

MC906/MO416 Introduction to Al

Lecture 4





Introduction to Al Lecture 4 - Problembased agents and search without information

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•••• Summary

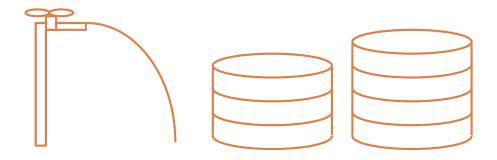
- Model building and problems
 - Goal-based agent
- Search methods without information
 - Breadth-first search
 - Search with uniform cost
 - Depth-first search
 - Depth-limited Search
 - Iterative deepening search
 - Bidirectional search
- Exercise



Model building and problems



There are two jugs: a 4-liter and a 3-liter jug, initially empty, and a fountain that spouts water in abundance.



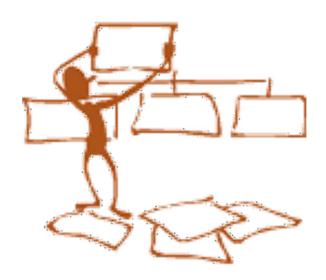
- □ Goal: Get 2 liters in any of the jugs.
- □ Possible actions:
 - Fill the vases
 - Empty the vases
 - Complete one vase with another
 - Throwing the vase content at another

••••• Review...

Agent types:

- Simple reactive agent: Select actions based on current perception, ignoring the rest of the perception history
- Model-based agent: Controls the part of the world that it cannot see now while maintaining an internal state that depends on the history of perceptions
- Goal-based agent: Combines your goal with information on the results of possible actions in order to choose actions that achieve its goals
- Utility-based agent: Use a performance measure (utility function) that allows a comparison between different states of the world, allowing to select the sequence of actions

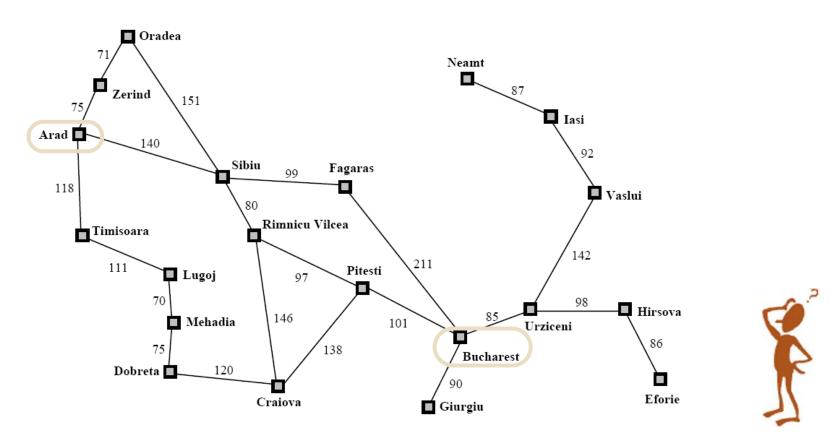
How can an agent find a sequence of actions that achieve its goals when no single action is able to do so?



□ Objective (Goal): set of states in the world

••••• Solving the problem with search methods

- Objective: to go from Arad to Bucharest
- Question: how to model the problem? Observable, discrete and deterministic environment!



•••• Problem definition

Zerind 9

Arad 140

Sibiu

Timisoara

Components:

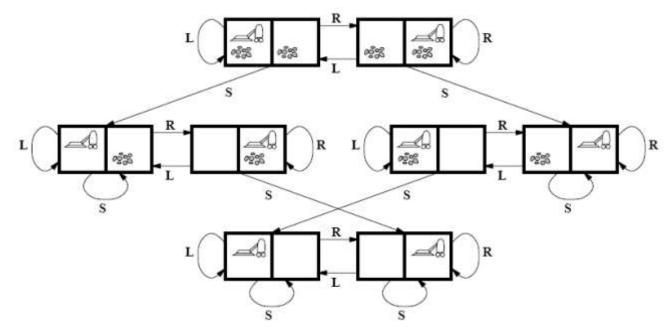
- 1. **Initial state:** where the agent starts.
 - Ex: In(Arad)
- 2. Agent's actions:

SUCCESSOR(x) returns a set of ordered pairs <action, successor>, in that each action is one of the valid actions (**Applicable action**) in state x

Example:

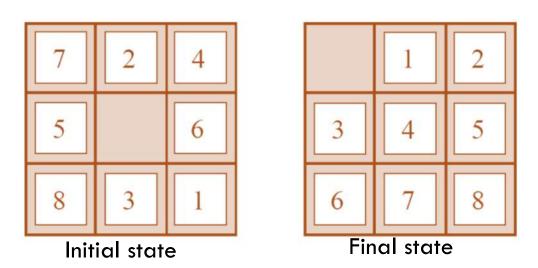
- Objective test: determines whether a given state is the objective state (or goal)
- 4. Path cost: function that assigns a numerical cost to each path

- Initial state: any state
- Successor function: generates valid states that result from trying to perform the three actions (L: Left, R: Right and S: Vacuum). The state space is:



- Objective test: check if all squares are clean
- Path cost: each step costs 1

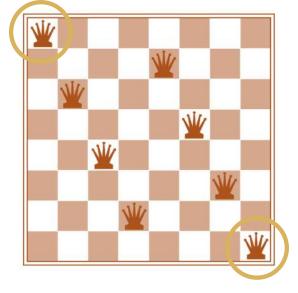
- Initial state: any state
- Successor function: generates the valid states that result from the attempt to perform the four actions: empty space moves to Left, Right, Up, Down.
- Objective test: check if the state corresponds to the final configuration
- Path cost: each step costs 1



- Initial state: empty board
- Successor function: place a queen in any empty square
- Objective test: 8 queens are on the board and none are attacked

Cost of the solution: no interest (only the final state is

important)





There are a total of 92 possible solutions and 12 distinct ones (excluding the symmetric solutions of rotation and reflection).

••••• Real world problems ...

- Problems in determining trajectories (computer networks, military action planning, air travel, ...)
- VLSI layout problems
- Robot navigation issues
- Automatic assembly sequence problems
- □ Protein design
- Internet searches
- ---

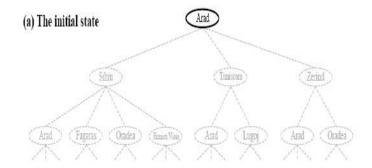
•••• A simple problem-solving agent

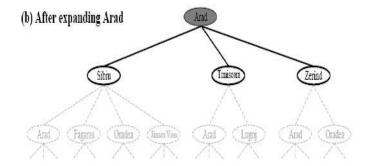
```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
          persistent: seq, an action sequence, initially empty
                     state, some description of the current world state
                    qoal, a goal, initially null
                     problem, a problem formulation
          state ← UPDATE-STATE(state, percept)
          if seq is empty then
                    qoal ← FORMULATE-GOAL(state)
                    problem ← FORMULATE-PROBLEM(state, goal)
                    sea ← SEARCH(problem)
                    if seg = failure then return a null action
          action \leftarrow FIRST(seq)
          seq \leftarrow REST(seq)
          return action
```

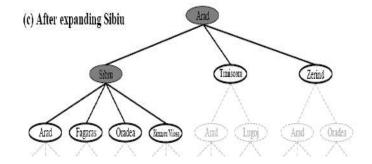
••••• In search of solutions ...

- State Space Search
 - Current state
 - 2. Expansion
 - Generation of new states
- Tree search techniques:
 - What is the best strategy?











How to represent the state of the game below?

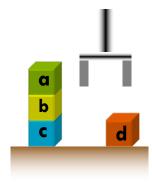


Vector: [6,3,8,*,6,7,1,4,2]

Logic:

on_place(1,6); on_place(2,3); on_place(3,8); on_place(4,0)...

□ Another Example → Blocks World



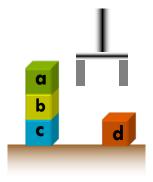
on(a,b)
on(b,c)
ontable(d)
ontable(c)
clear(a)
clear(d)
handempty

Characteristics of States

■ **Negation as Failure**: The state represents only what is true (TRUE) in the world. What is not represented is considered false.

