J. Astron. Space Sci. 25(4), 361–374 (2008)

Trajectory Planning of Satellite Formation Flying using Nonlinear Programming and Collocation

Hyung-Chu Lim[1 †] **and Hyochoong Bang**[2]
[1]Space Geodesy Research Division, KASI, Daejeon 305-348, Korea
[2]Department of Aerospace Engineering, KAIST, Daejeon 305-701, Korea
email: hclim@kasi.re.kr

(Received September 29, 2008; Accepted November 10, 2008)

Abstract

Recently, satellite formation flying has been a topic of significant research interest in aerospace society because it provides potential benefits compared to a large spacecraft. Some techniques have been proposed to design optimal formation trajectories minimizing fuel consumption in the process of formation configuration or reconfiguration. In this study, a method is introduced to build fuel-optimal trajectories minimizing a cost function that combines the total fuel consumption of all satellites and assignment of fuel consumption rate for each satellite. This approach is based on collocation and nonlinear programming to solve constraints for collision avoidance and the final configuration. New constraints of nonlinear equality or inequality are derived for final configuration, and nonlinear inequality constraints are established for collision avoidance. The final configuration constraints are that three or more satellites should form a projected circular orbit and make an equilateral polygon in the horizontal plane. Example scenarios, including these constraints and the cost function, are simulated by the method to generate optimal trajectories for the formation configuration and reconfiguration of multiple satellites.

Keywords: satellite formation flying, nonlinear programming, collocation, collision avoidance

1. Introduction

The primary goal of satellite formation flying (SFF) is to place multiple satellites into nearby orbits forming a satellite cluster to achieve a common mission. In the recent days, it has become a topic of significant focus in the aerospace engineering area. Formation flying system gives rise to several benefits compared to a large single spacecraft system for the same missions; 1) low cost for launch and mass production, 2) larger payload aperture size, 3) greater launch flexibility, 4) higher system reliability and 5) easier expandability. According to the characteristics of control purpose and design, the SFF problem can be categorized into three phases: 1) determination of initial conditions, 2) satellite formation-keeping, and 3) satellite formation configuration or reconfiguration. However the problem of determination of initial conditions may belong to the satellite formation-keeping problem because both problems are concerned with minimization of fuel consumption for the purpose of maintaining the formation against external disturbance sources such as the J_2 gravitational

[†]corresponding author

perturbation and air-drag effects. This study primarily addresses fuel optimal trajectory planning of satellites from the perspective of formation configuration or reconfiguration management.

When a satellite formation is deployed or needs to be modified, it is important to determine optimal maneuvers for fuel minimization due to limited lifetime of the satellite. Some techniques have been studied to construct optimal formation trajectories minimizing the fuel consumption. Convex optimization techniques were applied to derive fuel/time optimal control algorithm of formation-keeping and formation reconfiguration, which were consecutively extended into the time-varying linear dynamics of eccentric orbits (Tillerson & How 2001, Tillerson et al. 2002). The reconfiguration problem for fuel optimal maneuvers was modeled and analyzed as a multi-agent optimization problem in Yang et al. (2002). The mixed-integer linear programming (MILP) was applied to design fuel-optimal trajectories considering collision avoidance and plume impingement avoidance constraints in Richards et al. (2002). An open-loop control algorithm was derived to reorient a formation in the free space by minimizing a cost function with components of fuel consumption and fuel equalization (Beard et al. 2000).

The spatial separation between satellites could range from a few meters to several kilometers for some SFF missions. Thus it is important to avoid collisions between satellites as they move in the space under the configuration or reconfiguration mission. There have been previous attempts to solve the problem of trajectory optimization with the collision avoidance in collaborative systems. Especially, the collision avoidance problem has been extensively investigated in the field of robot motion planning. Some methods based on potential functions were developed for spacecraft applications to handle collision avoidance strategies (McInnes 1995, Johnston & McInnes 1997). Other approaches were also proposed for spacecraft path-planning with collision avoidance constraints by splines (Singh & Hadaegh 2001) and MILP (Richards et al. 2002) methods. For the configuration strategy, there are literatures dealing with final configurations. Some approaches evaluate the cost for many predefined sets to assign the final states, and then select a set that yields the lowest cost. The problem of trajectory planning and configuration selection is decoupled in those approaches (Wang & Hadaegh 1998, Tillerson et al. 2002). The MILP technique includes configuration selection in the trajectory optimization problem. Thus the selection and assignment are performed within the MILP to achieve the subset of final states that leads to the lowest cost and are recognized as a global configuration (Richards et al. 2002).

This paper is concerned with optimal trajectory planning of multiple satellites. For the optimal trajectory planning, an optimal control problem is converted into a parameter optimization problem (Hull 1997) that can be solved by a nonlinear programming (NLP). The NLP handles constraints of collision avoidance, final configuration and defect equations which are generated by collocation methods (Russell & Shampine 1972, Dickmanns & Well 1974, Hargraves & Paris 1987). In contrast to the aforementioned configuration strategies, new constraints of nonlinear equality or inequality are derived for the final configuration and applied to the NLP problem in this paper. The final configuration constraints are that three or more satellites should be placed in an equilateral polygon of the projected circular orbit (PCO) which was considered in the TechSat-21 mission (Martin & Stallard 1999).

2. Problem Formulation

This section presents dynamic equations describing the SFF and details of the NLP formulation to solve the trajectory optimization problems. Principal idea of the collocation approach is also elaborated.

