UML Diagram

```
Employee  0..*  worksIn  1..1  Department  partecipates  Project
   △                                  |
   |        <<subset>>               0..1              Role
 ┌─┴──┐                      directs
Workers  Director  1..1
   △        △
   └───┬────┘
    TempDir
```

Alphabet

$E(x)$, $D(x)$, $P(x)$, $R(x)$,

$W(x)$, $Dir(x)$, $TDir(x)$

$worksIn(x,y)$, $directs(x,y)$

$participates(x,y,z)$

$$\forall x,y . worksIn(x,y) \supset E(x) \wedge D(y)$$

$$\forall x . E(x) \supset 1 \le \#\{y \mid worksIn(x,y)\} \le 1$$

$$\text{i.e. } \forall x\, E(x) \supset (\exists y. worksIn(x,y) \wedge$$

$$\forall y,y'. worksIn(x,y) \wedge worksIn(x,y') \supset y=y'$$
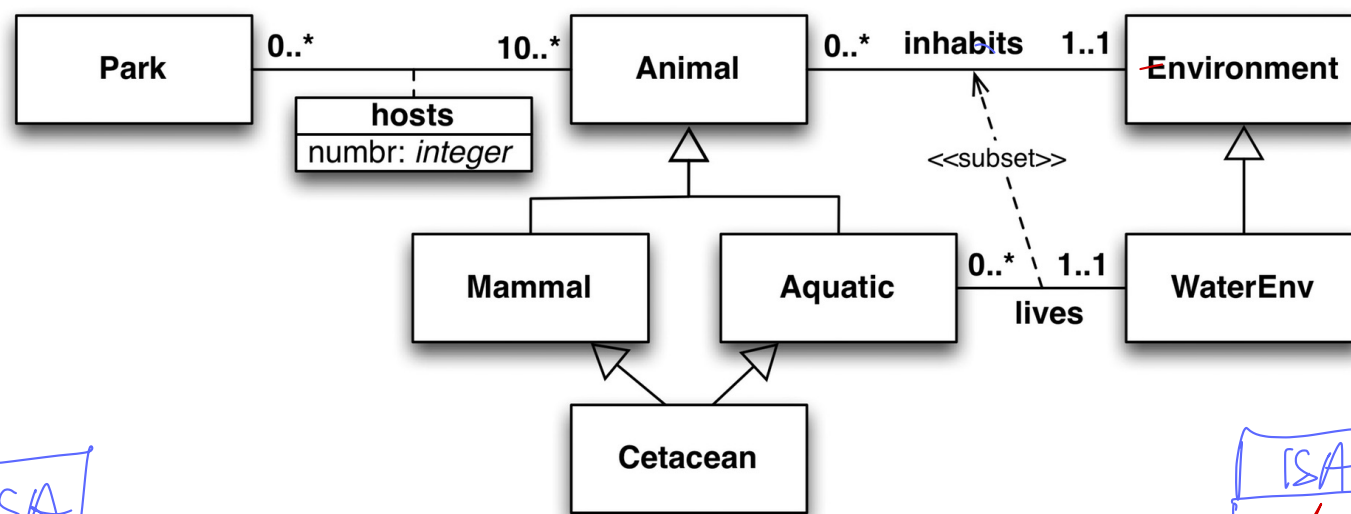
$$\forall x,y . directs(x,y) \supset Dir(x) \wedge D(y)$$

$$\forall x. Dir(x) \supset \#\{y \mid directs(x,y)\} \le 1$$

$$\forall x. D(x) \supset 1 \le \#\{y \mid directs(y,x)\} \le 1$$

subset

$$\forall x,y . directs(x,y) \supset worksIn(x,y)$$

$$\forall x,y,z . participate(x,y,z) \supset D(x) \wedge P(y) \wedge R(z)$$

$$\forall x. W(x) \supset E(x)$$

$$\forall x. Dir(x) \supset E(x)$$

$$\forall x. TDir(x) \supset W(x)$$

$$\forall x\, TDir(x) \supset Dir(x),$$

Park 0..* —— 10..* **Animal** 0..* **inhabits** 1..1 **Environment**

hosts
numbr: *integer*

Mammal **Aquatic** 0..* 1..1 **WaterEnv**
lives

<<subset>>

Cetacean

[ISA]

$$\forall x \; M(x) \supset A(x)$$
$$\forall x \; Aq(x) \supset A(x)$$
$$\forall x \; C(x) \supset M(x) \land Aq(x)$$
i.e. $\forall x. C(x) \supset M(x)$
$\forall x. C(x) \supset Aq(x)$

$$\forall x,y. \; \boxed{hosts}(x,y) \supset P(x) \land A(y)$$
$$\forall x. \; P(x) \supset 10 \leq \#\{y \mid hosts(x,y)\}$$

