LQR Control Design for Cessna 172

Model Parameters

I used the following equations to model the LQR controller:

$$\mu = -kx, \quad \dot{x} = Ax + B\mu$$

Here, A is the system matrix and B is the input matrix.

I then designed the Q and R matrices. These matrices penalize large values of each state (α, q, β) and large control inputs $(\delta_{elevator})$ respectively.

I started with diagonal matrices prioritizing pitch angle accuracy, allowing some pitch rate, and accounting for angle of attack.

To enable reference tracking, I added a feedforward gain N_{bar}. The pitch angle smoothly rises to a commanded 5 degrees (see examples below).

Longitudinal Dynamics

- <u>States:</u> $[\mu, w, q, \theta, z]$ (forward speed, vertical speed, pitch rate, pitch angle, altitude)
- <u>Input:</u> Elevator deflection $\delta_{elevator}$

The weighting matrices chosen were the following:

$$A_{long} = \begin{bmatrix} -0.890 & -0.343 & 0 \\ -0.106 & -0.117 & 0 \\ 0 & 83.86 & 0 \end{bmatrix}$$

$$B_{long} = \begin{bmatrix} -1.57 \\ -0.002 \\ 0 \end{bmatrix}$$

$$Q_{long} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

$$R_{long} = 1$$

The resulting LQR gain matrix K is:

$$K_{long} = [-7.3362, \quad 348.8960, \quad 10.0000]$$

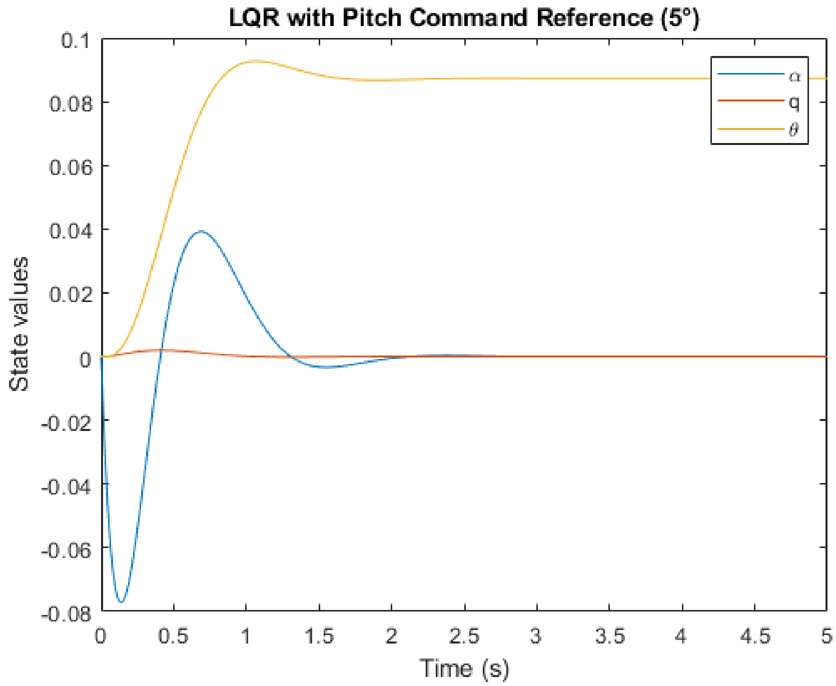

Figure 1: Longitudinal stability response to initial disturbance without reference input, tracking a 5° roll angle command.

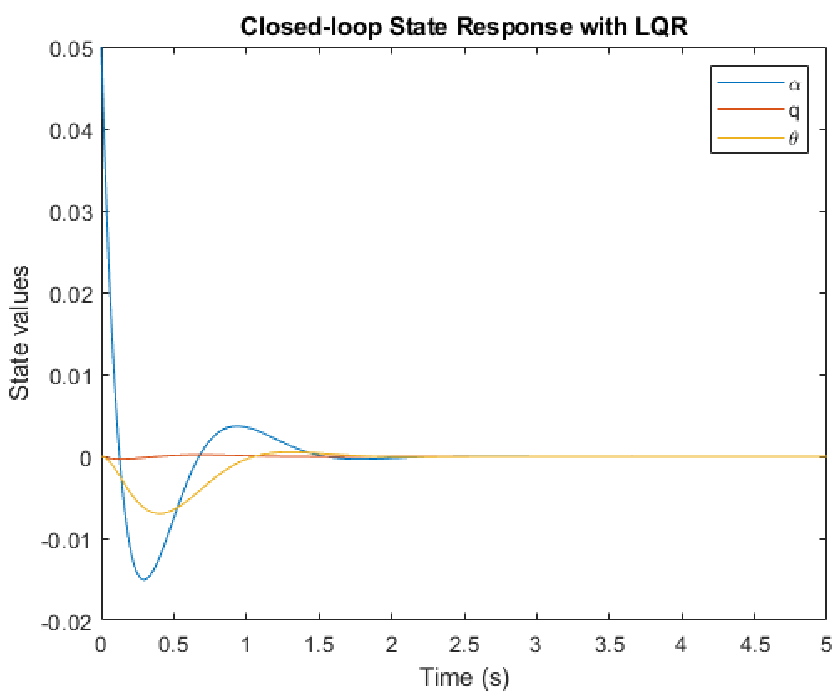

Figure 2: Closed-loop longitudinal-directional response.

