Q1: $y_i'' = P_i y_i' + q_i y_i + r_i \;\longrightarrow\; y_i'' = -a y_i' - w^2 y_i + \cos(w_1 t)$

$$y_i'' = \frac{1}{h^2}\left(y_{i+1} - 2y_i + y_{i-1}\right) \quad , \quad y_i' = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$\longrightarrow \quad \frac{1}{h^2}\left(y_{i+1} - 2y_i + y_{i-1} + \frac{a\, y_{i+1} h}{2} - \frac{a\, y_{i-1} h}{2} \right) = -w^2 y_i + \cos(w_1 t)$$

$$\longrightarrow \quad y_{i+1}\underbrace{\left(1 + \frac{ah}{2}\right)}_{\gamma} + y_i\underbrace{\left(w^2 h^2 - 2\right)}_{\beta} + y_{i-1}\underbrace{\left(1 - \frac{ah}{2}\right)}_{\alpha} = \cos(w_1 t)\, h^2$$

$y_0 = A$ & $y_0' = 0 \;\longrightarrow\; y_1 = y_0 + y_0' h = y_0$

$\hookrightarrow e_0 = -A\left(1 - \frac{ah}{2}\right) = A\left(\frac{ah}{2} - 1\right) \qquad e_1 = -y_1 \alpha = -A\alpha$

$$\begin{bmatrix} w^2h^2-2 & 1+\frac{ah}{2} & 0 & 0 & \cdots & 0 \\ \beta & \gamma & 0 & 0 & \cdots & 0 \\ \alpha & \beta & \gamma & 0 & \cdots & 0 \\ 0 & \alpha & \beta & \gamma & \cdots & 0 \\ \vdots & \vdots & \alpha & \beta & \ddots & 0 \\ \vdots & \vdots & & \ddots & \beta & \gamma \\ 0 & 0 & 0 & \cdots & \alpha & \beta \end{bmatrix} \times \begin{bmatrix} y_1 = A + y_0' h \\ y_2 \\ y_3 \\ \vdots \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} \cos(w_1 t_0) h^2 + e_0 \\ \cos(w_1(t+2h))\, h^2 + e_1 \\ \cos(w_1(t+3h))\, h^2 \\ \vdots \\ \\ \\ \end{bmatrix}$$

$h^2 \to y_{-1}$

Q2: $\nabla^2\Phi(x,y)=0$; $\Phi(0,y)=y^2$, $\Phi(x,0)=x$, $\Phi(L,y)=0$, $\Phi(x,L)=1$

300 × 300

$$\nabla^2\Phi(x,y)=\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0$$

$$\frac{\partial^2\Phi(x,y)}{\partial x^2}=\frac{\frac{\partial\Phi(x+\Delta x,y)}{\partial x}-\frac{\partial\Phi(x-\Delta x,y)}{\partial x}}{2h}$$

$$\frac{\partial\Phi(x,y)}{\partial x}=\frac{\Phi(x+\Delta x,y)-\Phi(x-\Delta x,y)}{2h}$$

$$\longrightarrow \frac{\partial^2\Phi(i,j)}{\partial x^2}=\frac{\Phi(i+1,j)-2\Phi(i,j)+\Phi(i-1,j)}{h^2}$$

$$\longrightarrow \frac{\partial^2\Phi(i,j)}{\partial y^2}=\frac{\Phi(i,j+1)-2\Phi(i,j)+\Phi(i,j-1)}{h^2}$$

$$\longrightarrow \nabla^2\Phi(i,j)=\frac{1}{h^2}\left(\Phi(i+1,j)+\Phi(i-1,j)+\Phi(i,j+1)+\Phi(i,j-1)-4\Phi(i,j)\right)=$$

$$\longrightarrow \Phi(i,j)=\frac{1}{4}\left[\Phi(i+1,j)+\Phi(i-1,j)+\Phi(i,j+1)+\Phi(i,j-1)\right]$$

Q4: $\frac{d^2\theta}{dt^2}=-\omega_0^2\sin\theta-\alpha\frac{d\theta}{dt}+f\cos(\omega t)$; $\omega_0=1$, $\alpha=0.2$, $f=0.52$, $\omega=0.666$

$\dot\theta=\frac{d\theta}{dt}$; $\frac{d\dot\theta}{dt}=-\omega_0^2\sin\theta-\alpha\dot\theta+f\cos(\omega t)$ $\qquad$ $\dot\theta_i=\frac{\theta_{i+1}-\theta_{i-1}}{2h}$

$\frac{\dot\theta_{i+1}-\dot\theta_{i-1}}{2h}=-\omega_0^2\sin\theta_i-\alpha\dot\theta_i+f\cos(\omega t_i)$ $\qquad$ $\frac{d\dot\theta}{dt}=\frac{1}{h^2}(\theta_{i+1}-2\theta_i+\theta_{i-1})$

$$\longrightarrow \frac{1}{h^2}(\theta_{i+1}-2\theta_i+\theta_{i-1})=-\omega^2 S\theta_i-\alpha\left(\frac{\theta_{i+1}-\theta_{i-1}}{2h}\right)+f\cos(\omega t_i)$$

$\longrightarrow$