Reinforcement Learning of Theorem Proving

a paper by

Cezary Kaliszyk, Josef Urban, Henryk Michalewski and Miroslav Olšák

presented by

Dobrik Georgiev

Overview

How tableau provers search

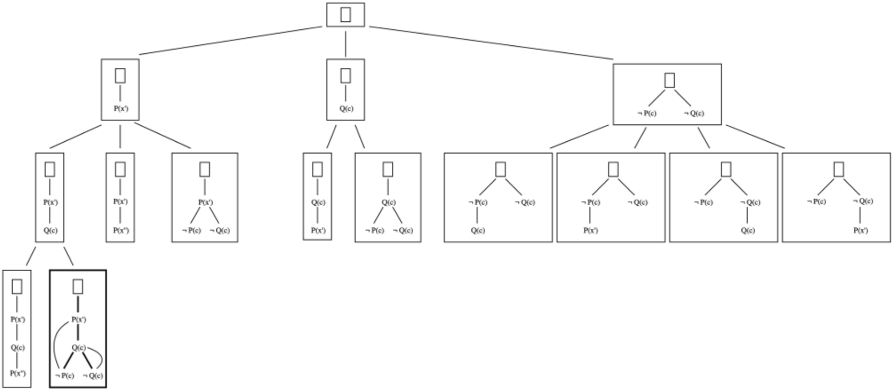


Figure: The search tree of a tableau based TP

RL Basics

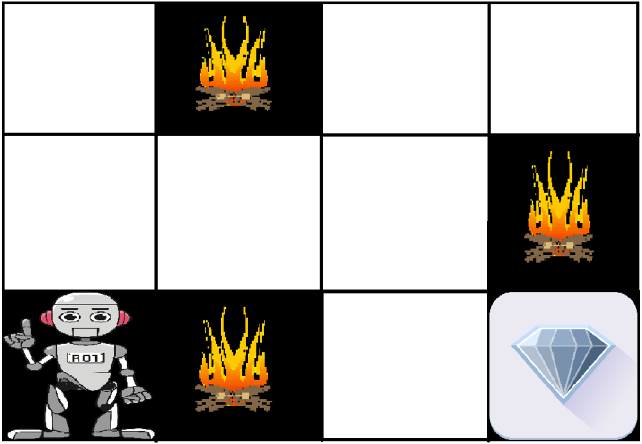


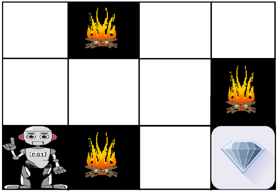
Figure: An agent has to reach a reward without burning

- policy learning
- value learning

Application to TP

RL to TP mapping:

- agent ↔ TP
- environment ↔ search tree
- actions ↔ extending search tree
- reward ↔ finding a closed tableau



Application to TP – the UCT formula

Tree search with RL – use the UCT formula!
For each node i:

$$f_i = \frac{w_i}{n_i} + c \cdot p_i \cdot \sqrt{\frac{\ln N_i}{n_i}}$$

On every step, take the node with highest f_i.

w_i: **total reward**
n_i: number node of visits
c: hyperparameter
p_i: **prior probability**
N_i: total parent visits

Application to TP – Extracting features (Literals)

- For each Literal L, e.g. $f(X, Y) = g(sk_1, sk_2(X))$
- Build it's feature tree
- Count term walks of length 3
- E.g. for L we get $\{(\oplus, =, f) : 1, (=, f, \circledast) : 2, ...\}$

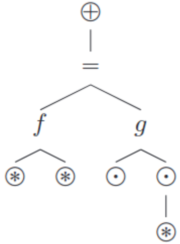


Figure: Feature Tree for L

Example and picture from Jakubuv and Urban (2017)

Application to TP – Extracting features

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- Feature vector for a *state*:
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  - additional metadata – No. of open goals, depth of node, etc.
- Features for an *action* contains:
  - features of the clause used
  - features of literal used

Application to TP – Learning the parameters

- Start with unrestricted Monte-Carlo TP runs:

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  - $r_a = \frac{overal\ frequency\ of\ a}{action\ frequency\ at\ node}$
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- Apply regression on the logits to learn prior probability and value
- **Deduce**, **Learn** from it, and **Loop** (Urban, 2007)

Results

- 90%-10% split on the Mizar Mathematical Library (Grabowski et al., 2010) .
- training set is $\approx 30K$ problems, testing – $\approx 3.2K$

Iteration	1	2	5	8
Training Proved	12325	13749	14403	14498
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Table: Proved problems per iterations of learning

Methodology	**Proved**	**IPS**
Heuristics tableaux	1143	64K
RL tableaux	1624	16K

Table: Using RL gives 40% more proves but slows down the inference speed

Summary

- ML (RL) application on Theorem Provers
- Helps ‘discover’ new proofs!
- ... but slows down inference speed

Questions?

Bibliography

Grabowski, A., Kornilowicz, A., and Naumowicz, A. (2010). Mizar in a nutshell. *Journal of Formalized Reasoning*, 3(2):153–245.

Jakubuv, J. and Urban, J. (2017). ENIGMA: efficient learning-based inference guiding machine. In *Intelligent Computer Mathematics - 10th International Conference, CICM 2017, Edinburgh, UK, July 17-21, 2017, Proceedings*, pages 292–302.

Urban, J. (2007). MaLARea: a metasystem for automated reasoning in large theories. In *Proceedings of the CADE-21 Workshop on Empirically Successful Automated Reasoning in Large Theories, Bremen, Germany, 17th July 2007*.