Lateral Dynamics

- States: $[v, p, r, \phi, \psi]$ (side velocity, roll rate, yaw rate, roll angle, heading)
- Inputs: $\delta_{aileron}, \delta_{rudder}$

Weighting matrices used:

$$A_{lat} = \begin{bmatrix} -0.322 & 0.052 & 0.028 & -1.12 & 0.002 \\ 0 & -0.429 & 0.804 & 0 & -0.001 \\ -10.6 & 0 & -2.87 & 0 & 0.46 \\ 6.87 & 0 & -0.04 & -0.32 & -0.02 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B_{lat} = \begin{bmatrix} 0 & 0.002 \\ 0.001 & 0 \\ -0.65 & 0.13 \\ -0.02 & 0.0001 \\ 0 & 0 \end{bmatrix}$$

$$Q_{lat} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 300 & 0 & 0 & 0 \\ 0 & 0 & 300 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 100 \end{bmatrix}$$

$$R_{lat} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The lateral LQR gain matrix K_{lat} computed is:

$$K_{lat} = \begin{bmatrix} 14.4437 & -5.5212 & -14.4331 & -3.5247 & -10.5225 \\ 6.3861 & 1.3214 & 2.9067 & -2.0134 & 1.9616 \end{bmatrix}$$

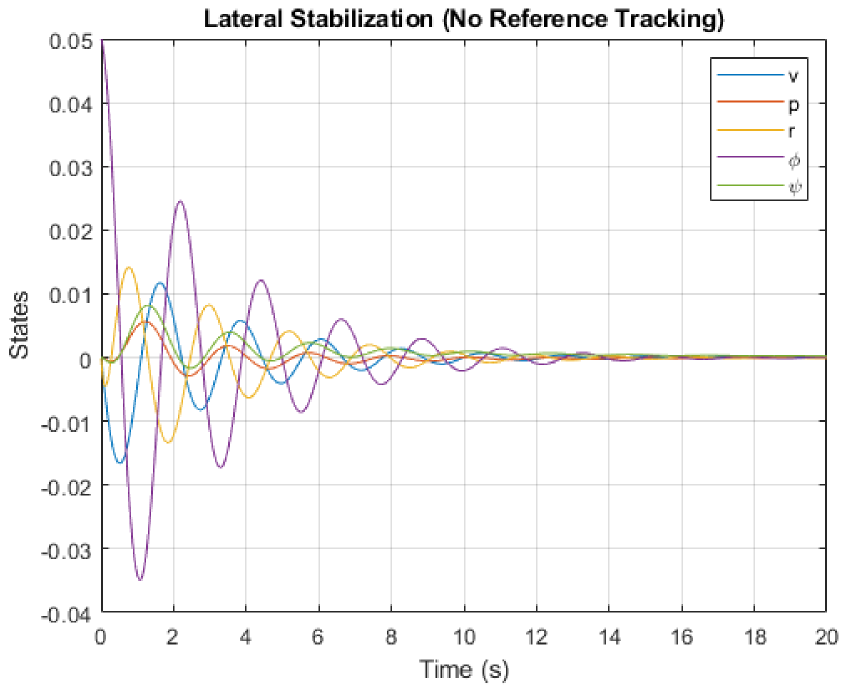

Figure 3: Lateral stability response to initial disturbance without reference input.

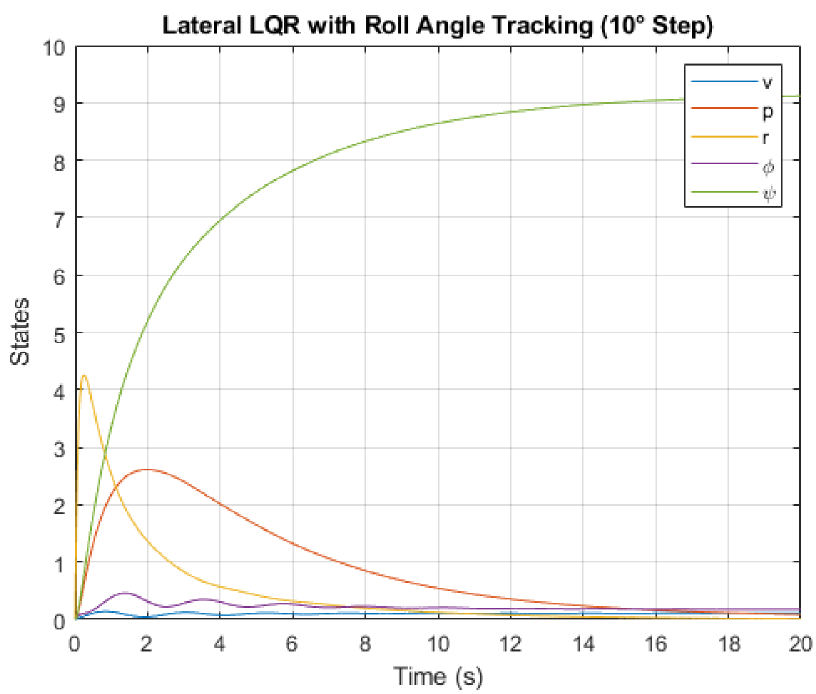

Figure 4: Closed-loop lateral-directional response tracking a 10° roll angle command.