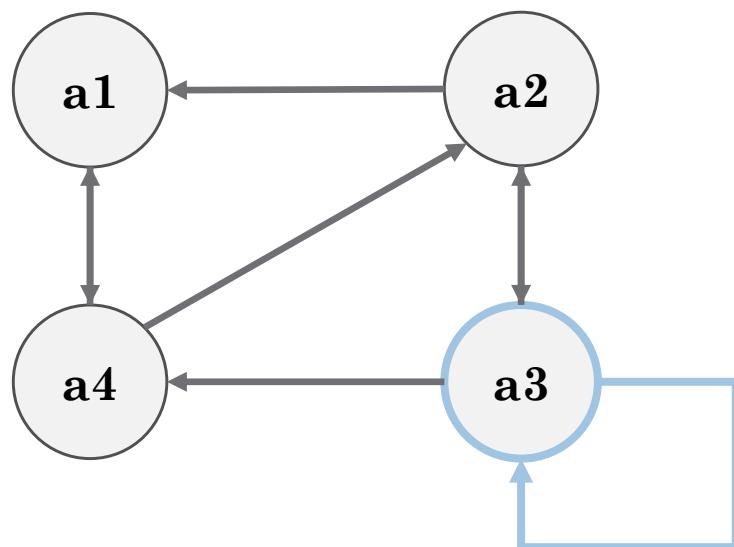


Network components

1. Nodes (vertices)
2. Edges (links) – self-loops

	a1	a2	a3	a4
a1	0	0	0	1
a2	1	0	1	0
a3	0	1	1	1
a4	1	1	0	0

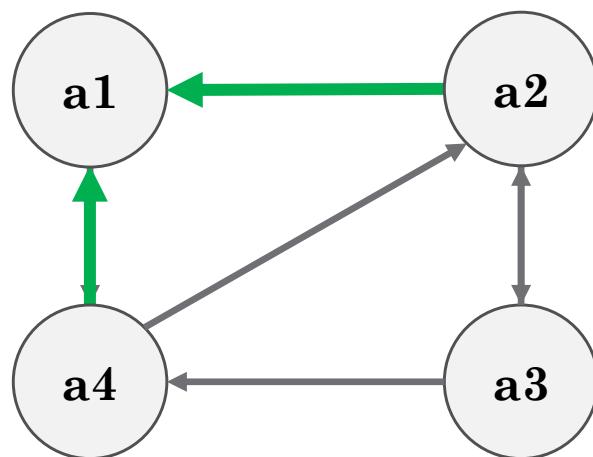
a: articles
l: citation



Network metrics

Degree: number of connections
In-degree vs **Out-degree**

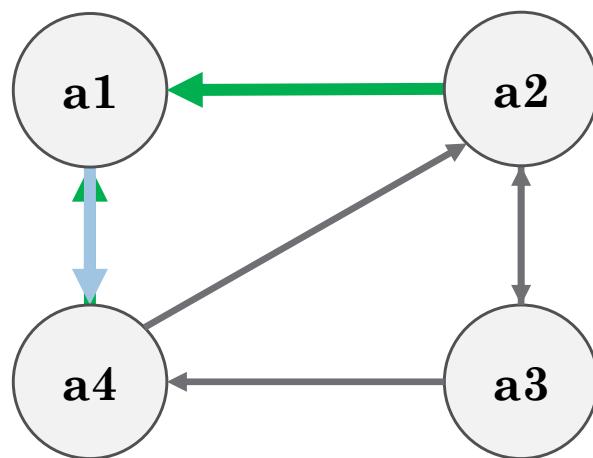
	a1	a2	a3	a4
a1	0	0	0	1
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a3	0	1	0	1
a4	1	1	0	0



Network metrics

Degree: number of connections
In-degree vs **Out-degree**

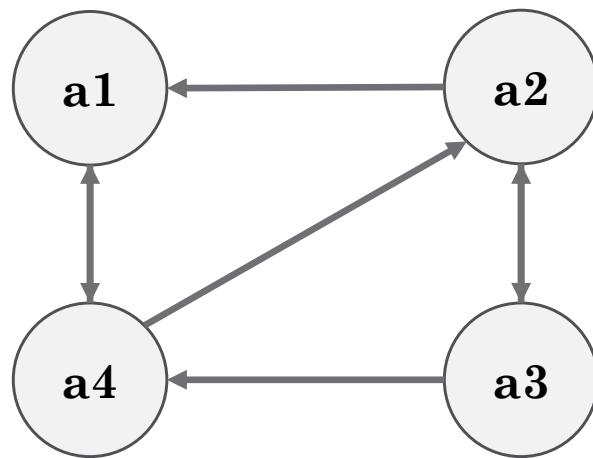
	a1	a2	a3	a4
a1	0	0	0	1
a2	1	0	1	0
a3	0	1	0	1
a4	1	1	0	0



Network metrics

Path length: number of hops

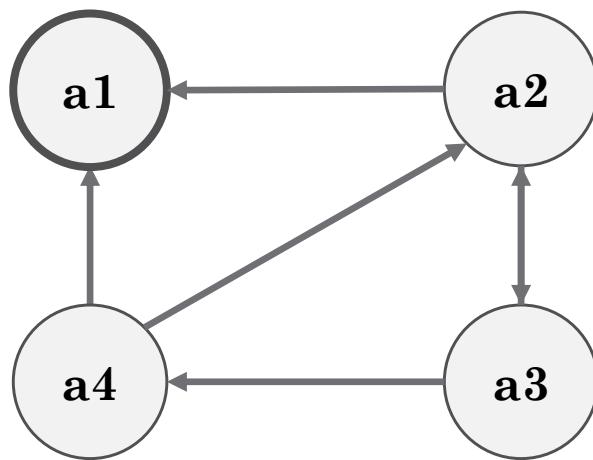
	a1	a2	a3	a4
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a3	0	1	0	1
a4	1	1	0	0



Network metrics

Path length: number of hops (can be infinite)

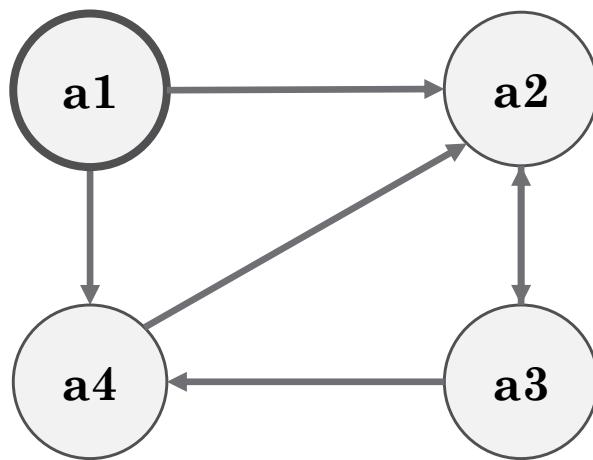
	a1	a2	a3	a4
a1	0	0	0	0
a2	1	0	1	0
a3	0	1	0	1
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Network metrics

Path length: number of hops (can be infinite)

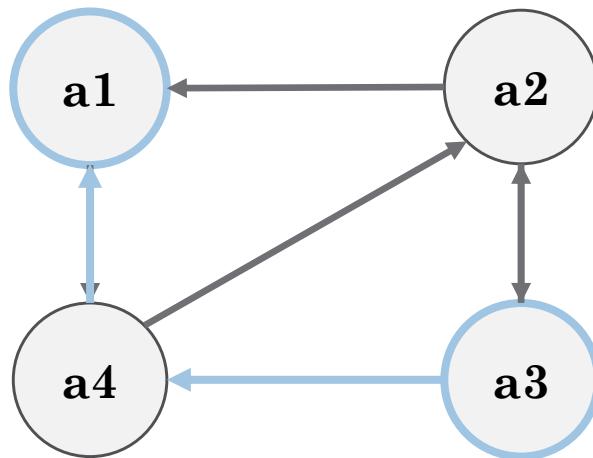
	a1	a2	a3	a4
a1	0	1	0	1
a2	0	0	1	0
a3	0	1	0	1
a4	0	1	0	0



Network metrics

Distance: shortest path

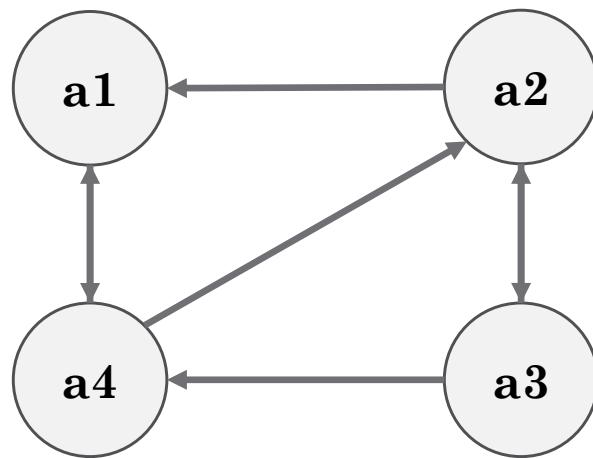
	a1	a2	a3	a4
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Network metrics

Diameter: longest distance

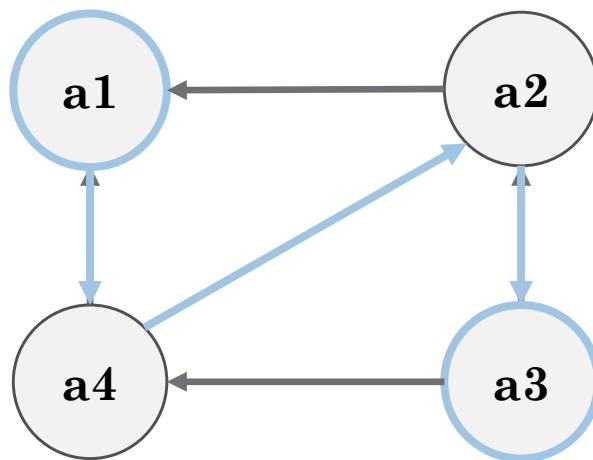
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Network metrics

Diameter: longest distance

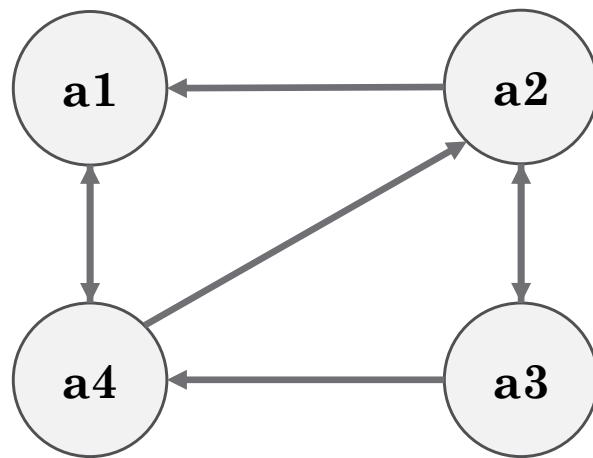
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Network metrics

Clustering coefficient: “density of connections”

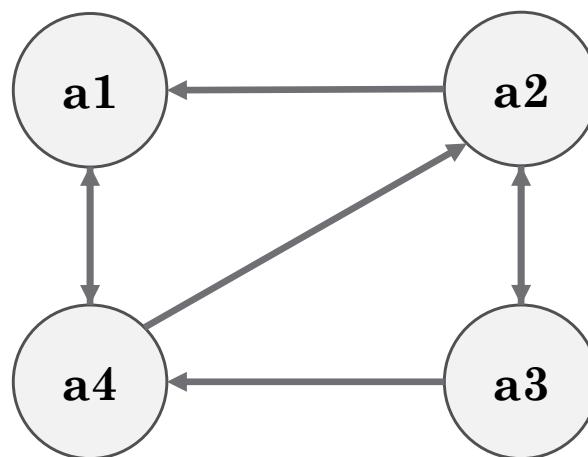
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Network metrics

