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Q-4: $x(t) = A\cos(\omega t + \phi)$, $\phi = [0, 2\pi]$
uniformly distributed

Energy Conservation:

$$E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m v^2$$

$$P(\phi) = \frac{1}{2\pi} \quad , \quad \phi \in [0, 2\pi]$$

we consider snapshots of x depend only on the phase ϕ in the stationary state. Hence:

$$x(t) = A\cos(\phi)$$

$$P(x) = P(\phi) \left| \frac{d\phi}{dx} \right| \quad , \quad \frac{dx}{d\phi} = -A\sin(\phi)$$

$$\rightarrow \left| \frac{d\phi}{dx} \right| = \left| \frac{1}{-A\sin(\phi)} \right| = \frac{1}{A|\sin(\phi)|}$$

$$\sin(\phi) = \sqrt{1 - \cos^2\phi} = \sqrt{1 - \frac{x^2}{A^2}}$$

$$\rightarrow P(x) = P(\phi) \cdot \frac{1}{A\sqrt{1 - \frac{x^2}{A^2}}} = \frac{1}{2\pi} \frac{1}{A\sqrt{1 - x^2/A^2}}$$

$$\rightarrow P(x) = \frac{1}{2\pi\sqrt{A^2 - x^2}} \quad , \quad x \in [-A, +A]$$

TAT

Q-5 :

$$P(x) = \frac{a}{x^{a+1}} \quad x \in [1, \infty)$$

A: $y = x^2 \rightarrow x = \sqrt{y} \quad y \in [1, \infty)$

$$\frac{dy}{dx} = 2x \rightarrow \frac{dx}{dy} = \frac{1}{2\sqrt{y}}$$

$$\rightarrow \quad P(y) = P(x)\left|\frac{dx}{dy}\right| = \frac{a}{(\sqrt{y})^{a+1}} \times \frac{1}{2\sqrt{y}}$$

$$P(y) = \frac{a}{2y^{\frac{1}{2}(a+2)}}$$

B: $Z = \frac{1}{x} \rightarrow x = \frac{1}{Z} \quad Z \in (0, 1]$

$$\frac{dz}{dx} = \frac{-1}{x^2} \rightarrow \frac{dx}{dz} = -\frac{1}{z^2}$$

$$P(Z) = P(x)\left|\frac{dx}{dz}\right| = \frac{a}{Z^{-(a+1)}} \times \frac{+1}{Z^2} = \frac{a}{Z^{-a+1}} = aZ^{a-1}$$

C: $T = \ln(x) \rightarrow x = e^T \quad T \in [0, \infty)$

$$\frac{dT}{dx} = \frac{1}{x} \quad , \quad \frac{dx}{dT} = x = e^T$$

$$P(T) = P(x)\left|\frac{dx}{dT}\right| = \frac{a}{e^{T(a+1)}} \times e^T = \frac{a}{e^{Ta}} = ae^{-aT}$$