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Atividade Semana 03 - 03B

Nise - Capítulo 2

Exercício (1) -

a) $u(t)$: $F(s)= \int_0^{\infty} e^{-st} \cdot f(t)\, dt = \int_0^{\infty} e^{-st} \cdot 1\, dt$

$$F(s) = \left.\frac{e^{-st}}{-s}\right|_0^{\infty} = \left(\frac{e^{-s.\infty}}{-s}\right) - \left(\frac{e^{-s.0}}{-s}\right) = \frac{1}{s}$$

$\to 0$

$$\boxed{\mathcal{L}\{u(t)\} = F(s) = \frac{1}{s}}$$

b) $t.u(t)$: $F(s)= \int_0^{\infty} e^{-st} \cdot f(t)\, dt = \int_0^{\infty} e^{-st} \cdot t\, dt$

Por partes: $\int f g' = f.g - \int f' g$ com $f = x \mid g = -\frac{e^{-st}}{s}$

$f' = 1 \mid g' = +e^{-st}$

Assim:

$$= -\frac{t.e^{-st}}{s} + \int_0^{\infty} \frac{e^{-st}}{s}\, dt$$

substituindo
$u = -st \to dt = -\frac{1}{s} du$

$$= -\frac{t.e^{-st}}{s} + \left(-\frac{1}{s^2}\int e^{u} du\right) = -\frac{t.e^{-st}}{s} + \left(-\frac{e^{-st}}{s^2}\right)$$

$$= \left.\frac{t.e^{-st}}{s} - \frac{e^{-st}}{s^2}\right|_0^{\infty} = \left[(0-0) - \left(0 - \frac{1}{s^2}\right)\right]$$

$$\boxed{\mathcal{L}\{t\,u(t)\} = \frac{1}{s^2}}$$

Exercício ① - Continuação:

c) $\sin wt\, u(t)$

$$F(s) = \int_0^\infty \sin wt\, e^{-st}\, dt \rightarrow \int fg' = fg - \int f'g$$

$$f = \sin(wt) \,\Big|\, g = \frac{e^{-st}}{s}$$

$$f' = w\cos(wt) \,\Big|\, g' = e^{-st}$$

$$1^{\underline{o}} = \frac{e^{-st}\sin(wt)}{s} - \int -\frac{we^{-st}\cos(wt)}{s}\, dt$$

$$2^{\underline{o}} = \frac{e^{-st}\sin(wt)}{s}\left(\frac{we^{-st}\cos(wt)}{s^2} - \int -\frac{w^2 e^{-st}\sin(wt)}{s^2}\right.$$

$$= \frac{e^{-st}\sin(wt)}{s} - \left(\frac{we^{-st}\cos(wt)}{s^2} + \frac{w^2}{s^2}\int e^{-st}\sin(wt)\, dt\right)$$

$$= \left.\frac{-se^{-st}\sin(wt) - we^{-st}\cos(wt)}{w^2+s^2}\right|_0^\infty$$

$$F(s) = \frac{w}{w^2+s^2}$$

d) $\cos wt\, u(t)$:

De maneira similar:

$$F(s) = \int_0^\infty \cos wt\, e^{-st}\, dt = \frac{s}{s^2+w^2}$$

Exercicio 2 -

a) Usando teorema da frequencia e Transformada de Laplace do sen wt:

$$F(s) = \frac{w}{(s+a)^2 + w^2}$$

b) Semelhantemente:

$$\frac{(s+a)}{(s+a)^2 + w^2}$$

c) Usando integração sucessiva

$$\int dt = t \rightarrow \int t\,dt = \frac{t^2}{2} \rightarrow \int \frac{t^2}{2} dt = \frac{t^3}{6}$$

$$\mathcal{L}\{t^3 u(t)\} = \frac{6}{s^4}$$

3 - Exercicio Referente ao c, e p d?

4 - :

8 - Um sistema descrito por:

$$\frac{d^3y}{dt^3} + \frac{3d^2y}{dt^2} + \frac{5dy}{dt} + y = \frac{d^3x}{dt^3} + \frac{4d^2x}{dt^2} + \frac{6dx}{dt} + 8x$$

Encontre a função de transferência $\frac{Y(s)}{X(s)}$:

Questão (8) Resolução:

Laplace:

$$\mathcal{L}\left\{ \frac{d^3y}{dt^3} + \frac{3d^2y}{dt^2} + \frac{5dy}{dt} + y = \frac{d^3x}{dt^3} + \frac{4d^2x}{dt^2} + \frac{6dx}{dt} + 8x \right\}$$

$$s^3y(s) + 3s^2y(s) + 5sy(s) + y(s) = s^3x(s) + 4s^2x(s) + 6\ldots$$

Evidência:

$$y(s)(s^3 + 3s^2 + 5s + 1) = X(s)(s^3 + 4s^2 + 6s + 8)$$

$$\boxed{\frac{y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}}$$

Questão (9):

a) $$\frac{X(s)}{F(s)} = \frac{7}{s^2 + 5s + 10} \rightarrow 7F(s) = X(s)(s^2 + 5s + 10)$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 10x = 7f}$$

b) $$\frac{X(s)}{F(s)} = \frac{15}{(s+10)(s+11)} = X(s)[s^2 + 21s + 110] = 15F(s)$$

$$\boxed{\frac{d^2x}{dt^2} + 21\frac{dx}{dt} + 110x = 15f}$$

Continuação (8).

Letra c) $\dfrac{X(s)}{F(s)} = \dfrac{s+3}{s^3+11s^2+12s+18}$

$$X(s)\,(s^3+11s^2+12s+18) = F(s)\,(s+3)$$

$$\frac{d^3x}{dt^3} + 11\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 18x = \frac{df}{dt} + 3f$$

Questão (10):

$$R(s) \rightarrow \frac{s^5+2s^4+4s^3+s^2+4}{s^6+7s^5+3s^4+2s^3+s^2+5} \rightarrow C(s)$$

$$\frac{C(s)}{R(s)} =$$

Assim:

$$\boxed{\frac{d^6c}{dt^6} + 7\frac{d^5c}{dt^5} + 3\frac{d^4c}{dt^4} + 2\frac{d^3c}{dt^3} + \frac{d^2c}{dt^2} + 5c = \frac{d^5R}{dt^5} + 2\frac{d^4R}{dt^4} + 4\frac{d^3R}{dt^3} + \frac{d^2R}{dt} + 4R}$$

Questão (11): $R(s) \rightarrow \underbrace{\dfrac{s^4+3s^3+2s^2+s+1}{s^5+4s^4+3s^3+2s^2+3s+2}}_{G(t)} \rightarrow C(s)$

$$\frac{C(s)}{R(s)} = G(t)$$

Assim: $\dfrac{d^5c}{dt^5} + 3\dfrac{d^4c}{dt^4} + 2\dfrac{d^3c}{dt^3} + 4\dfrac{d^2c}{dt^2} + 5\dfrac{dc}{dt} + 2c = \dfrac{d^4r}{dt^4} + 2\dfrac{d^3r}{dt^3} + 5\dfrac{dr^2}{dt^2} + \dfrac{dr}{dt} + r$

Continua →

Continuação questão (11):

Substituindo $r(t) = t^3$:

$$\frac{d^5c}{dt^5} + \frac{3d^4c}{dt^4} + \frac{2d^3c}{dt^3} + \frac{4d^2c}{dt^2} + \frac{5dc}{dt} + 2c = \underbrace{18k(t)}_{\text{substituição}}t^4 t(36+90t+9t^2+3t^3)$$

Questão (12):

$$F = \frac{d^2x}{dt^2} + \frac{4dx}{dt} + 5x = 1$$

$x(0) = 1$ e, $\dot{x}(0) = -1$.

$$\mathcal{L}\{F\} = X(s)\,(s^2 + 4s + 5) = g(s)$$

$$\frac{d^2x}{dt^2} + 4.1 + 5.(-1) = 1$$

$$\ddot{x} = 2$$

Exercícios Franklin ~~K~~ Capítulo 3.

3.2 : b) $f(t) = 3 + 7t + t^2 + \delta(t)$

$\mathcal{L}\{f(t)\} = \frac{3}{s} + 7\frac{1}{s^2} + \frac{2}{s^3} + \frac{1}{s}$

d) $f(t) = (t+1)^2 = t^2 + 2t + 1$

$\mathcal{L}\{f(t)\} = \frac{2}{s^3} + 2\frac{1}{s^2} + \frac{1}{s}$

e) $f(t) = \operatorname{senh} t \rightarrow \operatorname{senh}(at)$

$\mathcal{L}\{f(t)\} = \frac{a}{s^2 - a^2}$; $a = 1$

$\mathcal{L}\{f(t)\} = \frac{1}{s^2 - 1}$ //

$\sinh at = \left\{\frac{e^{at} - e^{-at}}{2}\right\}$

$\frac{1}{2}\mathcal{L}\{e^{at}\} - \mathcal{L}\{e^{-at}\}$

$\frac{1}{2}\left(\frac{1}{s-a} - \frac{1}{s+a}\right) = \frac{a}{s^2 - a}$

Com $a = 1$

$\mathcal{L}\{f(t)\} = \frac{1}{s^2 - 1}$ //

3.3 :

