Universitatea Tehnica a Moldovei
Facultatea Calculatoare, Informatica si Microelectronica
Catedra Tehnologii Informationale

RAPORT

despre lucrarea de laborator nr. 2

la disciplina Metode si modele de calcul

Tema: Puncte de extreme. Functii convexe. Matricea Hessiana. Algoritmul Hestenes – Stiefel. Rezolvarea grafica a problemei de programare liniara.

A efectuat: st. gr. TI-173 Heghea Nicolae

A verificat: Ghetmancenco S.

Chisinau 2018

Cuprins

1. Notiuni teoretice

O multime M, M $\subset R^n$ se numeste ***multine convexa*** daca, luand oricare 2 puncte ale sale $x^1, x^2 \in M$, segmentul de dreapta ce le unste apartine multimii date.

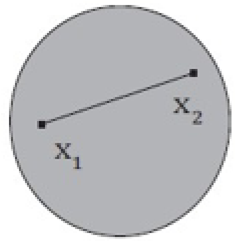


Fig. 1.1. Multime convexa.

Trosonul reprezinta intersectia unui numar finit de semispatii, si este o multime convexa :

$$S_i = \left\{ x \,\middle|\, \sum_{j=1}^{n} a_{ij} \leq b_i \right\}, i = 1,2, \dots , m$$

Matricea de dimensiune $n \, x \, n$

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \, \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \, \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \, \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \, \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \, \partial x_1} & \frac{\partial^2 f}{\partial x_n \, \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

se numeste ***matrice Hesse*** a functiei $\boldsymbol{f}$ sau ***matricea Hessiana***. Se noteaza si $\boldsymbol{\nabla^2 f(x)}$.

2. Multimi convexe. Determinarea trosonului

Problema 6. (pag. 6).

Conditia :

Sa se determine punctele extreme ale trosonului ***T***. definit de sistemul de inecuatii :

$$\begin{cases} x_1 + x_2 \leq 18 \\ \frac{1}{2}x_1 + x_2 \leq 6 \\ x_1 \leq 12 \\ x_2 \leq 9 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{cases}$$

Rezolvare :

Observam ca trosonul este creat de 5 virfuri, care apar la intersectia liniilor:

1. $\begin{cases} x_1 + x_2 = 18 \\ x_2 = 9 \end{cases} \Rightarrow \begin{cases} x_1 = 9 \\ x_2 = 9 \end{cases}$

2. $\begin{cases} x_1 + x_2 = 18 \\ x_1 = 12 \end{cases} \Rightarrow \begin{cases} x_1 = 12 \\ x_2 = 6 \end{cases}$

3. $\begin{cases} x_1 = 12 \\ \frac{1}{2}x_1 + x_2 = 6 \\ x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 12 \\ x_2 = 0 \end{cases}$

4. $\begin{cases} \frac{1}{2}x_1 + x_2 = 6 \\ x_1 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 6 \end{cases}$

5. $\begin{cases} x_1 = 0 \\ x_2 = 9 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 9 \end{cases}$

Cu ajutorul programului facut in aplicatia MatLab. Pun aceste functii pe axele XOY.