Clustering coefficient: “density of connections” = fraction of possible triangles through that node

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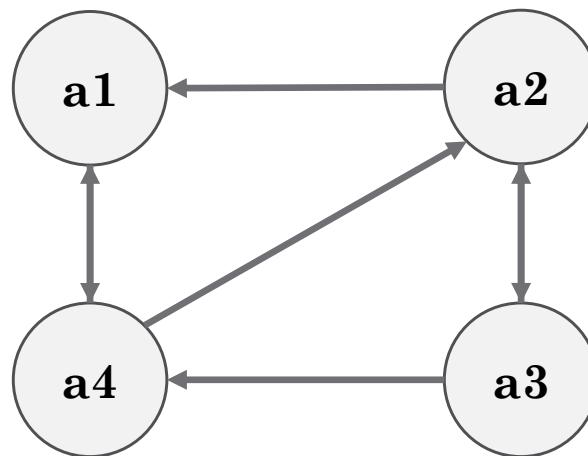


Undirected graph: $c_a = \frac{2 T_a}{d_a(d_a - 1)}$

Network metrics

Clustering coefficient: “density of connections” = fraction of possible triangles through that node

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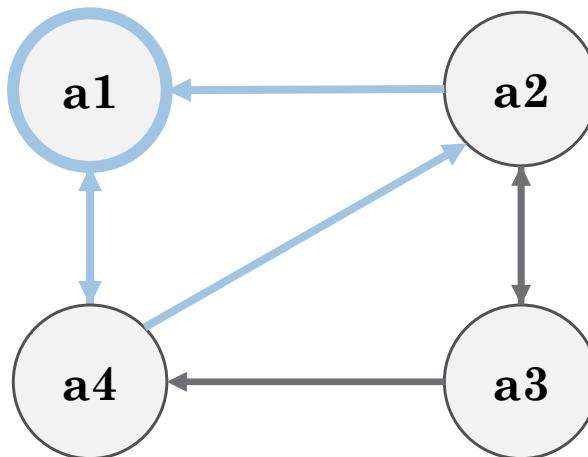


Directed graph: $c_a = \frac{t_a}{(d_a(d_a-1)-2d_a^{bid})}$

Network metrics

Clustering coefficient: “density of connections” = fraction of possible triangles through that node

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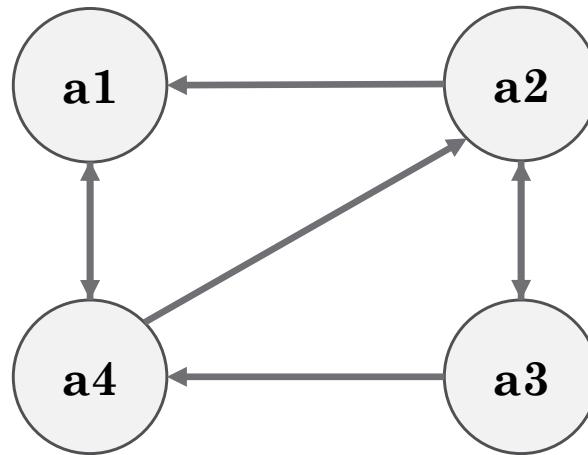


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Network metrics

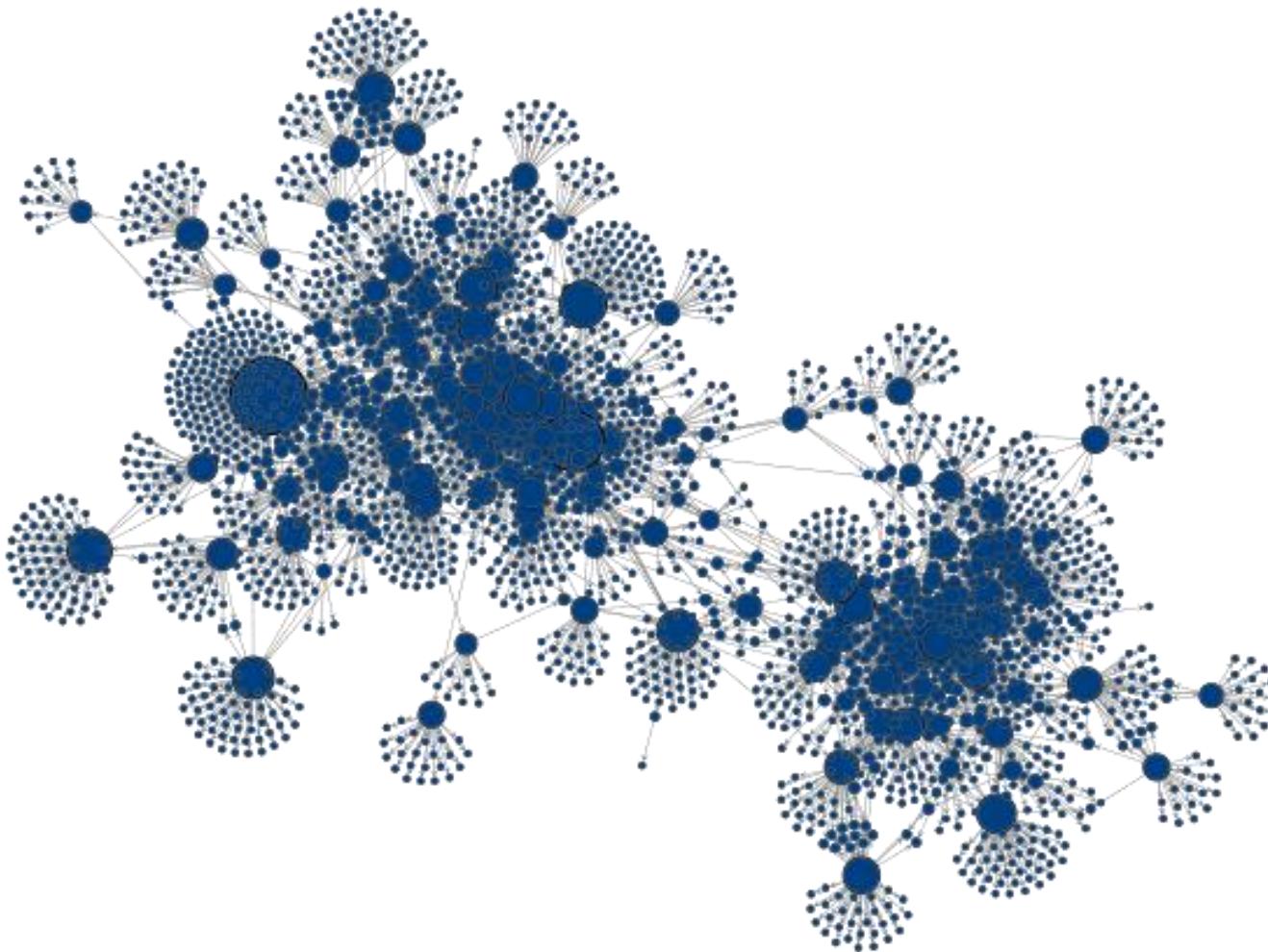
Centrality: “*node importance*” – but what is *important*?

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Network metrics

Centrality: “*node importance*” – but what is *important*?



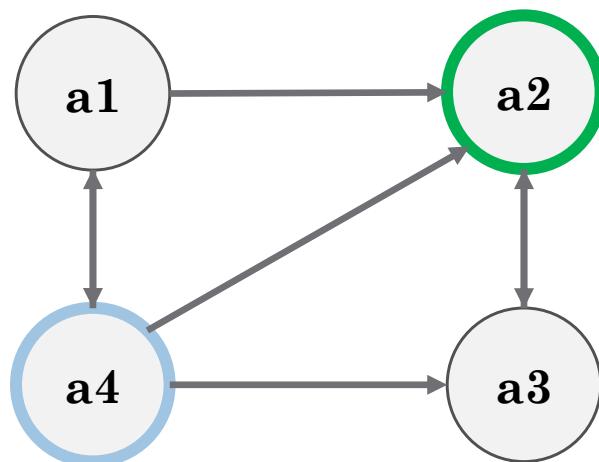
[Kaggle](#) (retrieved 2023-05-13)

Network metrics

Centrality: “*node importance*” – but what is *important*?

- Degree centrality
 - In-degree (*prestige*)
 - Out-degree

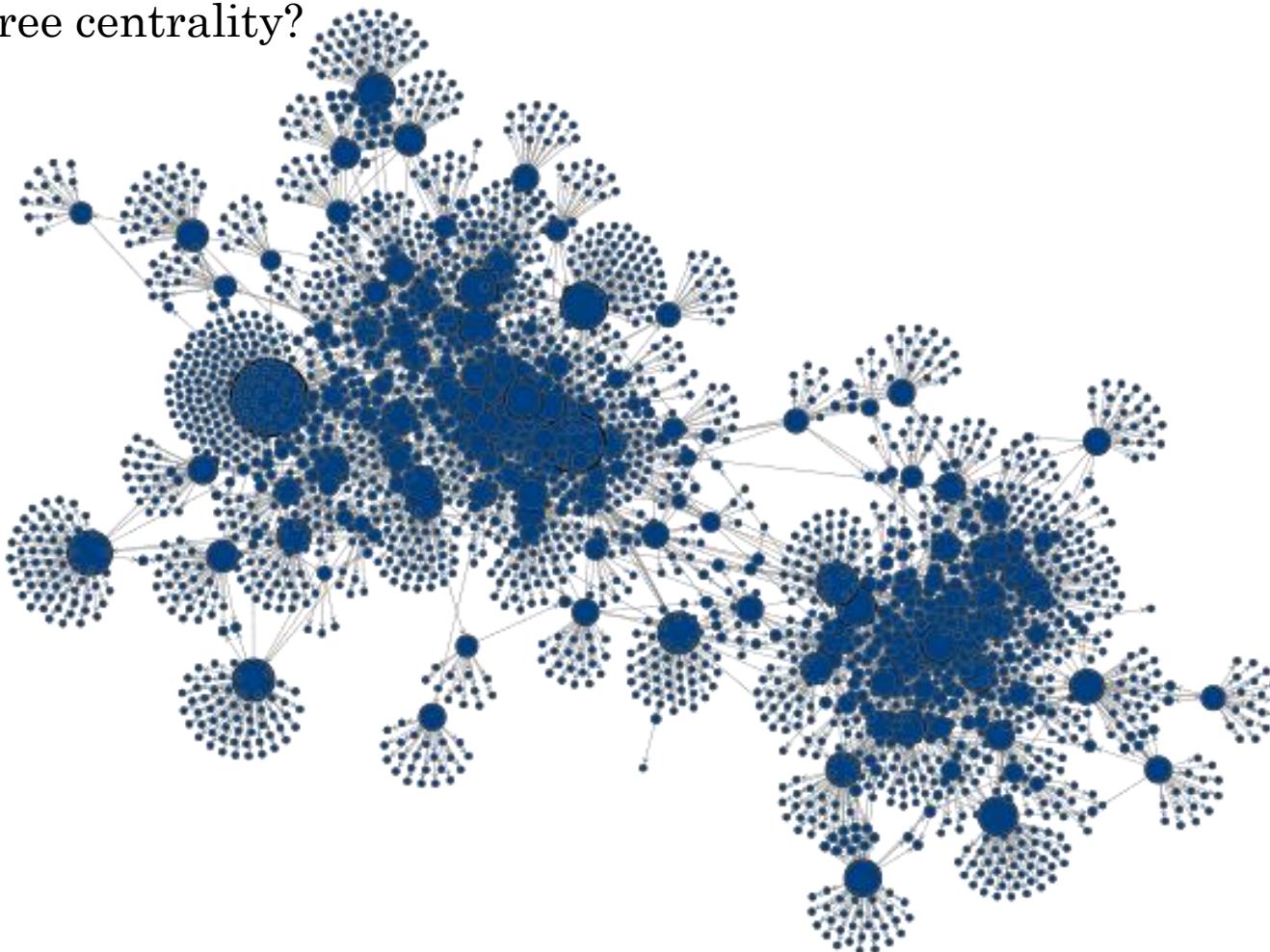
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Network metrics

Centrality: “*node importance*” – but what is *important*?

