Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Course Title	Control Systems
Course Number	ELE632
Semester/Year	W25

Instructor	Dr. Dimitri Androutsos

Lab #	4
Lab Title	Discrete-Time Fourier Transform

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Due Date	March 23, 2025

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A Discrete-Time Fourier Transform

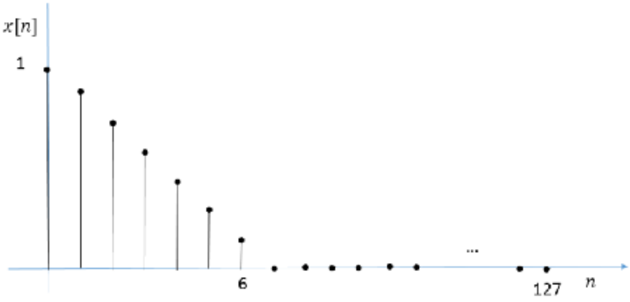

Figure 1: Main Signal for Part A

A.1 Computing DTFT With fft

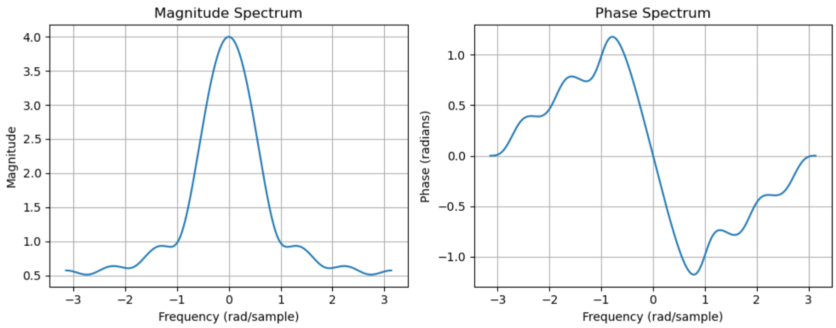

Figure 2: DTFT of original signal

A.2 Computing DTFT by Hand

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \tag{1}$$

By using the above equation, the DTFT of the signal above can be computed manually.

$$\begin{aligned} X(\Omega) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \\ &= \sum_{n=0}^{7} x[n]e^{-j\Omega n} \\ &= 1 + \frac{6}{7}e^{-j\Omega} + \frac{5}{7}e^{-j2\Omega} + \frac{4}{7}e^{-j3\Omega} + \frac{3}{7}e^{-j4\Omega}\frac{2}{7}e^{-j5\Omega} + \frac{1}{7}e^{-j6\Omega} \end{aligned}$$

When Plotting the above system, the following graph is obtained, matching the same as in Part A.1. This shows that Python's Scipy method of obtaining the DTFT match the equations method and both provide an accurate Fourier Transform.

