Heat distribution on a cooling fin

Reality Check 08

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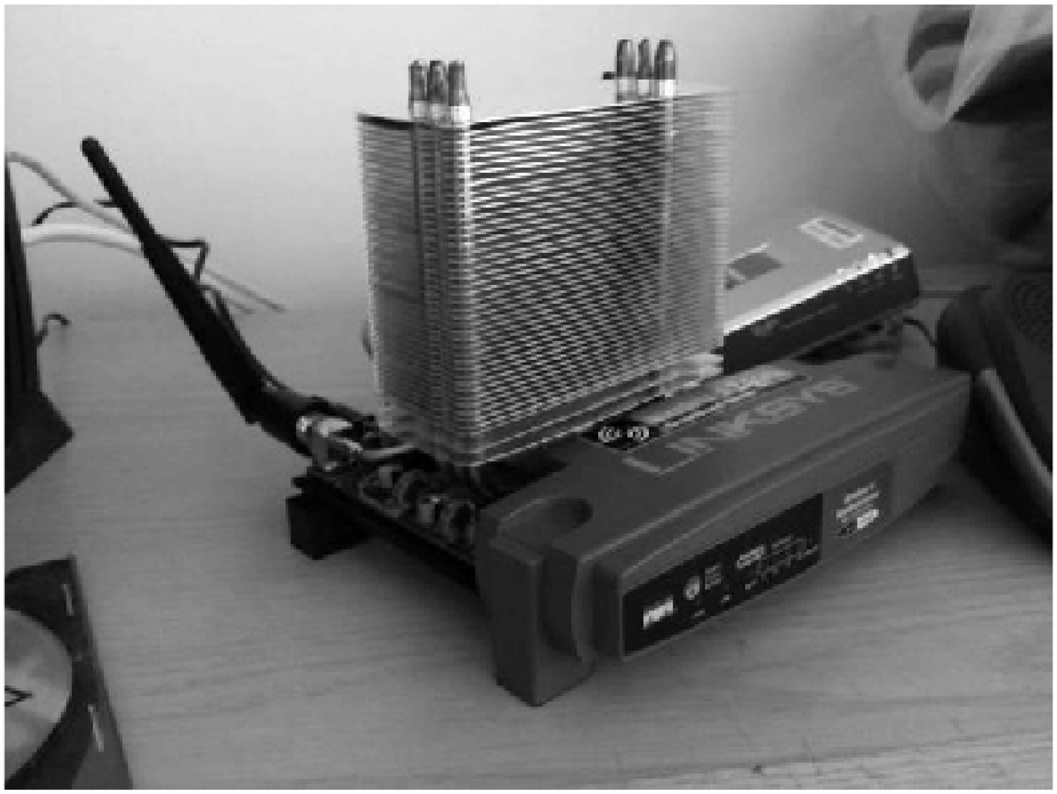
Math 412 : Scientific Computing



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Background

Heat sinks are used to move excess heat away from the point where it is generated. In this project, the steady-state distribution along a rectangular fin of a heat sink will be modeled. The heat energy will enter the fin along part of one side. The main goal will be to design the dimensions of the fin to keep the temperature within safe tolerances.

The fin shape is a thin rectangular slab, with dimensions $L_x \times L_y$ and width δ cm, where δ is relatively small. Due to the thinness of the slab, we will denote the temperature by $u(x, y)$ and consider it constant along the width dimension.

Heat moves in the following three ways: conduction, convection, and radiation. Conduction refers to the passing of energy between neighboring molecules, perhaps due to the movement of electrons, while in convection the molecules themselves move. Radiation, the movement of energy through photons, will not be considered here.

Conduction proceeds through a conducting material according to Fourier's first law

$$q = -KA\Delta u, \quad (8.41)$$

where q is heat energy per unit time (measured in watts), A is the cross-sectional area of the material, and Δu is the gradient of the temperature. The constant K is called the **thermal conductivity** of the material. Convection is ruled by Newton's law of cooling,

$$q = -HA(u - u_b), \quad (8.42)$$

where H is a proportionality constant called the **convective heat transfer coefficient** and u_b is the ambient temperature, or **bulk temperature**, of the surrounding fluid (in this case, air).