- Degree centrality?

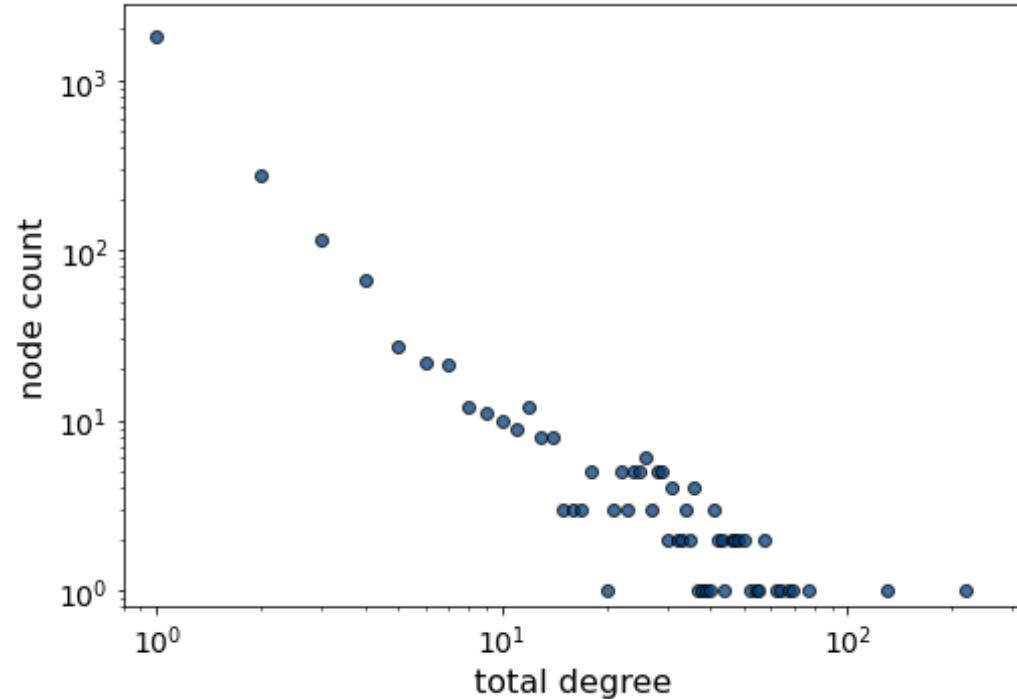
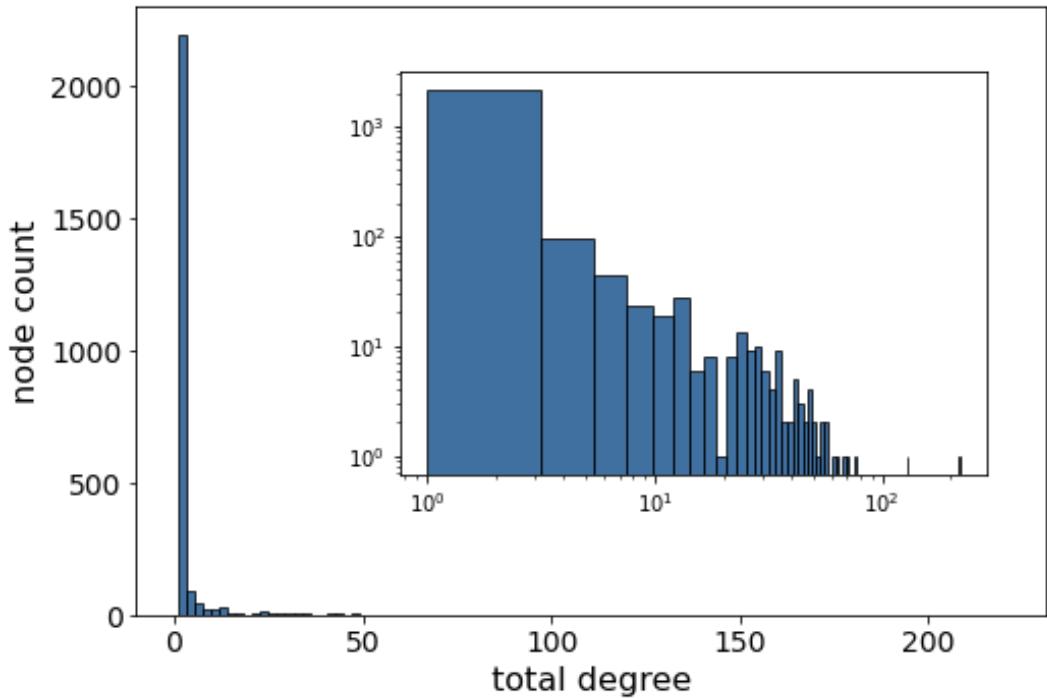


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Network metrics

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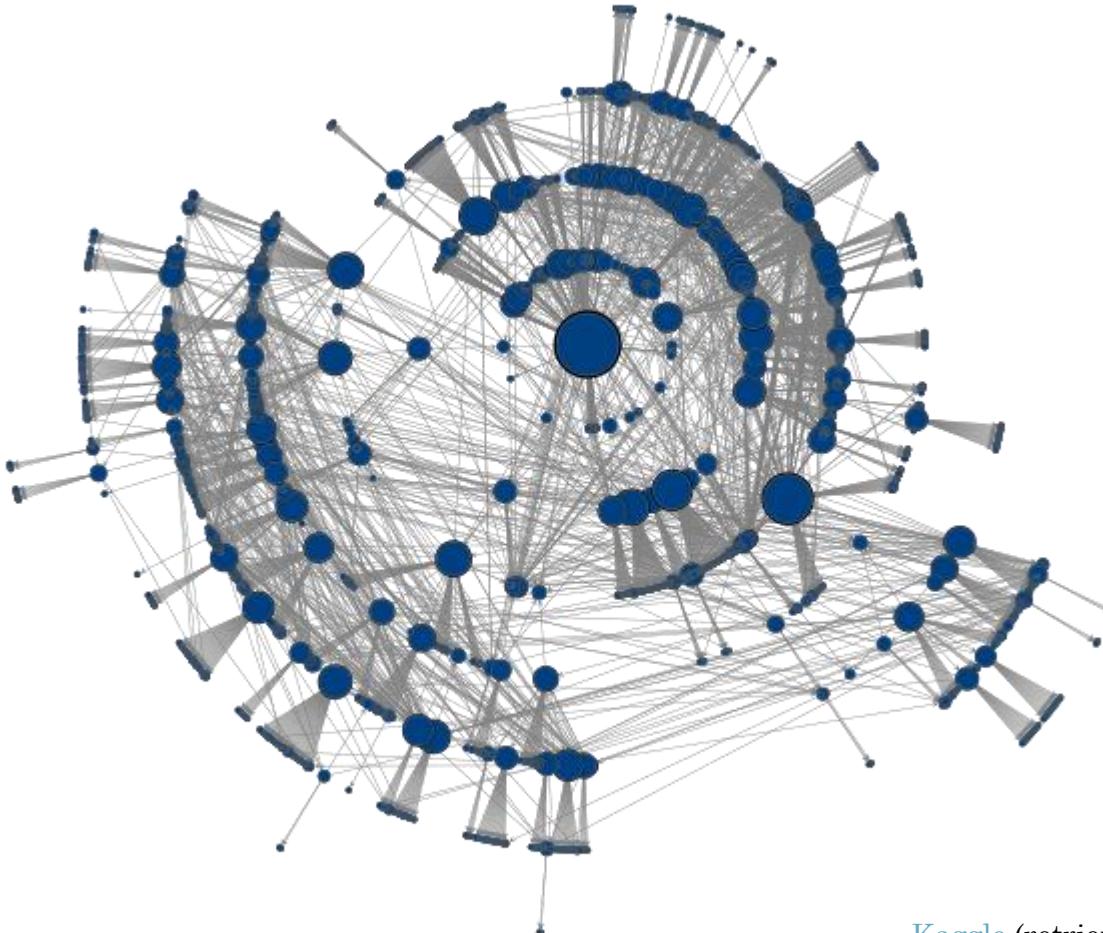
- Degree centrality → highly skewed degrees, maybe hierarchical?



Network metrics

Centrality: “*node importance*” – but what is *important*?

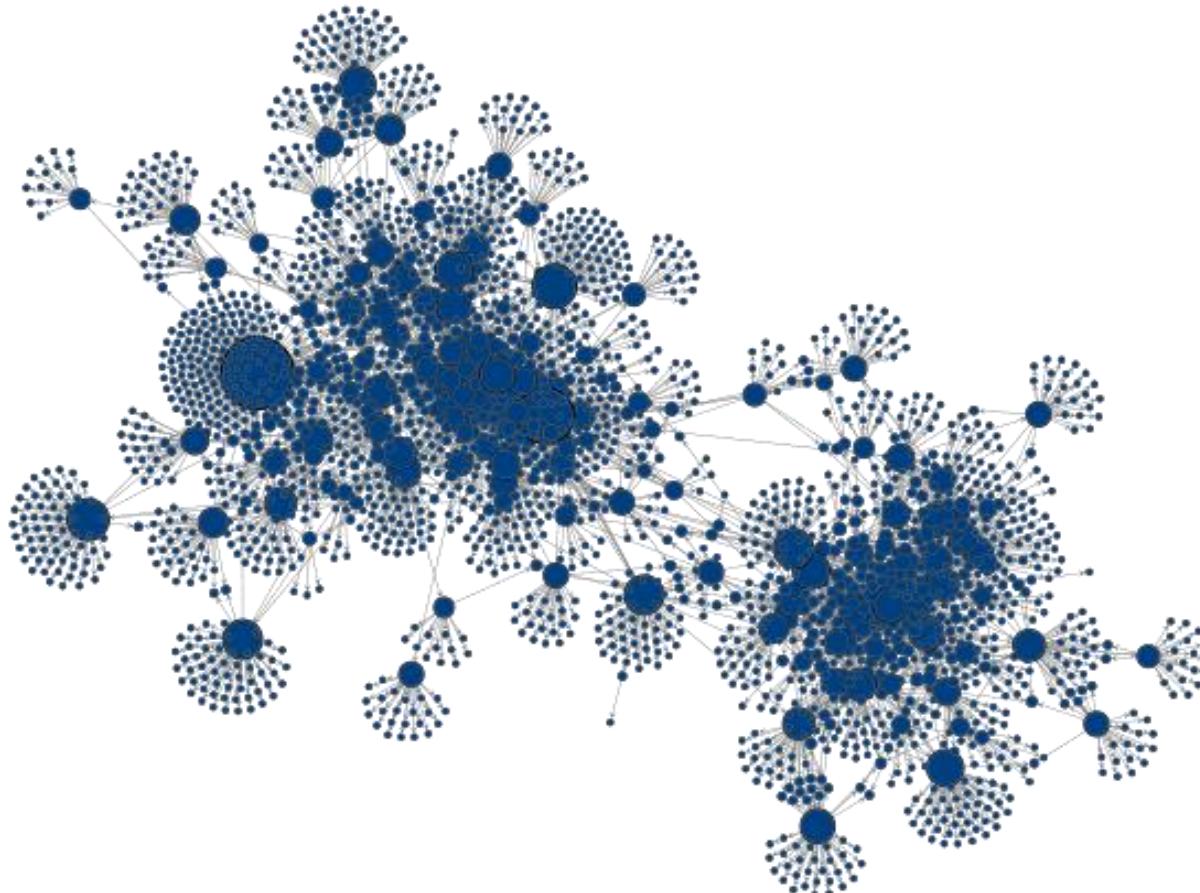
- Degree centrality → highly skewed degrees, maybe hierarchical?



Network metrics

Centrality: “*node importance*” – but what is *important*?