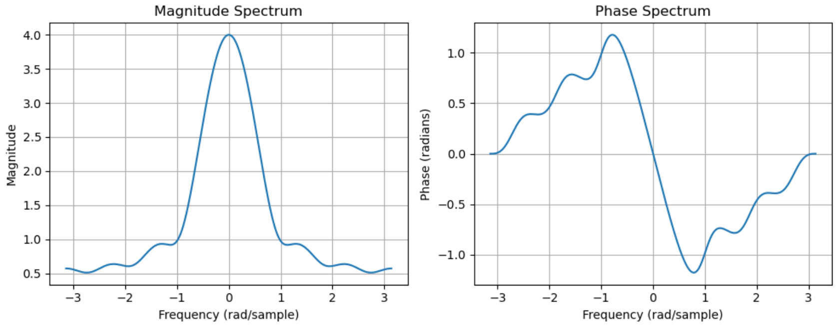

Figure 3: DTFT of Original Signal Calculated by Hand

A.3 Taking Inverse DTFT with ifft

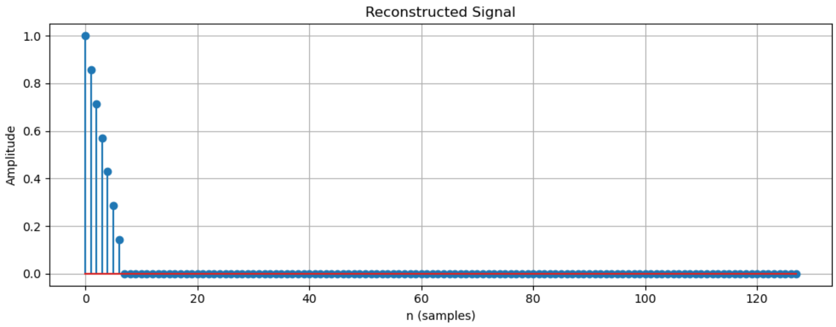

Figure 4: Inverse DTFT of $X(\Omega)$

By analyzing the graph above, it is identical to the original signal. This shows that it is possible to take the Fourier Transform of a Discrete-Time signal and recover it by taking the Inverse Fourier Transform.

B Time Convolution

B.1 Plotting DTFT of x[n]

$$x[n] = sin(\frac{2\pi n}{(4+1)})(u[n] - u[n-(4+1)]) \tag{2}$$

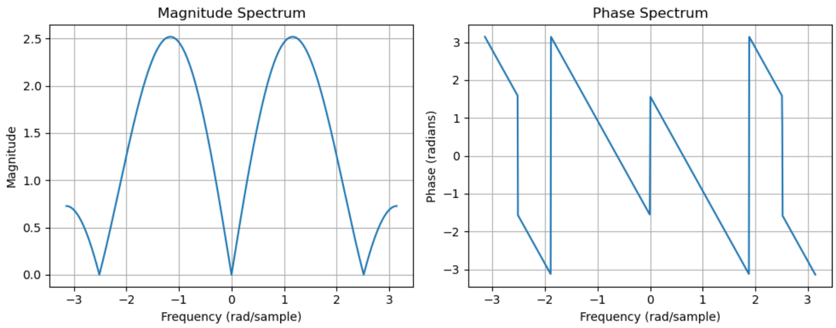

Figure 5: DTFT of the signal x[n]

B.2 Plotting DTFT of h[n]

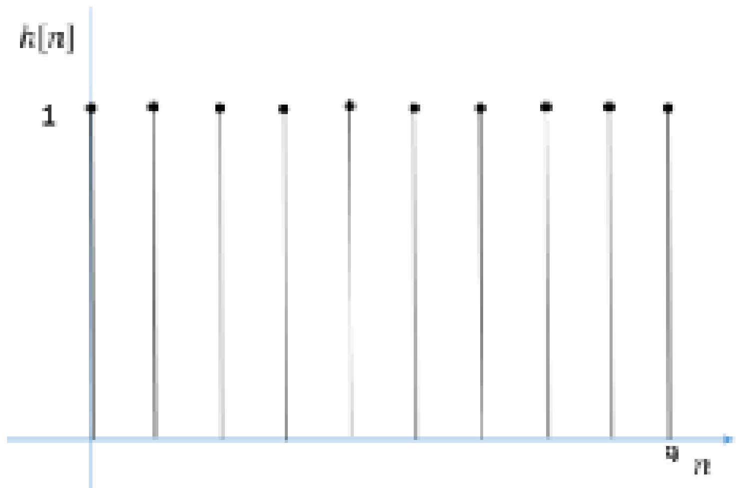

Figure 6: Plot of h[n]

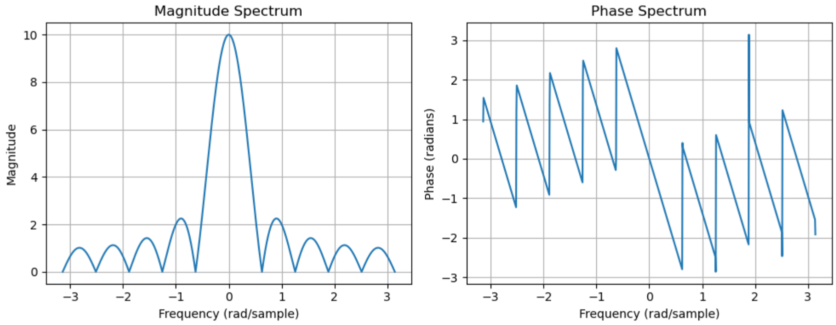

Figure 7: DTFT of h[n]