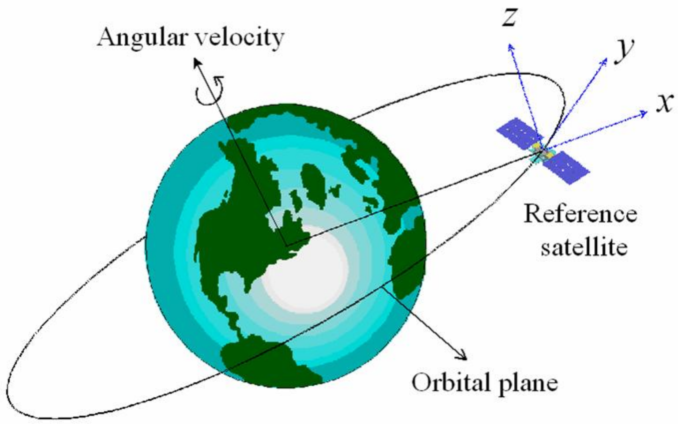


Figure 1. LVLH coordinate system for the relative motion.

2.1 Relative Formation Dynamics

A rotating local-vertical-local-horizontal (LVLH) frame is used to describe the relative dynamic motion with respect to the reference satellite. The x-axis points in the radial direction, the z-axis is perpendicular to the orbital plane and points in the direction of the angular momentum vector. Finally, the y-axis points the along-track direction as shown in Figure 1.

In general, the Clohessy-Wiltshire equation (Clohessy & Wiltshire 1960) based on the LVLH frame is utilized to describe the relative motion and control strategies between neighboring satellites. It is usually called as Hill's equation and expressed as:

$$\begin{aligned} \ddot{x} - 2\omega\dot{y} - 3\omega^2 x &= u_x \\ \ddot{y} + 2\omega\dot{x} &= u_y \\ \ddot{z} + \omega^2 z &= u_z \end{aligned} \tag{1}$$

where x, y and z are the relative position of the satellite about the reference satellite in the LVLH frame and ω represents the mean motion of the reference satellite. The Hill's equation is a set of linearized equations governing the relative motion between satellites. It has been used for the study of relative motion of rendezvous mechanics in the past. In recent years, the Hill's equation plays the role of baseline for various applications of satellite formation flying design. In this study, the Hill's equation derives the defect equation for collocation and the constraints of the final configuration. For the convenience of the NLP, we introduce a new time variable $(\tau = \omega t)$ using the mean motion (ω) of the reference orbit. From the relationship, $dx/dt = \omega(dx/d\tau)$ and $d^2x/dt^2 = \omega^2(d^2x/d\tau^2)$, the Hill's equation in state space form for the relative dynamics in terms of the new time variable (τ) can be rewritten as

$$\dot{\mathbf{x}} = \mathrm{A}\mathbf{x} + \mathrm{B}\mathbf{u} \tag{2}$$

where

$$\mathbf{x} = [x\ \ y\ \ z\ \ \dot{x}\ \ \dot{y}\ \ \dot{z}]^{\mathrm{T}}, \quad \mathbf{u} = [u_x\ \ u_y\ \ u_z]^{\mathrm{T}}$$

$$\mathrm{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $(\ddot{\ })$ and $(\dot{\ })$ denote derivatives with respect to (w.r.t.) the new time variable (τ). The state vector and control input will be represented as variables w.r.t. the new time variable in the subsequent derivations, i.e. $\mathbf{u}(t) = \omega^2\mathbf{u}(\tau)$. Let us note that the out-of-plane motion is decoupled from the in-plane motion as it is shown in Eq. (1).

2.2 Nonlinear Programming

Trajectory optimization problems basically seek to compute the control history and optimal trajectories for an objective function such as fuel and time minimization. It can be formulated into NLP by using parameter optimization techniques. There are four general classes of methods for converting a trajectory optimization problem into a NLP problem according to the unknowns: 1) control parameters, 2) control parameters and some state parameters, 3) control parameters and state parameters, and 4) state parameters (Hull 1997). The NLP problem in this paper is to find states and control inputs at nodes that minimize the cost function. Hence, this problem can be categorized into the third class, because state and control inputs are the unknowns in our problem. To formulate the NLP, nodes for discretization are defined as

$$t_0 < t_1 < ... < t_i < .... < t_N = t_f, \quad \forall i \in [0, 1, 2, ..., \mathrm{N}] \tag{3}$$

where the index, i denotes the ith node and $N + 1$ is the total number of nodes. The time interval $(\Delta t = t_{i+1} - t_i)$ between the nodes is spaced equally and t_f is a fixed final time. The variables for the NLP problem are the collected state vectors and control vectors of all satellites at the nodes. If the number of satellites is equal to S in the trajectory planning problem, state vector z for the NLP is given by

$$\mathbf{z} = \begin{pmatrix} \mathbf{X} \\ \mathbf{U} \end{pmatrix} = \begin{pmatrix} [\mathbf{x}_{11},\ \mathbf{x}_{12}, ...\mathbf{x}_{1N}...\mathbf{x}_{p1},\ \mathbf{x}_{p2}, ...\mathbf{x}_{pN}...\mathbf{x}_{S1},\ \mathbf{x}_{S2}, ...\mathbf{x}_{SN}]^{\mathrm{T}} \\ [\mathbf{u}_{10},\ \mathbf{u}_{11}, ...\mathbf{u}_{1N}...\mathbf{u}_{p0},\ \mathbf{u}_{p1}, ...\mathbf{u}_{pN}...\mathbf{u}_{S0},\ \mathbf{u}_{S1}, ...\mathbf{u}_{SN}]^{\mathrm{T}} \end{pmatrix}, \ \forall p \in [1, 2, 3, ..., S] \tag{4}$$