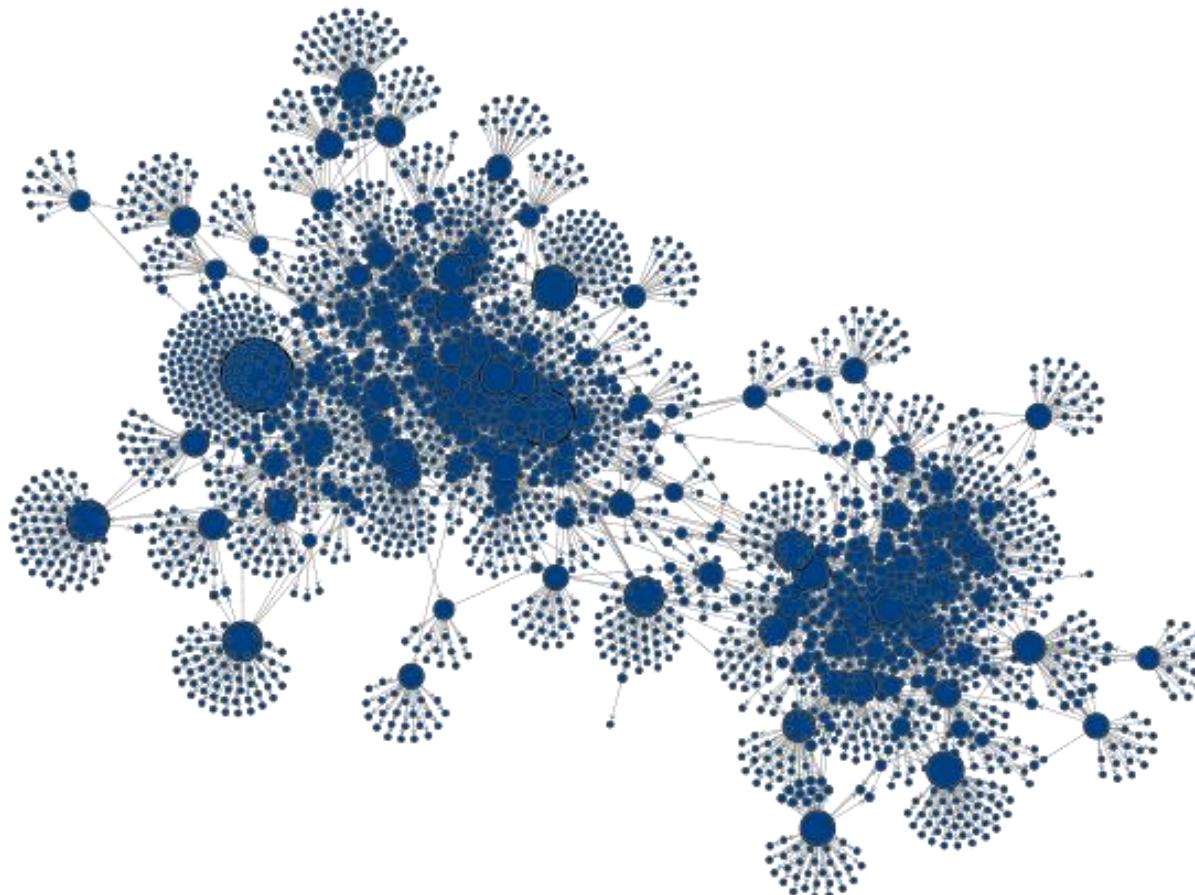
- Closeness centrality: best connected *within their own clusters*



Network metrics

Centrality: “*node importance*” – but what is *important*?

- Closeness centrality: best connected *within their own clusters*

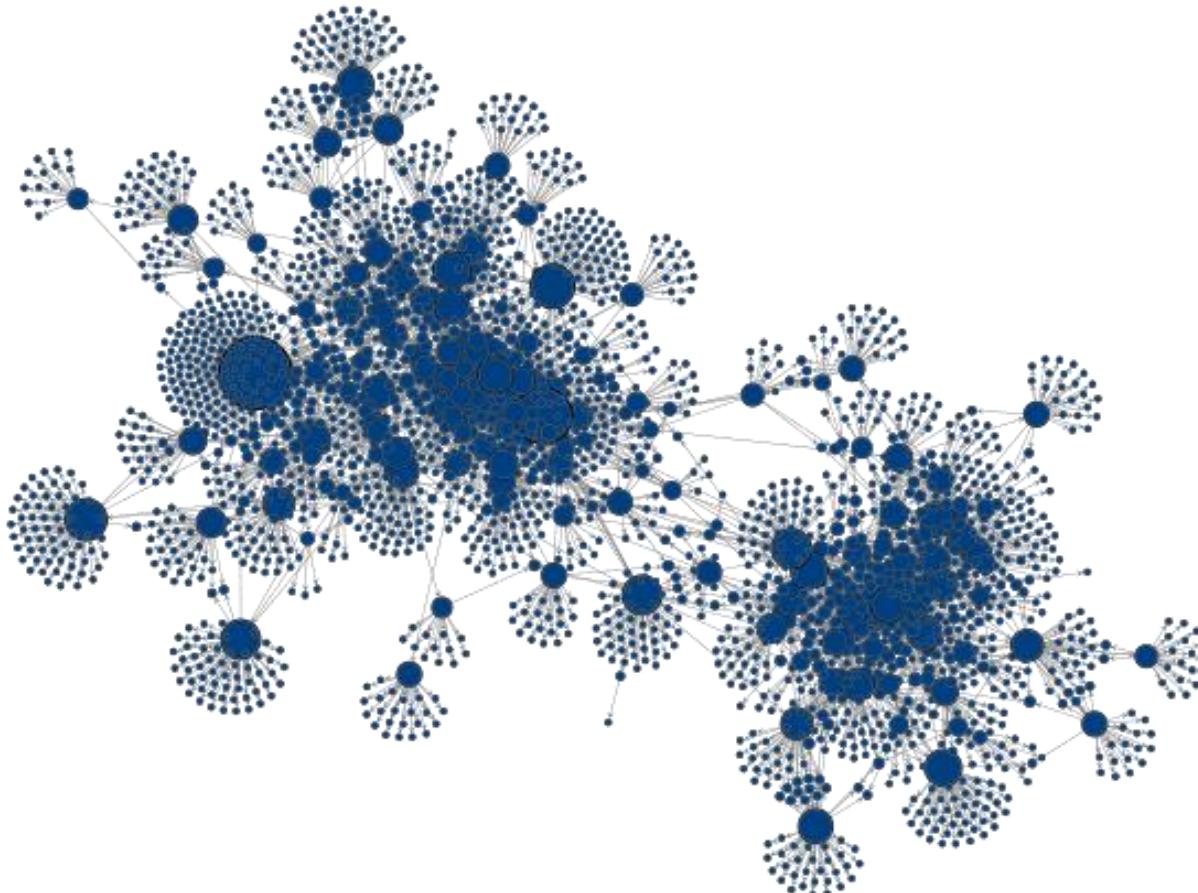


[Kaggle](#) (retrieved 2023-05-13)

Network metrics

Centrality: “*node importance*” – but what is *important*?

- Betweenness centrality: connectors / bridges



[Kaggle](#) (retrieved 2023-05-13)

Network metrics

Centrality: “*node importance*” – but what is *important*?

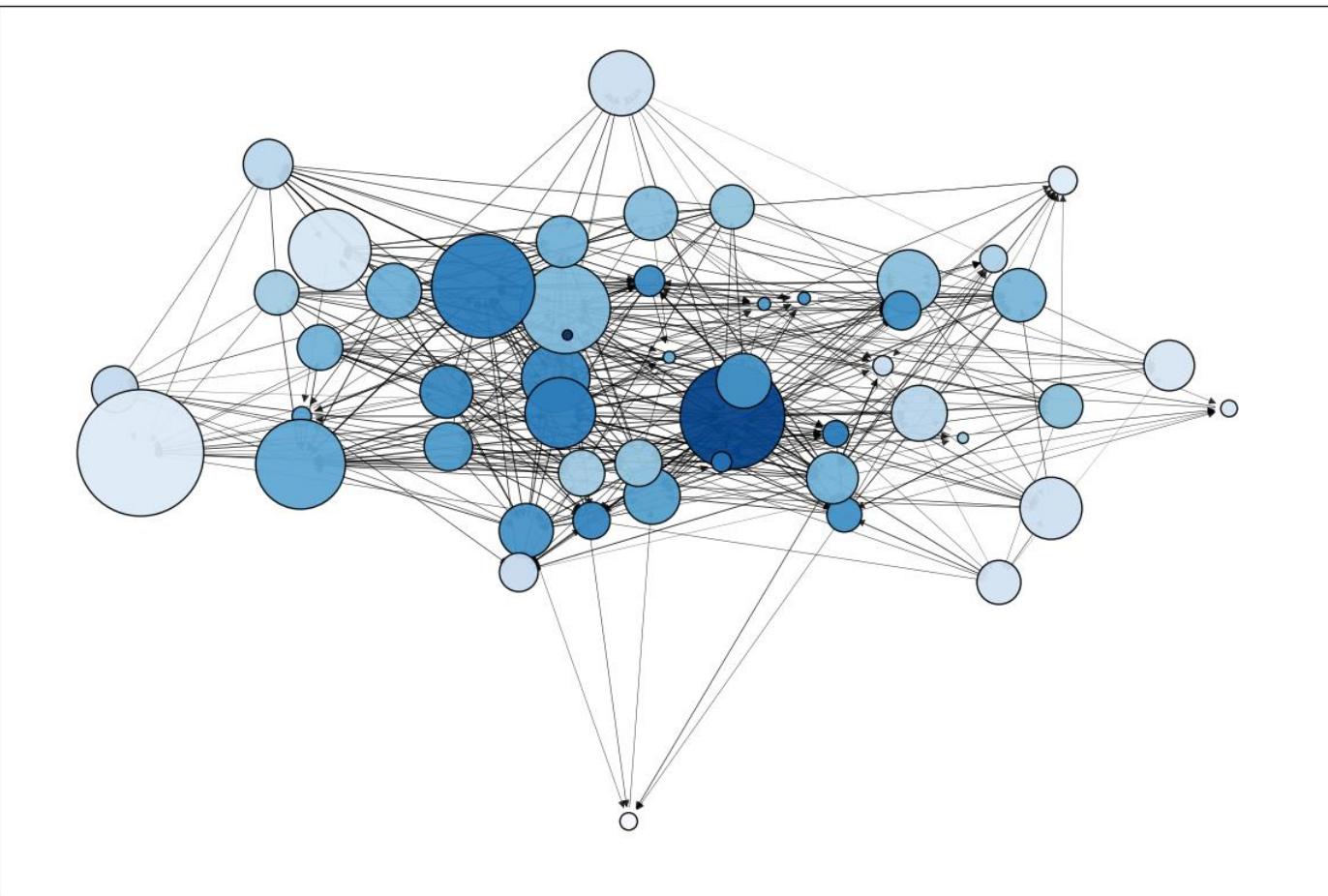
- Betweenness centrality: connectors / bridges



