Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Course Title	Control Systems
Course Number	ELE632
Semester/Year	W25

Instructor	Dr. Dimitri Androutsos

Lab #	5
Lab Title	Sampling and the Discrete Fourier Transform

Submission Date	April 6, 2025
Due Date	April 6, 2025

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A Discrete Fourier Transform and Zero Padding

A.1 Length-N DFT Magnitude of signals x1[n] and x2[n]

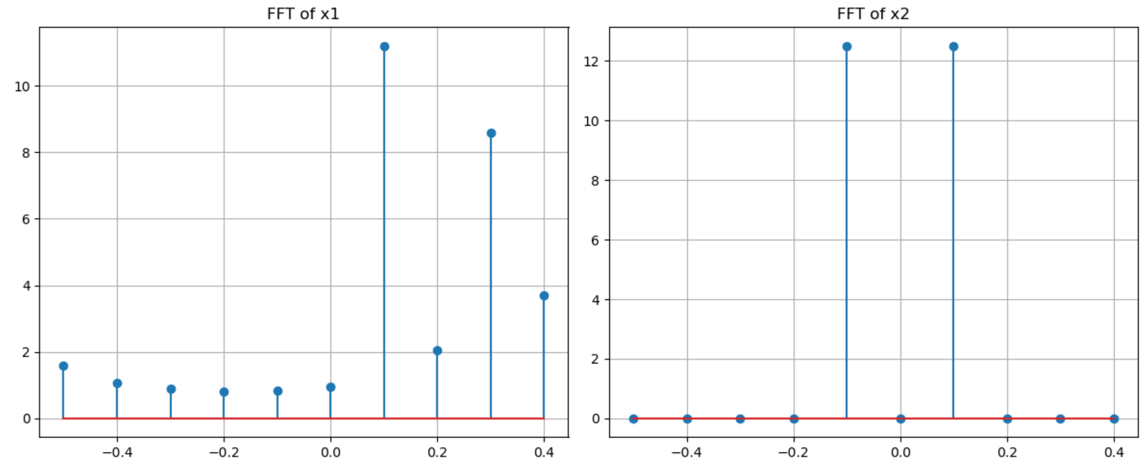

Figure 1: Fourier Transform of x1[n] and x2[n]

$$X_1(jw) = e^{j2\pi n(10(1))/100} + e^{j2\pi n33/100} \tag{1}$$

$$X_2(jw) = 2cos(2\pi n(10(1))/100) + 0.5cos(2\pi n(10(9))/100) \tag{2}$$

When analyzing the above spectrums, it appears that X2 has a synmmetrical signal while X1 is not. This is due to the second function being based on the sum of two cos functions, which are both real signals, while the first signal is of two complex exponential terms. The complex exponential terms are not real, as shown in the magnitude spectra not being symmetrical, while cos is symmetrical and real.

It is possible to distinguish frequency components in X2, as they occur at a specific frequency. for X2, there should be an impulse at both positive and negative 1/10 and 9/10. On the other hand, the 2 impulses for X1 seem to have been spread apart.

This is due to "spectral leakage" where the values leak to other frequencies since we are not sampling the value at the complex exponential (sampling from (-0.5 : 0.5: 0.1) but complex exp. exists at 3.3).