where the index p represents the pth satellite and $\mathbf{x}$ is the state vector, $\mathbf{x}_{pi} = [x_{pi}\ y_{pi}\ z_{pi}\ \dot{x}_{pi}\ \dot{y}_{pi}\ \dot{z}_{pi}]^{\mathrm{T}}$ and $\mathbf{u}$ represents the control input vector such that $\mathbf{u}_{pi} = [u_{pix}\ u_{piy}\ u_{piz}]^{\mathrm{T}}$. The variables x, y, z will be explained later when describing relative dynamics. The subscripts in $\mathbf{x}$ and $\mathbf{u}$ denote the pth satellite and the ith node, respectively. The number of state vector is smaller than that of the control input vectors because the initial state vectors for all satellites are fixed $a\ priori$. The state and control input vectors are confined within specified limits as boundary conditions as follows;

$$\begin{aligned} -\mathbf{x}_{max} \le \mathbf{x}_{pi} \le \mathbf{x}_{max} \\ -\mathbf{u}_{max} \le \mathbf{u}_{pi} \le \mathbf{u}_{max} \end{aligned} \tag{5}$$

Let M be the mass of a satellite and $\dot{m}$ the fuel mass consumption rate. In this study, it is assumed that the mass is same for all satellites and constant during the formation maneuvers and a variable specific impulse propulsion system is employed. Linear interpolation is used to assess the control between two nodes because the collocation method uses linear interpolation to obtain control vectors at the center of nodes. It follows such that

$$\mathbf{u}(\mathrm{t}) = \mathbf{u}_i + \frac{t - t_i}{\Delta t}(\mathbf{u}_{i+1} - \mathbf{u}_i) \qquad \text{for } \ t_i \le t \le t_{i+1} \tag{6}$$

Thus the fuel consumption, F_p of the pth satellite, can be expressed as

$$\begin{aligned} \mathrm{F}_p &= \int_{t_0}^{t_f} \dot{\mathrm{m}}\mathrm{d}\tau = \frac{\mathrm{M}^2}{2\mathrm{P}} \int_{t_0}^{t_f} (u_{px}^2 + u_{py}^2 + u_{pz}^2)\mathrm{dt} \\ &= \frac{\mathrm{M}^2}{2\mathrm{P}} \sum_{i=0}^{N-1} \sum_{k=x,y,z} \left\{ u_{pik}^2 + \frac{1}{3}(u_{p(i+1)k} - u_{pik})^2 + u_{pik}(u_{p(i+1)k} - u_{pik}) \right\} \Delta t \end{aligned} \tag{7}$$

where P is the power delivered to the propulsion system. The upper equation of Eq. (7) is motivated by Yang et al. (2002) and the lower one can be derived using Eq. (6) for control inputs. The cost function is defined as a combination of the total fuel consumption of all satellites and assignment of fuel consumption rate for each satellite. In Eq. (7), the first term $(\mathrm{M}^2/2\mathrm{P})$ be can omitted because it is constant and same for all satellites. Thus it follows as

$$\mathrm{J} = \sum_{p=1}^{S} \mathrm{f}_p + \mu \sum_{p=1}^{S} \sum_{q=p+1}^{S} |\lambda_q \mathrm{f}_p - \lambda_p \mathrm{f}_q| \tag{8}$$

where

$$\mathrm{f}_p = \sum_{i=0}^{N-1} \sum_{k=x,y,z} \left\{ u_{pik}^2 + \frac{1}{3}(u_{p(i+1)k} - u_{pik})^2 + u_{pik}(u_{p(i+1)k} - u_{pik}) \right\} \Delta t$$

which subjects to equality constraints such as the defect equations of the collocation

$$\mathrm{C}(\mathrm{z}) = 0 \tag{9}$$

and the inequality constraints such as the boundary conditions (Eq. (5))

$$\mathrm{D}(\mathrm{z}) \ge 0 \tag{10}$$

The first term in Eq. (8) represents the total fuel consumption for the formation while the second term is introduced to assign the distribution of fuel consumption to each satellite using the index λp. Also, μ in the second term denotes the weighting factor for the assignment of fuel consumption. For instance, let us consider the case of trajectory planning using three satellites. If fuel consumptions of the 2^{nd} and the 3^{rd} satellite are required to be 80 % and 120 % on the 1^{st} satellite, then λ_1, λ_2, and λ_3 will have 1.0, 0.8 and 1.2 with appropriate values of μ. This cost function thus comprises boundary conditions, path constraints, terminal constraints and defect equations. The collocation method is used in place of direct numerical integration for parameter optimization formulation in this work.

2.3 Direct Transcription Method

There are two approaches for solving trajectory optimization problems; direct and indirect methods. Indirect methods attempt to find solutions satisfying the necessary conditions in optimal control theory such as the Euler-Lagrange equations or the Pontryagin's maximum principle. They can be solved by numerical methods in general cases. Direct methods, on the other hand, recursively update the control and trajectory to reduce the cost function while satisfying the boundary conditions and the terminal constraints. However, it is rather difficult to categorize every technique into direct or indirect method. Betts (1998) provides more details of the aforementioned direct and indirect methods. In direct methods, control or state variables can be represented by polynomials and the discretized control or state variables at discrete points (nodes) are considered as parameters for optimization. A variety of direct methods have been proposed and can be categorized roughly according to the methods of handling the discretizations of the dynamic equation. The most common approach is the finite parameter representation of the control history with explicit integration of the dynamic equations. Another approach is to formulate the control and state variables by polynomials, retaining the constraints so that the integration formula can be satisfied at each node. This approach is referred to as the direct transcription or collocation. A new collocation method was introduced by Hargraves & Paris (1987) using piecewise Hermite cubic polynomials to solve several atmospheric trajectory optimization problems. It was also applied to the optimization of finite-thrust spacecraft trajectories control problem (Enright & Conway 1991). The same collocation method is adopted in this study to construct optimal trajectories of SFF because it is robust about initial guesses thus producing optimal trajectory even under poor initial guess.

