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Problem Set 03; Alireza Mayhparast

1. $\chi^2 = \sum_i w_i (y_i - (mx_i + c))^2$ $\quad w_i = \frac{1}{\sigma_{xi}^2 + \sigma_{yi}^2}$

$= \sum_i w_i (y_i^2 - 2y_i(mx_i + c) + (mx_i + c)^2)$

$= \sum_i w_i (y_i^2 - 2y_i m x - 2y_i c + m^2 x_i^2 + 2mcx_i + c^2)$

$$\frac{\partial \chi^2}{\partial m} = \sum_i w_i (-2y_i x_i + 2m x_i^2 + 2c x_i) = 0$$

$$= \sum w_i (-y_i x_i + m x_i^2 + c x_i) = 0$$

$$\frac{\partial \chi^2}{\partial c} = \sum_i w_i (-2y_i + 2mx_i + 2c) = 0$$

$$= \sum w_i (-y_i + mx_i + c) = 0$$

$\bar{x}_w = \frac{\sum_i w_i x_i}{\sum_i w_i}$, $\bar{y}_w = \frac{\sum w_i y_i}{\sum w_i}$, $\overline{x^2_w} = \frac{\sum w_i x_i^2}{\sum w_i}$, $\overline{xy}_w = \frac{\sum_i w_i x_i y_i}{\sum w_i}$

$\overline{xy}_w = m\overline{x^2}_w + c\bar{x}_w$, $\bar{y}_w = m\bar{x}_w + c \rightarrow c = \bar{y}_w - m\bar{x}_w$

$\Longrightarrow \overline{xy}_w = m\overline{x^2}_w + (\bar{y}_w - m\bar{x}_w)\bar{x}_w \Rightarrow m(\overline{x^2_w} - \bar{x}^2_w) = \overline{xy}_w - \bar{x}_w \bar{y}_w$

$\rightarrow m = \frac{\overline{xy}_w - \bar{x}_w \bar{y}_w}{\overline{x^2}_w - \bar{x}^2_w}$

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ignoring the assosiated uncertainties.

$$\chi^2 = \sum_i (y_i - (mx_i + c))^2$$

$$\frac{\partial \chi^2}{\partial m} = -2 \sum_i (y_i - (mx_i + c))\, x_i = 0$$

$$\frac{\partial \chi^2}{\partial c} = -2 \sum (y_i - (mx_i + c)) = 0$$

$$m = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} \quad , \quad c = \bar{y} - m\bar{x}$$

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(c) $$S_{xx} = \sum w_i (x_i - \bar{x}_w)^2 = \sum w_i x_i^2 - \frac{\left(\sum_i w_i x_i\right)^2}{\sum_i w_i}$$

$$\sigma_m = \sqrt{\frac{1}{S_{xx}}} \quad , \sigma_c = \sqrt{\frac{\overline{x^2}_w}{S_{xx}}}$$