B.3 Plotting the result of $X(\Omega)H(\Omega)$

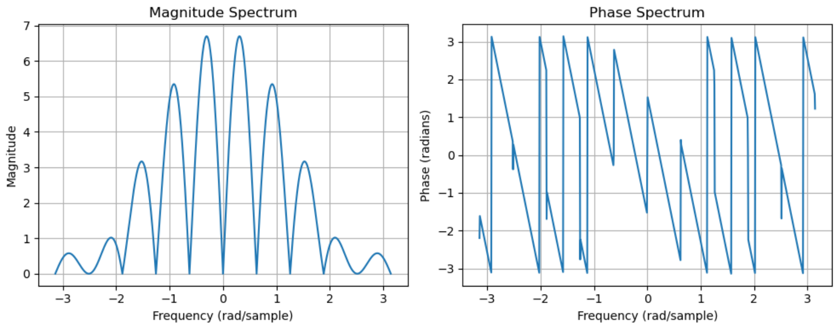

Figure 8: Plot of $X(\Omega)H(\Omega)$

B.4 Convolving x[n] and h[n], then taking DTFT

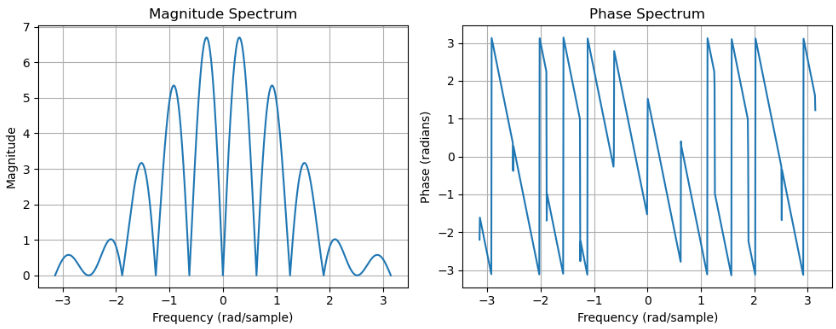

Figure 9: DTFT of Convolution Between x[n] and h[n]

When analyzing figure 9 and figure 8, they are both the exact same. This is because convolution in the time-domain is the same thing as multiplying in the frequency domain and vice versa. The convolution of x[n] and h[n] is the same thing as $X(\Omega)H(\Omega)$.

C FIR Filter Design by Frequency Sampling

C.1 Design a High Pass FIR Filter

When making a high pass fir filter with a cutoff frequency $\Omega_0 = \frac{2\pi}{3}$, this means that in the frequency domain, the absolute values under $\frac{2\pi}{3}$ must be 0 and are only 1 above. By taking this into account, the filter will be 1 when the frequency is above the cutoff, and 0 below in the frequency domain. Taking the inverse fourier transform provides the impulse response h[n] as shown below