Network metrics

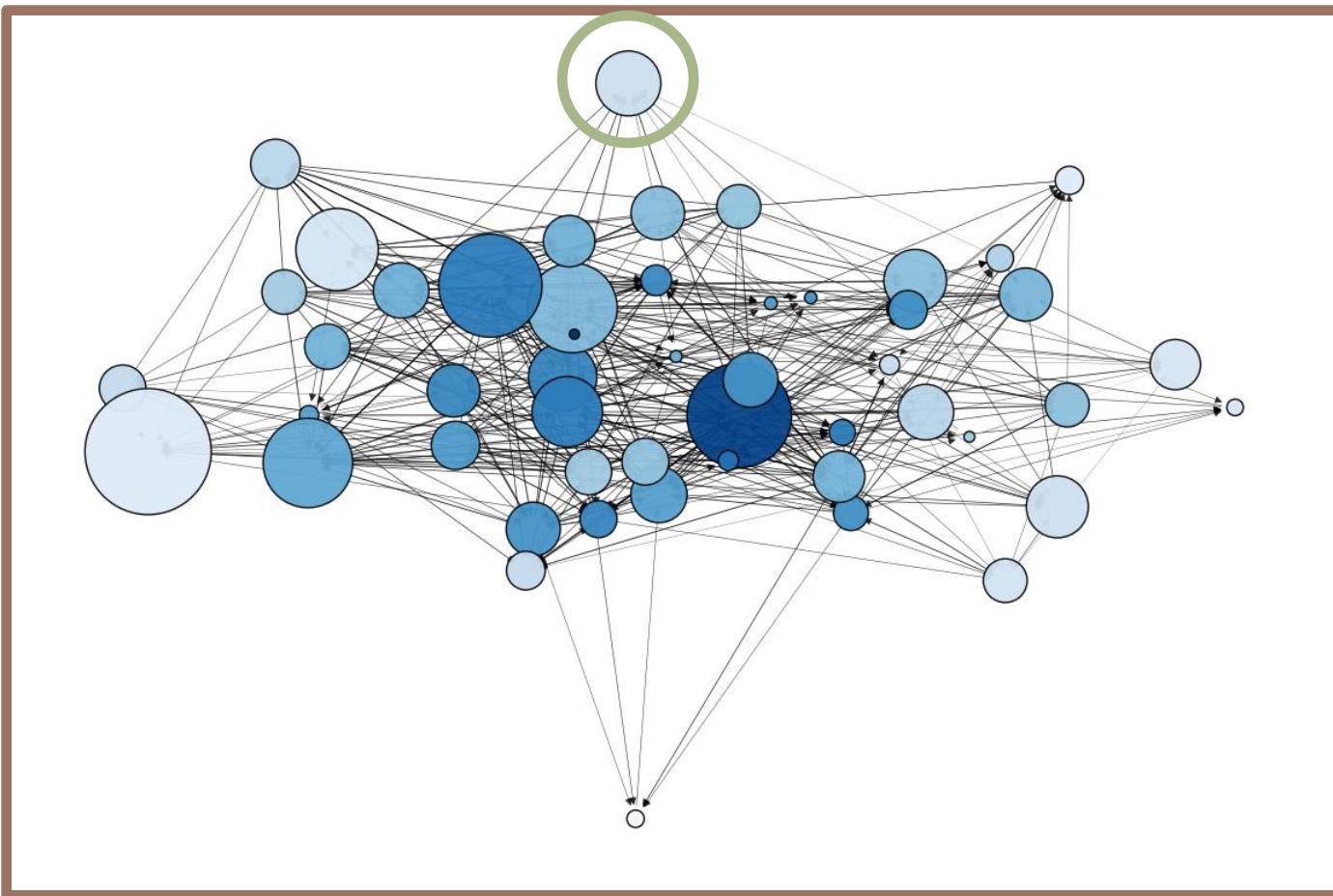
Centrality: “*node importance*” – but what is *important*?

- HITS centrality: authorities (in-degree) and hubs (out-degree)



Network metrics

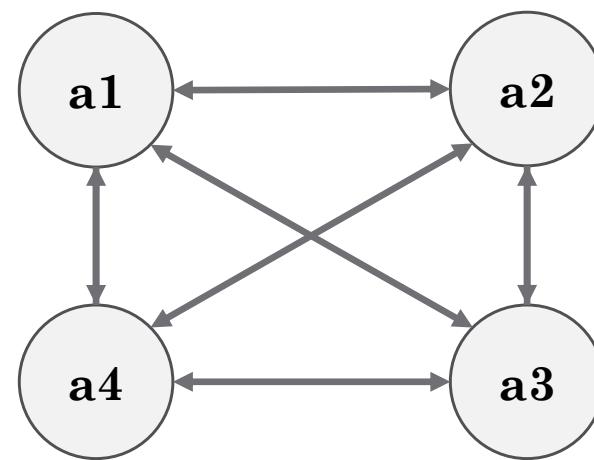
Local (e.g., node degree) vs global (e.g., assortativity)



Network topology

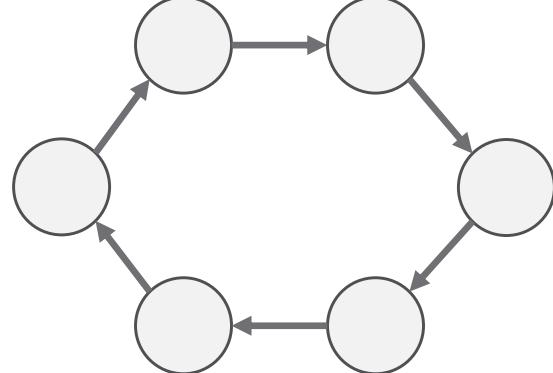
Complete graph = clique

	a1	a2	a3	a4
a1	0	1	1	1
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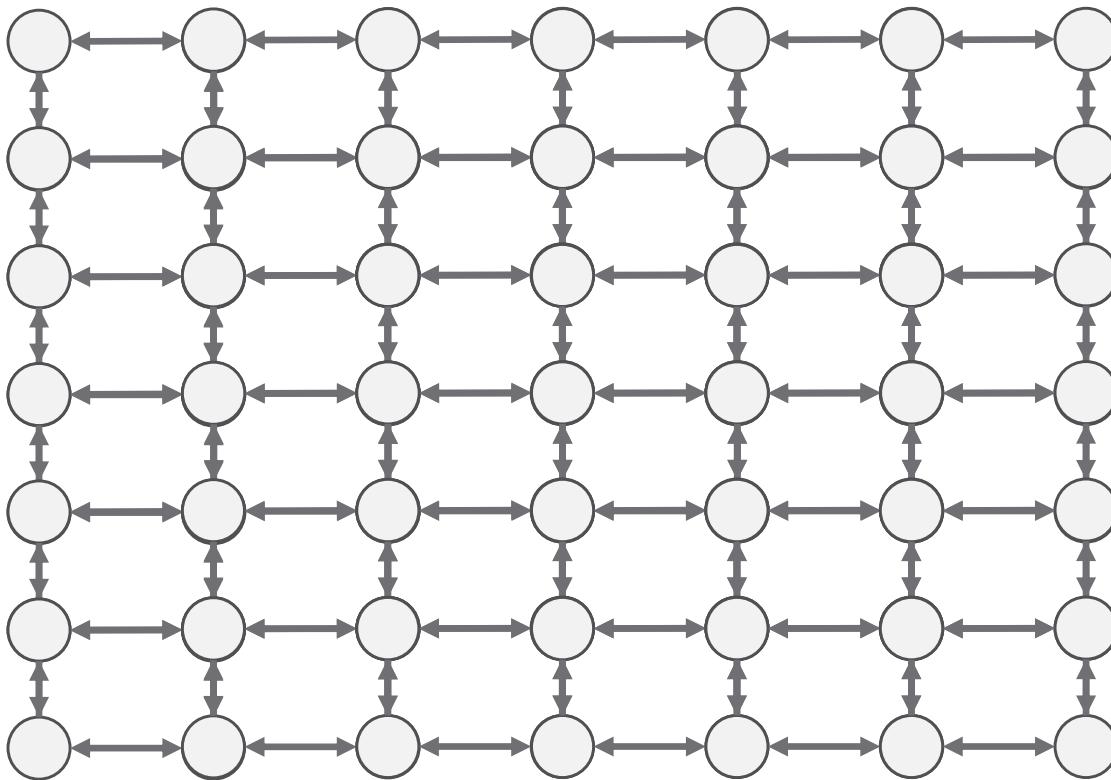


