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Parameter Identification Methods for Non-Linear Discrete-Time Systems

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Abstract— **This paper presents three techniques for parameter identification for non-linear, discrete-time systems. The methods presented are intended to improve the performance of adaptive control systems. The first two methods rely on system excitation and a regressor matrix, in either case, the true parameters are identified when the regressor matrix is of full rank and can be inverted. The third case uses a novel method developed in [3] to define a parameter uncertainty set. The uncertainty set is periodically updated to shrink around the true parameters. Each method guarantees convergence of the parameter estimation error, provided an appropriate persistence of excitation condition is met. Each method is subsequently demonstrated using a simulation example, displaying convergence of the parameter error estimation error.**

I. INTRODUCTION

Parameter identification is an important feature in many control situations. In many adaptive control algorithms the reference trajectory is unknown and dependent on system dynamics, which can rely on a set of unknown system parameters. For example, in adaptive extremum seeking control, the system is optimized using a cost function that may rely on unknown parameters [5][6]. The performance of the system is dependent on the performance of the parameter identification method. It has been shown that efficient parameter convergence increases the robustness properties of closed loop adaptive systems [4].

Few studies are available regarding parameter identification for discrete time systems in the context of adaptive control. The problem of output feedback systems and strict feedback systems ([10] [11]) have been proposed where the parameter identification algorithm is separated from some control task. This algorithm uses a two-phase approach to identify parameters then apply some appropriate control to achieve a desired objective. This approach is limited by the assumption that the system is free from noise, though robustness is shown for a small additive random noise. Several recent studies solve similar identification problems with a variety of methods. The application of a neural network identification approach to a system that has been parameterized is shown in [12]. Results from this study demonstrate convergence of the internal state of the neural network to the true system state. A similar problem with the addition of time varying parameters is solved in [13].

This paper adapts parameter estimation methods developed in [1] [2] and [3]. The first method, the finite time identification method, allows the direct and exact recovery of parameters immediately once a persistence of excitation condition is met. This method requires the online inversion and computation of rank of a regressor matrix. Since the parameters are identified in finite-time, it is possible to remove the excitation signal at the moment the parameters are recovered. Within the past five years, several results on intelligent excitation signals where the magnitude of the excitation signal is adjusted as needed have become available [8] [9]. Traditionally, the excitation signal is removed when the parameters are assumed to have converged. The method presented in the current paper allows the excitation signal to be removed once the parameters are guaranteed to have converged. The second method is a refinement of the first, it uses an adaptive compensator to eliminate the need to for online inversion and rank computation of a matrix. The parameter estimation error can be shown to be non-increasing once a persistence of excitation condition is met. The third method defines a parameter uncertainty set that evolves based on a worst case estimate. Further, the parameter estimates are not allowed to fall outside the uncertainty set. This method ensures convergence of the parameter uncertainty set to the true parameters provided the true parameters fall within the initial uncertainty set, as the update algorithm ensures non-exclusion of the true parameter estimate.

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II. Problem Description and Assumptions

The parameter identification methods discussed in this paper

$$x_{k+1} = x_k + F(x_k, u_k) + G(x_k, u_k)\theta \tag{1}$$

where x_k is a state at some time step k, u_k is a the control input at some time step k, and θ is a column vector of system parameters.

Assumption 1 : The state of the system x_k is accessible for measurement at any time step k.

Assumption 2 : There is some known, bounded control law, u_k that achieves some control objective.

Given the bounded control law, the objective of the three methods presented below is to determine the true values of the plant parameters.