The interpolated state vector at the center between any two nodes (τ_i and τ_{i+1}) using the Hermite interpolation and the Hill's equation in Eq. (2) is expressed as

$$\mathbf{X}_{ic} = (1/2)(\mathbf{x}_i + \mathbf{x}_{i+1}) + (\Delta\tau/8)[\mathrm{A}(\mathbf{x}_i - \mathbf{x}_{i+1}) + \mathrm{B}(\mathbf{u}_i + \mathbf{u}_{i+1})] \tag{11}$$

and the control vector at the same center by a linear interpolation is given by

$$\mathbf{u}_{ic} = (1/2)(\mathbf{u}_i + \mathbf{u}_{i+1}) \tag{12}$$

The derivative of the state vector also at the same location is described as

$$\dot{\mathbf{X}}_{ic} = -[3/(2\Delta\tau)](\mathbf{x}_i - \mathbf{x}_{i+1}) - (1/4)[\mathrm{A}(\mathbf{x}_i - \mathbf{x}_{i+1}) + \mathrm{B}(\mathbf{u}_i + \mathbf{u}_{i+1})] \tag{13}$$

Moreover, the defect vector at the center of nodes is introduced as

$$\Delta_i = (\mathrm{A}\mathbf{x}_{ic} + \mathrm{B}\mathbf{u}_{ic}) - \dot{\mathbf{x}}_{ic} \tag{14}$$

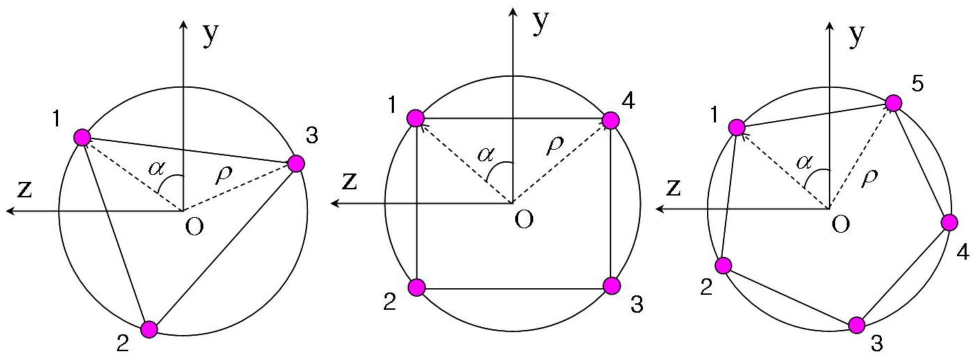
If the state and control vectors at each node is estimated such that the defect vector (Δ) becomes nearly equal to zero, those state and control vectors for all nodes will be an approximate solution of the Hill's equation. Thus the defect vector will be regarded as equality constraints for the NLP problem instead of direct numerical integration of the Hill's equation.

3. Constraints of Formation Trajectories

Two constraints can be considered here; collision avoidance and final formation configuration. The final formation configuration has the equality constraints for three and four satellites, and the inequality constraints for three or more satellites.

3.1 Collision Avoidance

Collision avoidance is a critical requirement for the configuration or reconfiguration maneuver of the SFF system which involves multiple satellites. Constraints of the collision avoidance among



(a) Equilateral triangle (b) Equilateral square (c) Equilateral pentagon

Figure 2. Configuration of satellites in the projected circular orbit (α: phase angle measured from y axis, ρ: radius of a circle).

different satellites are addressed in this section. They can be derived easily from a geometry based on the LVLH frame. A satellite should be placed at least outside a 3-dimensional sphere whose center is the position of other satellites at each node. Let the position vectors at the ith node be given by $[x_{pi}\ y_{pi}\ z_{pi}]^{\mathrm{T}}$ and $[x_{qi}\ y_{qi}\ z_{qi}]^{\mathrm{T}}$ for pth and qth satellites, respectively. Then the constraint for collision avoidance between two satellites can be stated as

$$(x_{pi} - x_{qi})^2 + (y_{pi} - y_{qi})^2 + (z_{pi} - z_{qi})^2 \geq R_a^2, \quad \forall p, \forall q \in [1, 2, ..., S];\ p < q \tag{15}$$

where R_a is the safety radius of the 3-dimensional sphere for collision avoidance. This equation should be satisfied at all nodes for collision avoidance. Hence, this equation is augmented to the NLP problem as an inequality constraint.

3.2 Final Configuration

There are some literatures (Wang & Hadaegh 1998, Tillerson et al. 2002, Richards et al. 2002) dealing with the final configuration, namely final states of the relative dynamics. However, most of them presume the final states of satellites as fixed or compute the cost of many sets including the final states. Then they select one set for the lowest cost among the candidate sets. So this selection of final states is decoupled from optimization problem in these approaches. Richards et al. (2002) defined many subsets of the final states and performed assessment of all subsets within the trajectory optimization. Then they selected only one subset with the lowest overall fuel cost as it is known as a global configuration. However, the proposed approach appends the constraints of the final configuration to the NLP problem instead of using the final state set to find the optimal trajectories.