Data structure

- A state can be a vector of strings (name of predicates attribute) Bad !!!
- A state can be a vector of integers where each integer represents a predicate (Mapping each predicate to an integer).
 - Ex: world of Blocks



predicate	Number
handempty	1
holding (a)	2
holding (b)	3
holding (c)	4
holding (d)	5

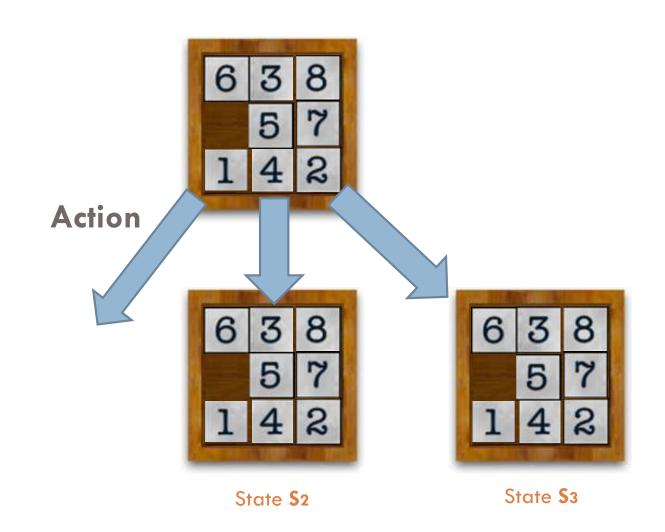
predicate	Number
on(a,a)	6
on(a,b)	7
on(a,c)	8
on(a,d)	9
on(b,a)	10

••••• States

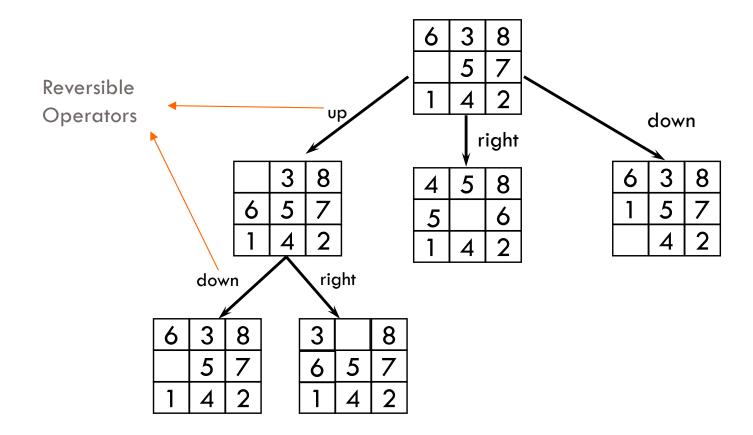
Examples of Data Structures

- Integers Vector
 - **[1,4,6,7,3,2]**
 - Querying a predicate is slow (traverses the entire vector)
 - Determining the set of predicates is quick (it is the vector itself)
- Representative string
 - **1111101100**
 - Quick query of a predicate (just check the position)
 - Determining the set of predicates is slow (go through the entire string)
- All of this must be defined at the time of structuring knowledge representation
- Define which representation to use (vector, matrix, Logic mapped to integers, etc.)
- Define representation in a coherent Data Structure

State So

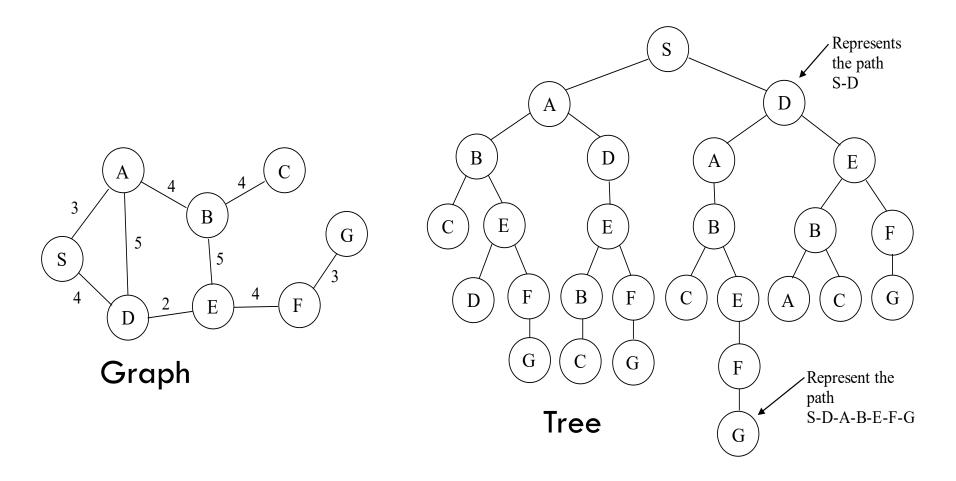


State **\$1**

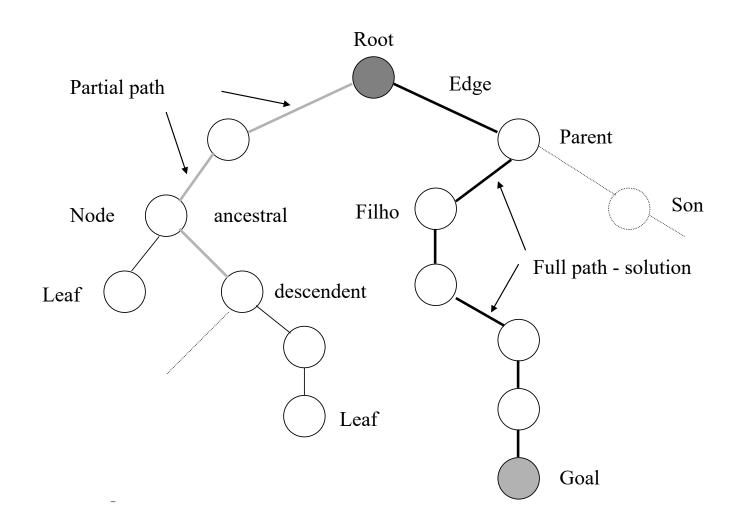


••••• State Space in Graph and Tree

A graph and its tree-like equivalent



Detailing a tree



••••• General tree search algorithm

function TREE-SEARCH(problem) returns a solution, or failure initialize the list using the initial state of problem

loop do

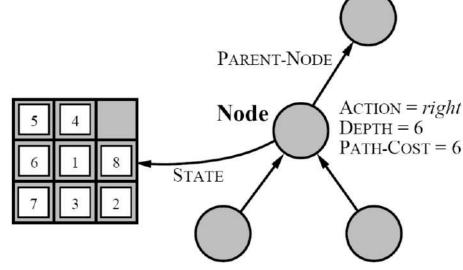
if the list is empty then return failure choose a leaf node and remove it from the list

if the node contains a goal state then **return** the corresponding solution

expand the chosen node, adding the resulting nodes to the list

- □ State: the state in the state space that the node corresponds to
- Parent node: the node in the search tree that generated this node
- Action: the action that was applied to the parent to generate the node
- Path cost: the cost, traditionally denoted by g(x), of the path from the initial state to the node

Depth: the number of steps along the path, from the initial state





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Lecture 4 - PART II





Blind Search Methods



••••• Representing a collection of nodes

- □ Node Collection: Queue
 - Possible operations on the queue:
 - CREATE-QUEUE (element, ...): creates a queue with the given element (s)
 - EMPTY?(queue): returns true only if there are no more elements in the queue
 - □ FIRST(queue): returns the first element of the queue
 - REMOVE-FIRST(queue): returns FIRST (queue) and removes it from the queue
 - INSERT (element, queue): inserts an element into the queue and returns the resulting queue
 - INSERT-ALL(elements, queue): inserts a set of elements into the queue and returns the resulting queue

••••• General tree search algorithm

```
function TREE-SEARCH(problem, list) returns a solution, or
list
  list ← INSERT(CRREATE-NODE(INITIAL-STATE[problem]),
  list)
  repeat
      if EMPTY?(list) then return fail
      node ← REMOVE-FIRST(list)
      if TEST-GOAL[problem] applyied STATE[node]
            sucessful then return SOLUTION(node)
      list ← INSERT-ALL(EXPAND(node, problem), list)
```

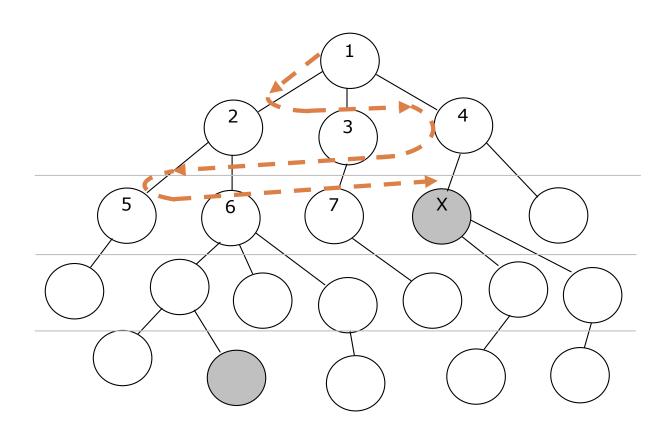
•••• Node expansion strategies

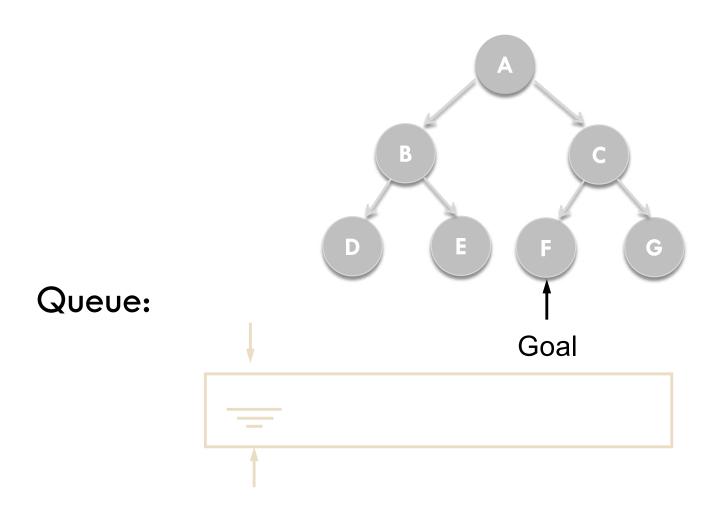
- Blind search strategies (without information about the problem):
 - Breadth-first search
 - Search with uniform cost
 - Depth-first search
 - Depth-limited Search
 - Iterative deepening search
 - Bidirectional search

- Completeness: Does the algorithm guarantee to find a solution when it exists?
- Optimization: Does the strategy find the optimal solution?
- Time complexity: How long does it take to find a solution?
- Memory complexity: How much memory is needed to perform the search?