The fin is a rectangle $[0, L_x] \times [0, L_y]$ by δ cm in the z direction, as illustrated in Figure 8.15(a). Energy equilibrium in a typical $x \times y \times \delta$ box interior to the fin, aligned along the x and y axes, says that the energy entering the box per unit time equals the energy leaving. The heat flux into the box through the two $y \times \delta$ sides and two $x \times \delta$ sides is by conduction, and through the two $x \times y$ sides is by convection, yielding the steady-state equation

$$-K\Delta y\delta u_x(x, y) + K\Delta y\delta u_x(x\Delta +, y) - K\Delta x\delta u_y(x, y)$$

$$-K\Delta x\delta u_y(x, y + \Delta y) - 2H\Delta x\Delta y u(x, y) = 0 \qquad (8.43)$$

Here, we have set the bulk temperature $u_b = 0$ for convenience; thus, u will denote the difference between the fin temperature and the surroundings. Dividing through by $\Delta x \Delta y$ gives

$$K\delta \frac{u_x(x+\Delta x, y) - u_x(x, y)}{\Delta x} + \frac{u_y(x, y+\Delta y) - u_y(x, y)}{\Delta y} = 2Hu(x, y) \text{ ,}$$

And in the limit as $\Delta x, \Delta y \to 0$, the elliptic partial differential equation

$$u_{xx} + u_{yy} = \frac{2H}{K\delta} u \qquad (8.44)$$

results,

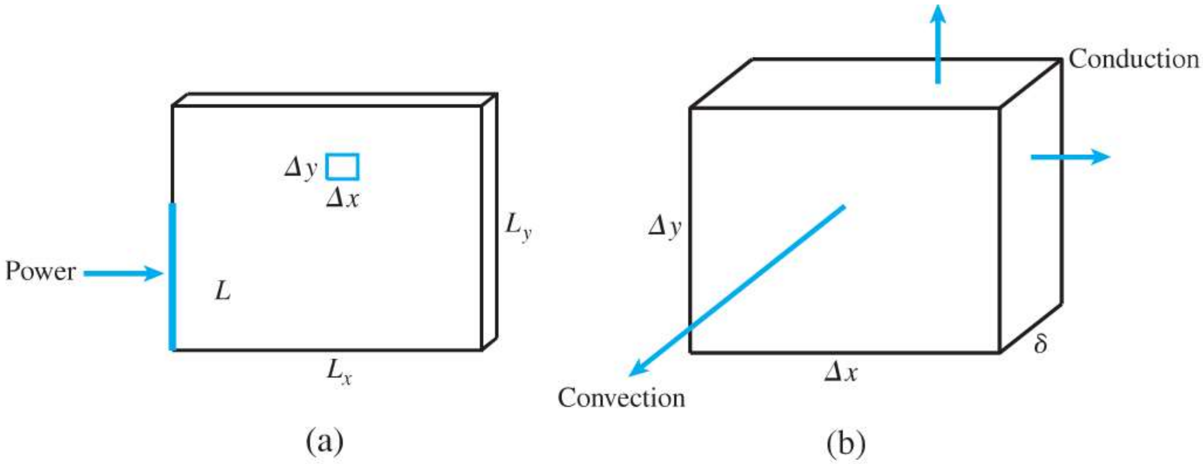


Figure 8.15 Cooling fin in Reality Check 8. (a) Power input occurs along interval $[0, L]$ on left side of fin. (b) Energy transfer in small interior box is by conduction along the x and y directions, and by convection along the air interface.

Similar arguments imply the **convective** boundary condition

$$Ku_{\text{normal}} = Hu$$

where u_{normal} is the partial derivative with respect to the outward normal direction n. The convective boundary condition is known as a **Robin** boundary condition, one that involves both the function value and its derivative. Finally, we will assume that power enters the fin along one side according to Fourier's law,

$$u_{\text{normal}} = \frac{P}{L\delta K},$$

where P is the total power and L is the length of the input.