A.2 Zero-Padding the Signals with 490 Zeros

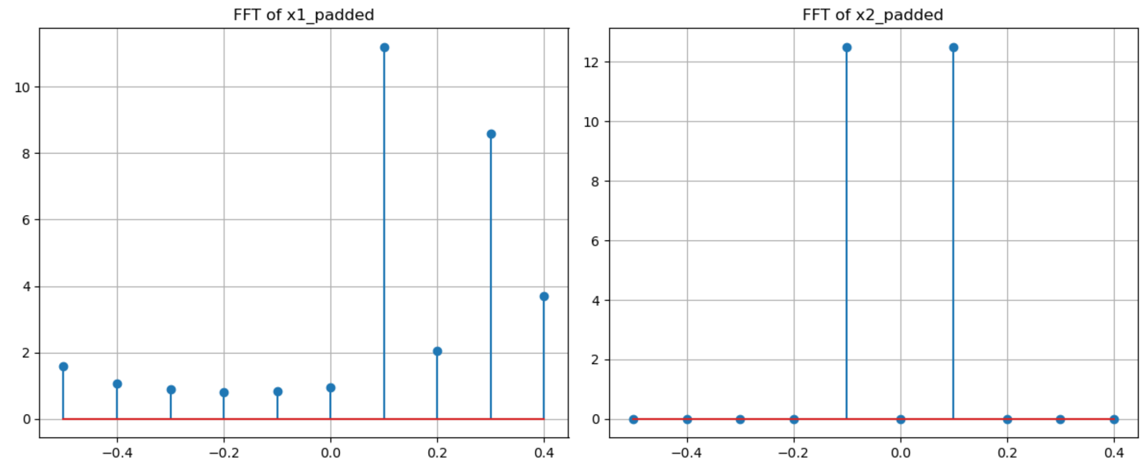

Figure 2: Fourier Transform of x1[n] and x2[n] with 490 zero padding

Even after padding, the signals' spectras have not seemed to change.

A.3 DFT of x1[n] and x2[n] with samplesize 100

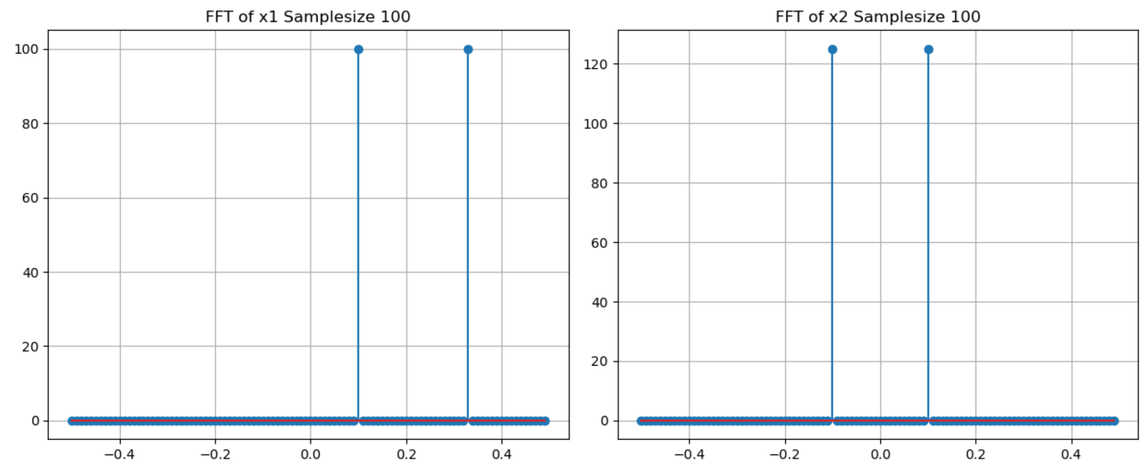

Figure 3: Fourier Transform of x1[n] and x2[n] with sample size 100

By increasing the sample size, the spectra for x1[n] has resulted in only 2 impulses located at the frequencies 1/10 and 3.3/10 as expected.