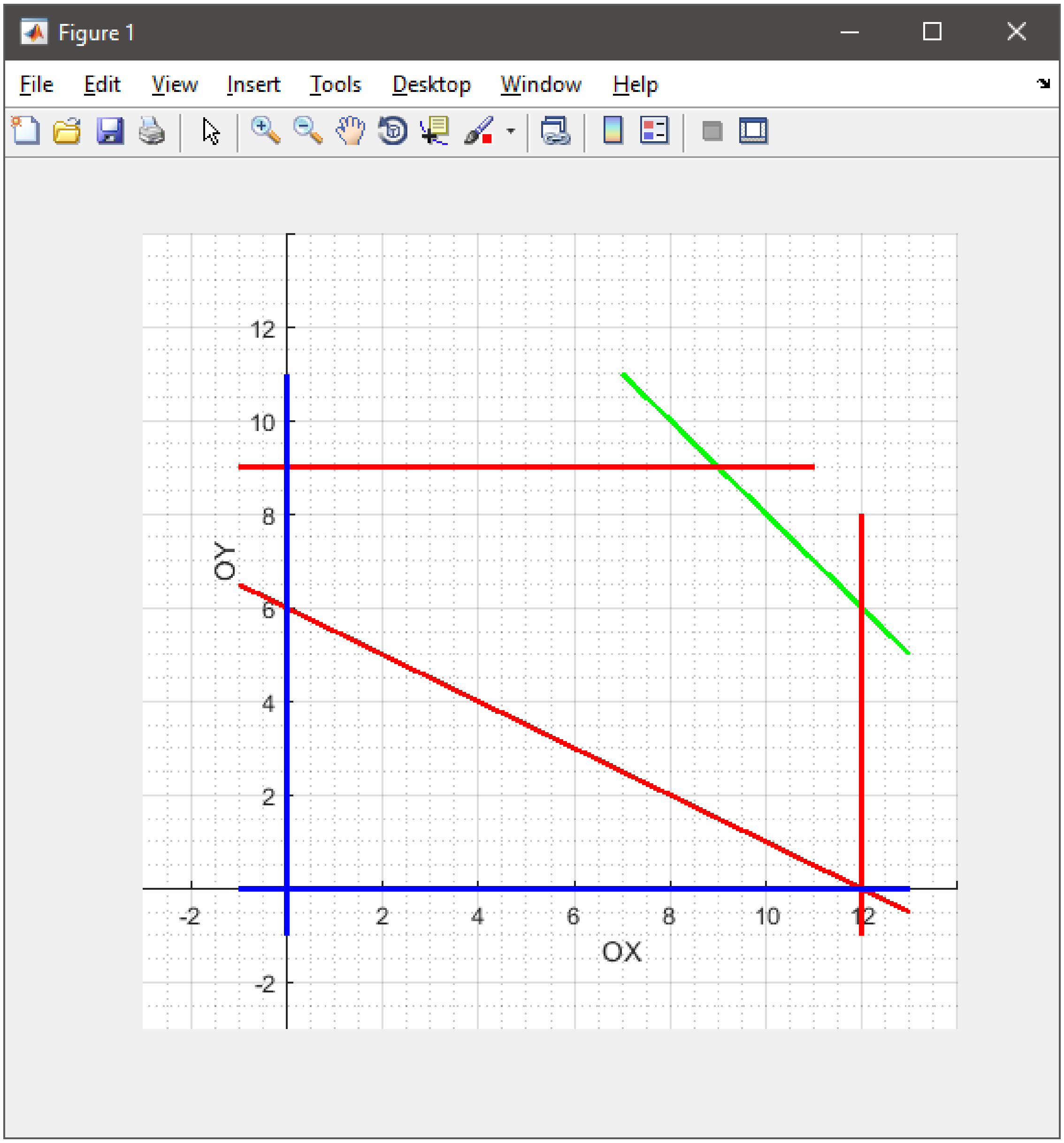


Fig. 2.1. Graficul functiilor.

3. Functii convexe. Matricea Hessiana.

3.1 Problema 9. (pag 11). Pe foaie.

P.9.1. :

$$f_1(x) = \sum_{k=1}^{n} |x_k|\,, x \in R^n$$

Sa presupunem ca $n = 1,2,3$, avem :

$n = 1$	$n = 2$	$n = 3$
$f(x) = \lvert x_1 \rvert$	$f(x) = \lvert x_1 \rvert + \lvert x_2 \rvert$	$f(x) = \lvert x_1 \rvert + \lvert x_2 \rvert + \lvert x_3 \rvert$
$f'(x) = 1$ $f''(x) = 0$	$f'(x) = 1$ $f''(x) = 0$	$f'(x) = 1$ $f''(x) = 0$

Rezulta ca functiile f_1 nu au Hessiana.

P.9.2. :

$$f_2(x) = -\sqrt{x}\,, \ \ x \geq 0, \ \ x \in R^1$$

$$f(x) = -\sqrt{x} \qquad f'(x) = -\frac{1}{2\sqrt{x}} \qquad f''(x) = \frac{1}{4\sqrt[3]{x^2}}$$

Rezulta ca f_2 nu este continua.