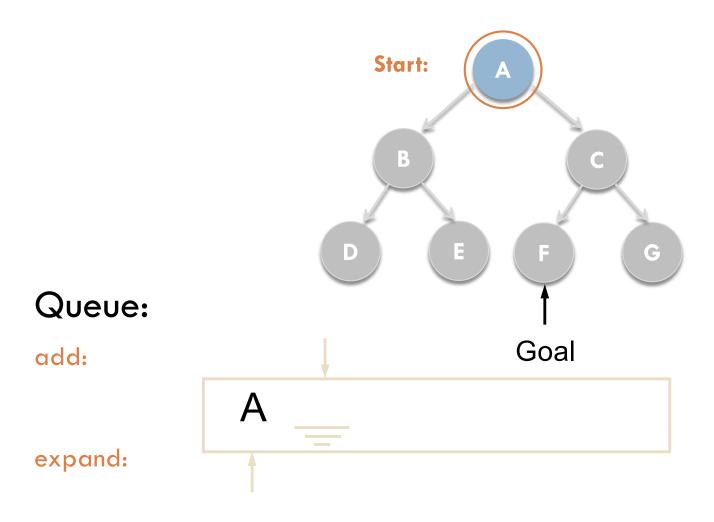
- The root node is expanded first, then all successors in the root only, then the successors of those nodes, and so on
- It can be implemented by doing:
 - SEARCH-IN-TREE (problem, QUEUE-FIFO())
- QUEUE "First In First Out":
- Place all newly arrived successors at the end of the queue
- Choose as successor the first node in the queue (shallow nodes expanded first)

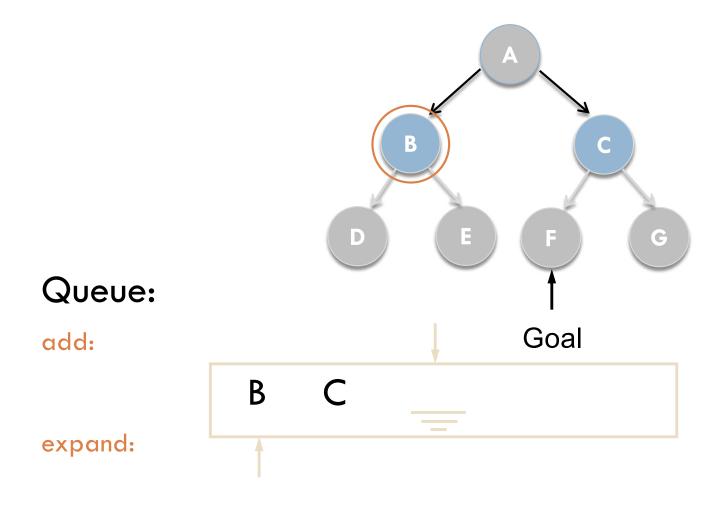
□ Path strategy in search of the solution:

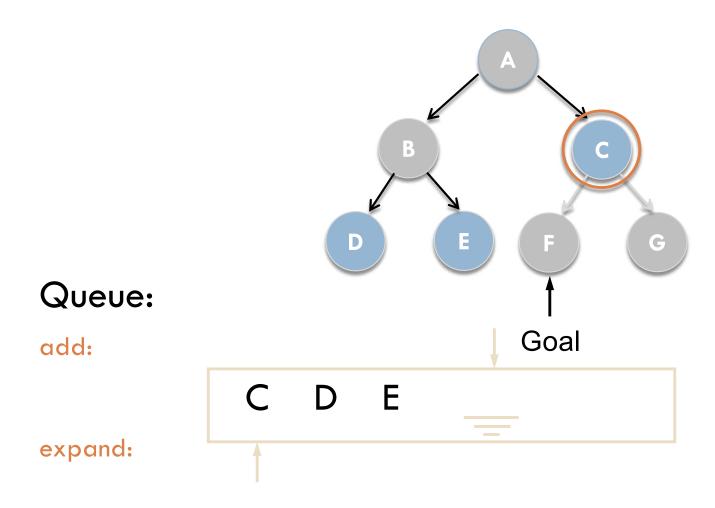


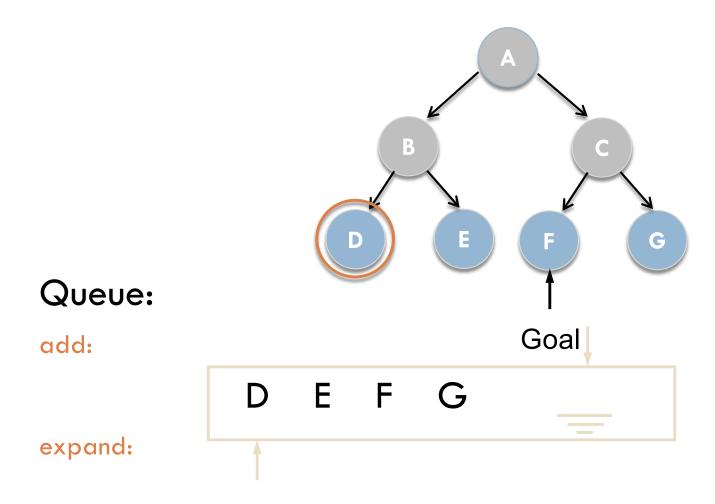


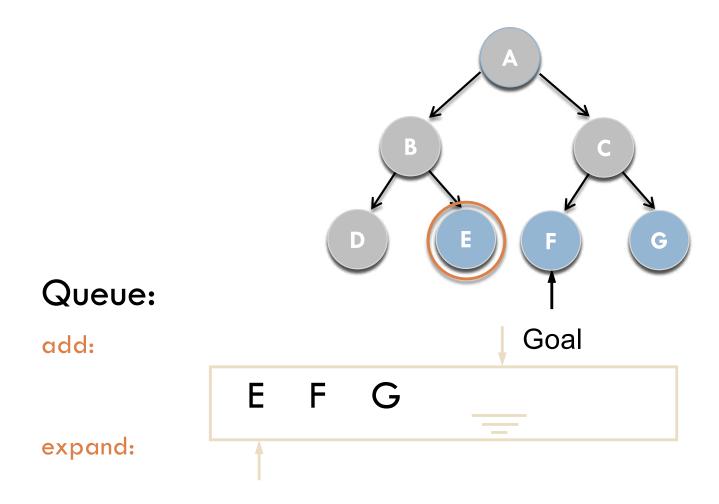
•••• Breadth-first Search: example



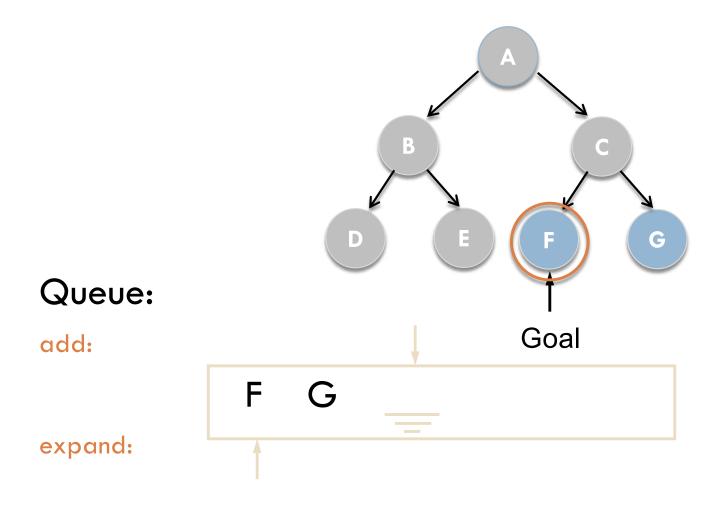


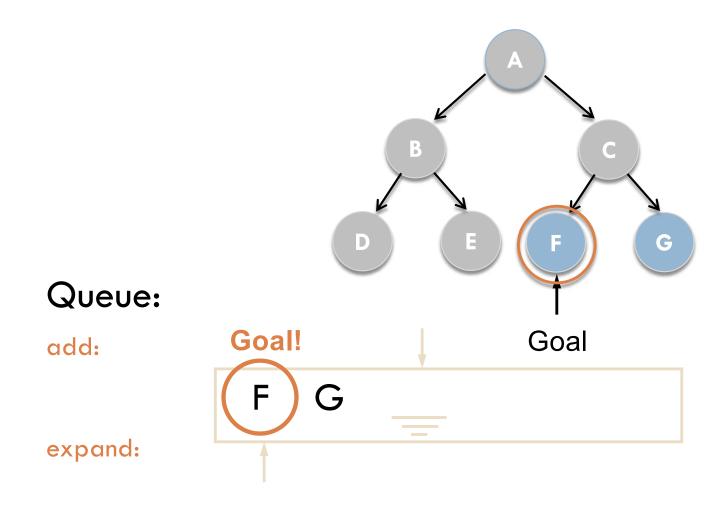






•••• Breadth-first Search: example





Consider:

