Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Course Title	Control Systems
Course Number	ELE632
Semester/Year	W25

Instructor	Dr. Dimitri Androutsos

Lab #	3
Lab Title	Discrete-Time Fourier Series

Submission Date	March 9, 2025
Due Date	March 9, 2025

Last Name	**First Name**	**Student Number**	**Section**	**Signature**
Kandasamy	Heitan	501184804	04	HK

*By signing above you attest that you have contributed to this written lab report and confirm that all work you have contributed to this lab report is your own work. Any suspicion of copying or plagiarism in this work will result in an investigation of Academic Misconduct and may result in a "0" on the work, an "F" in the course, or possibly more severe penalties, as well as a Disciplinary Notice on your academic record under the Student Code of Academic Conduct, which can be found online at: http://www.ryerson.ca/senate/current/pol60.pdf

A Discrete-Time Fourier Series

A.1 Fourier Series of an Equation

$$x[n] = 4sin(2.4\pi n) + 2cos(3.2\pi n) \tag{1}$$

A.1.1 Determining Fundamental Period & Fundamental Frequency

To determine the Fundmental period, the individual frequencies of the sin and cos need to be checked and made sure that they are both rational.

$$\begin{aligned} x[n] &= 4sin(2.4\pi n) + 2cos(3.2\pi n) \\ &= 4sin(0.4\pi n) + 2cos(1.2\pi n) \end{aligned}$$

Simplifying the equation as any frequency over 2π repeats.

$$N_0 = m\frac{2\pi}{\frac{2\pi}{5}} = 5m \tag{2}$$

$$N_0 = n\frac{2\pi}{\frac{6\pi}{5}} = \frac{5n}{3} \tag{3}$$

$$N_0 = LCM(5, \frac{5}{3}) = 5 \tag{4}$$

$$\Omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{5} \tag{5}$$

By taking the fundamental periods of the cosine and sine, they are compared and the lowest common multiple of the two is the fundamental period of the signal. Taking the fundamental period, the fundamental frequency can be calculated as above.

