Semana 7 - Vitor Carvalho Marx Lima
11821 - ECP015

Capítulo 4 - NISE

Questão 2 -

a) $G(s) = \frac{5}{s(s+5)}$

$$\frac{5}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} \rightarrow \frac{5}{s(s+5)} = \frac{A(s+5) + Bs}{s(s+5)}$$

$5 = As + 5A + Bs$

$5 = As + Bs + 5A$

$5 = (A+B)s + 5A$

$0 = (A+B)s \rightarrow A + B = 0$

$\rightarrow \boxed{A = 1} \rightarrow \boxed{5A = 5}$

$\boxed{B = -1}$

$$\rightarrow \frac{5}{s(s+5)} = \frac{1}{s} + \frac{(-1)}{s+5} \rightarrow \frac{1}{s} - \frac{1}{s+5}$$

$$c(t) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+5}\right) \rightarrow c(t) = 1 - e^{-5t}$$

$$c(t) = (1 - e^{-at})$$

$-5 = -a$

$\boxed{a = 5}$ $\rightarrow T = \frac{1}{a} \rightarrow T = \frac{1}{5}$

$\boxed{T = 0{,}2}$

$$T_r = \frac{2,2}{a} = \frac{2,2}{5} = 0,44\,s$$

$$T_s = \frac{4}{a} = \frac{4}{5} = 0,8\,s$$

b) $\frac{1}{s} \cdot \frac{20}{s+20} = C(s)$

$$\frac{20}{s(s+20)} = \frac{A}{s} + \frac{B}{s+20} \rightarrow 20 = As + 20\cancel{A} + Bs$$

$$20 = (A+B)s + 20A$$

$$0 = (A+B)s \rightarrow A+B=0 \rightarrow 20A = 20 \rightarrow \boxed{A=1}$$

$$\boxed{B=-1}$$

$$c(t) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+20}\right) = 1 - e^{-20t}$$

$$C(t) = 1 - e^{-at} = 1 - e^{-20t} \rightarrow a = 20$$

$$\boxed{\begin{array}{l} T = \frac{1}{a} = \frac{1}{20} = 0,05\,s \\ T_r = \frac{2,2}{a} = \frac{2,2}{20} = 0,11\,s \\ T_s = \frac{4}{a} = \frac{4}{20} = 0,2\,s \end{array}}$$

Questão (4)-

$$5 - 2\frac{dq}{dt} - \frac{q}{0,5} = 0$$

$$2q = V \rightarrow 2\frac{dq}{dt} + 2q = 5 \rightarrow \frac{dV}{dt} + V = 5$$

$$V = \frac{5}{s+1} \rightarrow T_c = \frac{1}{1} = 1s, \quad T_r = \frac{2,2}{1} = 2,2s$$

$$T_s = \frac{4}{1} = 4s$$

Questão (6)-

$$M\ddot{x} + 6\dot{x} = f$$

$$M\dot{v} + 6v = f$$

$$T(s) = \frac{1}{Ms+6} \rightarrow s = -\frac{6}{M}$$

$$T_r = \frac{2,2M}{6} = 0,366M$$

$$T_s = \frac{4M}{6} = \frac{2M}{3}$$

Questão (08) -

a) Resposta = $a + ke^{-2t}$

b) Resposta = $a + k_1 e^{-3t} + k_2 e^{-6t}$

c) Resposta = $a + k_1 e^{-10t} + k_2 e^{-20t}$

d) Resposta = $a + e^{-3t} (k_1 \cos(11{,}6t) + k_2 \sin(11{,}6t))$

e) Resposta = $a + k_1 \cos(3t) + k_2 \sin(3t)$

f) Resposta = $a + k_1 e^{-10t} + k_2 t e^{-10t}$

Questão (10) -

$T(s) = C(sI - A)^{-1} B + D$, onde B é uma coluna e C é um vetor linha, e $D = 0$.

$$A = \begin{bmatrix} 3 & -4 & 2 \\ -2 & 0 & 1 \\ 4 & 7 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}, \quad C = [1 \; 7 \; 1], \quad D = 0$$

Raízes = [1 2 -38 25]

Resposta = -7,5 4,81 0,69

Questão 12 -

$$\frac{V - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1}{sL} + \frac{sV_1}{1/C}$$

$$\frac{V_1}{V} = \frac{1}{R_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{sL} + sC\right)}$$

$$\frac{V_1}{V} = \frac{10}{s^2 + 20s + 500}$$

Polos complexos:

$(-10 \pm 20i)$

$$\boxed{\text{Forma geral} = e^{-10t}\left(k_1 \cos(20t) + k_2 \operatorname{sen}(20t)\right)}$$

14 - $F = (Ms^2 + f_v s + Ks)X$

$$\frac{X}{F} = \frac{1}{Ms^2 + f_v s + Ks}$$

$$\frac{X}{F} = \frac{1}{2s^2 + 6s + 2} \rightarrow X = \frac{1}{2s(s^2 + 3s + 1)} \rightarrow \boxed{X = \frac{1}{2s(s^2 + 3s + 1)}}$$