P.9.3. :

$$f_3(x) = 2x_1^2 + x_2^2 - 2x_1x_2\,, \ \ x \in R^2$$

$$f'_{x_1} = 4x_1 - 2x_2$$
$$f'_{x_2} = -2x_1 + 2x_2$$
$$f''_{x_1x_1} = 4$$
$$f''_{x_1x_2} = -2$$
$$f''_{x_2x_2} = 2$$

$$\begin{cases} 4x_1 - 2x_2 = 0 \\ -2x_1 + 2x_2 = 0 \end{cases} => \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases}$$

$$\nabla^2 f(x) = \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix} = 4$$

In $(0,0) -$ functia are punct de minim, si este functie convexa.

P.9.4. :

$$f_4(x) = x_1^4 + 2x_2^2 + 3x_3^2 - 4x_1 - 4x_2x_3\,, \quad x \in R^3$$

$$\begin{aligned} f'_{x_1} &= 4x_1^3 - 4 \\ f'_{x_2} &= 4x_2 - 4x_3 \\ f'_{x_3} &= 6x_3 - 4x_2 \end{aligned} \qquad \begin{cases} 4x_1^3 - 4 \\ 4x_2 - 4x_3 \\ 6x_3 - 4x_2 \end{cases} \Rightarrow \begin{cases} x_1 = 1 \\ x_2 = 0 \\ x_3 = 0 \end{cases}$$

$$\begin{aligned} f''_{x_1x_1} &= 12x_1^2 & f''_{x_2x_1} &= 0 & f''_{x_3x_1} &= 0 \\ f''_{x_1x_2} &= 0 & f''_{x_2x_2} &= 4 & f''_{x_3x_2} &= -4 \\ f''_{x_1x_3} &= 0 & f''_{x_2x_3} &= -4 & f''_{x_3x_3} &= 6 \end{aligned}$$

$$\nabla^2 f(x) = \begin{bmatrix} 12x_1^2 & 0 & 0 \\ 0 & 4 & -4 \\ 0 & -4 & 6 \end{bmatrix} = 288x_1^2 - 192x_1^2 = 96x_1^2 \Rightarrow 96$$

In $(1,0,0)$ – functia are punct de minim, si este functie convexa.

3.1 Problema 9. (pag 11). La calculator

P.9.2. :

```
Command Window

>> Hessiana

f =

-x1^(1/2)


Diferentiala_Ord_1 =

-1/(2*x1^(1/2))


Hessian =

1/(4*x1^(3/2))


Det =

1/(4*x1^(3/2))

>> 
```

P.9.3. :

```
Command Window

>> Hessiana

f =

2*x1^2 - 2*x1*x2 + x2^2


Diferentiala_Ord_1 =

 4*x1 - 2*x2
 2*x2 - 2*x1


R =

[ 0, 0]


Hessian =

[  4, -2]
[ -2,  2]


Det =

4


T =

    '(0, 0) = 4 -> (Convexa) => Punct de Minim'

>> 
```

P.9.4. :

Command Window

```
>> Hessiana

f =

x1^4 - 4*x1 + 2*x2^2 - 4*x2*x3 + 3*x3^2


Diferentiala_Ord_1 =

  4*x1^3 - 4
 4*x2 - 4*x3
 6*x3 - 4*x2


R =

[ 1, 0, 0]


Hessian =

[ 12*x1^2,  0,  0]
[       0,  4, -4]
[       0, -4,  6]


Det =

96*x1^2


T =

    '(1, 0, 0) = 96 -> (Convexa) => Punct de Minim'

>> 
```

4. Metode de directii conjugate. Algoritmul lui Hestenes – Stiefel.

Se da functia : $f(x,y) = x^2y^2$

Punctele alese abitrar sunt $M_0(2,1)$.

