APPENDIX

A: Nomenclature

ω_r	Rotor speed
β	Pitch angle
i_{dr}, i_{qr}	Rotor-side d- and q-axis currents
i_{dg}, i_{qg}	Grid-side d- and q-axis currents
V_{dc}	DC-link voltage
$i_{\text{int},dr}, i_{\text{int},qr}$	Integral states for rotor-side current controllers
$i_{\text{in},dg}, i_{\text{int},qg}$	Integral states for grid-side current controllers
$i_{\text{int},Vdc}$	Integral state for DC-link voltage controller
i_{int,ω_r}	Integral state for rotor speed controller
$i_{\text{int},\theta}$	Integral state for pitch controller
$i_{dr,ref}, i_{qr,ref}$	Reference values for rotor-side d- and q-axis currents
$i_{dg,ref}, i_{qg,ref}$	Reference values for grid-side d- and q-axis currents
$V_{dc,ref}$	Reference DC link voltage
β_{ref}	Reference pitch angle
$\omega_{r,ref}$	Reference rotor speed
K_{pr}, K_{ir}	PI gains of rotor-side current controller
K_{pg}, K_{ig}	PI gains of grid-side current controller
K_{ps}, K_{is}	PI gains of speed controller
K_{pp}, K_{ip}	PI gains of pitch controller
K_{pdc}, K_{idc}	PI gains of DC-link voltage controller
P_{rated}	Rated capacity
H	Inertia constant

B.1 PMSG based wind turbine state space modelling

1. Model of drivetrain aerodynamics:

$$\frac{d\omega_r}{dt} = \frac{1}{J}\left(T_{mech} - T_{em}\right)$$

2. Pitch controller dynamics:

$$\frac{d\beta}{dt} = K_{pp}\left(\omega_{ref} - \omega_r\right) + K_{ip}\int\left(\omega_{ref} - \omega_r\right)dt$$

$$\frac{d}{dt}\int\left(\omega_{ref} - \omega_r\right) = \omega_{ref} - \omega_r$$

3. Rotor side converter dynamics:

$$\frac{di_{dr}}{dt} = \frac{1}{L_d}\left[K_{p,dr}\left(i_{dr,ref} - i_{dr}\right) + K_{i,dr}\int\left(i_{dr,ref} - I_{dr}\right) - R_r i_{dr} + \omega_r L_q i_{qr}\right]$$

$$\frac{dI_{qr}}{dt} = \frac{1}{L_q}\left[K_{p,qr}\left(i_{qr,ref} - i_{qr}\right) + K_{i,qr}\int\left(i_{qr,ref} - i_{qr}\right) + R_r i_{qr} + \omega_r\left(L_d i_{dr} + \lambda_m\right)\right]$$

$$\frac{d}{dt}\int\left(i_{dr,ref} - i_{dr}\right) = i_{dr,ref} - i_{dr}$$

$$\frac{d}{dt}\int\left(i_{qr,ref} - i_{qr}\right) = i_{qr,ref} - i_{qr}$$

4. Grid side converter dynamics:

$$\frac{di_{dg}}{dt} = \frac{1}{L_g}\left[K_{p,dg}\left(i_{dg,ref} - i_{dg}\right) + K_{i,dg}\int\left(i_{dg,ref} - i_{dg}\right) + R_g i_{dg} - \omega_g L_g i_{qg} + V_{gd}\right]$$

$$\frac{di_{qg}}{dt} = \frac{1}{L_g}\left[K_{p,qg}\left(i_{qg,ref} - i_{qg}\right) + K_{i,qg}\int\left(i_{qg,ref} - i_{qg}\right) + R_g i_{qg} + \omega_g L_g i_{dg} + V_{gq}\right]$$

$$\frac{d}{dt}\int\left(i_{dg,ref} - i_{dg}\right) = i_{dg,ref} - i_{dg}$$

$$\frac{d}{dt}\int\left(i_{qg,ref} - i_{qg}\right) = i_{qg,ref} - i_{qg}$$

5. DC-link dynamics:

$$\frac{dV_{dc}}{dt} = \left(\frac{i_{\mathrm{dr}} - i_{dg}}{C_{dc}}\right)$$

$$\frac{d}{dt}\int\left(V_{dc,ref} - V_{dc}\right) = V_{dc,ref} - V_{dc}$$

This all defining the detailed physics of a single aggregated turbine cluster.