A.4 Zero-padding of 400 zeros with samplesize 100

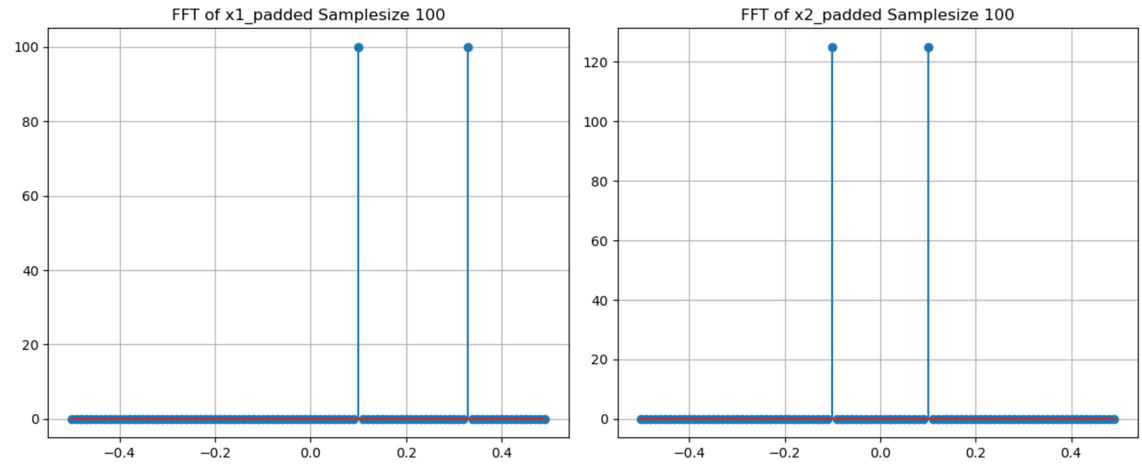

Figure 4: Fourier Transform of x1[n] and x2[n] with sample size 100 and 400 zero padding

Similar to before, there are no improvements to the spectra after adding padding to it (improving resolution)

B Sampling

B.1 N0, T0, and T for Y

In order to get the N_0, just taking the length of y (len(y)) will provide us the number of samples. for the Duration of the signal T_0, Take the number of samples (N_0) and divide by fs. Similarly, to get the Sampling Period T, just take the reciprocal of the Sampling Frequency fs.

$$N_0 = len(y) = 262094$$
$$T_0 = \frac{N_0}{fs} = 5.94 secs$$
$$T = \frac{1}{fs} = 2.26 * 10^-5 secs$$