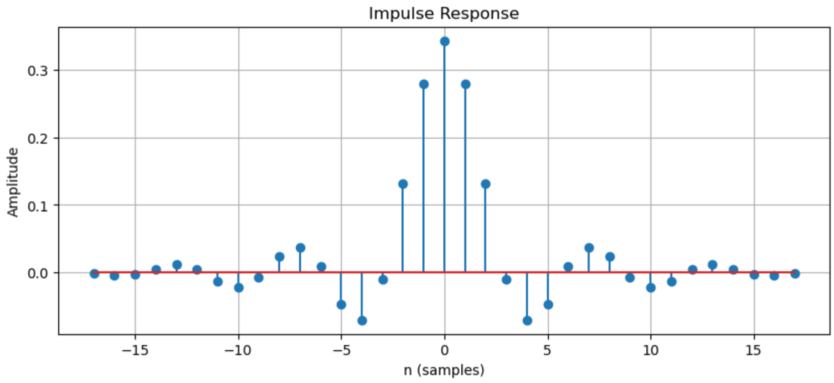

Figure 10: Impulse Response h[n] with Sample Size 35

C.2 Finding Frequency Response of The Filter

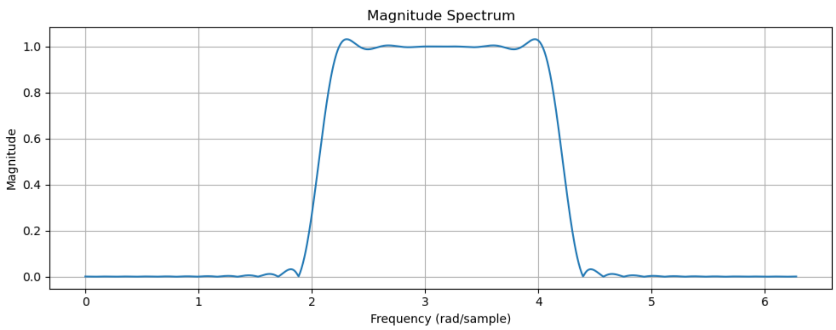

Figure 11: Frequency Response Magnitude $H(\Omega)$ with Sample size 35

Although the frequency response shows that frequencies around $2\pi/3$ are the only ones that make it through, the edges are not as sharp and seem to not have a very flat '1' at the top, where the edges have a little bump to them, resulting in potential noise.

C.3 Increasing Number of Points to 71

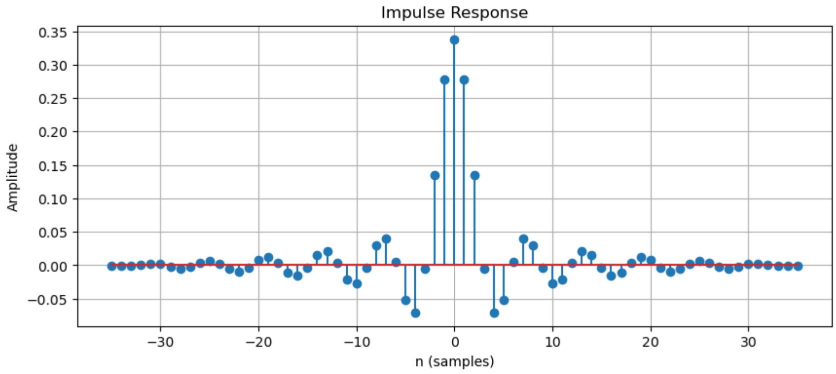

Figure 12: Impulse Response h[n] with Sample Size 71

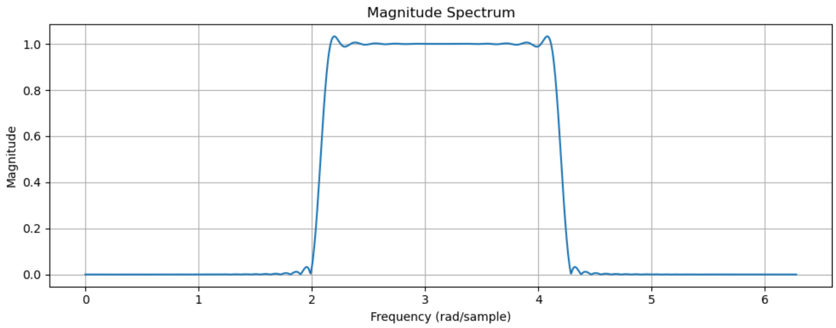

Figure 13: Frequency Response Magnitude $H(\Omega)$ with Sample Size 71

When increasing the sample size, there is not much difference besides the fact that the edges have become narrower as well as the bump at the corners being slightly less noticeable. This means that the greater the sample size, the more precise the high pass filter will be.