Pas 1.1:

$$f_x' = 2xy^2$$
$$f_y' = 2yx^2$$

Pas 1.2:

$$f_x'(2,1) = 4$$
$$f_y'(2,1) = 8$$

Pas 1.3:

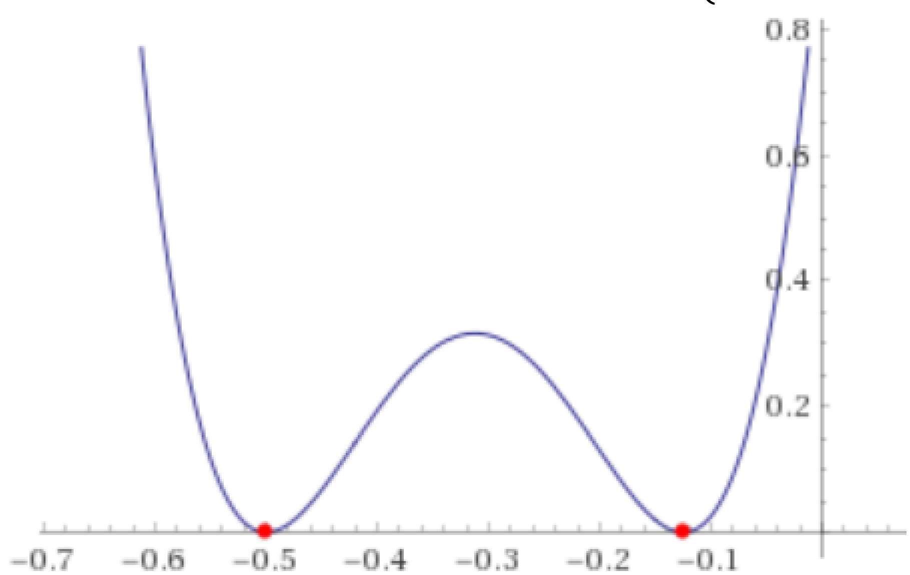
$$\nabla f = if_x' + jf_y' = 4i + 8j$$
$$x^{i+1} = x^i + h \cdot \nabla f$$

Pas 1.4:

$$\begin{bmatrix}2\\1\end{bmatrix} + h\begin{bmatrix}4\\8\end{bmatrix} = \begin{cases}x = 4h + 2\\ y = 8h + 1\end{cases}$$

Pas 1.5:

$$f(h) = (4h+2)^2 \cdot (8h+1)^2 =$$
$$=(16h^2 + 16h + 4) \cdot (64h^2 + 16h + 1) =$$
$$= 256h^4 + 320h^3 + 132h^2 + 20h + 1 => \begin{cases}h_1 = -\frac{1}{2}\\ h_2 = -\frac{1}{8}\end{cases}$$



$$=> \begin{cases}x = 4(-0.5) + 2\\ y = 8(-0.125) + 1\end{cases} = \begin{Bmatrix}0\\0\end{Bmatrix} => M_1(0,0) \rightarrow Punct\ optim.$$