On a discrete grid with step sizes h and k, respectively, the finite difference approximation (5.8) can be used to approximate the PDE (8.44) as

$$\frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{ij} + u_{i,j-1}}{k^2} = \frac{2H}{K\delta}u_{ij}.$$

This discretization is used for the interior points (x_i, y_i) where $1 < i < m$, $1 < j < n$ for integers $m, n.$ The fin edges obey the Robin conditions using the first derivative approximation

$$f'(x) = \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h} + O(h^2).$$

To apply this approximation to the fin edges, note that the outward normal direction translates to

$$\begin{aligned} u_{\text{normal}} &= -u_y \text{ on bottom edge} \\ u_{\text{normal}} &= u_y \text{ on top edge} \\ u_{\text{normal}} &= -u_x \text{ on left edge} \\ u_{\text{normal}} &= u_x \text{ on right edge} \end{aligned}$$

Second, note that the second-order first derivative approximation above yields

$$\begin{aligned} u_y &\approx \frac{-3u(x,y) + 4u(x,y+k) - u(x,y+2k)}{2k} \text{ on bottom edge} \\ u_y &\approx \frac{-3u(x,y) + 4u(x,y-k) - u(x,y-2k)}{-2k} \text{ on top edge} \\ u_x &\approx \frac{-3u(x,y) + 4u(x+h,y) - u(x+2h,y)}{2h} \text{ on left edge} \\ u_x &\approx \frac{-3u(x,y) + 4u(x-h,y) - u(x-2h,y)}{-2h} \text{ on right edge} \end{aligned}$$

Putting both together, the Robin boundary condition leads to the difference equations

$$\begin{aligned} \frac{-3u_{i1} + 4u_{i2} - u_{i3}}{2k} &= -\frac{H}{K}u_{i1} \text{ on bottom edge} \\ \frac{-3u_{in} + 4u_{i,n-1} - u_{i,n-2}}{2k} &= -\frac{H}{K}u_{in} \text{ on top edge} \\ \frac{-3u_{1j} + 4u_{2j} - u_{3j}}{2h} &= -\frac{H}{K}u_{1j} \text{ on left edge} \\ \frac{-3u_{mj} + 4u_{m-1,j} - u_{m-2,j}}{2h} &= -\frac{H}{K}u_{mj} \text{ on right edge.} \end{aligned}$$

If we assume that the power enters along the left side of the fin, Fourier's law leads to the equation