[ISA]

$$\forall x \;\; WE(x) \supset E(x)$$

$$\forall x,y. \; \boxed{inh}(x,y) \supset A(x) \land E(y)$$
$$\forall x \; A(x) \supset 1 \leq \#\{y \mid inh(x,y)\} \leq 1$$

$$\forall x,y. \; \boxed{lives}(x,y) \supset Aq(x) \land WE(y)$$
$$\forall x. \; Aq(x) \supset 1 \leq \#\{y \mid lives(x,y)\} \leq 1$$

SUBSET
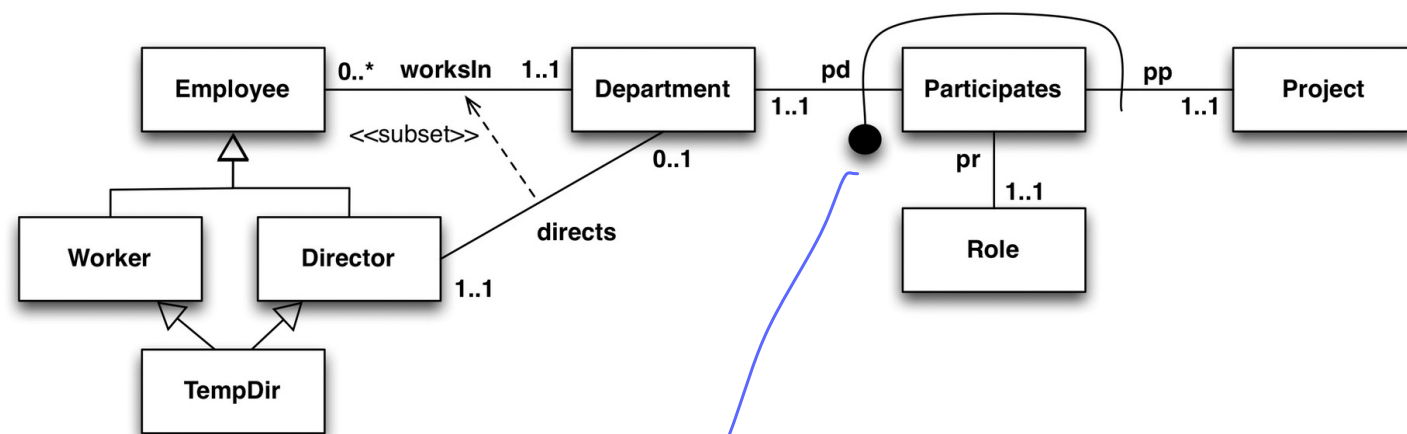$$\forall x,y \; \boxed{lives}(x,y) \supset inh(x,y)$$

attribute of association

$$\forall xyz \; \boxed{number}(x,y,z) \supset \underline{hosts(x,y)} \land$$
$$Int(z)$$

$$\forall x,y \; hosts(x,y) \supset$$
$$1 \leq \#\{z \mid number(x,y,z)\} \leq 1$$

UML diagram:

Employee — 0..* worksIn 1..1 — Department — pd 1..1 — Participates — pp 1..1 — Project

Worker, Director (subclasses of Employee via generalization), <<subset>>, directs 0..1 ... 1..1

TempDir (subclass of Worker and Director)