III. Finite Time Parameter Identification

Consider the following state predictor

$$\begin{aligned} \hat{x}_{k+1} &= \hat{x}_k + F(x_k, u_k) + G(x_k, u_k)\hat{\theta}_{k+1} \\ &+ K_k e_k - \omega_k(\hat{\theta}_k - \hat{\theta}_{k+1}) \\ &+ K_k \omega_k(\hat{\theta}_k - \hat{\theta}_{k+1}) \end{aligned} \tag{2}$$

where $\hat{\theta}_k$ is the vector of parameter estimates at time step k given by any update law, K_k is a correction factor at time step k, $e_k = x_k - \hat{x}_k$ is the state estimation error at time step k. The variable ω_k is the following output filter at time step k

$$\omega_{k+1} = \omega_k + G(x_k, u_k) - K_k\omega_k, \qquad \omega_0 = 0 \tag{3}$$

Let the parameter estimation error at some time step k be $\tilde{\theta}_k = \theta - \hat{\theta}_k$. Now from (2) and (1) the state estimation error at time step $k+1$ is given by

$$\begin{aligned} e_{k+1} &= e_k + G(x_k, u_k)\tilde{\theta}_{k+1} - K_k e_k \\ &+ \omega_k(\hat{\theta}_k - \hat{\theta}_{k+1}) - K_k\omega_k(\hat{\theta}_k - \hat{\theta}_{k+1}). \end{aligned} \tag{4}$$

Define the auxiliary variable

$$\eta_k = e_k - \omega_k\tilde{\theta}_k. \tag{5}$$

From (3),(4) and (5) it follows that

$$\eta_{k+1} = \eta_k - K_k\eta_k, \qquad \eta_0 = e_0. \tag{6}$$

Let $Q \in \mathbb{R}^{p\times p}$ and $C \in \mathbb{R}^p$ be defined as

$$\begin{aligned} Q_{k+1} &= Q_k + \omega_k^T\omega_k \\ Q_0 &= 0 \end{aligned} \tag{7}$$

$$\begin{aligned} C_{k+1} &= C_k + \omega_k^T(\omega_k\hat{\theta}_k + e_k - \eta_k) \\ C_0 &= 0. \end{aligned} \tag{8}$$

Lemma 3.1: If there exists some time step k_c such that Q_{k_c} is invertible, that is

$$Q_N = \sum_{i=0}^{N} \omega_i^T\omega_i \succ 0 \tag{9}$$

then the parameters are given by $\theta = Q_k^{-1}C_k$, $\forall k \geq k_c$.

Proof: This results can be shown from

$$Q_N\theta = \sum_{i=0}^{N} \omega_i^T\omega_i[\hat{\theta}_i + \tilde{\theta}_i]. \tag{10}$$

Upon substitution of (5), it follows that

$$\begin{aligned} \theta &= Q_k^{-1}\sum_{i=0}^{k}\omega_i^T(\omega_i\hat{\theta}_i - +e_i - \eta_i) \\ &= Q_k^{-1}C_k \qquad \forall k \geq k_c, \end{aligned} \tag{11}$$

which proves the result. ■

IV. Adaptive Compensation Design

Application of the finite-time identifier is problematic since it requires that one checks the nonsingularity of Q_k at all step k. In this section, an adaptive compensation design is proposed that recovers exponential stability of the parameter estimation error in finite-time without the need to test the matrix Q_k.

Consider the state predictor for system (1)

$$\begin{aligned} \hat{x}_{k+1} &= \hat{x}_k + F(x_k, u_k) + G(x_k, u_k)\theta^o \\ &\quad + K_k(x_k - \hat{x}_k) \end{aligned} \tag{12}$$

where $K_k > 0$ and θ^o is the vector of initial parameter estimates.

As above, define the auxiliary variable and the filter dynamic as:

$$\eta_k = x_k - \hat{x}_k - \omega_k(\theta - \theta^o) \tag{13}$$

$$\omega_{k+1} = \omega_k + G(x_k, u_k) - K_k\omega_k \quad \omega_0 = 0 \tag{14}$$

The auxiliary variable η_k can be generated from,

$$\eta_{k+1} = \eta_k - K_k\eta_k, \qquad \eta_0 = e_0 \tag{15}$$

Let Q and C be generated by:

$$Q_{k+1} = Q_k + \omega_k^T\omega_k, Q_0 = 0, \tag{16}$$

$$\begin{aligned} C_{k+1} &= C_k + \omega_k^T(\omega_k\theta^o + e_k - \eta_k), \\ &\qquad C_0 = 0. \end{aligned} \tag{17}$$

and let k_c be a time step at which $Q_{k_c} \succ 0$.
The parameter update law proposed in [2] is given by:

$$\hat{\theta}_{k+1} = \hat{\theta}_k + \Gamma_k(C_k - Q_k\hat{\theta}_k) \tag{18}$$