In this section, the final configuration constraints are derived in a general form for the PCO (Alfriend et al. 2000, Sabol et al. 2001). Three or more satellites need to be placed in an equilateral polygon of the horizontal $(y - z)$ plane as shown in Figure 2. Such orbits can be applied to the Earth observation mission due to the characteristics of the constant projected distance onto the horizontal plane between satellites. The PCO is inclined at $\pm 26.56°$ to the horizontal plane. Equality constraints are introduced for three and four satellites whereas equality/inequality constraints for three and more satellites for the purpose of the final configuration.

3.2.1 Equality constraints

Equality constraints are established for three and four satellites because it is rather difficult to derive general equality constraints of the final configuration for more than four satellites. Final position and velocity of satellites have to be chosen to constitute an equilateral polygon in the horizontal plane and to maintain the initial formation. First, all the satellites have to be positioned in a circle in the horizontal plane to generate a PCO. Eq. (16) can be derived from the simple geometry that the distance of the pth satellite from the origin is equal to ρ in the horizontal plane. Second, they should build up an equilateral polygon in the horizontal plane. This constraint is expressed in Eq. (17) which can be derived from the principle of center of mass.

$$y_p^2(\tau_f) + z_p^2(\tau_f) = \rho^2 \tag{16}$$

$$\sum_{p=1}^{S} y_p(\tau_f) = 0 \ \text{ and } \ \sum_{p=1}^{S} z_p(\tau_f) = 0 \tag{17}$$

However, one constraint needs to be added to construct an equilateral square in the case of four satellites formation because the number of constraints in Eqs. (16) and (17) is smaller than unknown parameters. For example, four satellites placed in a rectangle configuration inside a circle can satisfy the constraints of Eq. (16) and (17). To avoid this rectangle configuration, an additional constraint can be derived from the fact that the area of any triangle made of three out of four satellites located in an equilateral square is equal to the half area of an equilateral square (See Figure 2b). In other words,

$$\sqrt{r(r-a)(r-b)(r-c)} = \rho^2 \tag{18}$$

where each parameter is defined as

$$\begin{aligned}
r &= (1/2)(a+b+c) \\
a &= \sqrt{(y_1(\tau_f) - y_2(\tau_f))^2 + (z_1(\tau_f) - z_2(\tau_f))^2} \\
b &= \sqrt{(y_1(\tau_f) - y_3(\tau_f))^2 + (z_1(\tau_f) - z_3(\tau_f))^2} \\
c &= \sqrt{(y_2(\tau_f) - y_3(\tau_f))^2 + (z_2(\tau_f) - z_3(\tau_f))^2}
\end{aligned}$$

Equation (18) implies the area of a triangle made of any three satellites. It can be derived from the Heron's formula that the area of a triangle can be obtained using lengths of three sides (a, b, and c) of the triangle. Even though four satellites are placed in a rectangle inside the circle, if the area of any triangle by three satellites among them is equal to ρ^2, then four satellites form a square without any exception. Thus Eq. (18) poses an additional constraint for four satellites located in a rectangle form.

Even though all the satellites are placed in an equilateral polygon in the horizontal plane, additional constraints are necessary for the x components and velocities of satellites to maintain the equilateral polygon as time elapses. These constraints can be derived from periodic solutions (Alfriend et al. 2000) of the Hill's equation. Periodic solutions of the Hill's equation can be derived by requiring the periods of a reference or mother satellite and daughter satellites to be equal. Corresponding periodic solutions for the PCO are given by

$$\begin{aligned}
x &= (\rho/2)\sin(\tau+\alpha), & \dot{x} &= (\rho/2)\cos(\tau+\alpha) \\
y &= \rho\cos(\tau+\alpha), & \dot{y} &= -\rho\sin(\tau+\alpha) \\
z &= \pm\rho\sin(\tau+\alpha), & \dot{z} &= \pm\rho\cos(\tau+\alpha)
\end{aligned} \tag{19}$$

Table 1. Number of unknown parameters and constraints.

Category	3 satellites	4 satellites	5 satellites
Uuknown parameters	441	588	735
Boundary conditions	441	588	735
Defects	288	384	480
Constraints of collision avoidance	48	96	160
Constraints of final configuration*	17(18)	23(26)	(35)

* represents the equality constraints, but values in the parentheses are the number for the case that Eqs. (17) and (18) are exchanged with Eq. (22)

where α represents phase angle as shown in Figure 2. The + sign in Eqs. (19) is for the PCO inclined at $26.56°$ while – sign is for one inclined at $-26.56°$. If y and z components of all the satellites are obtained using Eqs. (16), (17) and (18), constraints for velocity and x components of all the satellites can be derived as Eqs. (20) and Eq. (21) by using Eq. (19).

$$\begin{aligned} &\dot{y}_p(\tau_f) \pm z_p(\tau_f) = 0 \\ &\dot{z}_p(\tau_f) \mp y_p(\tau_f) = 0 \\ &2\dot{x}_p(\tau_f) \mp \dot{z}_p(\tau_f) = 0 \end{aligned} \tag{20}$$

$$2x_p(\tau_f) + \dot{y}_p(\tau_f) = 0 \tag{21}$$

Equations (16), (17), (18), (20) and (21) are added to the NLP problem as constraints for the final configuration of the PCO. The upper and lower signs imply again the PCOs are inclined at $26.56°$ and $-26.56°$, respectively in Eq. (20).

