Number of Divisors(NOD) & Sum of Divisors(SOD)

Saturday, February 3, 2024 9:18 PM

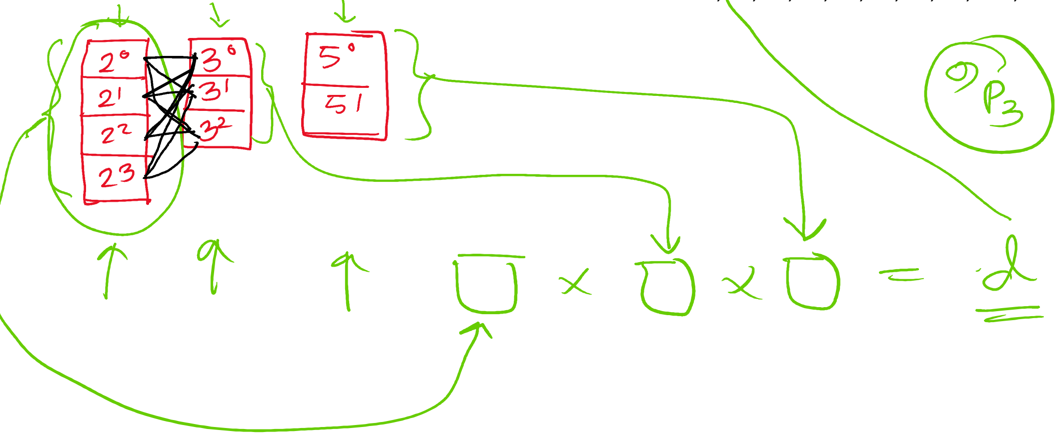
problem 1: $NOD(N) = ?$

$O(\sqrt{N}) \rightarrow \quad N = P_1^{a_1} \cdot P_2^{a_2} \cdot P_3^{a_3} \cdot \;\; \cdots \;\; P_K^{a_K}$

$360 = 2^3 \cdot 3^2 \cdot 5^1$

360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180 and 360

$\boxed{{}^nP_r,\ {}^nC_r}$ ⊗



d

$d\,P_3$

12

$(4 \times 3) \times 2 = 24$

(12×2)

$N = P_1^{a_1} \cdot P_2^{a_2} \cdot P_3^{a_3} \cdot \;\; \cdots \;\; P_K^{a_K} \longrightarrow O(\sqrt{N})$

$NOD(N) = (a_1+1)(a_2+1)(a_3+1) \cdots (a_K+1)$

$N = 10^{12},\ T = 10^3 \qquad O(T \cdot \sqrt{N}) = O(10^3 \cdot 10^6)$

$\;\;\hookrightarrow NOD(N) \qquad\qquad = O(10^9)$

Sieve: $[1 - \sqrt{N}] \rightarrow$ prime divisors ⊗ $O(N \log N)$

$\sqrt{N} \rightarrow 10^6$

$360 = 2^{\square} \; 3^{\square} \; 5^{\square}$

$N = 2^{32} \approx 10^{10}$

$\boxed{2^{40}} > 10^{12}$

$2^{\square} \; 3^{\square} \; 5^{\square} \; 3^{\boxed{40}}$

$5^{\boxed{40}}$

$$\boxed{O(\sqrt{N} + T \cdot K_m)} \qquad \boxed{K_m = 40}$$

$(10^6 + 4 \times 10^4) \approx 10^6$

1.01×10^6

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	3	2		2		2		2	11	2	13	2		2	17	2	19	2		2	23	2	
				3			3			3			3			3			3	22		3	
			5					5					5					5					5
					7							7							7				

26	27	28	29	30
2		2	29	2
13	3			3
				5
		7		

problem 2: $SOD(N) = ?$

360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180 and 360

$360 = 2^3 \cdot 3^2 \cdot 5^1$

$(2^0 + 2^1 + 2^2 + 2^3) \cdot (3^0 + 3^1 + 3^2) \cdot (5^0 + 5^1)$ Sum = $\square$

$1 \quad 4$ $3^3 \quad 1$ $5^2 - 1$ $p^{e+1} - 1$

$$\left(\frac{2^4-1}{2-1} \times \frac{3^3-1}{3-1} \times \frac{5^2-1}{5-1}\right) \qquad \frac{p^{e+1}-1}{p-1}$$

Discrete Math → H.N Rosen

$\boxed{\text{Counting}}$