Reinforcement Learning of Theorem Proving

a paper by

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presented by

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Overview

1. How tableau provers work

2. Reinforcement Learning and Application to TP
    - Basics
    - Application to TP
    - Results

3. Summary

How tableau provers work

Clauses:

$c_1 \ : \ P(X)$

$c_2 \ : \ R(X, Y) \vee \neg P(X) \vee Q(Y)$

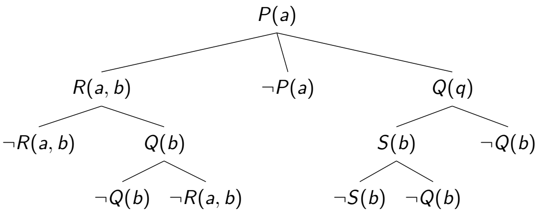
$c_3 \ : \ S(X) \vee \neg Q(b)$

$c_4 \ : \ \neg S(X) \vee \neg Q(X)$

$c_5 \ : \ \neg Q(X) \vee \neg R(a, X)$

$c_6 \ : \ \neg R(a, X) \vee Q(X)$

Figure: A closed tableau



RL Basics

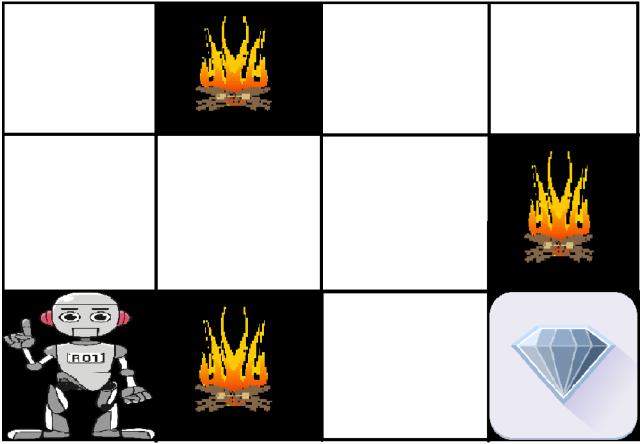


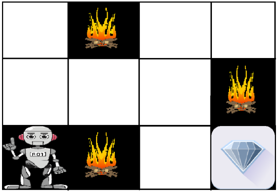
Figure: An agent has to reach a reward without burning

- policy learning
- value learning

Application to TP

RL to TP mapping:

- agent $\leftrightarrow$ TP
- environment $\leftrightarrow$ search tree
- actions $\leftrightarrow$ extending search tree
- reward $\leftrightarrow$ finding a closed tableau



Application to TP – the UCT formula

Tree search with RL – use the UCT formula!
For each node i:

$$f_i = \frac{w_i}{n_i} + c \cdot p_i \cdot \sqrt{\frac{\ln N_i}{n_i}}$$

On every step, take the node with highest f_i.

w_i: **total reward**

n_i: number node of visits

c: hyperparameter

p_i: **prior probability**

N_i: total parent visits

Application to TP – Extracting features (Literals)

- For each Literal L, e.g. $f(X, Y) = g(sk_1, sk_2(X))$
- Build it's feature tree
- Count term walks of length 3
- E.g. for L we get $\{(\oplus, =, f) : 1, (=, f, \circledast) : 2, \ldots\}$

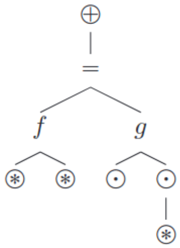


Figure: Feature Tree for L

Example and picture from Jakubuv and Urban (2017)

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- Feature vector for a *state*:
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  - additional metadata – No. of open goals, depth of node, etc.
- Features for an *action* contains:
  - features of the clause used
  - features of literal used

Application to TP – Learning the parameters

- Start with unrestricted Monte-Carlo TP runs:

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- Apply regression on the logits to learn prior probability and value
- **Deduce**, **Learn** from it, and **Loop**

Results

- 90%-10% split on the Mizar Mathematical Library (Grabowski et al., 2010) .
- training set is $\approx 30K$ problems, testing – $\approx 3.2K$

Iteration	1	2	5	8
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Methodology	**Proved**	**IPS**
Heuristics tableaux	1143	64K
RL tableaux	1624	16K

Table: Using RL gives 40% more proves but slows down the inference speed

Summary

- ML (RL) application on Theorem Provers
- Helps ‘discover’ new proofs!
- ... but slows down inference speed

Questions?

Bibliography

Grabowski, A., Kornilowicz, A., and Naumowicz, A. (2010). Mizar in a nutshell. *Journal of Formalized Reasoning*, 3(2):153–245.

Jakubuv, J. and Urban, J. (2017). ENIGMA: efficient learning-based inference guiding machine. In *Intelligent Computer Mathematics - 10th International Conference, CICM 2017, Edinburgh, UK, July 17-21, 2017, Proceedings*, pages 292–302.