It follows from (18) that the dynamics of the parameter estimation error are

$$\tilde{\theta}_{k+1} = \tilde{\theta}_k - \Gamma_k(C_k - Q_k\hat{\theta}_k) \tag{19}$$

For all time steps $k \geq k_c$, $Q_k\theta = C_k$, it follows that $\forall k \geq k_c$

$$\tilde{\theta}_{k+1} = \tilde{\theta}_k - \Gamma_k(Q_k\theta - Q_k\hat{\theta}_k). \tag{20}$$

Define the variable $\Gamma_k = \frac{1}{\|Q_k\|+\epsilon}$, where ϵ is some small positive number

$$\tilde{\theta}_{k+1} = (I - \frac{Q_k}{\|Q_k\|+\epsilon})\tilde{\theta}_k \tag{21}$$

It follows from (21) that for all time steps $k \geq k_c$ that $\tilde{\theta}$ is non increasing, and

$$\lim_{k\to\infty} \tilde{\theta} = 0. \tag{22}$$

V. Parameter Uncertainty Set Estimation

The adaptive compensator design provides an effective mechanism to recover the finite-time identifier performance. However, the properties of this design can be lost in the presence of exogenous disturbance variables and model mismatch. In this section, a parameter estimation technique is proposed to handle nonlinear systems subject to exogenous variables. The technique relies on aan uncertainty set update formulation that provides robust performance.

A. Parameter Update

Consider the uncertain non-linear system

$$x_{k+1} = x_k + F(x_k,u_k) + G(x_k,u_k)\theta + \vartheta_k \tag{23}$$

where ϑ is a bounded disturbance that satisfies $\|\vartheta_k\| \leq M_\vartheta < \infty$. It is assumed that θ is uniquely identifiable and lies within an initially known compact set defined by the ball function $\Theta^0 = B(\theta_0, z_\theta)$ where θ_0 is an initial estimate of the unknown parameters and z_θ is the radius of the parameter uncertainty set.

Using the state predictor defined in (2) and the output filter defined in (3), the prediction error $e_k = x_k - \hat{x_k}$ is given by

$$\begin{aligned} e_{k+1} &= e_k + G(x_k,u_k)\tilde{\theta}_{k+1} - K_k e_k \\ &+ \omega_k(\hat{\theta}_k - \hat{\theta}_{k+1}) - K_k\omega_k(\hat{\theta}_k - \hat{\theta}_{k+1}) + \vartheta_k \\ e_0 &= x_0 - \hat{x}_0. \end{aligned} \tag{24}$$

The auxiliary variable η_k dynamics are as follows

$$\begin{aligned} \eta_{k+1} &= e_{k+1} - \omega_{k+1}\tilde{\theta}_{k+1} + \vartheta_k \\ \eta_0 &= e_0. \end{aligned} \tag{25}$$

Since ϑ_k is unknown, it is necessary to use an estimate, $\hat{\eta}$, of η . The estimate is generated by the recursion:

$$\hat{\eta}_{k+1} = \hat{\eta}_k - K_k\hat{\eta}_k \tag{26}$$

The resulting dynamics of the η estimation error are:

$$\tilde{\eta}_{k+1} = \tilde{\eta}_k - K_k\tilde{\eta}_k + \vartheta \tag{27}$$

Let the identifier matrix Σ_k be defined as

$$\Sigma_{k+1} = \Sigma_k + \omega_k^T\omega_k, \quad \Sigma_0 = \alpha I \succ 0 \tag{28}$$

with an inverse generated by the recursion

$$\Sigma_{k+1}^{-1} = \Sigma_k^{-1} - \Sigma_k^{-1^T}\omega_k^T\omega_k\Sigma_k^{-1}, \quad \Sigma_0 = \frac{1}{\alpha}I \succ 0. \tag{29}$$