- b: average number of successors for each node
- d: shallowest depth of solution





Time Complexity:

- Nodes per level: $1 \rightarrow b^1$, $2 \rightarrow b^2$, $3 \rightarrow b^3$, ..., $n \rightarrow b^n$
- □ All nodes are expanded to level d, except the last (solution). Like: $b+b^2+b^3+...+b^d+(b^{d+1}-b)=O(b^{d+1})$

Memory Complexity:

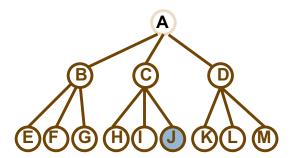
Nodes that have already been visited need to be stored in another vector in memory, as it may be necessary to consult them after finding the goal to determine the path from the initial state to the goal. Therefore, every node remains in memory (memory complexity is equal to time complexity)

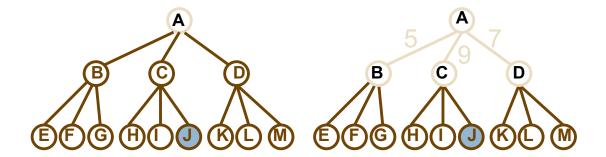
- Always expands the shallowest node
- Memory and time requirements are a big problem

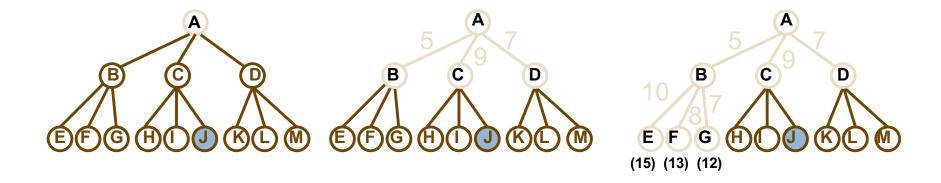
Depth	Nodes	Time	Memory
2	1,100	0,11 seg	1 megabyte
4	111,100	11 seg	106 megabyte
6	107	19 min	10 gigabyte
8	109	31 hours	1 terabyte
10	1011	129 days	101 terabyte
12	1013	35 years	10 pentabyte
14	1015	3.523 years	1 exabyte

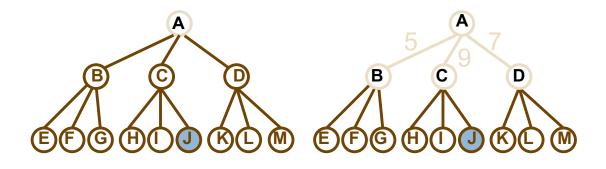
Suppose b=10, analyse of 10,000 nodes/sec and 1,000 bytes/node

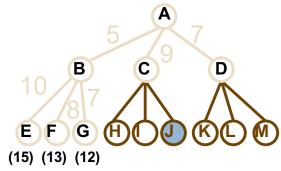
- Breadth search: optimal when the cost of all steps are equal
- Uniform cost search: instead of expanding the shallowest node, expand the node with the lowest cost path. Expands nodes in order of increasing cost
- □ Uses g(n)
 - "It doesn't matter how many steps the path takes, but only its total cost"
 - If all step costs are the same, it is identical to the search in extension
 - Use a priority Queue!

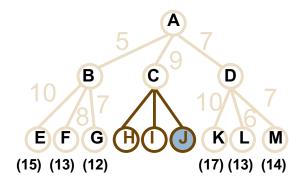


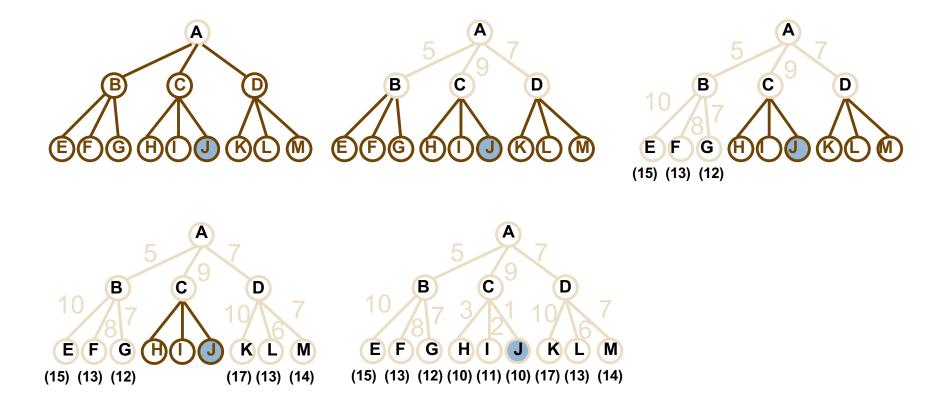


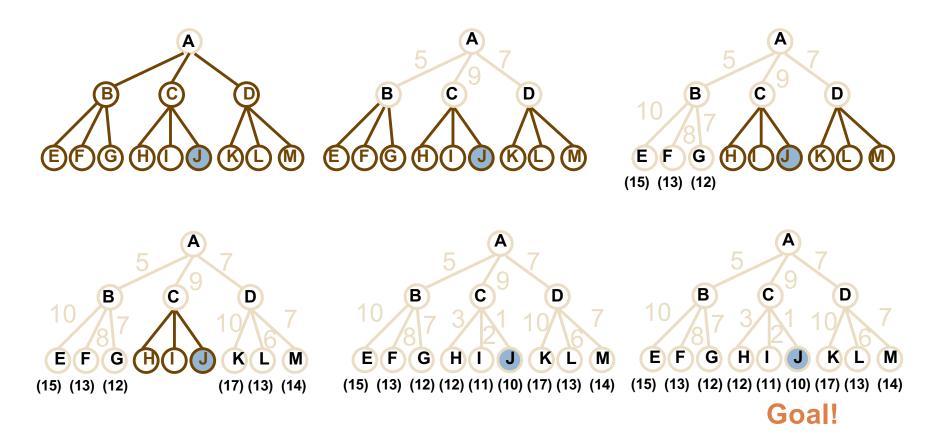






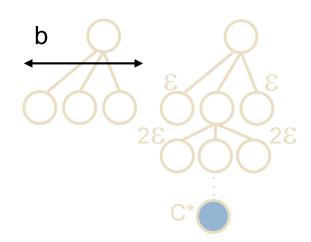






Explore large trees with small steps before exploring paths involving large steps.

- Difficult to characterize in terms of b and d, since the search is path-oriented
- Since the algorithm can explore deeper solutions than d first, its complexity can be greater than O(b^d)
- Performance:
 - **Completeness:** complete if step cost $c \ge \epsilon$
 - Optimization: optimal
 - Memory and Time Complexity: $O(b^{[C^*/\epsilon]})$
 - Optimal solution cost: C*
 - Cost of each step \geq = ϵ
 - The tree can be reinterpreted as dependent on cost. In the worst case, all nodes costing less than C* will be analyzed first



•••• Depth-first

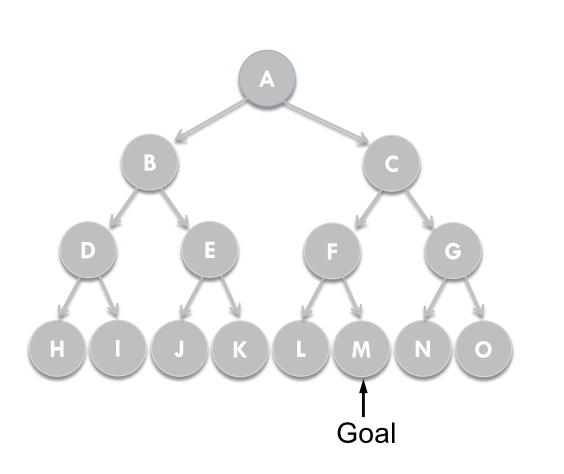
- Always expands the deepest node in the search tree.
- It can be implemented by doing:
 - TREE SEARCH(problem, STACK)



- Stack (Last in First Out):
 - Puts all newly arrived successors at the top of the stack
- Choose as successor the first node in the stack (last arrived nodes expanded first)