a) $f(t) = 3\cos 6t$

$$\mathcal{L}\{f(t)\} = 3\,\mathcal{L}\{\cos 6t\}$$

$$\mathcal{L}\{f(t)\} = 3\left(\frac{s}{s^2+6^2}\right) = \frac{3s}{s^2+36}$$

b) $f(t) = \operatorname{sen}(2t) + 2\cos(2t) + e^{-t}\operatorname{sen} 2t$

$$\mathcal{L}\{f(t)\} \; \frac{2}{s^2+4} + \frac{\cancel{2}s}{s^2+4} + \frac{2}{(s+1)^2+4}$$

c) $f(t) = t^2 + e^{-2t}\operatorname{sen} 3t$

$$\mathcal{L}\{f(t)\} = \frac{2}{s^3} + \frac{3}{(s+2)^2+9}$$

3.5 -

a) $\operatorname{sen} t \cdot \operatorname{sen} 3t$

$$\sin(s)\sin(t) = \frac{1}{2}\left(-\cos(s+t) + \cos(s-t)\right)$$

$$\mathcal{L}\{f(t)\} = -\frac{1}{2}\mathcal{L}\{\cos(4t)\} + \frac{1}{2}\mathcal{L}\{\cos(-2t)\}$$

$$\mathcal{L}\{f(t)\} = -\frac{1}{2}\cdot\frac{s}{s^2+16} + \frac{1}{2}\,\frac{s}{s^2+4}$$

$$= \frac{6s}{(s^2+4)(s^2+16)}$$

3.5

b) $f(t) = \operatorname{sen}^2 t + 3\cos^2 t$

$$\mathcal{L}\{f(t)\} \Rightarrow \operatorname{sen}^2(x) = \frac{1}{2} - \cos(2x)\frac{1}{2}$$

$$= \frac{1}{2} - \cos(2t)\frac{1}{2} + 3\cos^2(t)$$

$$= \mathcal{L}\left\{\frac{1}{2}\right\} - \mathcal{L}\left\{\cos(2t)\cdot\frac{1}{2}\right\} + 3\mathcal{L}\{\cos^2(t)\}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2s} - \frac{1}{2}\frac{s}{s^2+4} + 3\left(\frac{1}{2s} + \frac{s}{2(s^2+4)}\right)$$

c) $f(t) = \dfrac{\operatorname{sen} t}{t}$

se $\mathcal{L}\{f(t)\} = F(s) \to \mathcal{L}\left\{\dfrac{f(t)}{t}\right\} = \displaystyle\int_s^\infty F(u)\,du$

$$= \int_s^\infty \mathcal{L}\{\sin(t)\}_u\,du$$

$$= \int_s^\infty \frac{1}{u^2+1}\,du = \frac{\pi}{2} - \arctan(s)$$

3.7

a) $F(s) = \dfrac{2}{s(s+2)}$

$$\mathcal{L}^{-1}\left\{\frac{2}{s(s+2)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+2}\right\}$$

$$= u(t) - e^{-2t}$$

d) $F(s) = \dfrac{3s^2 + 9s + 12}{(s+2)(s^2+5s+11)}$

$$\mathcal{L}^{-1}\{F(s)\} =$$

l) $\dfrac{1}{s^2+4} = \dfrac{1}{s^2+2^2} = \dfrac{1}{2}\,\dfrac{1}{s^2+2^2}$

$$\mathcal{L}^{-1}\{f(s)\} = \frac{1}{2}\,\mathrm{sen}(2t)$$

i) $F(s) = \dfrac{4}{s^4+4} =$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{s+2}{2(s^2+2s+2)} + \frac{-s+2}{2(s^2-2s+2)}$$

$$\Rightarrow \frac{s+2}{2(s^2+2s+2)} = \frac{1}{2}\cdot\frac{s+1}{(s+1)^2+1} + \frac{1}{2}\frac{1}{(s+1)^2+1}$$

$$\rightarrow \frac{-s+2}{2(s^2-2s+2)} = \frac{-1}{2}\cdot\frac{s-1}{(s-1)^2+1} + \frac{1}{2}\frac{1}{(s-1)^2+1}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+1}\right\}$$

$$= \frac{1}{2}e^{-t}\cos(t) + \frac{1}{2}e^{-t}\operatorname{sen}(t) - \frac{1}{2}e^{t}\cos(t) + \frac{1}{2}e^{t}\operatorname{sen}(t)$$

j) $F(s) = \dfrac{e^{-s}}{s^2}$ se $\mathcal{L}^{-1}\{F(s)\} = f(t)$

então $\mathcal{L}^{-1}\{e^{-as}F(s)\} = \mu(t-a).$
$f(t-a)$

$$\mu(t-1)\cdot\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}(t-1)$$

$$= \mu(t-1)(t-1)$$