From equations (2), (3), and (26) and based on the the prefered parameter update law proposed in [3], the parameter update law is

$$\hat{\theta}_{k+1} = \text{Proj}\{\hat{\theta}_k + \gamma\Sigma_k^{-1}\omega_k^T(e_k - \hat{\eta}_k), \Theta_k\} \tag{30}$$

where $Proj$ represents an orthogonal projection onto the surface of the uncertainty set applied to the parameter estimate.The parameter uncertainty set is defined by the ball function $B(\hat{\theta}_c, z_{\hat{\theta}c})$, where $\hat{\theta}_c$ and $z_{\hat{\theta}c}$ are the parameter estimate and set radius found at the latest set update. Define

$$\hat{\theta}'_{k+1} = \hat{\theta}_k + \gamma\Sigma_k^{-1}\omega_k^T(e_k - \hat{\eta}_k) \tag{31}$$

then the projection algorithm is given by:

$$\hat{\theta}_{k+1} = \begin{cases} \hat{\theta}'_{k+1} & \text{if } \hat{\theta}'_{k+1} \in B(\hat{\theta}_c, z_{\hat{\theta}c}) \\ \hat{\theta}_c + \frac{\sqrt{z_{\hat{\theta}c}}(\hat{\theta}'_{k+1} - \hat{\theta}_c)}{\|\hat{\theta}'_{k+1} - \hat{\theta}_c\|^2} & \text{if } \hat{\theta}'_{k+1} \notin B(\hat{\theta}_c, z_{\hat{\theta}c}) \end{cases} \tag{32}$$

It can be shown that the parameter update law defined in (30) guarantees convergence of parameter estimates to the true values.

Lemma 5.1: [7] Consider the system

$$x_{k+1} = Ax_k + Bu_k \tag{33}$$

where A is a stable matrix with eigenvalues inside the unit circle and B is a matrix of appropriate dimension. Then it can be shown that

$$\sum_{k=0}^{K-1} x_{k+1}^T x_{k+1} \leq \delta^2 \sum_{k=0}^{K-1} u_k^T u_k \tag{34}$$

for some $\delta > 0$ and $K - 1 > 0$.

Let l_2 denote the space of square finitely summable signals and consider the following lemma.

Lemma 5.2: The identifier (28) and parameter update law (30) are such that $\tilde{\theta}_k = \theta_k - \hat{\theta}_k$ is bounded. Furthermore, if

$$\vartheta_k \in l_2 \quad or \quad \sum_{k=0}^{\infty}[\|\tilde{\eta}_k\|^2 - \gamma\|e_k - \hat{\eta}_k\|^2] < +\infty \tag{35}$$

and

$$\lim_{k\to\infty} \Sigma_k = \infty \tag{36}$$

are satisfied, then $\tilde{\theta}_k$ converges to 0 asymptotically.

Proof: Let $V_{\tilde{\theta}k} = \tilde{\theta}_k^T \Sigma_k \tilde{\theta}_k$ It follows that

$$V_{\tilde{\theta}k+1} - V_{\tilde{\theta}k} = \tilde{\theta}_{k+1}^T \Sigma_{k+1} \tilde{\theta}_{k+1} - \tilde{\theta}_k^T \Sigma_k \tilde{\theta}_k$$

Upon substitution of the parameter update law, the identifier matrix dynamics, the filter dynamics and the auxiliary variable dynamics, the rate change of the $V_{\tilde{\theta}k}$ is given by:

$$\begin{aligned} V_{\tilde{\theta}k+1} - V_{\tilde{\theta}k} &= (1-2\gamma)(e_k - \hat{\eta}_k)^T(e_k - \hat{\eta}_k) \\ &+(2\gamma-2)\tilde{\eta}_k^T(e_k - \hat{\eta}_k) + \tilde{\eta}_k^T\tilde{\eta}_k \\ &+(-2+2\gamma^2)(e_k - \hat{\eta}_k)^T\omega_k\Sigma_k^{-1}\omega_k^T(e_k - \hat{\eta}_k) \\ &-\gamma^2(e_k - \hat{\eta}_k)^T\omega_k\Sigma_{k+1}^{-1}\omega_k^T(e_k - \hat{\eta}_k) \end{aligned} \tag{37}$$