Network topology

Regular network

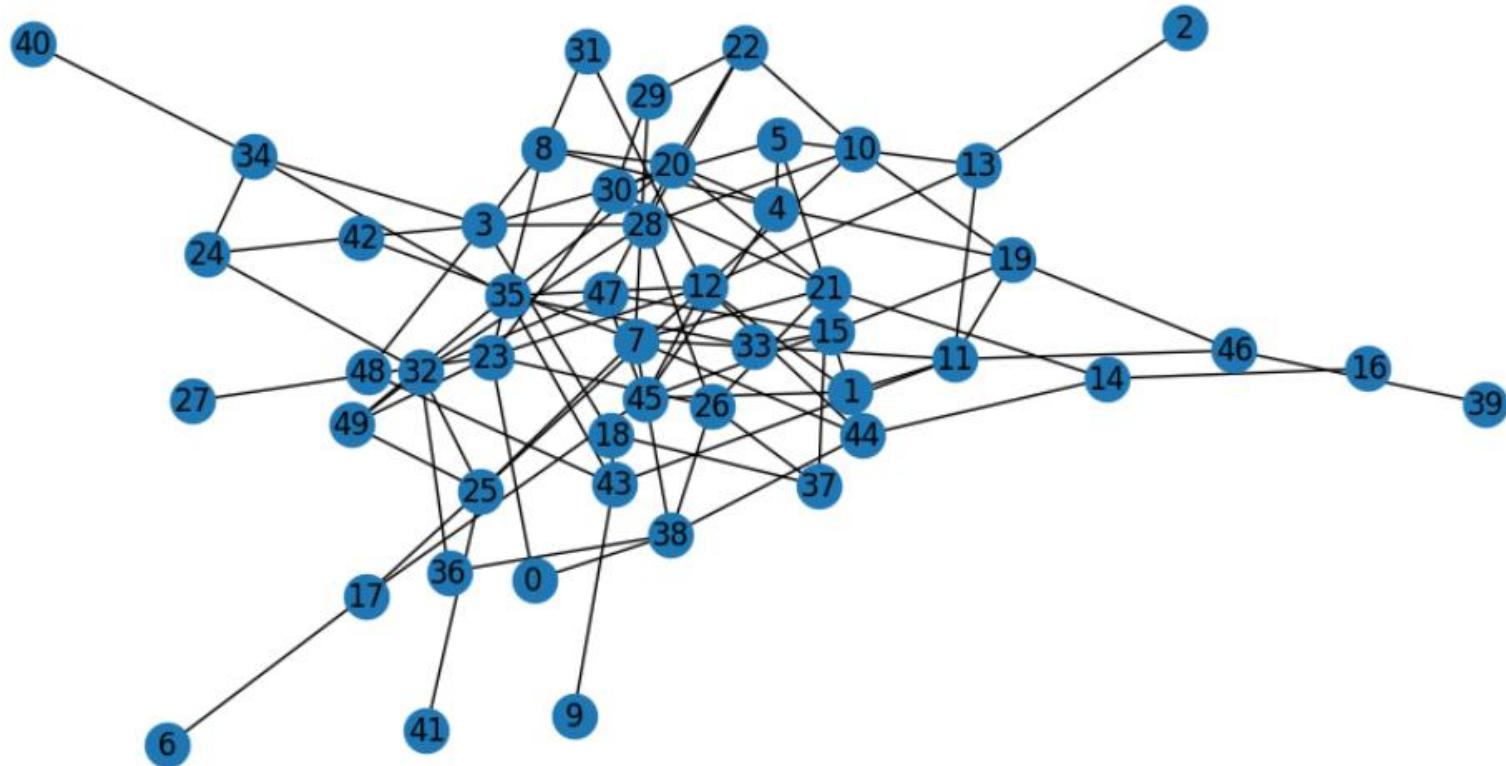


Grid

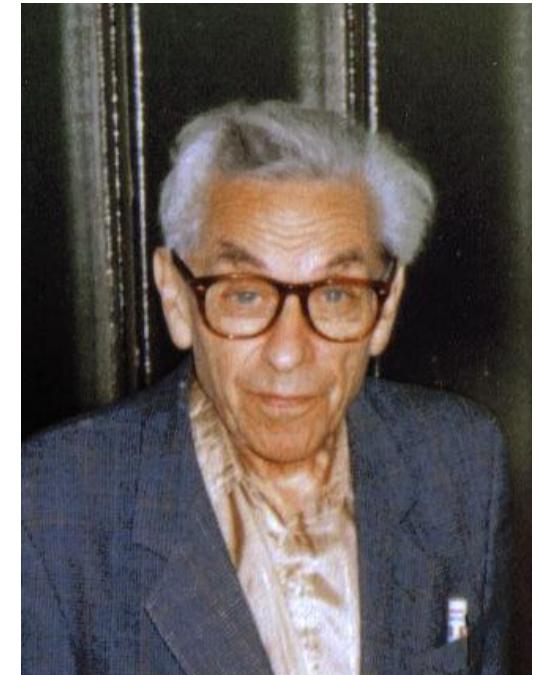


Network topology

Erdős-Rényi network



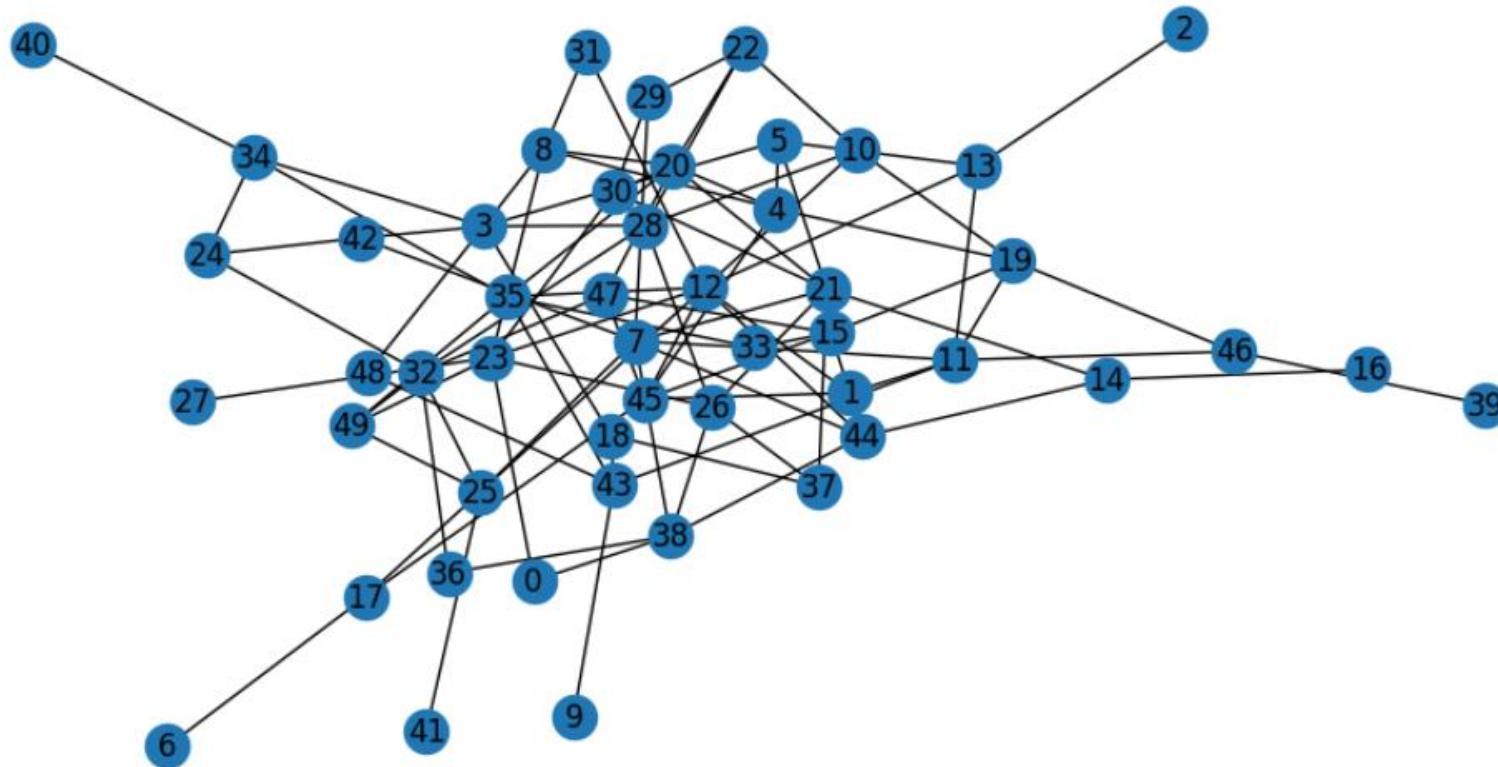
Paul Erdős



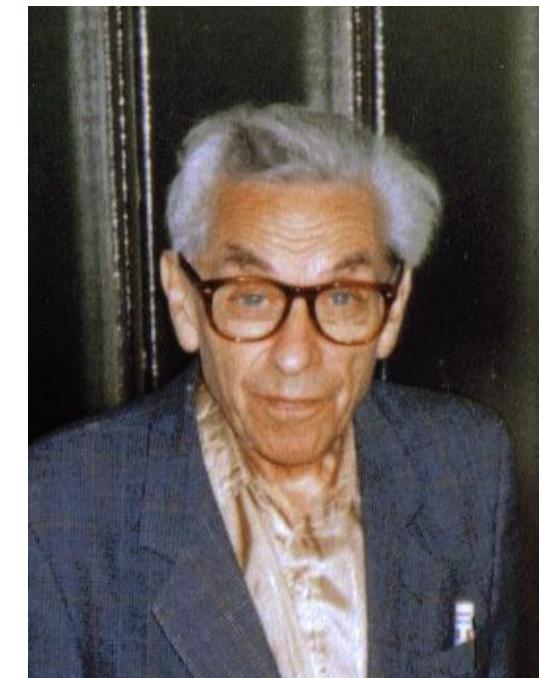
Network topology

Erdős-Rényi network

- n nodes
- connect two nodes with probability p

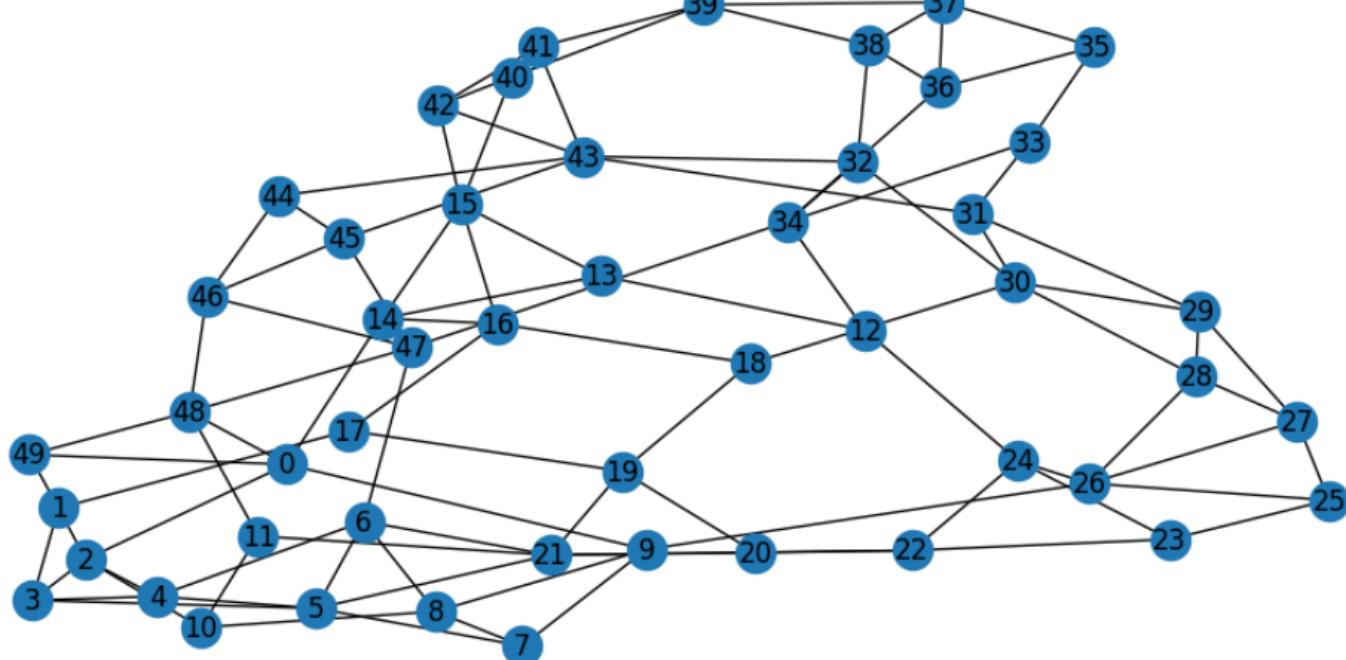


Paul Erdős



Network topology

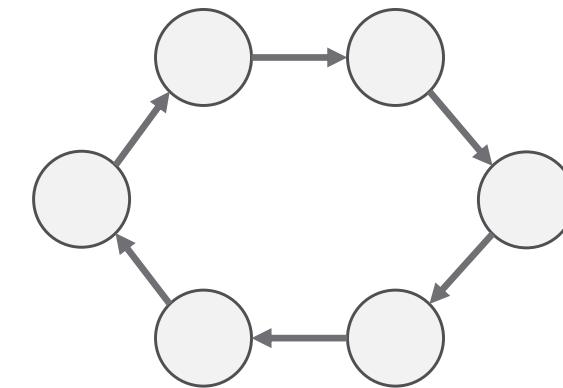
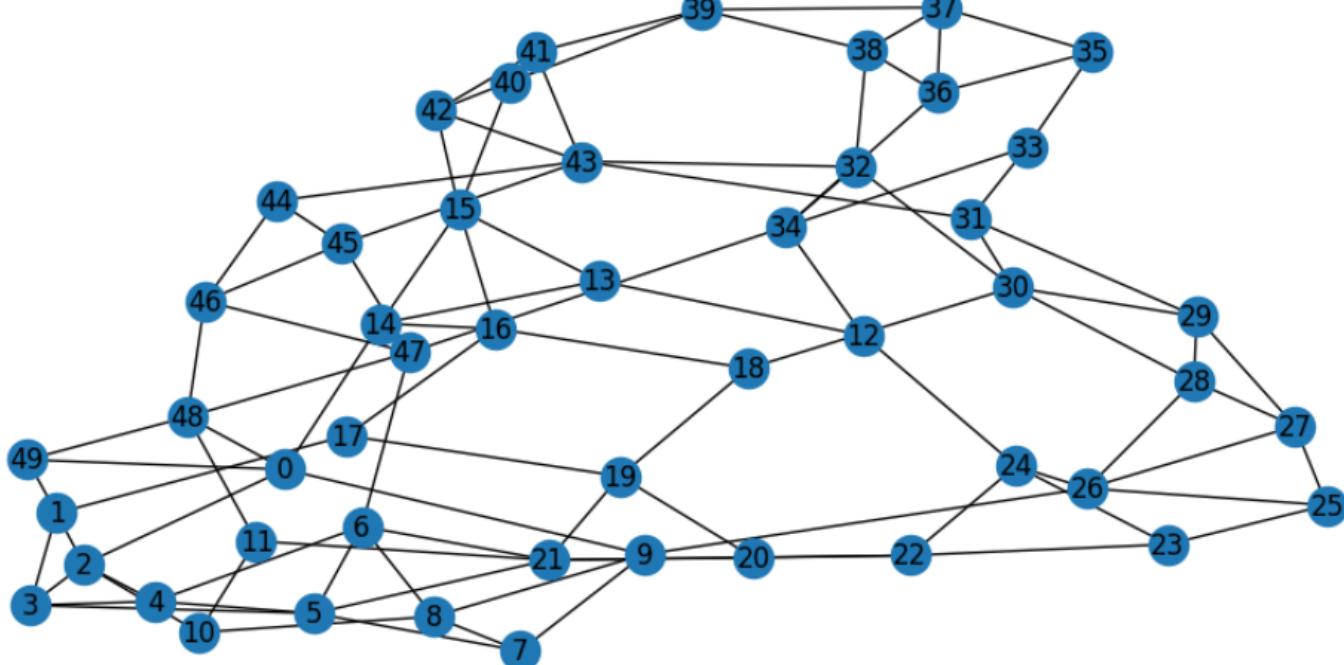
Watts-Strogatz (aka. Small World) network