3.2.2 Inequality constraints

Equality constraints for the final configuration were derived for three and four satellites in the previous section. Eqs. (17) and (18) can be replaced with inequality constraints which can be used for three, four and more satellites involved in the formation. Those inequality constraints are derived from the very simple principle that the length between two satellites should be larger than or equal to the side length of an equilateral polygon in the horizontal plane. Thus inequality constraint in Eq. (22) can be added to the NLP problem with Eqs. (16), (20) and (21) for the final configuration instead of Eqs. (17) and (18).

$$\mathrm{D}_{pq} \geq \rho\sqrt{2 - 2\cos(2\pi/S)}, \quad \forall p, \forall q \in [1, 2, ..., S]; p < q \tag{22}$$

where the parameter D_{pq} represents the distance between two satellites in the horizontal plane defined as

$$\mathrm{D}_{pq} = \sqrt{(y_p(\tau_f) - y_q(\tau_f))^2 + (z_p(\tau_f) - z_q(\tau_f))^2} \tag{23}$$

The right hand side term of Eq. (22) defines the length of the base for an isosceles triangle made of two adjacent satellites on the equilateral polygon inside the circle and the origin. Thus if the radius of the circle and the equilateral polygon inside the circle are given, the length of two adjacent satellites can be easily computed from the right term of Eq. (22). When the inequality constraint of Eq. (22) is satisfied for all the satellites on the circle in the horizontal plane, the formation is ensured to make an equilateral polygon.

Table 2. Results of some examples for case I (unit: $10^3\mathrm{m}^2/\mathrm{sec}^3$).

Fuel	$\mu = 0.0$	$\mu = 1.2, \lambda p^1$	$\mu = 2.0, \lambda p^2$	$\mu = 2.0, \lambda p^3$
Satellite 1	1.354	2.564	2.859	2.454
Satellite 2	2.872	2.821	2.859	2.945
Satellite 3	1.159	2.564	2.859	2.454
Total	5.385	7.949	8.577	7.853

[1] $(\lambda_1, \lambda_2, \lambda_3)$ = (1.0, 1.0, 1.0), [2] $(\lambda_1, \lambda_2, \lambda_3)$ = (1.0, 1.0, 1.0), [3] $(\lambda_1, \lambda_2, \lambda_3)$ = (1.0, 1.2, 1.0)

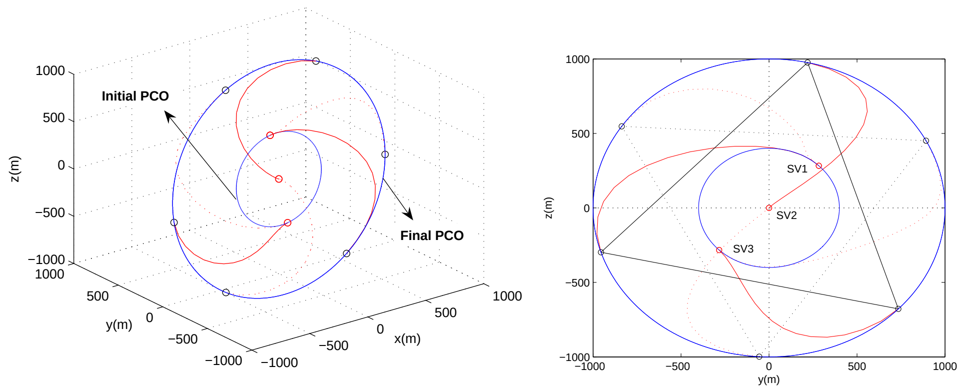
4. Numerical Simulation and Results

In this section, an optimal formation trajectory planning is examined using the NLP and collocation method with constraints of collision avoidance and final configuration. Both equality and inequality constraints were derived in the previous section to describe the SFF mission goal. As mentioned in the previous section, Eqs. (17) and (18) are applicable only for three or four satellites. They can be exchanged with Eq. (22) for three, four and more satellites. To solve the NLP problem numerically in this study, the CFSQP (Lawrence 1997) package is used, in which the sequential quadratic programming (SQP) algorithm is implemented. The CFSQP especially generates feasible iterations throughout the optimization process.

Numerical simulation is performed for 3, 4 and 5 satellites by using 17 discrete nodes. The final time is fixed as π and the PCO inclined at $26.56°$ to the horizontal plane is considered for the final configuration. Three cases are considered in this simulation; one configuration and two reconfigurations. Mean motion (ω) is determined from reference orbit at 550 km high altitude. The radius of sphere (R_a) is set to be 50 m for collision avoidance and the radius of a circle in the horizontal plane (ρ) is 1 km for the final configuration. The boundary conditions are $[-0.003/\omega^2, 0.003/\omega^2]$ for the control inputs in Eq. (3). The boundaries of state vector are not significant, but it is necessary to bound the NLP results and obtain a unique solution, so they can be set up with reasonable values. Table 1 shows the number of unknown parameters and constraints according to the number of satellites. The NLP problems were solved on a 2.6 GHz personal computer with 512MB of RAM. In general, the processing time increases as the number of nodes increases. It took from a few minutes to a few hours to obtain optimal trajectories on the personal computer.

4.1 Case I: Reconfiguration for 3 Satellites

In this case, we assume that two satellites form a PCO with 400 m radius and their phase angles are 45 and 225 degrees, respectively in the horizontal plane. It is also assumed that a satellite is initially located at the origin of the LVLH frame. Now, the main purpose is to generate optimal trajectories such that the cost function is minimized according to the given values of μ and λ. The final formation should be the PCO in the form of an equilateral triangle in the horizontal plane with the radius of 1 km. Four results are listed in Table 2 for different values of μ and λ. Even though a new time variable is used in the NLP problem, the resultant fuel cost is changed to the unit of $[\mathrm{m}^2/\mathrm{sec}^3]$. Optimal trajectories with the lowest total fuel cost were obtained with μ equal to 0.0 because the rate of fuel consumption is not assigned for reconfiguration. Total fuel cost increases as μ becomes greater with the fuel cost rate assigned for all satellites. However, the desired trajectories cannot be obtained under the assignment of fuel rate if μ is too small. One can notice two examples in the 3^{rd} and the 4^{th} columns of Table 2. The total cost for $\mu = 1.2$ is lower than for $\mu = 2.0$ but the fuel cost of each satellite does not coincide with λ_p assigned. Figure 3 shows the optimal



(a) 3-dimensional space (b) Horizontal plane

Figure 3. Optimal trajectories of a reconfiguration for 3 satellites (The solid lines are for $\mu = 0.0$ and the dotted lines for $\mu = 2.0$ and $\lambda p = (1.0, 1.0, 1.0)$. SV1 indicates satellite 1 and so on).