From the inequality

$$\tilde{\eta}_k^T(e_k - \hat{\eta}_k) \le \frac{1}{2}\tilde{\eta}_k^T\tilde{\eta}_k + \frac{1}{2}(e_k - \hat{\eta}_k)^T(e_k - \hat{\eta}_k) \tag{38}$$

it can be shown that

$$\begin{aligned} V_{\tilde{\theta}k+1} - V_{\tilde{\theta}k} &\le -\gamma(e_k - \hat{\eta}_k)^T(e_k - \hat{\eta}_k) \\ &+(-2+2\gamma^2)(e_k - \hat{\eta}_k)^T\omega_k\Sigma_k^{-1}\omega_k^T(e_k - \hat{\eta}_k) \\ &-\gamma^2(e_k - \hat{\eta}_k)^T\omega_k\Sigma_{k+1}^{-1}\omega_k^T(e_k - \hat{\eta}_k) + \gamma\tilde{\eta}_k^T\tilde{\eta}_k \end{aligned} \tag{39}$$

This implies that

$$\begin{aligned} &V_{\tilde{\theta}k+1} - V_{\tilde{\theta}k} \le \\ &-\gamma(e_k - \hat{\eta}_k)^T(e_k - \hat{\eta}_k) + \sum_{k=0}^{\infty}[\|\tilde{\eta}_k\|^2] \end{aligned} \tag{40}$$

for $\frac{1}{2} \le \gamma \le 1$. From the $\tilde{\eta}_k$ dynamics given in (27), it follows from Lemma 5.1 if $\vartheta_k \in l_2$ then $\tilde{\eta}_k \in l_2$. Taking the limit as $k \to \infty$, the inequality becomes

$$\lim_{k\to\infty} V_{\tilde{\theta}k} = V_{\tilde{\theta}0} + \sum_{k=0}^{\infty} V_{\tilde{\theta}k+1} - V_{\tilde{\theta}k} \tag{41}$$

$$\le \sum_{k=0}^{\infty} -\gamma[\|e_k - \hat{\eta}_k\|^2] + \sum_{k=0}^{\infty}[\|\tilde{\eta}_k\|^2] \tag{42}$$

Therefore if the conditions (35) are met, the right hand side of (42) is finite and

$$\lim_{k\to\infty} \tilde{\theta}_k = 0 \tag{43}$$

as required. ■

B. Set Update

An update law that measures the worst-case progress of the parameter update law is adapted from the one proposed in [3]

$$z_{\hat{\theta}k} = \sqrt{\frac{V_{z\hat{\theta}k}}{\lambda_{min}(\Sigma_k)}} \tag{44}$$

$$\begin{aligned} V_{z\theta k+1} &= V_{z\hat{\theta}k} - \lambda_{min}(\gamma)(e_k - \hat{\eta}_k)^T(e_k - \hat{\eta}_k) \\ &+ \left(\frac{M_\vartheta}{K_k}\right)^2 \end{aligned} \tag{45}$$

$$V_{z\hat{\theta}0} = \lambda_{max}(\Sigma_0)(z_{\hat{\theta}0})^2 \tag{46}$$

The parameter uncertainty set, defined by the ball function $B(\hat{\theta}_c, z_c)$ is updated using the parameter update law (30) and the error bound (44) according to the following algorithm:

Algorithm 5.1: beginning at time step $k = 0$, the set is adapted according to the following iterative process

1) **Initialize** $z_{\hat{\theta}c} = z_{\hat{\theta}0}, \hat{\theta}_c = \hat{\theta}_0$
2) at time step k, using equations (30) and (44) perform the update

$$(\hat{\theta}_c, z_{\hat{\theta}c}) = \begin{cases} (\hat{\theta}_k, z_{\hat{\theta}k}) & \text{if} \quad z_{\hat{\theta}k} \le z_c - \|\hat{\theta}_k - \hat{\theta}_c\| \\ (\hat{\theta}_c, z_{\hat{\theta}c}) & \text{otherwise} \end{cases} \tag{47}$$

3) Return to step two and iterate, incrementing to time step $k + 1$

Lemma 5.3: The algorithm ensures that

1) the set is only updated when updating will yield a contraction,
2) the dynamics of the set error bound described in (44) are such that they ensure the non-exclusion of the true value $\theta \in \Theta_k$, $\forall k$ if $\theta_0 \in \Theta_0$.