•••• Depth-first Search

Put the right nodes in the stack first

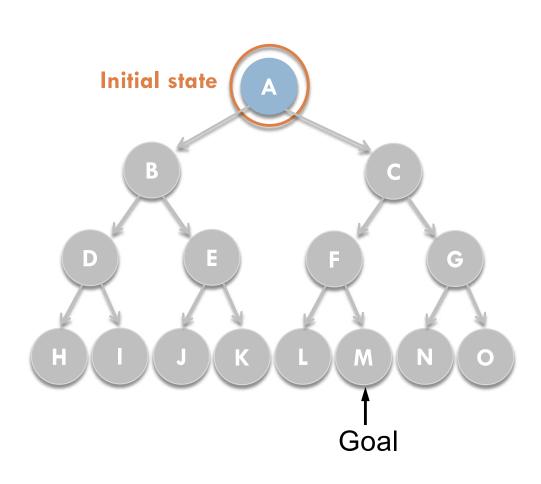


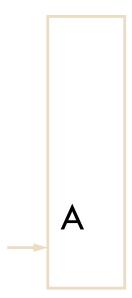


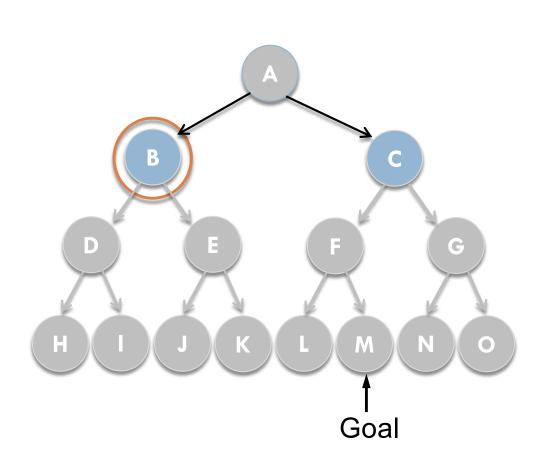


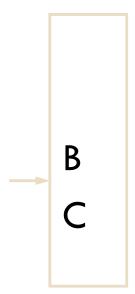
•••• Depth-first Search

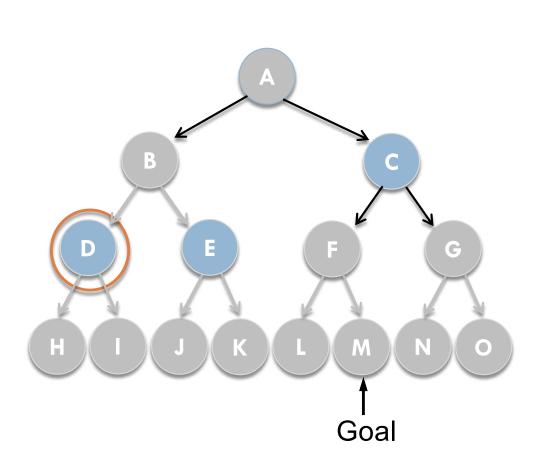
Put the right nodes in the stack first

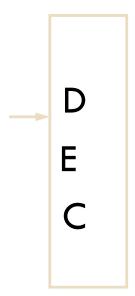


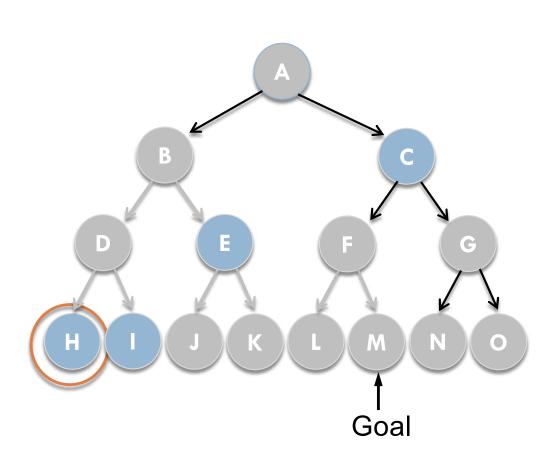




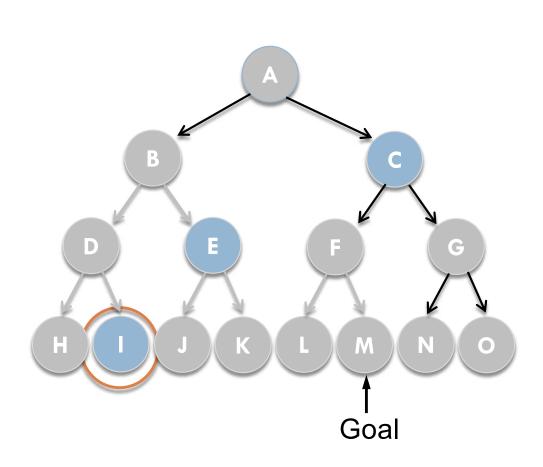




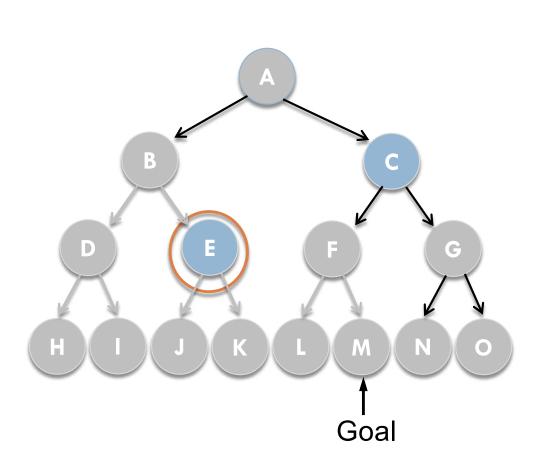




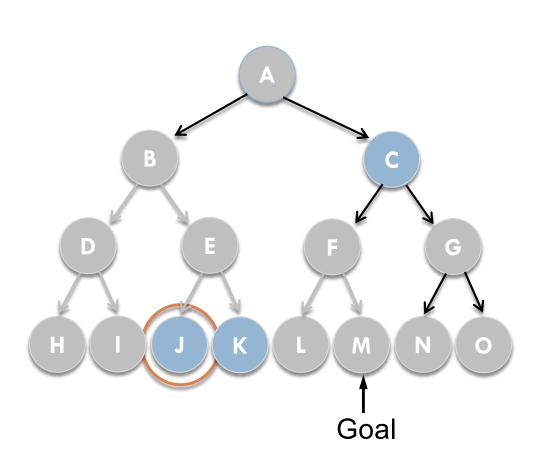


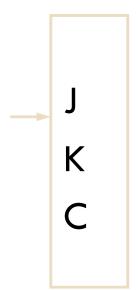


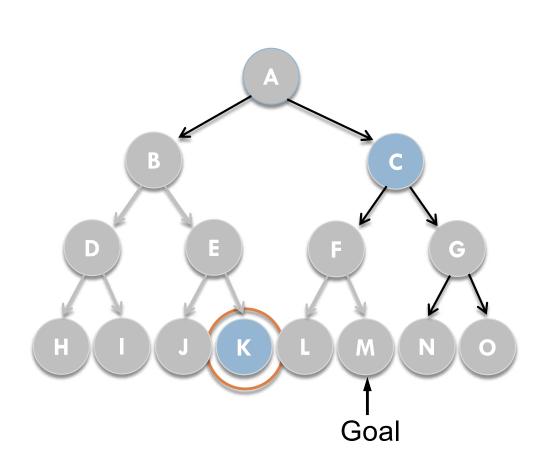


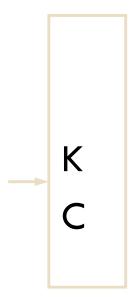


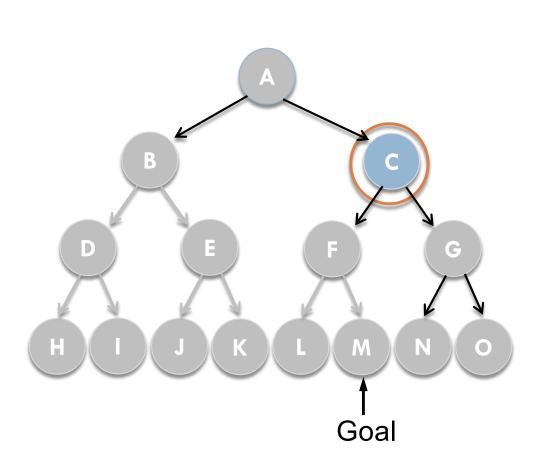




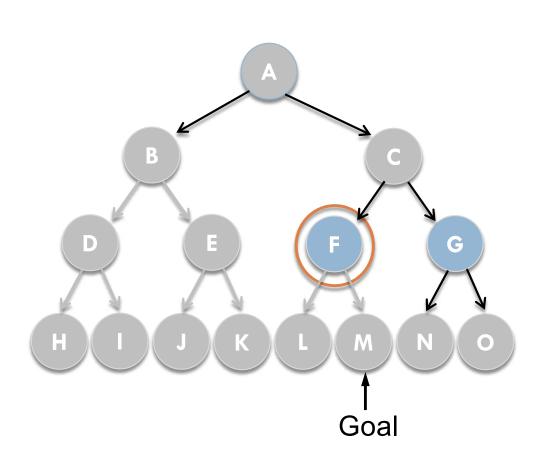


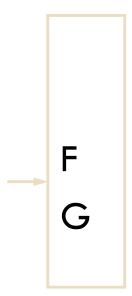


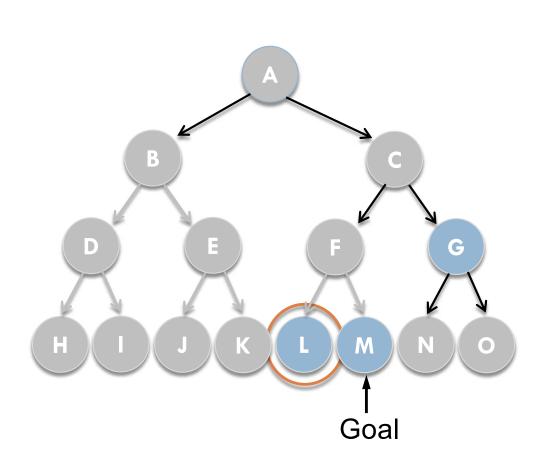


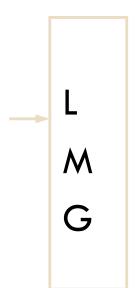


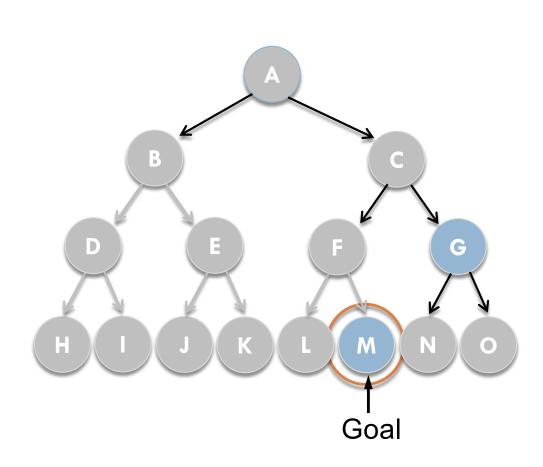


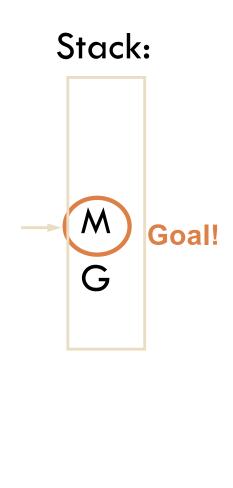








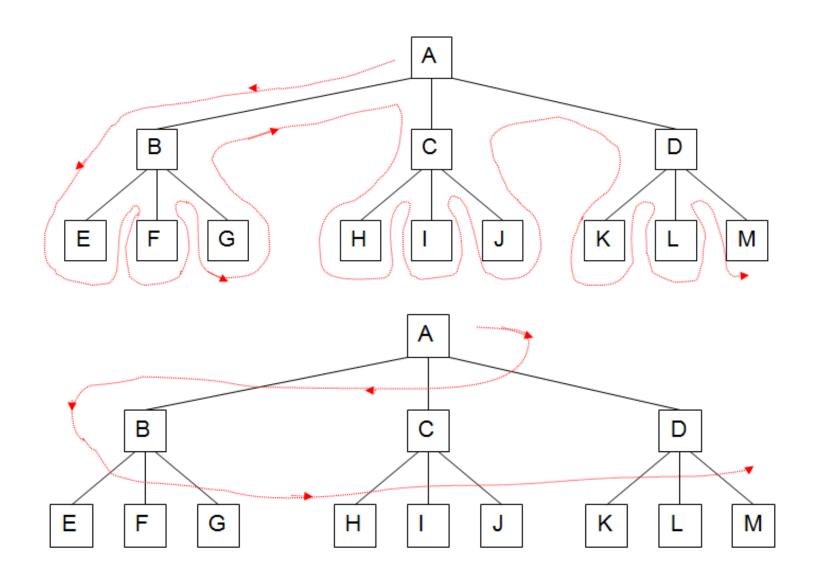




•••• Depth-first Search: performance

- Consider:
 - b: average number of successors for each node
 - d: maximum depth
- Completeness: is not complete (if a branch has no end, the depth search never ends)
- Optimization: not optimal (does not always return the lowest cost node)
- □ Time Complexity: $O(b^m)$
- Memory Complexity: Once a branch of memory (a node and its descendants) has been fully explored, that part of the path can be removed from memory. It is essential that only a single path from the root to the leaf node being analyzed remain in memory, along with its non-expanded sibling nodes.

Hence: it stores bm+1 nodes, i.e.: O(bm)



- Depth search with limited depth limit in L
- □ Performance:
 - Completeness: incomplete if L<d
 - Optimization: Not optimal if L>d
 - Time complexity: $O(b^l)$
 - Memory complexity: O(bL)



•••• Depth-limited Search: algorithm

```
function DEPTH-LIMITED-SEARCH(problem, limit) returns a solution, or failure/cutoff return RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns a solution, or failure/cutoff
```

```
if GOAL-TEST(node) then return SOLUTION(node)

else if limit = 0 then return cutoff

else

cutoff occurred? ← false

for each action in ACTIONS(STATE) do

child ← CHILD-NODE(problem, node, action)

result ← RECURSIVE-DLS(child, problem, limit − 1)

if result = cutoff then cutoff_occurred? ← true

else if result!= failure then return result

if cutoff occurred? then return cutoff else return failure