Participates — pr 1..1 — Role
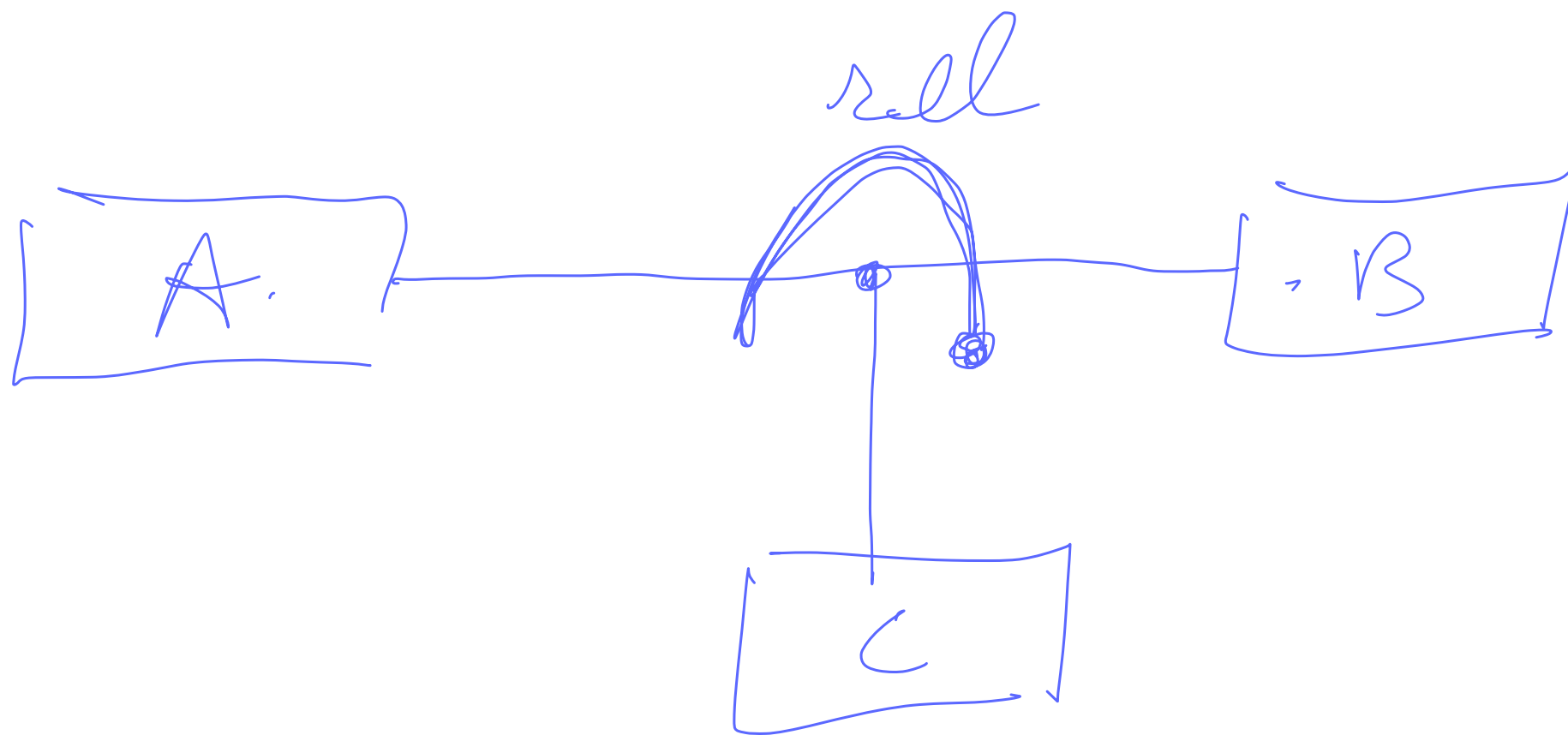
Logical constraints:

$$\forall x,y.\, pd(x,y) \supset P(x) \wedge D(y)$$

$$\forall x.\, P(x) \supset 1 \le \#\{y \mid pd(x,y)\} \le 1$$

$$\forall x,y.\, pp(x,y) \supset P(x) \wedge P_2(y)$$

$$\forall x.\, P(x) \supset 1 \le \#\{y \mid pp(x,y)\} \le 1$$

$$\forall x,y.\, p_2(x,y) \supset P(x) \wedge R(y)$$

$$\forall x.\, P(x) \supset 1 \le \#\{y \mid p_2(x,y)\} \le 1$$

$$\forall x,x',y,z. \; pd(x,y) \wedge pd(x',y) \wedge$$
$$pr(x,z) \wedge pp(x',z)$$
$$\supset x = x'$$

Key of a class

x = x'
pd → y, pr → z, pp, pd → x', pp → z

rel

A.   B

C

$$\forall x, y, z \cdot rel(x, y, z) \supset A(x) \wedge B(y) \wedge C(z)$$

Typing
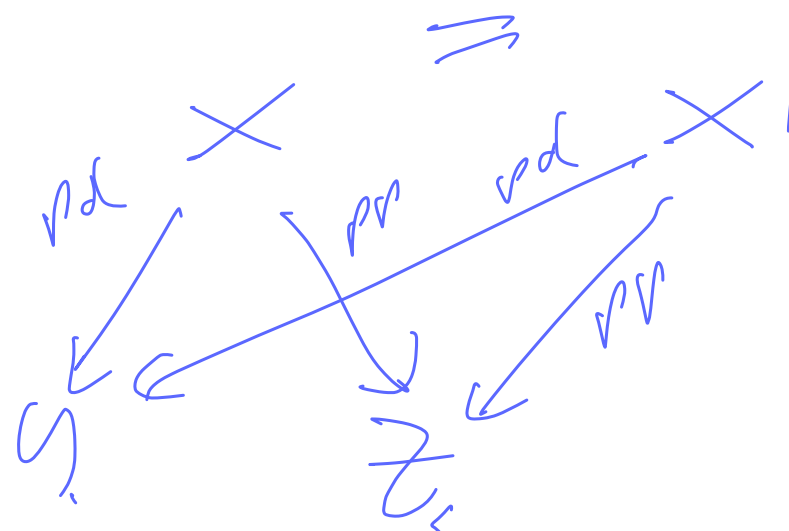
$$\forall x, y, z, z' \ rel(x, y, z) \wedge rel(x, y, z'') \supset z = z'$$

key of a rel
(association)