$$\frac{-3u_{1j} + 4u_{2j} - u_{3j}}{2h} = -\frac{P}{L\delta K}. \tag{8.45}$$

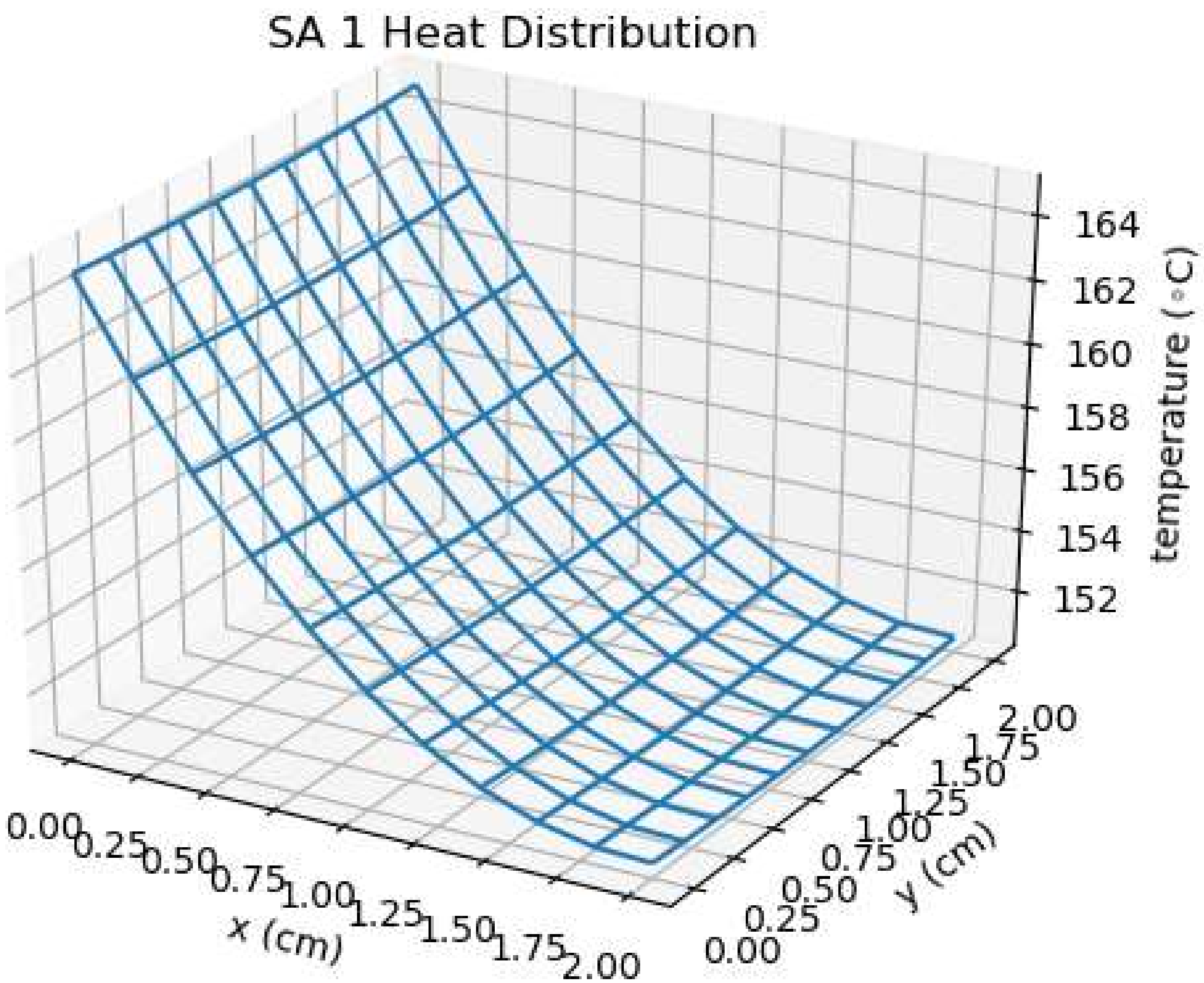
There are $m\,n$ equations in the $m\,n$ unknowns $u_{ij},\ 1 \le i \le m,\ 1 \le j \le n$ to solve. Assume that the fin is composed of aluminum, whose thermal conductivity is $K = 1.68\frac{W}{cm}\,°C$ (watts per centimeter-degree Celsius). Assume that the convective heat transfer coefficient is $H = 0.005\frac{W}{cm^2}°C$, and that the room temperature is $u_b = 20°C$.

Suggested Activities

1. Begin with a fin of dimensions $2 \times 2\ cm$, with $1\ mm$ thickness (δ). Assume that 5W of power is input along the entire left edge, as if the fin were attached to dissipate power from a CPU chip with $L = 2\ cm$ side length. Solve the PDE (8.44) with $M = N = 10$ subintervals in the x and y directions (11 points). Use the mesh command to plot the resulting heat distribution over the $xy-$plane. What is the maximum temperature of the fin, in $°C$?

Answer:

We found the maximum temperature across the xy plane given the conditions to be $164.936°C$.



2. Increase the size of the fin to $4 \times 4\ cm$. Input $5W$ of power along the interval $[0, 2]$ on the left side of the fin, as in the previous step. Plot the resulting distribution, and find the maximum temperature. Experiment with increased values of M and N (use 10, 40, 100). How much does the solution change?

Answer:

We found the best overall temperature reading to be 52.945 when $m = n = 100$. We feel that this is because there is less error when the partition is smaller, however we have to cap out our partition somewhere so we chose to do that at 100.