Questão 19 - $\rightarrow \dfrac{a}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\omega_n = 20$

$\zeta = 0{,}15 \rightarrow \dfrac{a}{s^2+6s+400}$

$s=0 \rightarrow a = 400$

$\rightarrow T(s) = \dfrac{400}{s^2+6s+400}$

$C(s) = \dfrac{400}{s(s^2+6s+400)} \rightarrow \dfrac{1}{s} - \dfrac{s+3}{(s+3)^2+391} - \dfrac{3}{(s+3)^2+391}$

$$\boxed{C(t) = 1 - e^{-3t}\left[\cos(\sqrt{391}\,t) + \frac{3}{\sqrt{391}}\sin(\sqrt{391}\,t)\right]}$$

Questão 20 -

$T_s = \dfrac{4}{\zeta\omega_n}$, $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ e $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \cdot 100$

a) $\omega_n = 4$ rad/s | $\zeta = 0{,}375$

$T_s = 2{,}66$ s, $T_p = 0{,}847$ s, $\%OS = 28\%$

b) $\omega_n = 0{,}2$ rad/s, $\zeta = 0{,}05$

$T_s = 400$ s, $T_p = 15{,}72$ e $\%OS = 85{,}4\%$

c) $\omega_n = 3240{,}37$ rad/s, $\zeta = 0{,}247$

$T_s = 5$ ms, $T_p = 1$ ms, $\%OS = 44{,}9\%$

Questão (23) - $\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$$Re = \omega_n \xi \quad e \quad Im = \omega\sqrt{1-\xi^2}$$

$$Tan(\theta) = \frac{\xi}{\sqrt{1-\xi^2}}$$

$T_s = \dfrac{4}{\xi\omega_n}$, $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$, $\%OS = e^{-\xi\pi/\sqrt{1-\xi^2}}$

a) $\tan(\theta) = 0{,}675$ e $Re = -6{,}67$

Polos $= -6{,}67 \pm 9{,}88i$

b) $\tan(\theta) = 0{,}733$ e $Im = 0{,}628$

Polos $= -0{,}46 \pm 0{,}628i$

c) $\tan(\theta) = -0{,}571$ e $Im = 1{,}045$

Polos $= -0{,}571 \pm 1{,}045i$

Questão (25) - $F - 2sX - 20X = 5s^2X$

$$T(s) = \frac{X}{F} = \frac{1}{5s^2+2s+20}$$

b) $\omega_n = \sqrt{\dfrac{20}{5}} = 2$ | $\xi = \dfrac{2}{5 \cdot 2 \cdot 2} = 0{,}1$

$$\%OS = e^{-\xi\pi/\sqrt{1-\xi^2}} \cdot 100 \cong 72{,}92\%$$

$$T_s = \frac{4}{0,2} = 20\,s \quad \Big| \quad T_p = \frac{\pi}{2\sqrt{1-0,1^2}} = 1,58\,s$$

$$T_r = \frac{1,104}{2} = 0,552\,s$$

$$C_{final} = \frac{1}{20} = 0,05 \quad (S \to 0)$$

Questão 26 - Valores conhecidos: $\omega_n^2 = 0,5$ | $\omega_n = 0,707\,rad/s$

$$2\xi\omega_n = 1 \rightarrow \xi = 0,707$$

Equações: $$T(t) = \frac{2d^2\theta_1(t)}{dt^2} + \frac{d[\theta_1(t) - \theta_2(t)]}{dt}$$

e $$0 = (1) \cdot \frac{d[\theta_2(t) - \theta_1(t)]}{dt} + (1)\,\theta_2(t)$$

a) Laplace 1: $$S\theta_1(S) = (S+1)\,\theta_2(S)$$

$$\theta_1(S) = \left(\frac{S+1}{S}\right)\theta_2(S)$$

Laplace 2: $$T(S) = (2S^2+1)\,\theta_1(S) - S\theta_2(S)$$

Então: $$\frac{\theta_2(S)}{T(S)} = \frac{0,5}{S^2+S+0,5}$$

b) $$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \Big| \quad T_s = \frac{4}{\xi\omega_n} \quad \Big| \quad T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\% = e^{-(\xi\pi/\sqrt{1-\xi^2})} \cdot 100\%$$

$$T_s = \frac{4}{0{,}707 \cdot 0{,}707} = \frac{4}{0{,}49} = 8{,}16s$$

$$T_p = \frac{\pi}{0{,}505} = 6{,}22s$$

$$\% = e^{-(0{,}707\pi/\sqrt{1-(0{,}707)^2})} \cdot 100\% = 4{,}32\%$$

$$\boxed{T_s = 8{,}16\,s \mid T_p = 6{,}22\,s \mid \% = 4{,}32\%}$$