Proof:

1) If $\Theta_{k+1} \not\subseteq \Theta_k$ then

$$\sup_{s\epsilon\Theta_{k+1}} \|s - \hat{\theta}_k\| \ge z_{\hat{\theta}k} \tag{48}$$

However, it is guaranteed by the set update algorithm presented, that Θ, at update times, obeys the following

$$\begin{aligned} &\sup_{s\epsilon\Theta_{k+1}} \|s - \hat{\theta}_k\| \\ \le \; &\sup_{s\epsilon\Theta_{k+1}} \|s - \hat{\theta}_{k+1}\| + \|\hat{\theta}_{k+1} - \hat{\theta}_k\| \end{aligned} \tag{49}$$

$$\le \; z_{\hat{\theta}k+1} + \|\hat{\theta}_{k+1} - \hat{\theta}_k\| \le z_{\hat{\theta}k} \tag{50}$$

This contradicts (48). Therefore, $\Theta_{k+1} \subseteq \Theta_k$ at time steps where Θ is updated.

2) It is known, by definition, that

$$V_{\tilde{\theta}0} \leq V_{z\theta 0}, \qquad \forall k \geq 0 \tag{51}$$

Since, $V_{\tilde{\theta}k} = \tilde{\theta}_k^T \Sigma_k \tilde{\theta}_k$,

$$\|\tilde{\theta}_k\| \leq \frac{V_{z\hat{\theta}k}}{\lambda_{min}(\Sigma_k)} = z_{\hat{\theta}k}^2, \qquad \forall k \geq 0 \tag{52}$$

Therefore, if $\theta \ \epsilon \ \Theta_0$, then $\theta \ \epsilon \ \Theta_k \ \ \forall \ k \ \geq \ 0$. ∎

VI. Simulation Examples

Consider the following nonlinear system

$$\begin{aligned} x_{1,k+1} &= 0.01 \times -x_{2,k} + u_{3,k} + x_{3,k}\theta_1 + \vartheta_{1,k} \\ x_{2,k+1} &= 0.01 \times (1 + x_{3,k})u_{1,k} - x_{1,k}\theta_2 + \vartheta_{2,k} \\ x_{3,k+1} &= 0.01 \times -x_{1,k} + u_{2,k} + x_{2,k}\theta_3 + \vartheta_{3,k} \end{aligned} \tag{53}$$

where $\theta^T = [\theta_1, \theta_2, \theta_3]$ The input is taken as constant, $u_k = [-0.1 \ 0.1 \ 0.2]^T$. The true parameter values are $\theta = [1.5 \ 3 \ 0.02]^T$.

In the first two examples (fig.1, fig.2), we consider system (53) with $\vartheta_{i,k} = 0$, i =1,2,3. To demonstrate the parameter uncertainty set approach (fig.4), a bounded noise term is added to the state equation, as shown in (23). The bounded noise term is

$$\vartheta_k = [\vartheta_{1,k}, \vartheta_{2,k}, \vartheta_{3,k}] = [\sin(k) \ \ \sin(k) \ \ \sin(k)]^T.$$

The input is taken as constant, $u_k = [-0.1 \ 0.1 \ 0.2]^T$. The true parameter values are $\theta = [34 \ 3 \ 0.02]^T$.

A. Finite Time Parameter Identification

In the first simulation study, we consider the finite-time parameter identification. Fig. 1 shows the parameter estimates converging to the true values almost immediately at the time step when the regressor matrix Q_k has full rank.

B. Adaptive Compensation Design

Consider the system described by (53). We consider the application of the adaptive compensator. The results are shown in Figure 2. Consistent with the result shown in (21), Figure 2 shows that after time step k_c the parameter estimate errors converge to their true values at an exponential rate.