Table 3. Results of some examples for case II (unit: $10^3\mathrm{m}^2/\mathrm{sec}^3$).

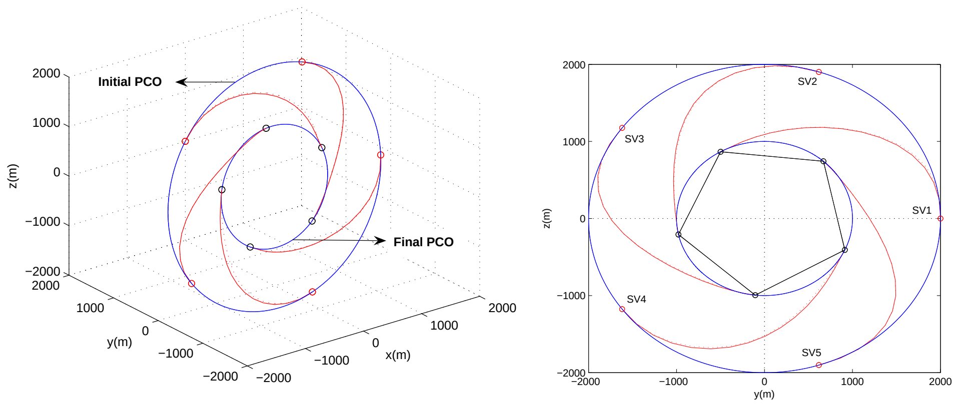
Fuel	$\mu = 0.0$	$\mu = 0.5, \lambda p^1$	$\mu = 2.0, \lambda p^2$	$\mu = 2.0, \lambda p^3$
Satellite 1	3.556	3.717	3.848	4.120
Satellite 2	3.776	3.775	3.848	4.531
Satellite 3	3.711	3.717	3.848	3.708
Satellite 4	3.595	3.717	3.848	4.120
Satellite 5	3.848	3.848	3.848	4.531
Total	18.486	18.774	19.240	21.010

$^1(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ = (1.0, 1.0, 1.0, 1.0, 1.0), $^2(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ = (1.0, 1.0, 1.0, 1.0, 1.0),
$^3(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ = (1.0, 1.1, 0.9, 1.0, 1.1)

trajectories of reconfiguration in 3-dimensional space and horizontal plane. In the final formation, all satellites constitute a PCO and an equilateral triangle in the horizontal plane.

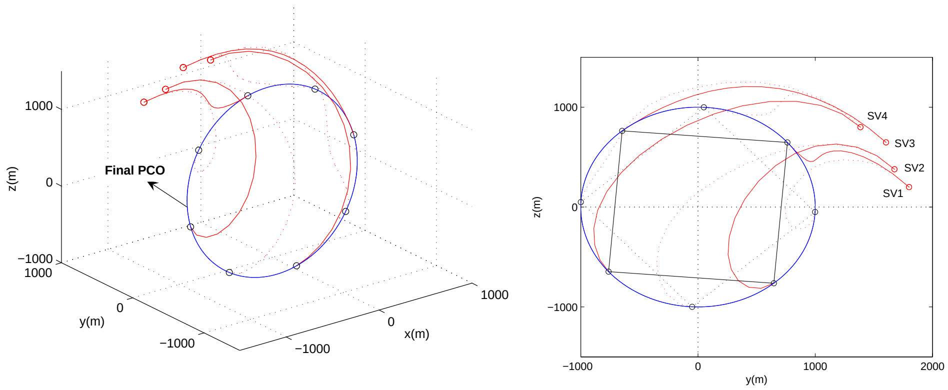
4.2 Case II: Reconfiguration for 5 Satellites (formation re-sizing)

In this case, five satellites form a 2 km radius PCO and are evenly spaced in the horizontal plane at the initial time. They develop into the PCO of 1 km radius and make an equilateral pentagon in the horizontal plane at the final time. This kind of problem is usually called as formation re-sizing. As the Case I, four examples are simulated for optimal trajectories using different μ and λ values. Those results are presented in Table 3 whereas the optimal trajectories are displayed in Figure 4. In the example of $\mu = 0.0$, each fuel cost of five satellites does not result in big difference because of the formation re-sizing problem when compared with the case I. The total fuel shows only a small increase of 4 % over the example of $\mu = 0.0$ in the example of $\mu = 2.0$ with $\lambda_p = (1.0, 1.0, 1.0, 1.0, 1.0)$, but big increase of 59 % in Case I. Thus the optimal trajectories of $\mu = 0.0$ are similar to those of $\mu = 2.0$ with $\lambda_p = (1.0, 1.0, 1.0, 1.0, 1.0)$ in Figure 4b.



(a) 3-dimensional space (b) Horizontal plane

Figure 4. Optimal trajectories of a reconfiguration for 5 satellites (The solid lines are for $\mu = 0.0$ and the dotted lines for $\mu = 2.0$ and $\lambda p = (1.0, 1.0, 1.0, 1.0, 1.0)$).



