4.1

Allgemeiner Code, der für alle folgenden Aufgaben verwendet wurde:

```
Python console for SymPy 1.0 (Python 2.7.12)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)

>>> a, b, c, d = symbols("a, b, c, d")
>>> def implication( x, y):
...     return satisfiable( Not(Implies( x, y ))) == False
>>> def equivalence( z, t ):
...     return implication( z, t) & implication( t, z )
```

a) i)

```
>>> phi = (Not(a) | b) & (Not(b) | c) & (Not(c) | d) & (Not(d) | a)
>>> psi = 1 ^ a ^ b ^ c ^ d

>>> implication( phi, psi )
```
True

```
>>> implication( psi, phi )
```
False

```
>>> equivalence( phi, psi )
```
False

ii)

```
>>> phi = Not(a | b | c) | (Not(a) & b & c) | (a & (b ^ c))
>>> psi = (a | (b ^ c)) & (a >> Equivalent(b, c))

>>> implication( phi, psi )
```
False
```
>>> implication( psi, phi )
```
False
```
>>> equivalence( phi, psi )
```
False

b) i)

```
>>> alpha = ((a >> b) & (b >> c))
>>> beta = (a >> c)

>>> implication( alpha, beta )
```
True

ii)

```
>>> gamma = ((a >> b) & b)

>>> implication( gamma, a )
```
False

iii)

Verwendung von neuen Funktionen (allg. Code gilt nicht mehr)

```
>>> a, b, c, p, q = symbols("a, b, c, p, q")
>>> def implication(x, y):
...     return satisfiable( Not(Implies( x, y ))) == False
>>> def equivalence( p, q, z):
...     return implication(p, q) & implication(q, p) & implication(q, z) &
implication(z, q) & implication(p, z) & implication(z, p)
>>> Equivalent((satisfiable(Not(Equivalent(a, b, c))) == False),
(satisfiable(Not(equivalence(a, b, c))) == False))
```
True
```
>>> satisfiable(Not(Equivalent(a, b, c))) == False
```
False
```
>>> satisfiable(Equivalent(a, b, c)) == False
```
False

4.4
b)

```
Python console for SymPy 1.0 (Python 2.7.12)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)

>>> a, b, c, d, e, f, g, h = symbols("a, b, c, d, e, f, g, h")
>>> psi1 = Equivalent(h, (Or(Not(e), c)))
>>> psi2 = e >> Or(a, b, c, d, f, g, h)
>>> psi3 = b & Or(Not(g), Not(c))
>>> psi4 = Xor(f, c)
>>> psi5 = Not(Xor(a, b, c, d, e, f, g, h))
>>> psi6 = Xor(a, c)
>>> psi = And(phi1, phi2, phi3, phi4, phi5, phi6)
```

c)

```
>>> satisfiable(And(psi, Not(d)))
```

{a:True,b:True,c:False,d:False,e:False,f:True,g:False,h:True}

```
>>> for x in satisfiable(psi, all_models=True):
...     pretty_print(x)
{a: True, b: True, c: False, d: False, e: False, f: True, g: False, h:
True}
{a: True, b: True, c: False, d: True, e: False, f: True, g: True, h: True}
{a: True, b: True, c: False, d: False, e: True, f: True, g: False, h:
False}
{a: True, b: True, c: False, d: True, e: True, f: True, g: True, h: False}
{a: False, b: True, c: True, d: True, e: False, f: False, g: False, h:
True}
{a: False, b: True, c: True, d: False, e: True, f: False, g: False, h:
True}
```

d)

i)

```
>>> satisfiable(And(phi, Or(e, f, h)))
```

{a:True,b:True,c:False,d:False,e:False,f:True,g:False,h:True}

ii)

```
>>> em, fm, fy, hm, ay, gy = symbols("em, fm, fy, hm, ay, gy")
>>> psi_e = em >> e
>>> psi_f = Or(fm, fy) >> f
>>> psi_h = hm >> h
>>> psi_a = ay >> a
>>> psi_g = gy >> g
>>> psi_f2 = And((fy >> Not(fm)), (fm >> Not(fy)))
>>> psi_M = Or(em, fm, hm)
>>> psi_Y = Or(ay, fy, gy)
>>> psi_all = And(psi_e, psi_f, psi_h, psi_a, psi_g, psi_f2, psi_M, psi_Y)
>>> satisfable(psi_all)

>>> satisfiable(And(phi, psi_all))
```

{a:True,ay:True,b:True,c:False,d:False,e:False,em:False,f:True,fm:False,fy:False,g:False,gy:False,h:True,hm:True}