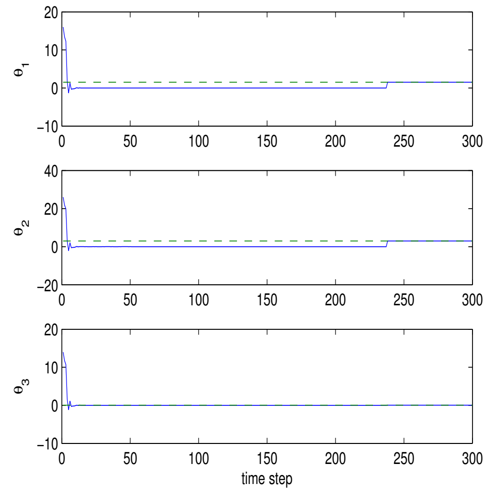

Fig. 1. Time course plot of the parameter estimates and true values, under the finite time estimation algorithm, the dashed lines (- -) represent the true parameter values, the solid lines (–) represent the parameter estimates

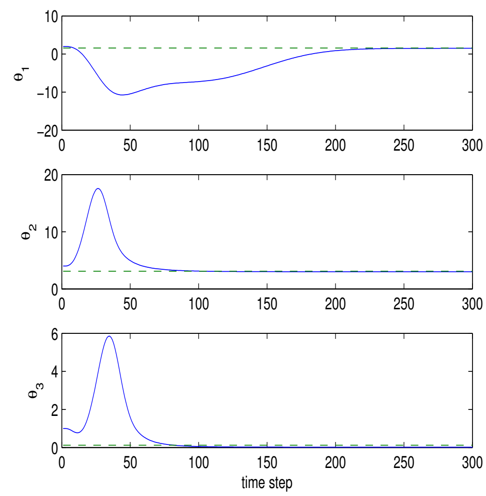

Fig. 2. Time course plot of the parameter estimates and true values, under the adaptive compensatory algorithm, the dashed lines (- -) represent the true parameter values, the solid lines (–) represent the parameter estimates

C. Parameter Uncertainty Set Estimation

In the third simulation, we consider the system (53) with the bounded noise term is ϑ. Fig. 3 shows the trajectories of the state prediction error. As expected, the error is shown to converge to a neighbourhood of zero. The parameter estimates are shown to converge to a neighbourhood of their true values in Figure 4. The ability of the estimation routine to recover the true parameter value in the presence of exogenous disturbances is clearly demonstrated. The size of this neighbourhood is limited by the magnitude of the injected noise. The size of the uncertainty set, z_θ is shown in Figure 5.

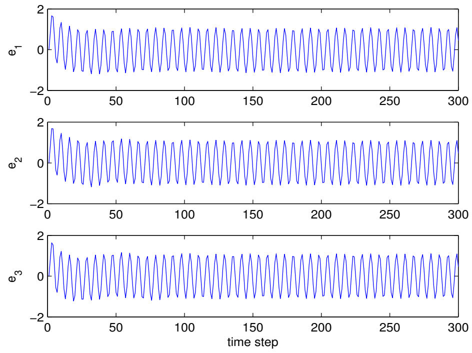

Fig. 3. Time course plot of the state prediction error $e_k = x_k - \hat{x}_k$

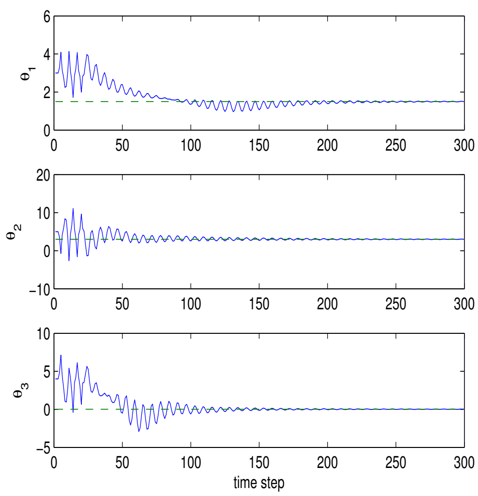

Fig. 4. Time course plot of the parameter estimates and true values under the parameter uncertainty set algorithm, the dashed lines (- -) represent the true parameter values, the solid lines (–) represent the parameter estimates

VII. Conclusions

This paper has presented three methods for parameter identification for nonlinear systems. Each method presented guarantees convergence of the parameter estimation error to zero, provided an appropriate persistence of excitation condition is met. Each identification algorithm has been implemented to demonstrate its performance, each algorithm demonstrates convergence of the parameter estimation error.