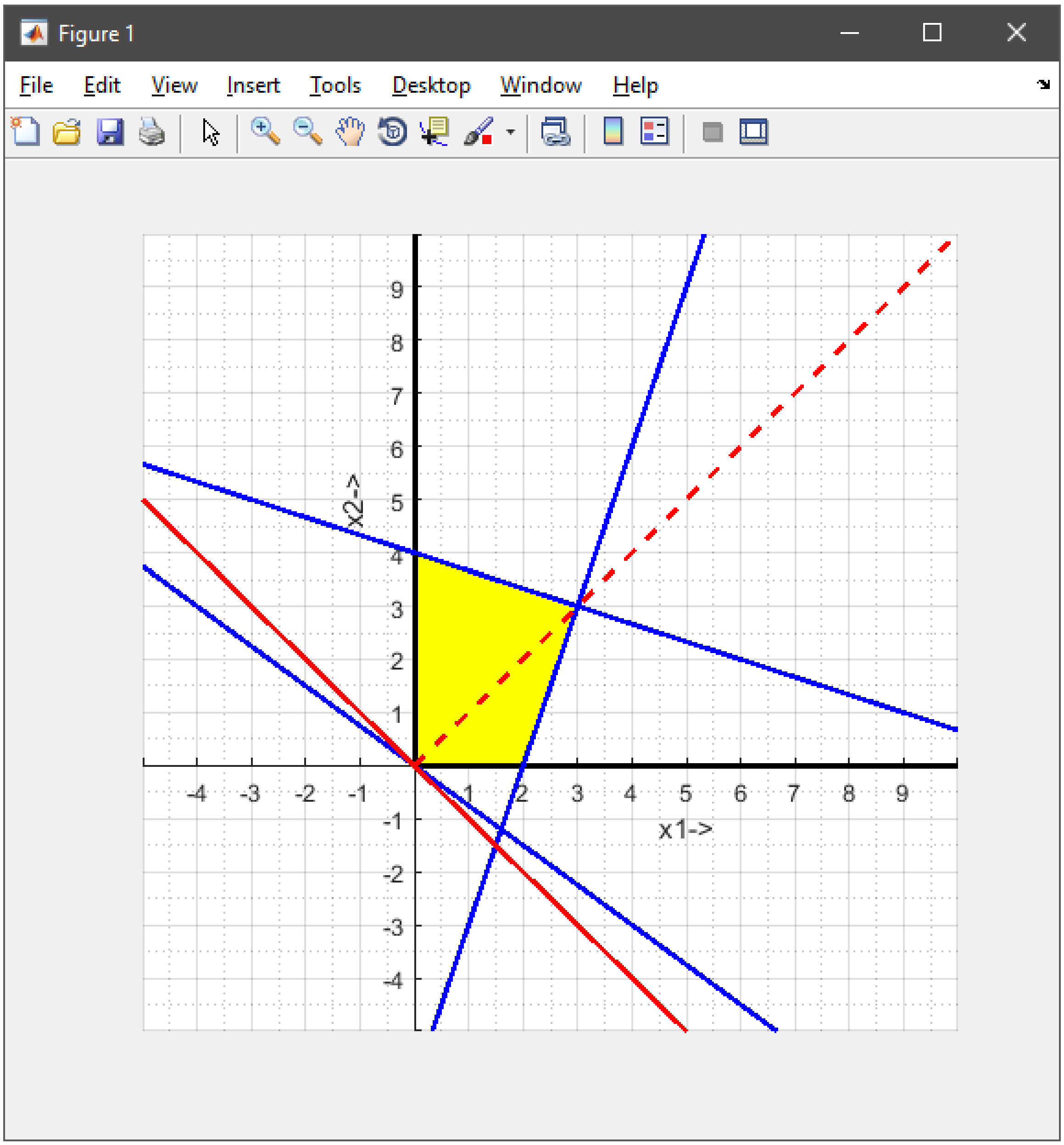
5. Rezolvarea grafica a poblemei de programare liniara.

Problema 1. (pag 28)

P.1.

$$Z = x_1 + x_2 \text{ (min.)}$$
$$x_1 + 3x_2 \leq 12$$
$$3x_1 - x_2 \geq 6$$
$$3x_1 + 4x_2 \geq 0$$
$$x_1 \geq 0, x_2 \geq 0$$

Aceasta problema am rezolvato in MatLab.