[Wikimedia](#) (retrieved 2023-04-04)

Network topology

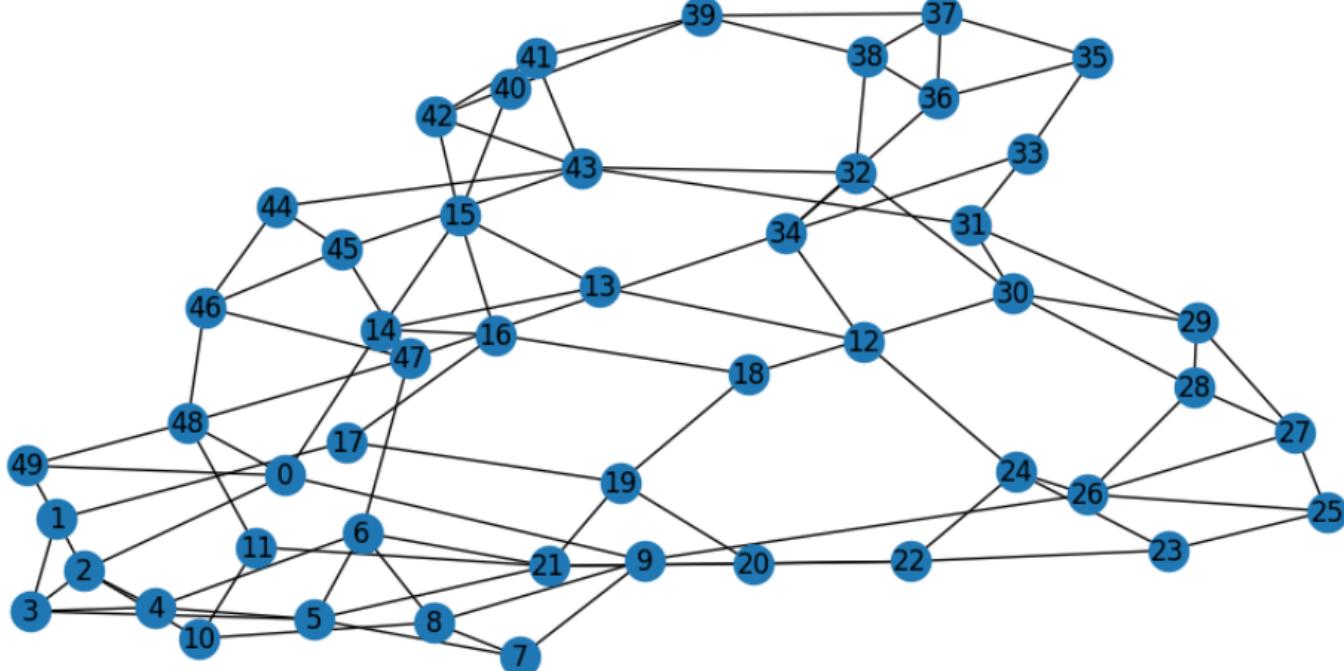
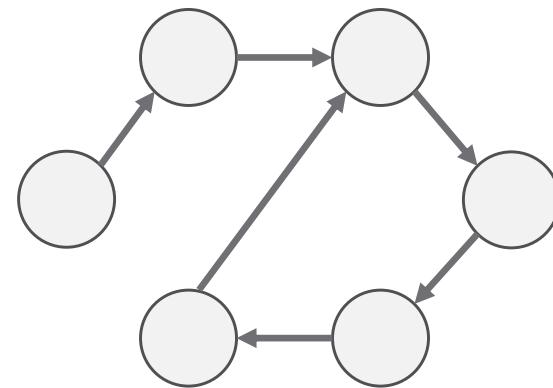
Watts-Strogatz (aka. Small World) network



[Wikimedia](#) (retrieved 2023-04-04)

Network topology

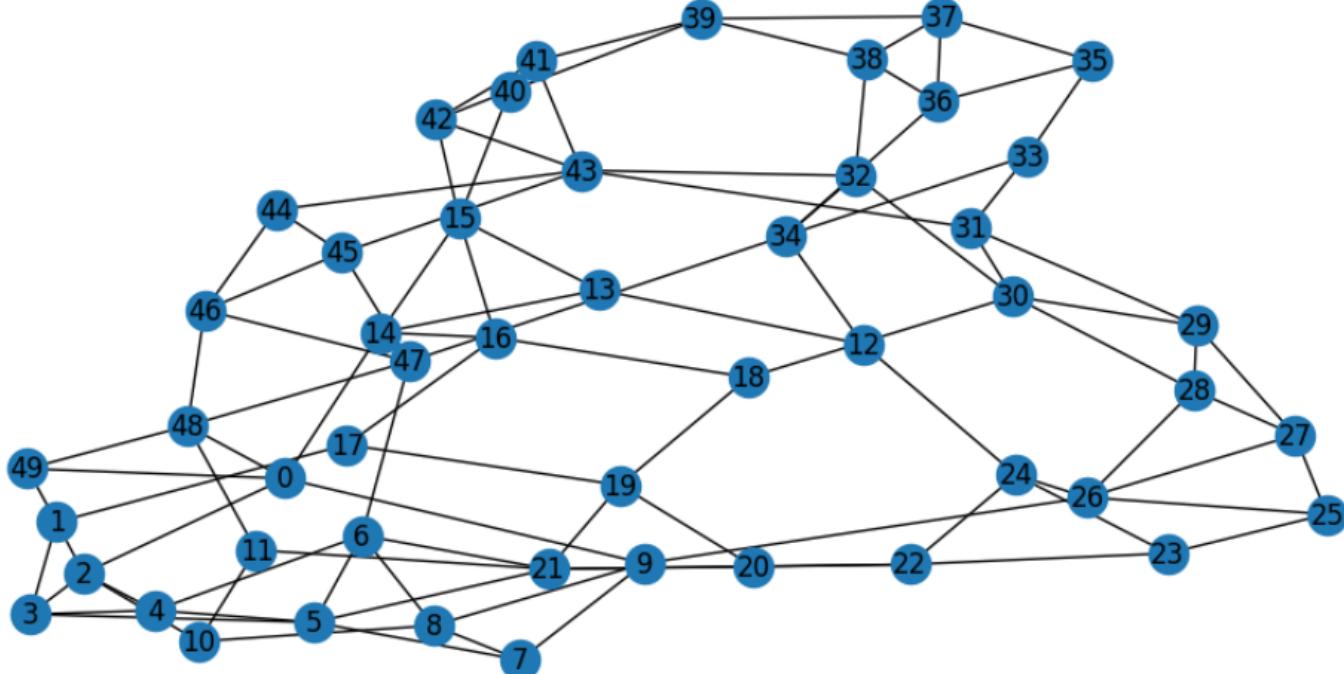
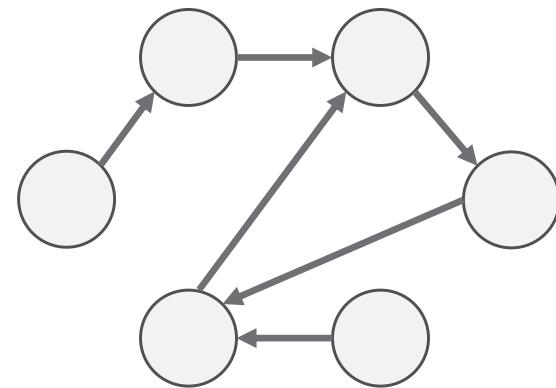
Watts-Strogatz (aka. Small World) network



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Network topology

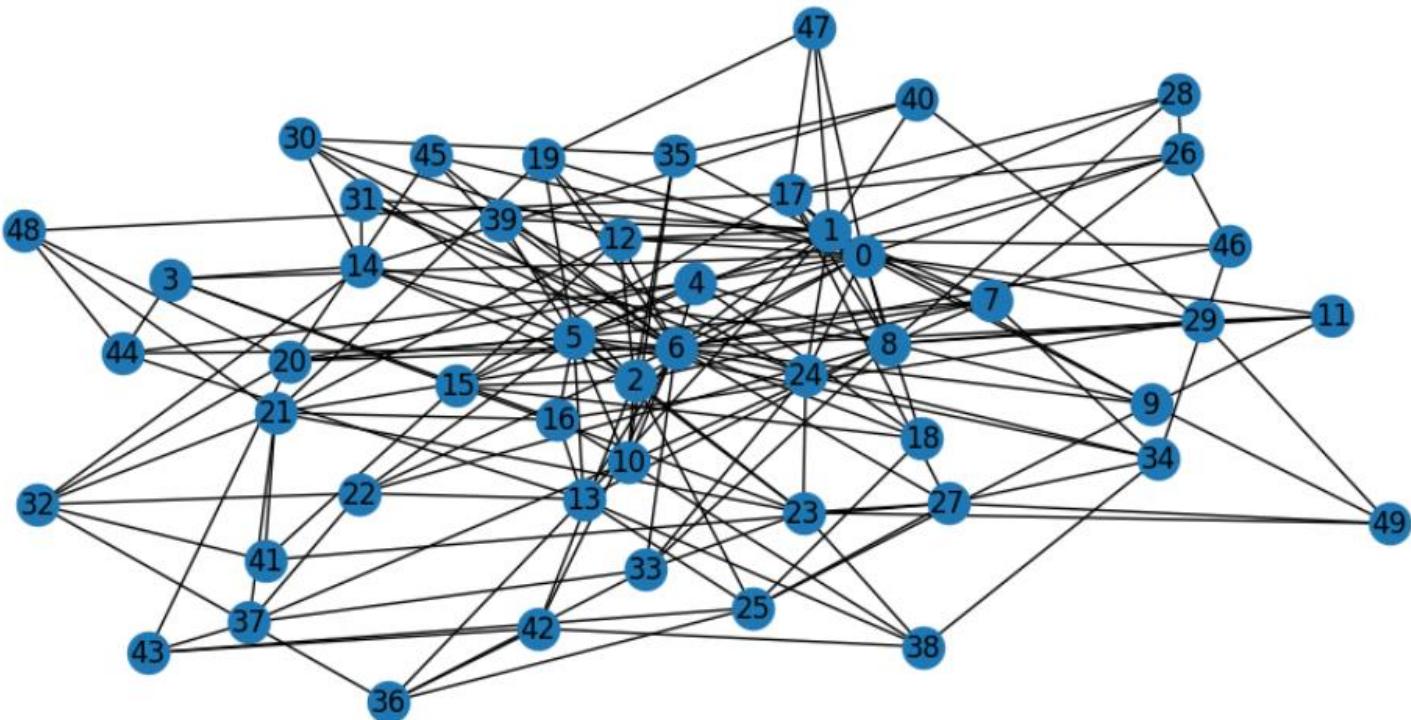
Watts-Strogatz (aka. Small World) network