```

- □ Gradually increase the depth limit (0,1,2,...) until you find the objective (L = d)
- Combines the advantages of searching in width and depth
- Algorithm:

function ITERATIVE-DEEPENING-SEARCH(*problem*) **return** a solution or failure

for depth ← 0 to ∞ do
 result ← DEPTH-LIMITED-SEARCH(problem, depth)
 if (result≠cut) then return result

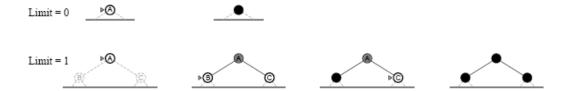
••••• Iterative Deepening Search



Limit = 0 ▶⊗

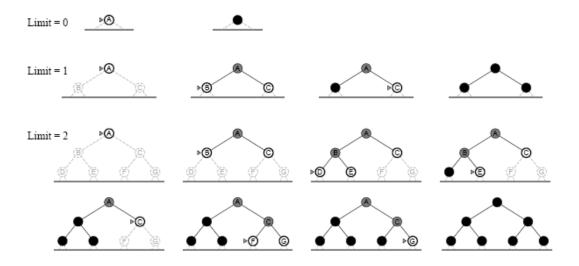
••••• Iterative Deepening Search

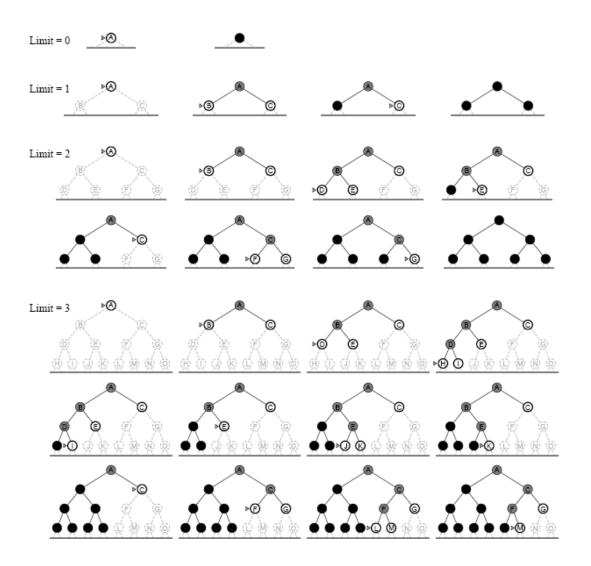




••••• Iterative Deepening Search





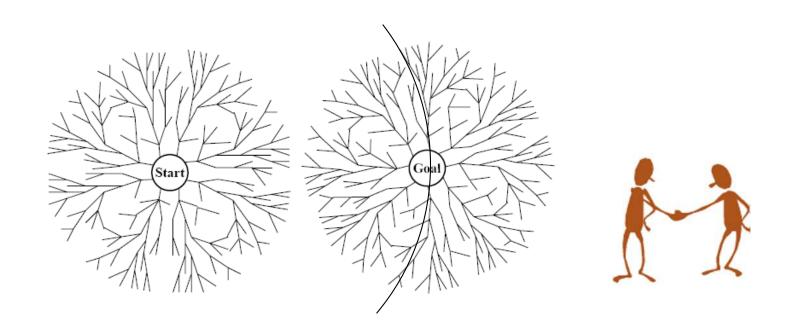


- Completeness: complete, as in the breadth-first search
- Optimization: optimal, as the breadth-first search
- □ Time complexity:

It may seem like a waste, but ...

- The first level nodes are generated **d** times, the second level nodes are generated (**d-1**) times, and so on ...
- Hence: $(d)b + (d-1)b^2 + ... + (1)b^d \rightarrow O(b^d)$
- Memory complexity: it is a depth-depth search. Hence: O(b.d)

- Perform two simultaneous searches: one from the initial state to the objective, the other from the objective to the initial state, stopping when the searches are in the intermediate state
- \square Motivation: $b^{d/2}+b^{d/2}$ is much smaller than b^d



- □ Performance:
 - Completeness: complete
 - Optimization: optimal (for uniform cost steps)
 - Time complexity: O(b^{d/2})
 - Memory complexity: O(b^{d/2})
- □ Restrictions:
 - The goal needs to be known
 - The predecessors of a node must be computable

Search strategies without information: comparative table

Criteria	Breadth-first	Uniform cost	Depth-first	Depth Limited	Iterative Deepening	Bidirec. (if aplicable)
Complete	Yes	Yes	No	No	Yes	Yes
Time	O(b ^{d+1})	O(b ^[C*/ε])	O(b ^m)	O(b ^L)	O(b ^d)	O(b ^{d/2})
Memory	O(b ^{d+1})	O(b ^[C*/ε])	O(bm)	O(b ^L)	O(bd)	O(b ^{d/2})
Optimal	Yes	Yes	No	No	Yes	Yes

b branching factor

d depth of solution

m maximum depth of the search tree

I depth limit

C* cost of the optimal solution

••••• Repeated states

- The search process can waste time expanding nodes already explored before
 - Repeated states can lead to infinite loops
 - Repeated states can turn a linear problem into an exponential problem

Strategy

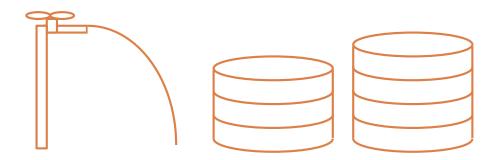
- Compare nodes about to be expanded with nodes already visited.
- If the node has already been visited, it will be discarded.
- A list stores nodes already visited.