(a) 3-dimensional space (b) Horizontal plane

Figure 5. Optimal trajectories of a reconfiguration for 3 satellites(The solid lines are for $\mu = 0.0$ and the dotted lines for $\mu = 2.0$ and $\lambda p = (1.0, 1.0, 1.0, 1.0)$).

4.3 Case III: Configuration for 4 Satellites

Unlike the Case I and Case II, this case tries configuration of four satellites. The position and velocity of the four satellites may have any values at the initial time. However, they will also complete a PCO with the radius of 1 km and make an equilateral square in the horizontal plane at the final time. The results of examples are presented in Table 4 while optimal trajectories are shown in Figure 5. In the case of $\mu = 2.0$ with $\lambda_p = (1.0, 1.0, 1.0, 1.0)$, the total fuel cost is about twice

Table 4. Results of some examples for case III (unit: $10^3\mathrm{m}^2/\mathrm{sec}^3$).

Fuel	$\mu = 0.0$	$\mu = 2.0, \lambda p^1$	$\mu = 1.2, \lambda p^2$	$\mu = 2.0, \lambda p^3$
Satellite 1	8.387	15.032	12.874	12.021
Satellite 2	15.790	15.032	14.161	14.692
Satellite 3	1.462	15.032	11.586	12.021
Satellite 4	5.665	15.032	14.161	13.357
Total	31.304	60.128	53.082	52.091

$^1(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ = (1.0, 1.0, 1.0, 1.0), $^2(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ = (1.0, 1.1, 0.9, 1.1),
$^3(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ = (0.9, 1.1, 0.9, 1.0)

the case of $\mu = 0.0$ for which the fuel cost of each satellite shows significant difference. Thus after separation of satellites from a launch vehicle, it could be so important to assign the fuel consumption rate to satellites from the consideration of formation configuration. In the previous examples of Case I and II, the desired optimal trajectories could not be obtained with a lower value of μ than 2.0 in assigning the fuel cost rate of each satellite. However, $\mu = 1.2$ could produce desirable solutions as in the 4$^{\mathrm{th}}$ column of Table 4. The 5$^{\mathrm{th}}$ column of Table 4 shows the fuel cost of all satellites on the basis of the 4$^{\mathrm{th}}$ satellite because λ_4 is 1.0 but others are not.

5. Conclusions

A direct optimization approach was introduced for the path planning of a PCO configuration of multiple satellites. Collocation method in conjunction with NLP was applied to solve the optimization problems that minimize the cost function including the fuel cost and the fuel assignment. The new constraints were derived for the final configuration to make all satellites form a PCO and an equilateral polygon in the horizontal plane. The method was simulated with some problems including reconfiguration of three and five satellites and configuration of four satellites. It was shown from the simulation results that the new method could provide desired solutions for trajectory planning of SFF and appeared to be considerably robust. However, it took from a few minutes to a few hours to solve them because of many constraints for the collision avoidance and final configuration. Even though the method cannot generate the real-time trajectories in general, it can be considered as a viable tool for the path planning of spacecraft formation flying controls.

References

Alfriend, K. T., Schaub, H., & Gim, D. W. 2000, Advances in Astronautical Sciences (Springfield VA: AAS), AAS-00-12

Beard, R. W., McLain, T. W., & Hadaegh, F. Y. 2000, JGCD, 23, 2

Betts, J. T. 1998, JGCD, 21, 2

Clohessy, W. H. & Wiltshire, R. S. 1960, Journal of the Aerospace Sciences, 27, 9

Dickmanns, E. D. & Well, K. H. 1974, Proceedings of IFIP Technical Conference, ed. G. I. Marchuk (London: Springer-Verlag), pp.158-166

Enright, P. J. & Conway, B. A. 1991, JGCD, 14, 5

Hargraves, C. R. & Paris, S. W. 1987, JGCD, 10, 4

Hull, D. G. 1997, JGCD, 20, 1

Johnston, A. G. Y. & McInnes, C. R. 1997, Advances in Astronautical Sciences (Springfield VA:

AAS), AAS-97-104
Lawrence, C., Zhou, J. L., & Tits, A. L. 1997, User's Guide for CFSQP Version 2.5 (College Park MD: University of Maryland), pp.2-70
Martin, M. & Stallard, M. 1999, Proceeding of the AIAA Space Technology Conference and Exposition (Reston VA: AIAA), AIAA-99-4479
McInnes, C. R. 1995, Advances in Astronautical Sciences (Springfield VA: AAS), AAS-95-447
Richards, A., Schouwenaars, T., How, J. P., & Ferson, E. 2002, JGCD, 25, 4
Russell, R. D. & Shampine, L. F. 1972, Numerical Mathematics, 19, 1
Sabol, C., Burns, R., & McLaughlin, C. A. 2001, Journal of Spacecraft and Rockets, 38, 2
Singh, G. & Hadaegh, F. Y. 2001, Proceeding of the AIAA Guidance, Navigation, and Control Conference (Reston VA: AIAA), AIAA-2001-4088
Tillerson, M. & How, J. 2001, Proceeding of the AIAA Guidance, Navigation, and Control Conference (Reston VA: AIAA), AIAA-2001-4092
Tillerson, M., Inalhan, G., & How, J. 2002, International Journal of Robust and Nonlinear Control, 12, 2
Wang, P. K. C. & Hadaegh, F. Y. 1998, Proceeding of the AIAA Guidance, Navigation, and Control Conference (Reston VA: AIAA), AIAA-98-4226
Yang, G., Yang, Q., Kapila, V., Palmer, D., & Vaidyanathan, R. 2002, International Journal Robust Nonlinear Control, 12, 2