B.2 Plot of signal y with respect to time

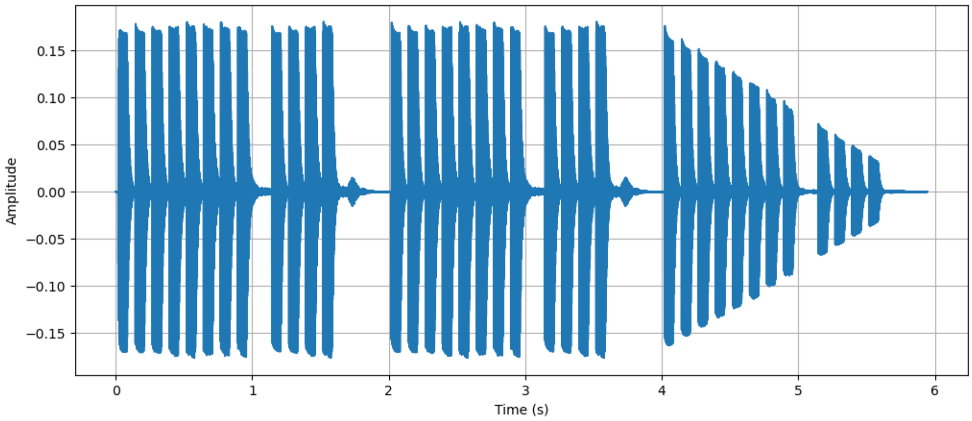

Figure 5: Signal y with respect to Time

B.3 DFT of signal Y

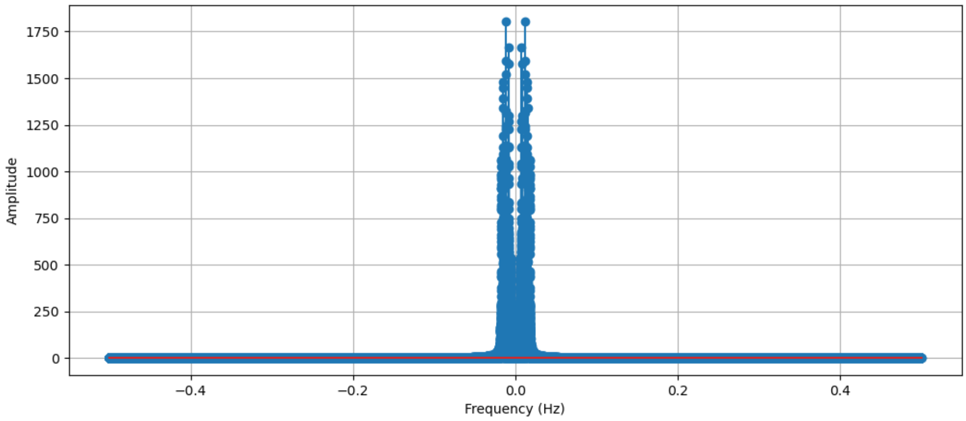

Figure 6: DFT of Signal y

B.4 N0, T0, and T for Y1

In order to get the N_0, just taking the length of y (len(y)) will provide us the number of samples. for the Duration of the signal T_0, Take the number of samples (N_0) and divide by fs. Similarly, to get the Sampling Period T, just take the reciprocal of the Sampling Frequency fs.

$$N_0 = len(y1) = 131047$$
$$T_0 = \frac{N_0}{fs} = 5.94secs$$
$$T = \frac{1}{fs/2} = 4.54 * 10^-5secs$$

