Class 19: Sieve of Eratosthenes

Sunday, January 28, 2024 8:40 PM

Problem 1: N → number $\leq 10^5$. Is N is a prime? $O(N)$

$(1, N)$ → test N/x p $q = \frac{N}{p}$

$\sqrt{45} = 6$ [2-6]

1×45

3×15

5×9

$p \cdot q = 45\ (N)$

$p \leq q$

$p \times q = N$

① $p < \sqrt{N}$ and $q < \sqrt{N}$ ✗

② $p > \sqrt{N}$ and $q < \sqrt{N}$ ✗

③ $p > \sqrt{N}$ and $q > \sqrt{N}$ ✗

④ $p < \sqrt{N}$ and $q > \sqrt{N}$ ✓

⑤ $p = \sqrt{N}$ and $q = \sqrt{N}$ ✓

$p \leq \sqrt{N}$

$p^2 \leq (\sqrt{N})^2$

$p^2 \leq N$

Problem 2: [1 - N] → print list of primes in the range

$N \leq 10^5$ $10^5 \times 10^3 = 10^8$

✗ [1 → N]: → $O(N\sqrt{N})$

primality test (x) → $O(\sqrt{N})$

$N = 30$

2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

$i = 2$, Step $= \frac{N}{2}$

$i = 3$, Step $= \frac{N}{3}$

$i = 4, \quad Step = \frac{N}{4}$

$\vdots$

$i = N \qquad Step = \frac{N}{N}$

$$\sum_{i=0}^{N} Step_i = \frac{N}{2} + \frac{N}{3} + \frac{N}{4} + \cdots + \frac{N}{N}$$

$$= N\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{N}\right)$$

→ Hermonic Series

$$= 1 + N\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{N-1}\right)$$

$\ln(N)$

$$= 1 + N \ln(N)$$

$O(N \ln N)$ ✓

$10^7 \rightarrow 5 \times 10^6$ sec

see

$10^5 \rightarrow (1.1 \times 10^5)/10^8 \rightarrow 1.1 \times 10^{-3}$ Sec

$10^6 \rightarrow 1.6 \times 10^6 / 10^8 \rightarrow 1.6 \times 10^{-2}$ see

$10^7 \rightarrow \quad \rightarrow$

① If a number is not prime, do not cut it's multiples.

② Handle all evens at first then just do odd jumps.

③ $i \times j = \text{multiple} \quad [i \leq j]$