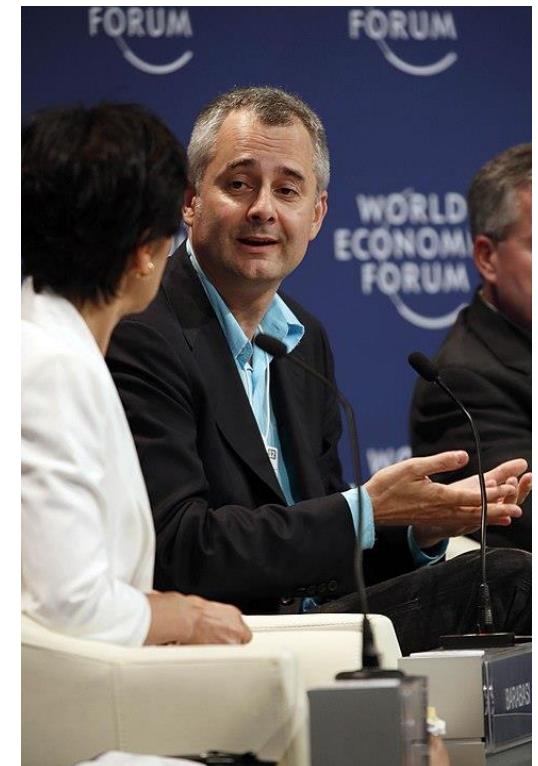
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Network topology

Barabási-Albert (aka. Scale-Free) network



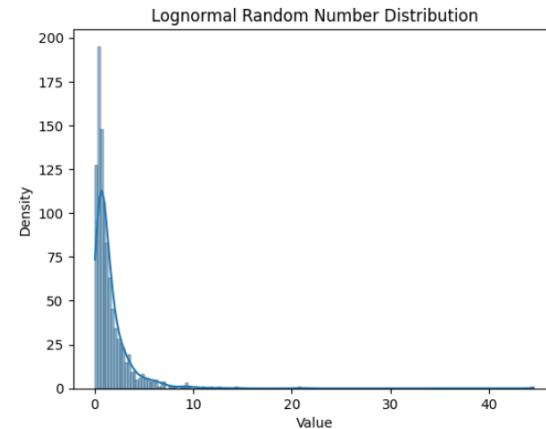
Albert-László Barabási



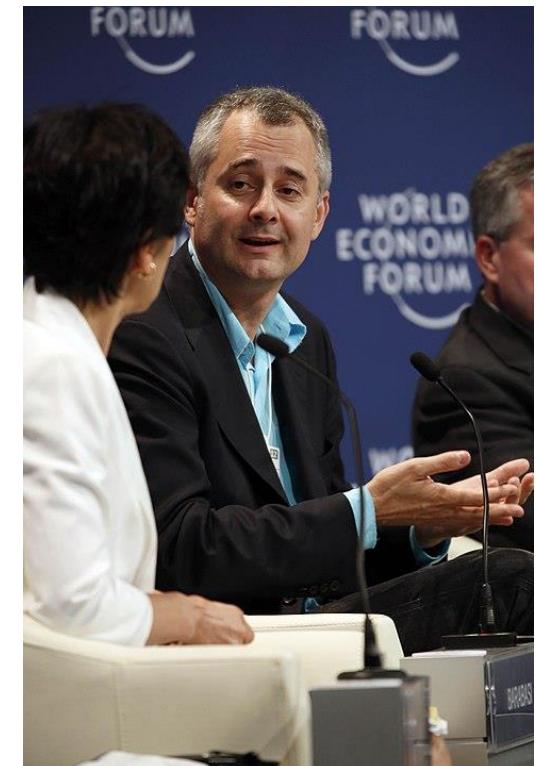
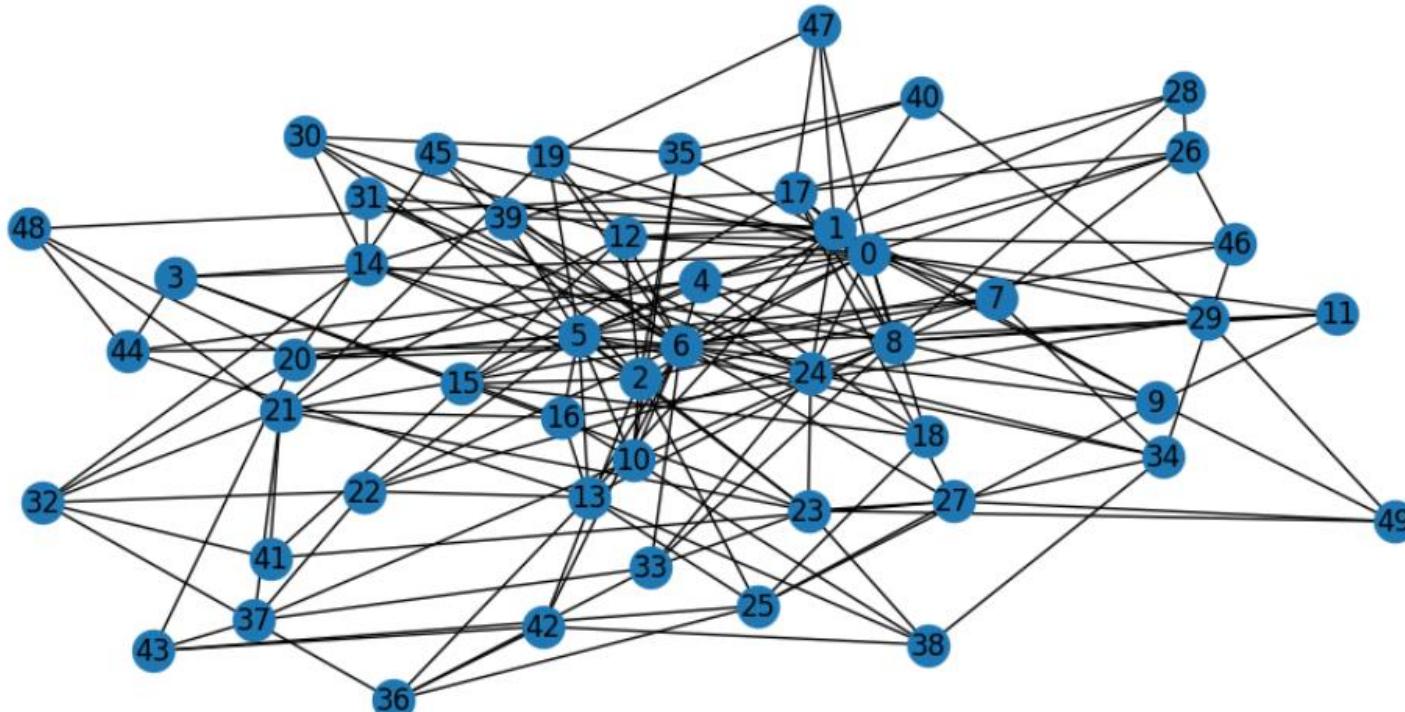
Preferential attachment = rich get richer

Network topology

Barabási-Albert (aka. Scale-Free) network

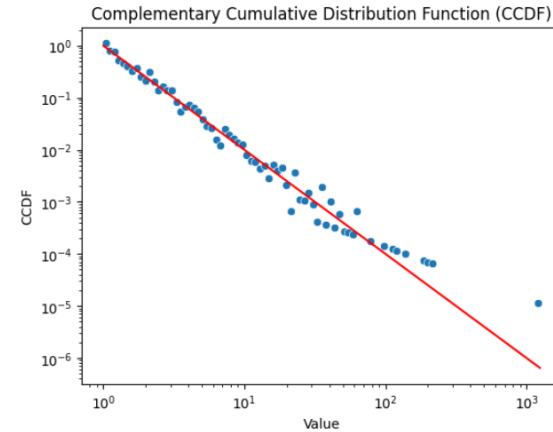


Albert-László Barabási

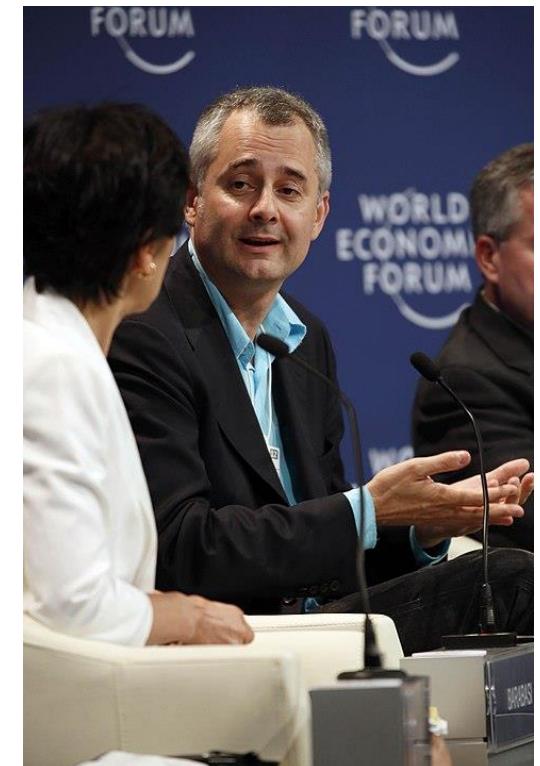
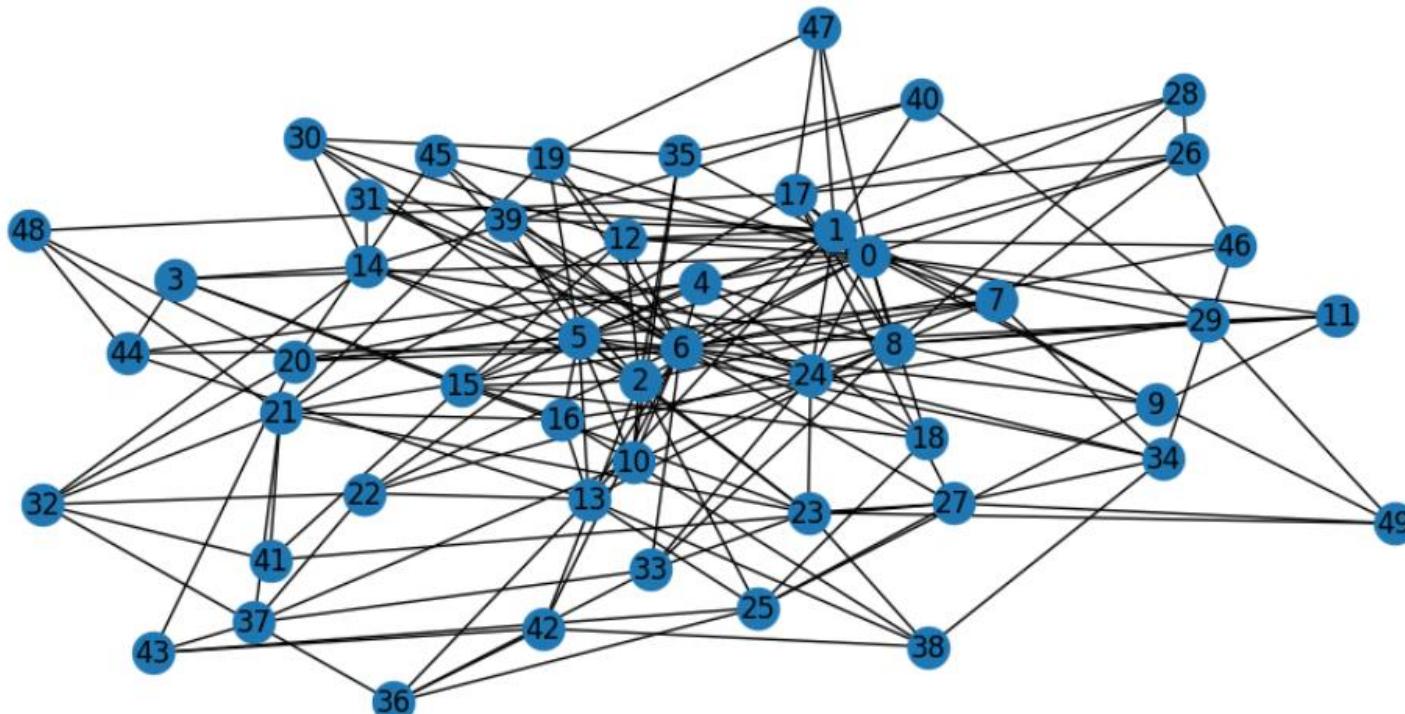


Network topology

Barabási-Albert (aka. Scale-Free) network
Scale-free = no characteristic scale



Albert-László Barabási



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