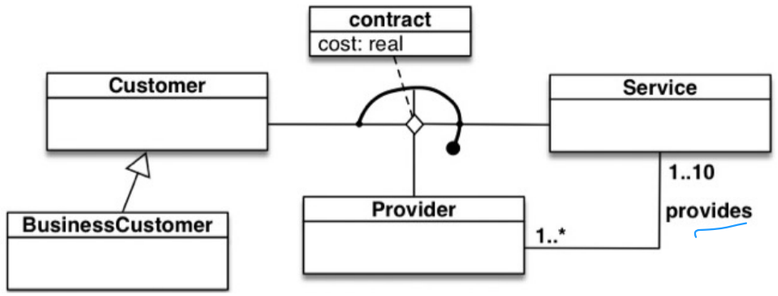
Exercise 1. Express the following UML class diagram in FOL:



Exercise 2. Consider the above UML class diagram and the following (partial) instantiation:

Customer

c1
c2
c3
c4

BCustumers

b1
b2
b3

Services

s1
s2
s3

Provider

p1
p2

provides

p1	s1
p1	s2
p1	s3
p2	s2

contacts/cost

c1	s1	p1	90.0
c1	s2	p1	80.0
c1	s3	p1	50.0
b2	s1	p2	170,0
b2	s2	p2	100,0

1. Check whether the above instantiation, once completed, is correct, and explain why it is or it is not.
2. Express in FOL the following queries and evaluate them over the completed instantiation:
   (a) Check whether there is a customer with contract with two providers for the same service.
   (b) Return those customers that have contracts only for one service.
   (c) Return those customers that have a contracts ~~with the same provider for all their services.~~

$C(x), B(x), P(x), S(x)$

$prov(x,y)$,

$contr(x,y,z)$,

$cost(x,y,z,w)$

$real(x)$

alphabet

[ISA]

$\forall x . B(x) \supset C(x)$ ✓

$\forall x,y . prov(x,y) \supset P(x) \wedge S(y)$ ✓

$\forall x . P(x) \supset |\leq \#\{y \mid prov(x,y)\} \leq 10$ ✓

$\forall x \; S(x) \supset |\leq \#\{y \mid prov(y,x)\}$ ✓

[prov]

cost

$\forall x,y,z,w . contr(x,y,z,w) \supset contr(x,y,z) \wedge real(w)$

$\forall x,y,z . contr(x,y,z) \supset |\leq \#\{w \mid contr(x,y,z,w)\} \leq 1$ ✓

[contr]

$\forall x,y,z . \; contr(x,y,z) \supset C(x) \wedge S(y) \wedge P(z)$ ✓

$\forall x,y,z,z' . \; contr(x,y,z) \wedge contr(x,y,z') \supset z = z'$

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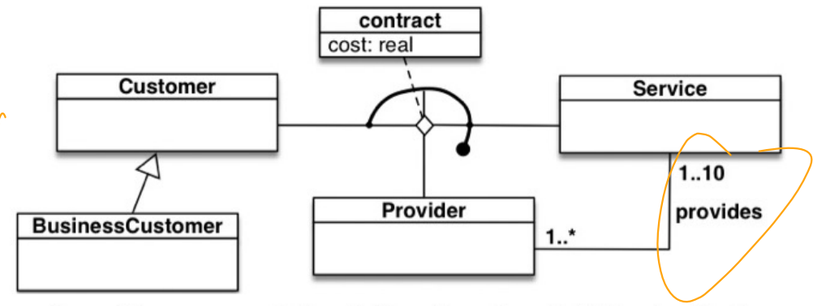

Γ in FOL

$I \models \Gamma$ ✓

(completed)

complete the instantiation is correct because it makes true all axioms in Γ.

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$\exists y, z, z'.\ contr(x, y, z) \wedge contr(x, y, z') \wedge z \neq z'$

ans: $\emptyset$

$\exists y, z.\ contr(x, y, z) \wedge$

$\forall y y'.\ \exists z\ contr(x, \boxed{y}, z) \wedge \exists z'.\ contr(x, \boxed{y'}, z') \supset y = y'$

ans: $\emptyset$

$$\exists p.\ \forall y.\ \left(\left(\exists z.\ contr(\boxed{x}, \boxed{y}, z)\right) \supset contr(x, y, p)\right)$$

$$\forall y.\ \underbrace{\phi(x, y) \supset contr(x, y, p)}_{\exists p.\ \phi(x, p)}$$

ans: $\{c1, b2\}$

"If you don't play you don't win."

$$\forall x \,.\, \neg p(x) \supset \neg w(x)$$

$$\equiv$$

$$\forall x \,.\, p(x) \vee \neg w(x)$$

$$\equiv$$

$$\forall x \,.\, w(x) \supset p(x)$$

$$\forall x \,.\, \neg w(x)$$