MC906/MO416 Introduction to Al

Lecture 4 - PART III





There are two vessels: a 4-liter and a 3-liter vessel, initially empty, and a fountain that spouts water in abundance.

Objective: Get 2 liters in any of the jugs.

Possible actions:

- Fill the vases
- Empty the vessels
- Complete one vase with another
- □ Throwing a vase at another

- \square State: (x, y), where:
 - x is the amount of water in the 4 Liter container;
 - y is the amount of water in the 3 Liter vessel
- □ Initial state: (0,0)
- Target States: (X, 2) or (2, X)
- □ Rules:
 - R0: Fill_jug_4
 - R1: Fill_jug_3
 - R2: Empty_jug_4
 - R3: Empty_jug_3
 - R4: Complete_3_with_4
 - R5: Complete_4_with_3
 - R6: Empty_4_in_3
 - R7: Empty_3_in_4

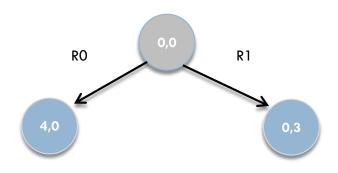
••••• Water jug problem: other solutions



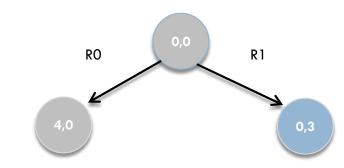
Breadth-first search:



```
RO: Fill_jug_4
```



```
RO: Fill_jug_4
```



Is it a goal? No, expand

```
RO: Fill_jug_4
```

R1: Fill_jug_3

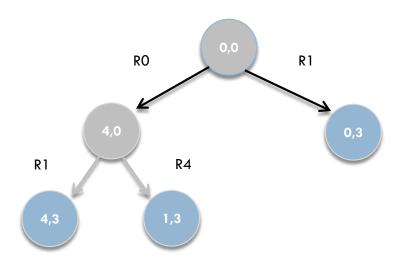
R2: Empty_jug_4

R3: Empty_jug_3

R4: Complete_3_with_4 R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



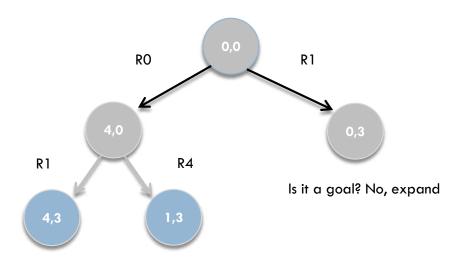
RO: Fill_jug_4

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



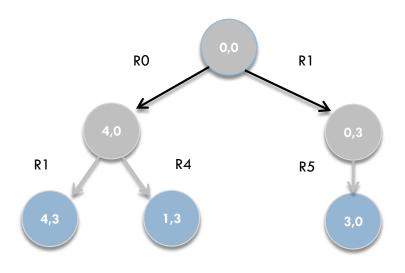
```
RO: Fill_jug_4
```

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



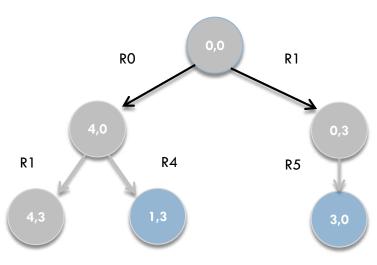
RO: Fill_jug_4

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



Is it a goal? No, expand

Nothing to do as all possible actions lead to a state that has been visited (4,0) ou (0,3)