3. Find the maximum power that can be dissipated by a $4 \times 4\ cm$ fin while keeping the maximum temperature less than $80\ °C$. Assume the power input is the same as in steps 1 and 2.

Answer:

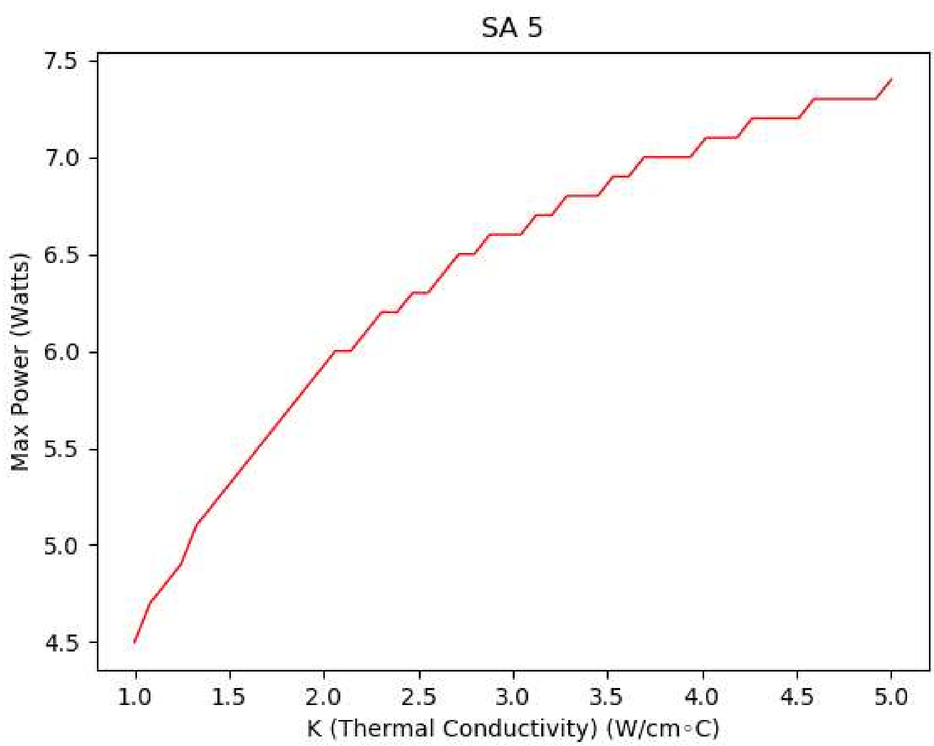
We found the maximum power that can be dissipated by a $4 \times 4\ cm$ fan while maintaining a maximum temperature less than 80 $°C$ was 5.666 watts. The highest temperature that our cooling fin reached given these conditions and at this power was 79.997 $°C$. To find this value we ran an iterative process that would recalculate the maximum temperature given a power, and would increment the power value and recalculate the maximum temperature reached. This iterative process terminated once the temperature exceeded 80 $°C$.

4.Replace the aluminum fin with a copper fin, with thermal conductivity $K = 3.85 \frac{W}{cm}$ °C. Find the maximum power that can be dissipated by a 4×4 cm fin with the 2 cm power input as in steps 1 and 2, while keeping the maximum temperature below 80 °C.

Answer:

After replacing the aluminum fin with a copper fin, we found the maximum power that can be dissipated by a 4×4 cm fan while maintaining a maximum temperature less than 80 °C was 7.150 watts. The highest temperature that our cooling fin reached given these conditions and at this power was 79.997 °C. To find this value we ran the same iterative process as in SA3.

5. Plot the maximum power that can be dissipated in step 4 (keeping maximum temperature below 80 degrees) as a function of thermal conductivity, for $1 \leq K \leq 5$ $\frac{W}{cm}$ °C.



Answer: We see that as we increase the value for Thermal Conductivity, that our heat fin can take in more Power in watts and still remain under a threshold of 80 $°C$. The corresponding graph of the maximum power that can be dissipated in step 4 (keeping maximum temperature below 80 degrees) as a function of thermal conductivity, for $1 \le K \le 5 \frac{W}{cm}$, follows a graph similar of $y = \sqrt{x}$.