A.1.2 Plotting DTFS

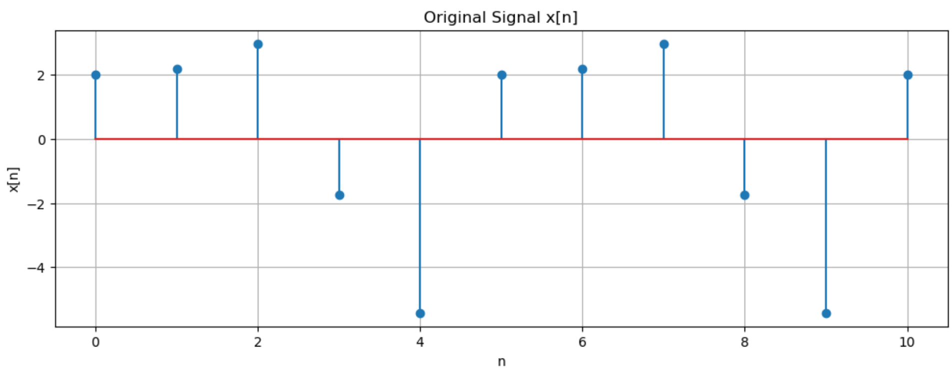

Figure 1: Original Graph for A1

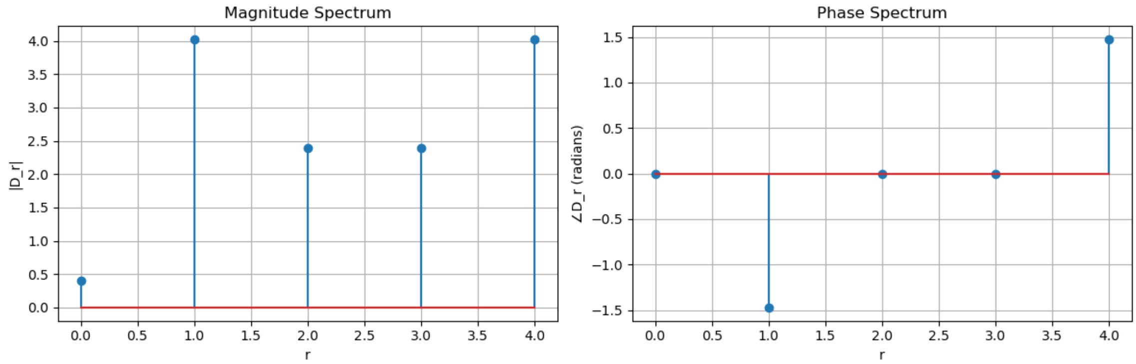

Figure 2: Spectrum Graphs for A1

A.2 Fourier Series of a Graph

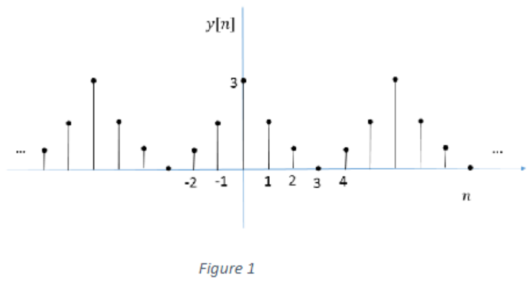

Figure 3: Graph of which DTFS will be taken

A.2.1 Determining Fundamental Period & Fundamental Frequency

Analyzing the graph, the signal repeats every 6n. This means that the **fundamental period is 6**, and can also be used to determine the fundamental frequency.

$$\Omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{6} = \frac{\pi}{3} \tag{6}$$