RO: Fill_jug_4

R1: Fill_jug_3

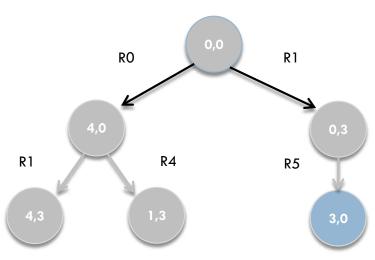
R2: Empty_jug_4

R3: Empty_jug_3

R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



Is it a goal? No, expand

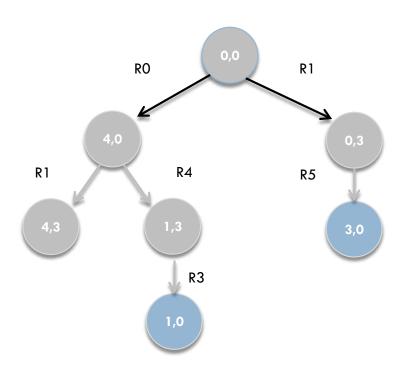
RO: Fill_jug_4

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



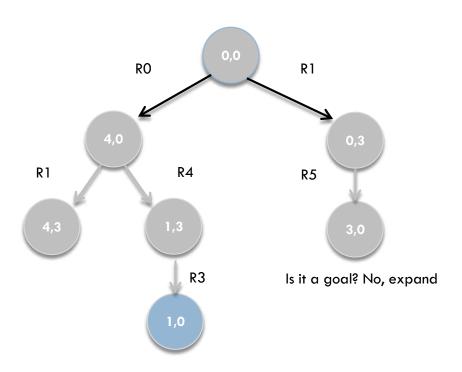
```
RO: Fill_jug_4
```

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



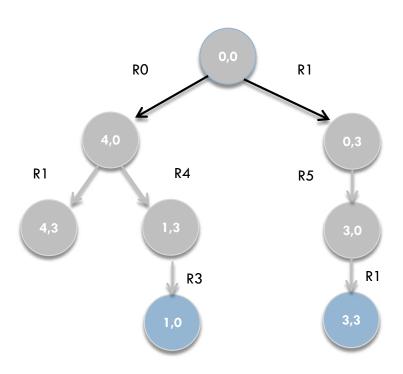
```
RO: Fill_jug_4
```

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



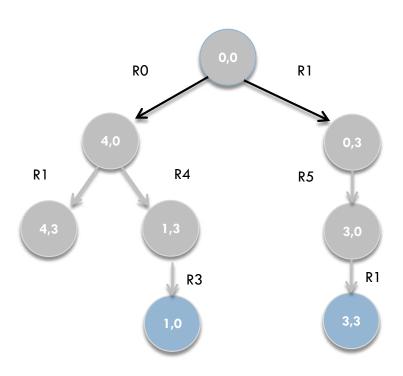
RO: Fill_jug_4

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



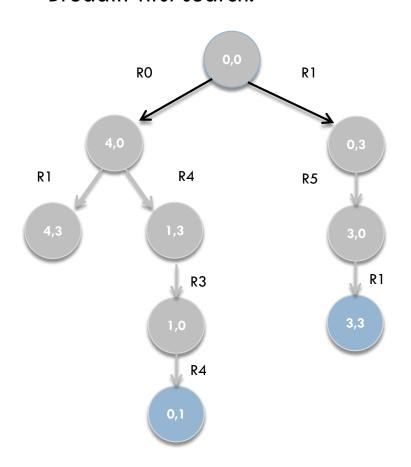
RO: Fill_jug_4

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



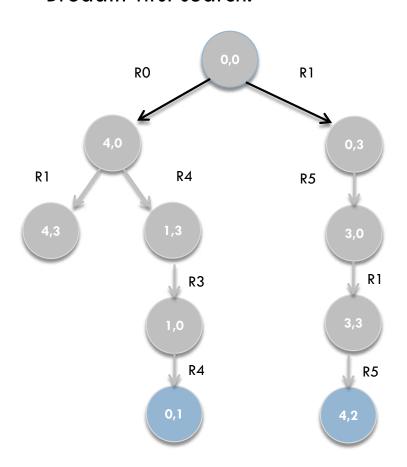
```
RO: Fill_jug_4
```

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



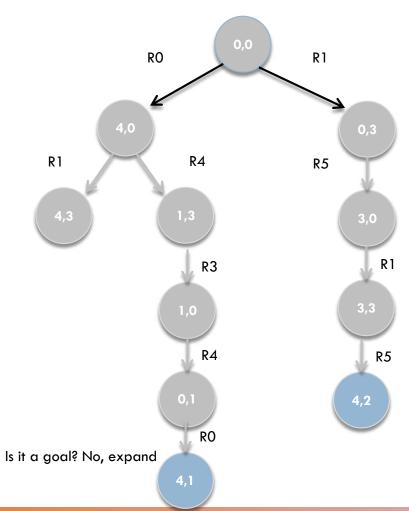
```
RO: Fill_jug_4
```

R1: Fill_jug_3 R2: Empty_jug_4

R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



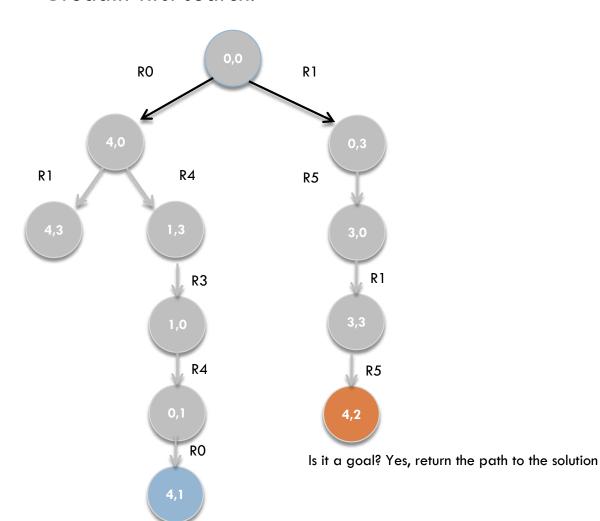
```
RO: Fill_jug_4
```

R1: Fill_jug_3 R2: Empty_jug_4

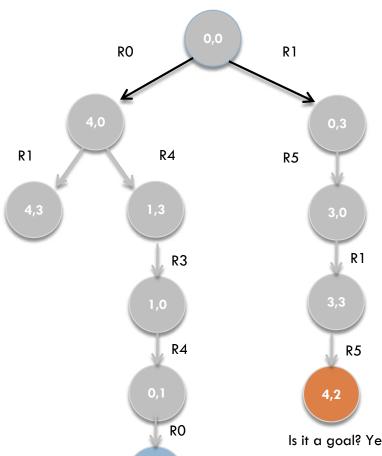
R3: Empty_jug_3
R4: Complete_3_with_4
R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4



```
RO: Fill_jug_4
```



RO: Fill_jug_4

R1: Fill_jug_3

R2: Empty_jug_4

R3: Empty_jug_3

R4: Complete_3_with_4

R5: Complete_4_with_3

R6: Empty_4_in_3

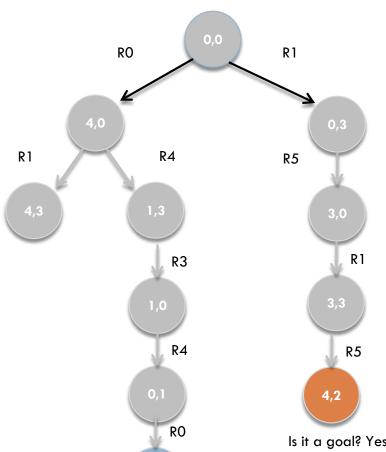
R7: Empty_3_in_4

From RO->R7

Path to solution: R1, R5, R1, R5

Is it a goal? Yes, return the path to the solution

4,1



RO: Fill_jug_4

R1: Fill_jug_3

R2: Empty_jug_4

R3: Empty_jug_3

R4: Complete_3_with_4

R5: Complete_4_with_3

R6: Empty_4_in_3

R7: Empty_3_in_4

From RO->R7

What is the order of expansion of the nodes?

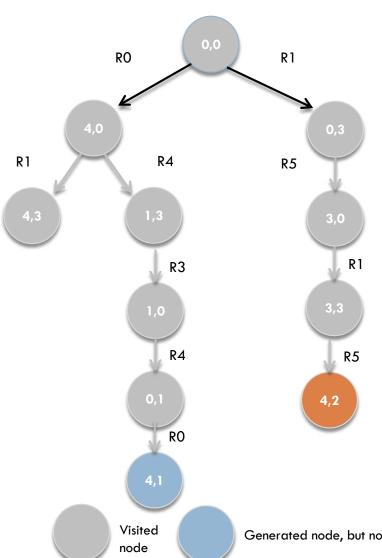
(0,0), (4,0), (0,3), (4,3), (1,3), (3,0), (1,0), (3,3), (0,1), (4,2)

Which nodes were created?

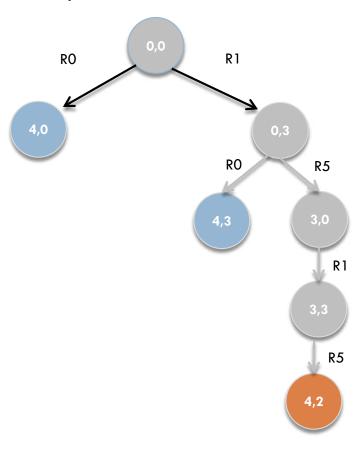
(0,0), (4,0), (0,3), (4,3), (1,3), (3,0), (1,0), (3,3), (0,1), (4,1), (4,2)

Is it a goal? Yes, return the path to the solution

4,1



Depth-first search:

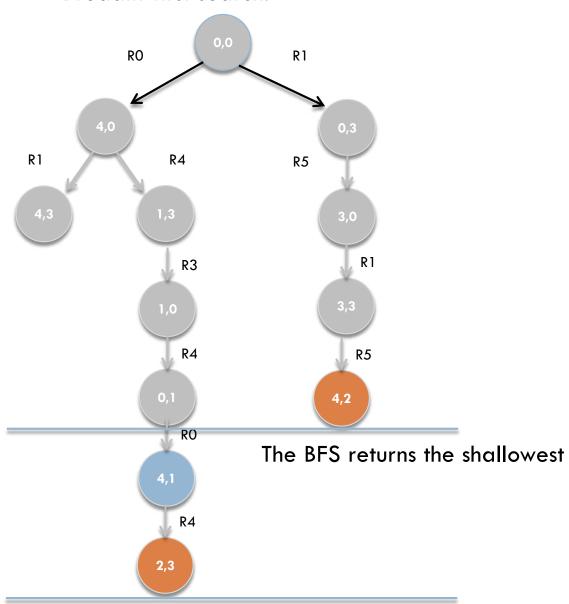


Generated node, but not visited. Considering the rules executed from R0 to R7

••••• Water jug problem: other solutions

If we could look deeper?

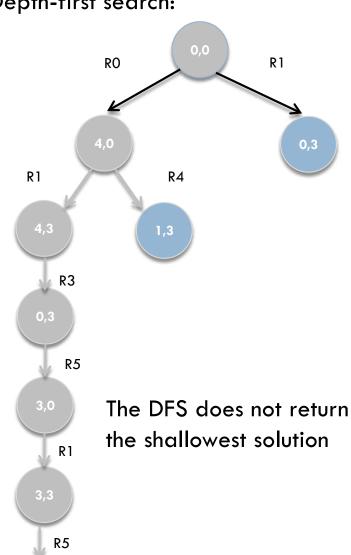
Breadth-first search:



Considering the we are opening the leftmost node

Depth-first search:

4,2

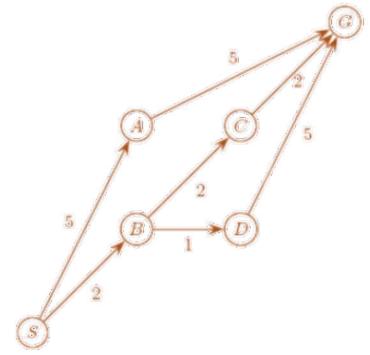


••••• Path problem

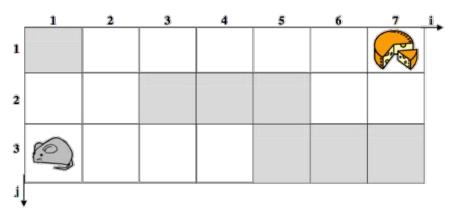


Consider node S as the initial state, G as the sole objective. The labels on the edges indicate the cost of traversing them. Consider the order of opening the nodes to be alphabetical. Compare the generated trees and paths found by one:

- Breadth-first search
- Depth-first search
- Search with uniform cost



- The drawing below shows a mouse in a maze looking for a piece of cheese. In this maze, the mouse can perform the following actions:
 - R1: walk to the right
 - R2: walk to the left
 - R3: walk up
 - R4: walk down



- \Box The rules are always applied in this order (R1, R2, R3, R4).
- □ White cells represent possible pathways and dark cells represent walls that the mouse cannot pass through. The cost of traveling from one square to another is given by g = 1, for each square covered in the maze. Admit that the mouse maintains a control to avoid repeated positions.
- Admit that the mouse performs the search offline, that is, it processes the entire search tree before going out in search of cheese. Admit that the mouse runs the entire algorithm to determine the best path for the cheese and only then begins its locomotion.
- Indicate the path the mouse will take in its search and display the search tree for the BFS and DFS algorithms.

Lecture 4

- Activities
 - Reading:
 - RUSSELL, S. NORVIG, P. Inteligência Artificial. 3° edição. Capítulo 3.
 - Recommended exercises:
 - 3.1 to 3.15

Lecture 4

- RUSSELL, S. NORVIG, P. Inteligência
 Artificial. 3^a edição.
- Simões, A. S. Slides de aula: lA para Controle e Automação.