6. Codul sursa

6.1 Determinarea matricei Hessiane

```
function Hessiana
%#ok<*NOPRT>%#ok<*AGROW>%#ok<*NBRAK>
%%%%%%%%%%-Exemple test-%%%%%%%%%%%%%%%%%%%%%%%%
%%%-Ex 3
%     n = 2;
%     x = sym('x',[1 n]);
%     f = x(1)^3 -12*x(1)*x(2) + 8*x(2)^3;

%%%-Ex 4
%     n = 2;
%     x = sym('x',[1 n]);
%     f = x(1)^4 + x(1)^2*x(2)^2 - 2*x(1)^2;

%%%%%%%%%%-Pe Acasa-%%%%%%%%%%%%%%%%%%%%%%%%
%%%-Exemplul 9.2.pag.11.
    n = 1;
    x = sym('x',[1 n]);
    f = -(x(1)^(1/2));

%%%-Exemplul 9.3.pag.11.
%     n = 2;
%     x = sym('x',[1 n]);
%     f = 2*x(1)^2 + x(2)^2 - 2*x(1)*x(2);

%%%-Exemplul 9.4.pag.11.
%     n = 3;
%     x = sym('x',[1 n]);
%     f = x(1)^4 + 2*x(2)^2 + 3*x(3)^2 - 4*x(1) - 4*x(2)*x(3);

    Diferentiala_Ord_1 = diff(f, x(1));
    for i = 2:n
        Diferentiala_Ord_1 = [Diferentiala_Ord_1, diff(f, x(i))];
    end
    Diferentiala_Ord_1 = Diferentiala_Ord_1.';
    Hessian = hessian(f,x);
    Det = det(Hessian);

    r = solve(Diferentiala_Ord_1, x, 'Real', true);
    rs = size(r);
    if rs > 0
        r = struct2cell(r);
        R = sym('q',[1 rs]);

        R = cell2sym(r(1));
        for i = 2:n
           R = [R, cell2sym(r(i))];
        end

        nr = size(R);
        nr = nr(1,1);
        p = zeros( nr , 1 );

        for i = 1:nr
            s = Det;
            for j = 1:n
```

```
                s = subs(s, x(j), R(i,j));
            end
            p(i) = s;
        end

        T = repmat({''}, nr, 1);
        for i = 1:nr
            q = [ '(' ];
            for j = 1:n
                q = [q, char(R(i,j))];
                if j < n
                    q = [q, ', '];
                end
            end
            q = [q, ') = ', num2str(p(i)), ' -> ' ];
            if p(i) <= 0
                q = [q '(Concava) => Punct de Maxim' ];
            else
                q = [q '(Convexa) => Punct de Minim' ];
            end
            T(i) = {q};
        end
    end

%%%%-Afisarea-%%%%%%%%%%%
    f

    Diferentiala_Ord_1

    if rs > 0
        R
    end

    Hessian

    Det

    if rs > 0
        T
    end
end
```

6.2 Metoda Grafica

```
function MetodaGrafica
    [yx, xy, z] = exemplul_1;

    deseana(yx, xy, z);
end
```

```
function [yx, xy, z] = exemplul_1
    setFigureProprietes;
    syms x;

    %cu x2
    yx = [
        (12 - x)/3
        3*x - 6
        (-3*x)/4
    ];

    %fara x2
    xy = [
    ];

    z = [
        (-x)
    ];

    x = [-5 10];
    y = [-5 10];
    axis([x y])

    x = [0 0 3 2];
    y = [0 4 3 0];
    fill(x,y,'y');
end
```

```
function deseana (yx, xy, z)
    cond_nenegativ

    fx(yx);
    fy(xy);
    Z(z);
end
```

```
function cond_nenegativ
    syms x;
    int = [0 100];
    oxy = 0*x;

    fplot(oxy, int, '-k', 'Linewidth', 2); hold on;
    fplot(oxy, x, int, '-k', 'Linewidth', 2); hold on;
end
```

```
function fx(y)
    int = [-100 100];
    n = size(y);
    n = n(1,1);

    for i = 1:n
        fplot(y(i), int, '-b', 'Linewidth', 2); hold on;
    end
end
```

```
function fy(y)
    int = [-100 100];
    syms x;
    n = size(y);
    n = n(1,1);

    for i = 1:n
        fplot(y(i), x, int, '-b', 'Linewidth', 2); hold on;
    end
end
```

```
function Z(y)
    syms x;
    if size(y) > 0
        fplot(y(1), [-100 100], '-r', 'Linewidth', 2); hold on;
        fplot(-y(1), [0 100], '--r', 'Linewidth', 2); hold on;
    end
end
```

```
function setFigureProprietes
    fig = figure(1);
    set(fig,'units','points','position',[400,125,430,400]);

    x = [-5 15];
    y = [-5 15];

    set(gca, 'xtick', x(1):1:x(2));
    set(gca, 'ytick', y(1):1:y(2));
    hold on

    ax = gca;
    ax.XAxisLocation = 'origin';
    ax.YAxisLocation = 'origin';

    xlabel('x1->');
    ylabel('x2->');
    grid on
    grid minor
    hold on
end
```

7. Concluzia

Aceste metode de rezolvare a problemelor, mai ajutat sa inteleg asa fel de probleme sub un al unchi. Acesti algoritmi si metode de rezolvare au un spectru larg de tipuri de probleme. Algoritmii si metodele la gasirea solutiei optime.