B.5 Plot of signal y1 with respect to time

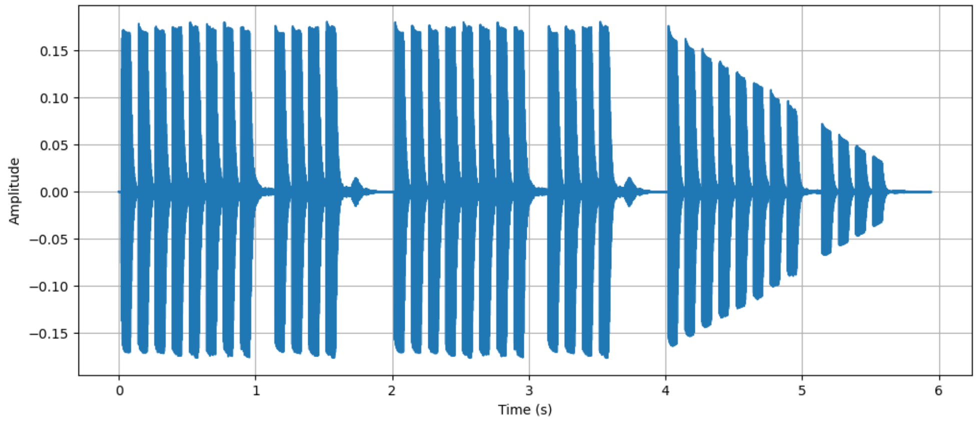

Figure 7: Signal y1 with respect to Time

B.6 DFT of signal Y1

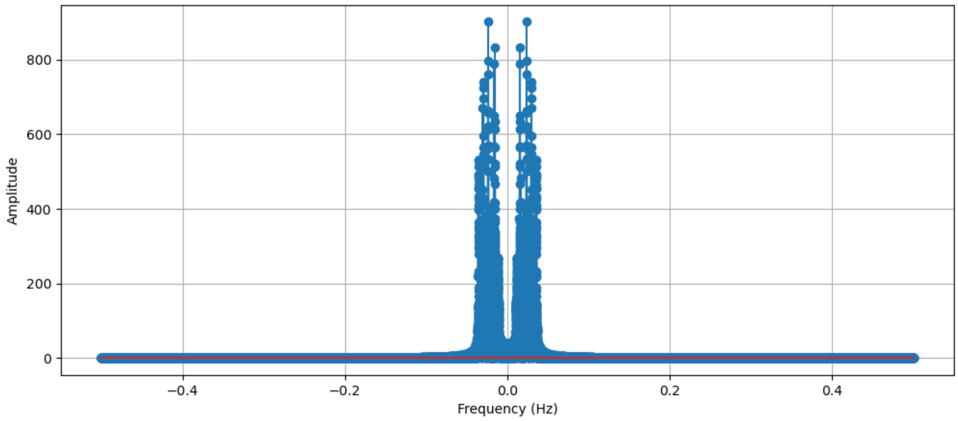

Figure 8: DFT of Signal y1

When looking at the two graphs, they don't appear to look very different until zooming in. What can be found is that the DFT of the signal Y1 has half as many impulses as compared to Y2. This is because by taking subsamples at a rate of 2, we are taking half as much samples, and resulting in a lower resolution of our sound without losing too much important details.

B.7 Comparing audio

When comparing both audios, they sound very similar and have very minimal differences.

B.8 Sub sampling at a rate of '5'

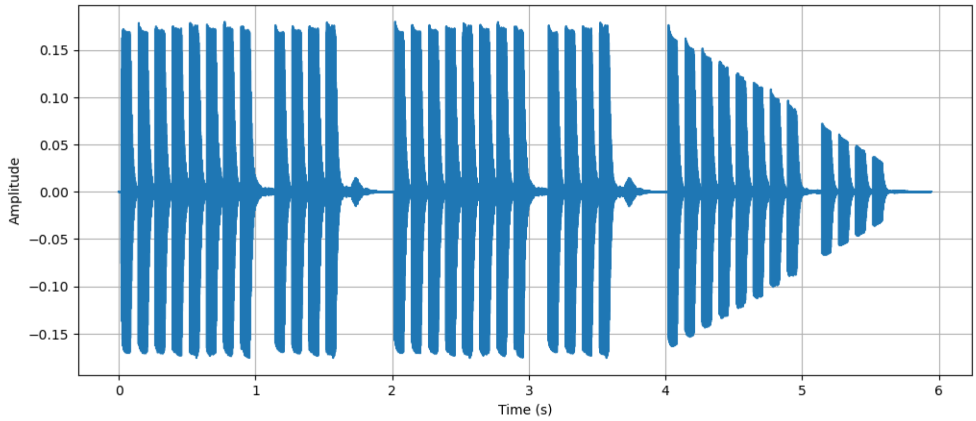

Figure 9: Signal y2 with respect to Time

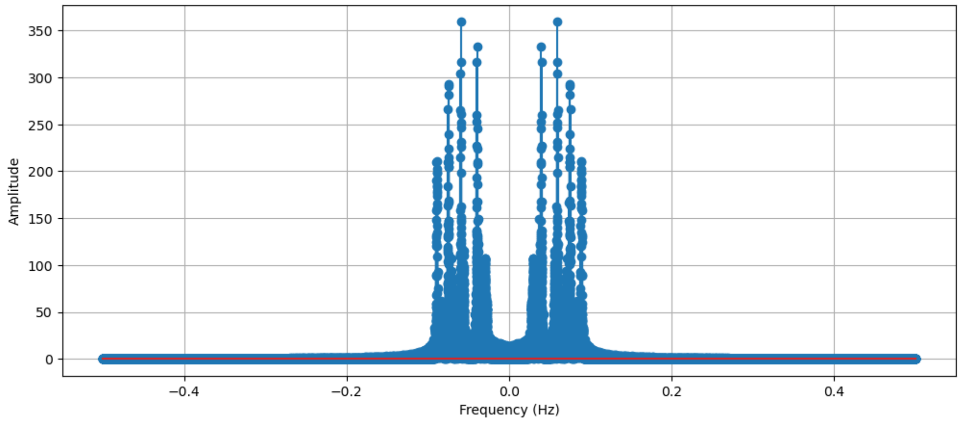

Figure 10: DFT of Signal y2

Although it is again very hard to tell the difference in the signal y2 with respect to time, the magnitude spectra again shows a decrease in impulses of the system. This results in lower audio quality as it is missing more frequencies that are part of the audio.

The audio is also much faster and comes off very high-pitched.

C Filter Design