1. Multi dimensional random walk

Definition 1 (Type II random walk in $\mathbb{Z}^m$)
Let $m \in \mathbb{N}$. $\forall n \in \mathbb{N}$, *let* $X_n = \begin{pmatrix} x_1^x & x_2^x & \dots, x_m^x \end{pmatrix}^T$, *where* $\{x_i^x\}_{i=1}^m$ *are* $\forall n \in \mathbb{N}$ *independent.* ***check english*** *NEVÍM JAK TO NAPSAT LÍP Let* $\forall i \in \{1, 2, \dots, m\} x_i^x$ *have values in* $\{-1, +1\}$ *with probabilities* $\mathsf{P}(x_i^x = +1) = p_i \in (0,1)$ *and* $\mathsf{P}(x_i^x = -1) = q_i = 1 - p_i \in (0,1)$.

Let $\{X_n\}_{n=0}^{+\infty}$ *be a sequence of independent and identically distributed random variables. Let* $S_0 = \mathbf{0}$ *and* $\forall n \in \mathbb{N} : \mathbf{S_n} = \sum\limits_{i=1}^{n} X_i$ *and* $\mathbf{p} = \left(p_1, p_2, \dots, p_m\right)^{\mathbf{T}}$. *Then the pair* $\left(\{\mathbf{S_n}\}_{n=0}^{+\infty}, \mathbf{p}\right)$ *is called* Type II random walk in $\mathbb{Z}^m$.

If $\forall i \in \{1, 2, \dots, m\} : p_i = q_i = \frac{1}{2}$ *we call the* ***check english*** *element* $\{\mathbf{S_n}\}_{n=0}^{+\infty}$ Symmetric type II random walk $\mathbb{Z}^m$.

Remark. Type II random walk can be interpreted as m simple random walks in $\mathbb{Z}$ happening at a time, each of them parallel to an axis of $\mathbb{Z}^m$.

Remark. Due to the the aim of this thesis which is reasearching occupation time of a set of random walks we are going to concern only on symmetric random walks.

Theorem 1
Let $m \in \mathbb{N}$ *and* $\left(\{\mathbf{S_n}\}_{n=0}^{+\infty}, \mathbf{p}\right)$*be a Type II random walk in* $\mathbb{Z}^m$. *Let* $\mathbf{y} = \left(y_1, y_2, \dots, y_m\right)^T \in \mathbb{Z}^m$. *Then following equation stands:*

$$\mathsf{P}(S_n = x) = \begin{cases} \prod_{i=1}^m \binom{n}{\frac{y_i+n}{2}} p_i^{\frac{n+y_i}{2}} q_i^{\frac{n-y_i}{2}}, & \text{if } \forall i \in \{1, 2, \dots, m\} : y_i \in A_n, \\ 0, & \text{if } \exists i \in \{1, 2, \dots, m\} : y_i \notin A_n. \end{cases}$$

Proof. $\mathsf{P}(\mathbf{S_n} = \mathbf{y}) = \mathsf{P}(S_1^x = y_1, S_2^x = y_2, \dots, S_m^x = y_m) \overset{\perp\!\!\!\perp}{=} \prod i = 1^m \, \mathsf{P}(S_n^x = y_i)$
$= \prod\limits_{i=1}^{m} \binom{n}{\frac{y_i+n}{2}} p_i^{\frac{n+y_i}{2}} q_i^{\frac{n-y_i}{2}}$ □