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HW2, Q1 and 2

1a) $5n^3 + 2n^2 + 3n = O(n^3)$

$5n^3 + 2n^2 + 3n \leq (5+2+3)n^3$

$5n^3 + 2n^2 + 3n \leq Cn^3$

$5(1)^3 + 2(1)^2 + 3(1) \leq 10(1)^3$

$10 \leq 10$ ✓

$C = 10$
$n_0 = 1$

b) $\sqrt{7n^2+2n-8} = \Theta(n)$

$C_2(n) \leq \sqrt{7n^2+2n-8} \leq C_1(n)$

$1n \leq \sqrt{7n^2+2n-8} \leq 9n$

$1(1) \leq \sqrt{7(1)^2+2(1)-8} \leq 9(1)$

$1 \leq 1 \leq 9$ ✓

$C_1 = 9$
$C_2 = 1$
$n_0 = 1$

c) $d(n) = O(f(n)) \rightarrow d(n) \leq C_1 f(n)$

$e(n) = O(g(n)) \rightarrow e(n) \leq C_2 g(n)$

$\therefore d(n) \cdot e(n)$ must be $\leq C_1 f(n) \cdot C_2 g(n)$ where $C_1 \cdot C_2 = C$

↳ $d(n)e(n) \leq C(f(n) \cdot g(n))$

↳ $d(n)e(n) = O(f(n)g(n))$

2 Example 2 ← (Example 2 first, Example 1 is in next page)

```
θ(1) { n = len(lst)
       prefix = 0
       total = 0
θ(n) { for j in range(n):
θ(1)   { prefix += lst[j]
         total += prefix
θ(1) { return total
```

Runtime / asymptotic order = $\Theta(n)$

2 Example 1

$\theta(1)$ { n = len(lst)
total = 0

$\theta(\frac{1}{2}n^2)$ { for j in range(n):
$= \theta(n^2)$ $\theta(\frac{1}{2}n)$ { for k in range(1+j):
total += lst[k]

$\theta(1)$ { return total

Runtime = $\theta(n^2)$

2 Example 3

$\theta(1)$ { i = 1
sum = 0
while (i < n*n):
$\theta(1)$ { i *= 2
sum += i
$\theta(1)$ { return sum

$2^i < n^2$

$\log 2^i < \log n^2$

$i < 2 \log n$

Runtime = $\theta(\log n)$

3 Example 4

$\theta(1)$ { i = n
sum = 0
while (i > 1):
for j in range(i):
sum += i * j
i //= 2
$\theta(1)$ { return sum

If n = 32

i = 32 → 32 iterations
i = 16 → 16 iterations
i = 8 → 8 iterations
i = 4 → 4 iterations
i = 2 → 2 iterations

$\log_2 n$ 62 iterations = $2^6 - 2$

$2^{\log_2 n + 1} - 2$

$2^{\log_2 n} \cdot 2 - 2$

$2n - 2 \Rightarrow n$

Runtime = $\theta(n)$