A.2.2 Plotting DTFS

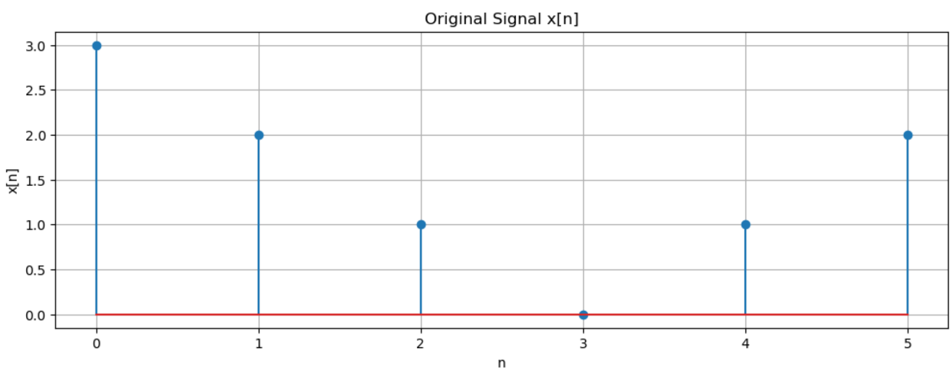

Figure 4: Original Graph for A1

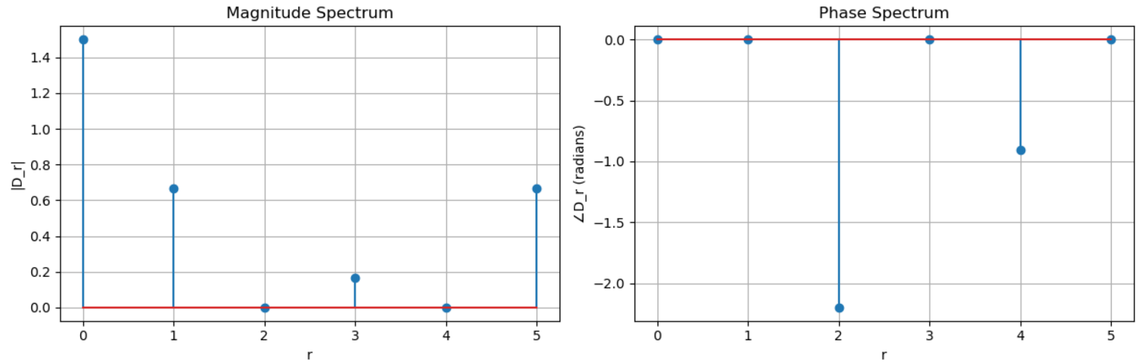

Figure 5: Spectrum Graphs for A1

B Inverse DTFS and Time Shifting Property

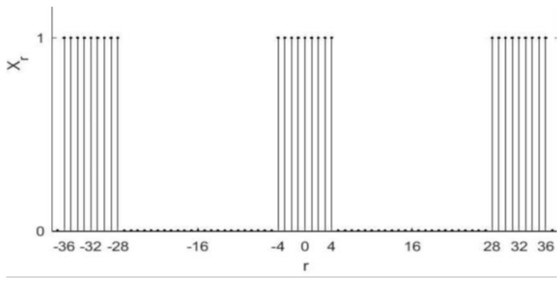

Figure 6: D_r coefficients

B.1 Taking Inverse DTFS

In order to calculate the inverse DTFS, eq. 9.3 from the textbook is used. Instead of having to multiply by negative e^j, it is multiplied by a positive exponent.

$$x[n] = \sum_{n=0}^{N_0-1} D_r e^{jr\Omega_0 n} \tag{7}$$

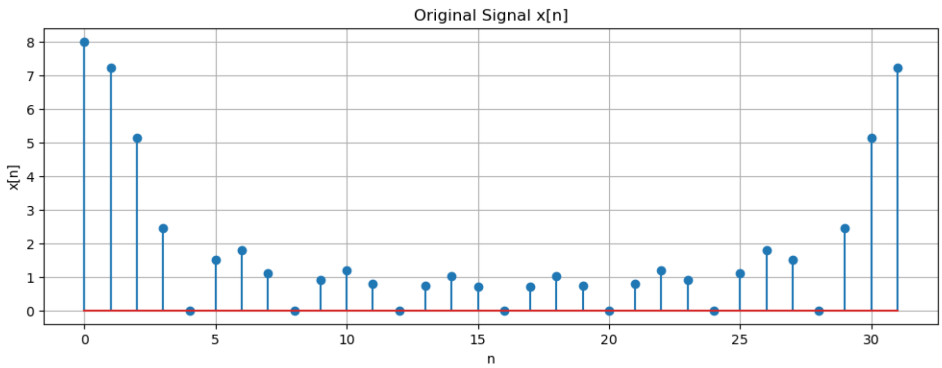

Figure 7: x_n After DTFS

B.2 Applying Shift and Then Taking Inverse DTFS

Once multiplying the Fourier series coefficients by $e^{-j(1+1)\Omega_0 r}$ and then taking the Inverse DTFS, the following signal is produced. When comparing this signal with the one in the previous section (figure 7), they appear to be the exact same but Figure 8 has a shift to the right by 2 n terms. This is due to the $e^{-j(1+1)\Omega_0 r}$ that was multiplied into the Fourier series coefficients and is shifted by a value of 2 to the right because the coefficient in the exponent is -2.