6. Redo step 4 for a water-cooled fin. Assume that water has a convective heat transfer coefficient of $H = 0.1 \frac{W}{cm^2}$ $°C$, and that the ambient water temperature is maintained at 20 $°C$.

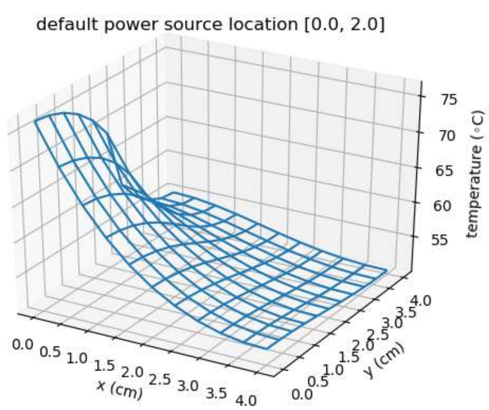
Answer:

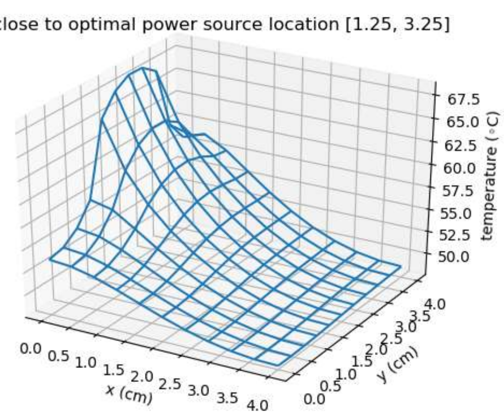
Under a water cooled fin, we found the maximum power that can be dissipated by a 4x4cm fan while maintaining a maximum temperature less than 80 $°C$ was 36.517 watts. The highest temperature that our cooling fin reached given these conditions and at this power was 80.000 $°C$. To find this value we ran the same iterative process as in SA3.

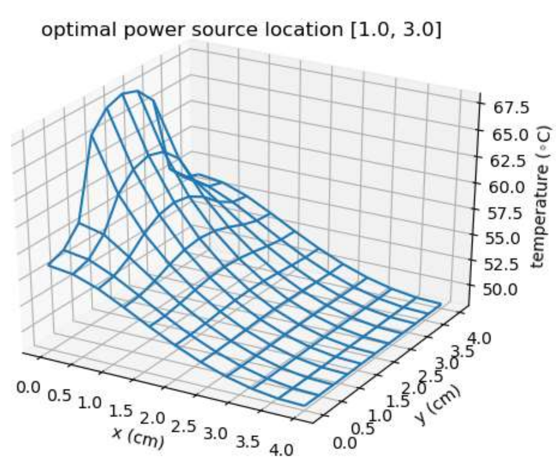
7.Using the $4 \times 4\ cm$ fin from step 2, try various positions for the $2\ cm$ power source along the left edge. What is the best location for the power source? How much is the maximum temperature reduced compared to the configuration in step 2?

Answer:

After much trial and error, we found that the optimal location for the power source to be applied is right in the middle of the left side. We found that this location optimizes the dissipation across the entire cooling fin, thus minimizing the maximum temperature found on the cooling fin.







NOTE:

The design of cooling fins for desktop and laptop computers is a fascinating engineering problem. To dissipate ever greater amounts of heat, several fins are needed in a small space, and fans are used to enhance convection near the fin edges. The addition of fans to complicated fin geometry moves the simulation into the realm of computational fluid dynamics, a vital area of modern applied mathematics.

Further Ideas and concepts to pursue

1.Change Shape of fin

2. Modeling a set of fins

3. Adding cut's and holes

4. Modeling fans

5.Modeling submerged heat sinks

6. Streamline evaluation process

7. Learn other stencils and methods

8.Optimize calculation process(GPU/Threading)