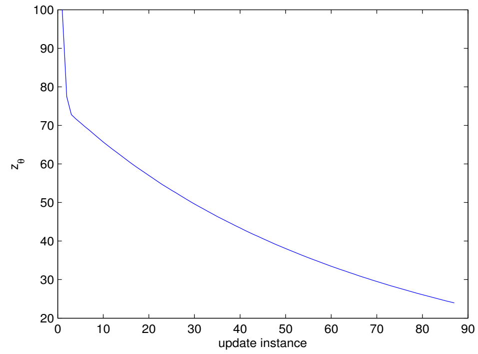

Fig. 5. The progression of the radius of the parameter uncertainty set at time steps when the set is updated

References

[1] Veronica Adetola and Martin Guay, "Finite-time Parameter Estimation in Adaptive Control of Nonlinear Systems", in *Proc. American Control Conference*, New York, NY, 2007, pp. 1920.

[2] Veronica Adetola and Martin Guay, "Performance Improvement in Adaptive Control of Nonlinear Systems", in *Proc. American Control Conference*, St. Louis, MO, 2009, pp. 5610

[3] V. Adetola and M. Guay, "Robust Adaptive MPC for Systems with Exogeneous Disturbances",in *Proc. IFAC ADCHEM*, Istanbul, Turkey, 2009

[4] Jung-Shan Lin and Ioannis Kanellakopoulos, Nonlinearities Enhance Parameter Convergence in Output-Feedback Systems *IEEE Transactions on Automatic Control*, vol. 43, No. 2, 1998, pp. 204-222.

[5] M. Guay, D. Dochain, and M. Perrier, Adaptive Extremum Seeking Control of Continuous Stirred Tank Bioreactors *Automatica*, vol. 40, 2004, pp 881-888.

[6] Hsin-Hsiung Wang, Miroslav Krstic, and Georges Bastin, Optimizing Bioreactors by Extremum Seeking *Int. J. of Adaptive Control and Signal Processing*, vol. 13, 1999, pp 651-669.

[7] Wassim M. Haddad, *Nonlinear Dynamical Systems and Control*, Princeton University Press, Princeton, NJ; 2008.

[8] V. Adetola and M. Guay, "Excitation Signal Design for Parameter Convergence in Adaptive Control", in *Proc.of the 45th IEEE Conference on Decision and Control*, San Diego, CA, 2006, pp. 447 - 452

[9] Chengyu Cao, Jiang Wang and N. Hovakimyan, "Adaptive Control with Unknown Parameters in Reference Input", in *Proc. of the 13th Mediterranean Conference on Control and Automation*, Limassol, Cyprus, 2005, pp. 225 - 230

[10] Jiaxiang Zhao and Ioannis Kanellakopoulos, "Active Identification for discrete time Nonlinear Control - Part I: Output-Feedback Systems", *IEEE Transactions on Automatic Control*, vol. 47, 2002, pp 210-224.

[11] Jiaxiang Zhao and Ioannis Kanellakopoulos, "Active Identification for discrete time Nonlinear Control - Part II: Strict-Feedback Systems", *IEEE Transactions on Automatic Control*, vol. 47, 2002, pp 225-240.

[12] Wen Yu, "Nonlinear system identification using discrete-time recurrent neural networks with stable learning algorithms", *Infomation Sciences*, vol. 158, 2004, pp 131-147.

[13] Michel Fliess, Stefan Fuchshumer Markus Schberl, Kurt Schlacher, Herbertt Sira-Ramirez, An introduction to algebraic discrete-time linear parametric identificaion with a concrete application *J. Europen des Systmes Automatiss*, vol. 42, 2008, pp 210-232.