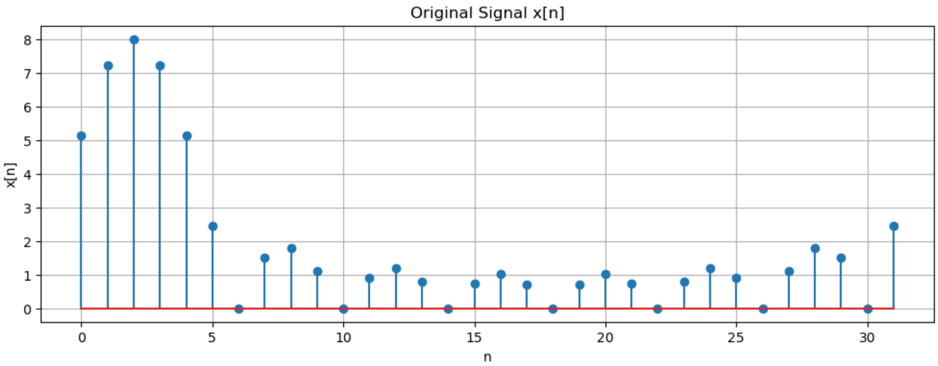

Figure 8: x_n After DTFS and Shift

C System response

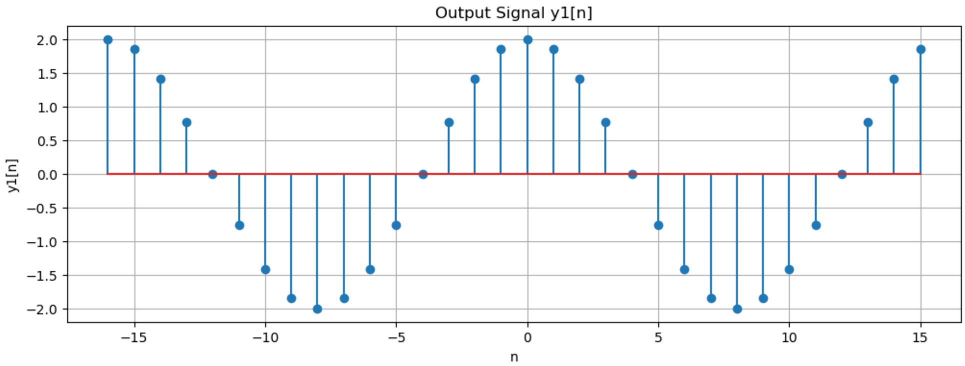

Figure 9: $y_1[n]$

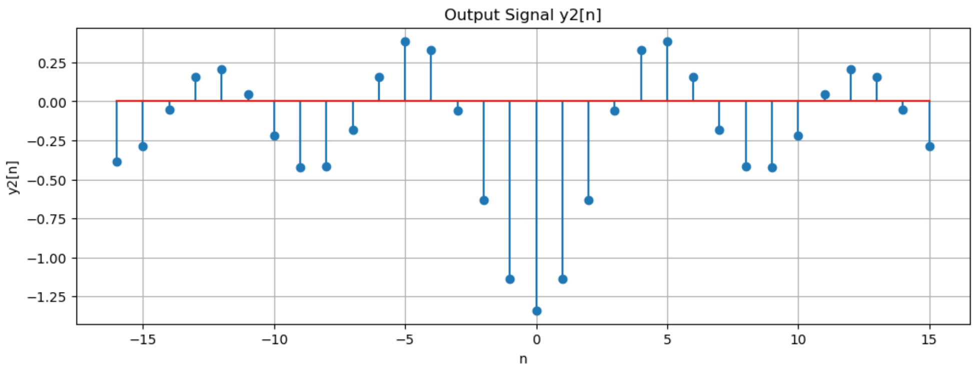

Figure 10: $y_2[n]$

Given the two graphs, y1 looks very similar to a normal cosine function whereas y2 is a little disoriented. This has to do with the individual frequencies of the inputs x1 and x2.

Since the frequency of x1 is $\frac{\pi}{8}$, the period is 16 ($\frac{2\pi}{\pi/8}$). Given that this signal exists for 16 terms, when multiplying it with the H_R, values between -4 and 4 are taken, while the rest are removed, essentially a low-pass filter. With this filter, x1 has no problem because the period of 16 allows it to get past the filter with minimal distortion.

On the other hand, x2 has a frequency of $\frac{\pi}{3}$, resulting in a period of 6 ($\frac{2\pi}{\pi/3}$). 6 does not go easily into [-4, 4], and results in some of the signal being cutoff. It is getting sampled at a different rate, resulting in some values overlapping where they shouldn't be, and others being lost, thus returning the signal shown in figure 10.