B.2 Aggregated Wind Farm State-Space Model

For each cluster $k = 1, \dots, K$ (one PMSG-based equivalent per cluster):

- $X^{(k)} \in \mathbb{R}^{n_k}$: state vector ($\omega_r, \beta, i_{dr}, i_{qr}, i_{dg}, i_{qg}, V_{dc}$ and controller integrators).
- $U^{(k)} \in \mathbb{R}^{m_k}$: input vector ($i_{dr,\text{ ref}}, i_{qr,\text{ ref}}, i_{dg,\text{ ref}}, i_{qg,\text{ ref}}, V_{dc,\text{ ref}}, \beta_{\text{ref}}, \omega_{r,\text{ ref}}$).
- $V^{(k)}, I^{(k)}$: terminal voltage and current of the cluster at its collector node
- System matrices of the cluster:
  $A^{(k)}$ (state), $B^{(k)}$ (input), $E^{(k)}$ (voltage coupling),
  $C^{(k)}$ (state-to-current), $D^{(k)}$ (input-to-current), $F^{(k)}$ (voltage-to-current).

Step 1 — Per-cluster PMSG model

$$\dot{X}^{(k)} = A^{(k)}X^{(k)} + B^{(k)}U^{(k)} + E^{(k)}V^{(k)} \tag{36}$$
$$I^{(k)} = C^{(k)}X^{(k)} + D^{(k)}U^{(k)} + F^{(k)}V^{(k)} \tag{37}$$

Here, system matrices for the cluster:

$A^{(k)}$ (state), $B^{(k)}$ (input), $E^{(k)}$ (voltage coupling),
$C^{(k)}$ (state-to-current), $D^{(k)}$ (input-to-current), $F^{(k)}$ (voltage-to-current).

Step 2 — Stack all Clusters

Build farm-level stacked vectors:

$$X = \left[X^{(1)}; \dots; X^{(n)}\right], U = \left[U^{(1)}; \dots; U^{(n)}\right], V = \left[V^{(1)}; \dots; V^{(K)}\right], I = \left[I^{(1)}; \dots; I^{(K)}\right]$$

Using (36)-(37), the farm-level plant is:

$$\dot{X} = AX + BU + EV \tag{38}$$
$$I = CX + DU + FV \tag{39}$$

Step 3 — Collector-network coupling (PCC included)

Let Y_{col} be the nodal admittance matrix of the collector system (strings, cables, transformers, PCC model).

$$I = Y_{col}V \tag{40}$$

Step 4 — Closed DAE for the farm

Combine (39) and (40):

$$(Y_{\text{col}} - F)V = CX + DU$$

Together with (39), the farm is a differential-algebraic system

$$\dot{X} = AX + BU + EV \tag{41}$$
$$0 = (Y_{\text{col}} - F)V - (CX + DU) \tag{42}$$

Step 5 — Eliminate algebraic voltages

If $(Y_{\text{col}} - F)$ is nonsingular:

$$V = (Y_{\text{col}} - F)^{-1}(CX + DU) \quad (43)$$

Substitute (43) into (42) to obtain a pure ODE:

$$\dot{X} = A_{\text{agg}} X + B_{\text{agg}} U \quad (44)$$

with the aggregated matrices

$$A_{\text{agg}} = A - E(Y_{\text{col}} - F)^{-1}C \quad (45)$$

$$B_{\text{agg}} = B - E(Y_{\text{col}} - F)^{-1}D \quad (46)$$

This $(A_{\text{agg}}, B_{\text{agg}})$ is the final aggregated wind-farm state-space.

C. Parameters of the model

TABLE I: WT AND CABLE PARAMETERS FOR CASE STUDY [4], [19]

Parameters	Values	Parameters	Values
Rated Power PMSG (MW)	1.5	Rated frequency	50 Hz
Stator resistance (p.u.)	0.0272	35/0.69 kV tran. rated power (MVA)	2.5
Stator reactance (p.u.)	0.5131	35/0.69 kV tran. impedance (p.u.)	0.05
220/35 kV tran. rated Power (MVA)	50	220/35 kV tran. impedance (p.u.)	0.1
Flux linkage (p.u.)	1.1884	Leakage reactance of T1 (p.u.)	0.0415
Inertia constant (s)	1.4393	DC Bus Voltage (V)	1150
Filter resistance (p.u.)	0.03	Filter reactance (p.u.)	0.3
DC capacitor (F)	0.6	Length of MV cable (km)	3.0
WT cable resistance (Ω/km)	0.124	WT cable reactance (mH/km)	0.39
MV cable resistance (Ω/km)	0.36	MV cable reactance (mH/km)	0.32
Submarine cable resistance (Ω/km)	0.34	Submarine cable reactance (mH/km)	0.17