AIR WATER AIR TEST CASE

Author
Madison Sheridan

Contents

1   Problem Description

We describe the air-water-air test that was introduced in [2], and lated tested in [1, 3]. In this test, we consider a radially symmetric two-phase configuration consisting of an inner air region, a middle water layer, and an outer air region. The computational domain corresponds to a cylinder of radius $r \in [0, 1.2]$, where the material distribution is defined as follows:

$$(\rho, \boldsymbol{v}, p) = \begin{cases} (0.001, \mathbf{0}, 1000), & 0 < r < 0.2, \\ (1, \mathbf{0}, 1), & 0.2 < r < 1.0, \\ (0.001, \mathbf{0}, 0.001), & 1.0 < r < 1.2. \end{cases} \tag{1}$$

The first and last regions correspond to air and are modeled by the ideal gas EOS with $\gamma = 1.4$. The middle region contained water and is modeled by the Stiffened Gas EOS with $\gamma = 7$ and $p_s = 3000$. The problem thus models the interaction between compressible air and nearly incompressible water layers under Lagrangian motion.

Because of the radial symmetry of the configuration, we restrict computations to a quarter of the domain and apply symmetry boundary conditions along the bottom, left, and right boundaries.

This test is designed to assess the robustness and positivity-preserving properties of the Lagrangian scheme in the presence of large material density contrasts. In particular, it is known that without a positivity-preserving limiter, the internal energy may become negative in the air regions, leading to numerical instability. Therefore, this case provides a stringent test for evaluating the ability of the limiter to maintain the physical admissibility of density and internal energy throughout the evolution.

References

[1] Juan Cheng and Chi-Wang Shu. Positivity-preserving lagrangian scheme for multi-material compressible flow. J. Comput. Phys., 257(PA):143–168, January 2014. ISSN 0021-9991.

[2] Keh-Ming Shyue. A fluid-mixture type algorithm for barotropic two-fluid flow problems. Journal of Computational Physics, 200(2):718–748, 2004. ISSN 0021-9991. doi: https://doi.org/10.1016/j.jcp.2004.05.003. URL `https://www.sciencedirect.com/science/article/pii/S002199910400186X`.

[3] François Vilar, Chi-Wang Shu, and Pierre-Henri Maire. Positivity-preserving cell-centered lagrangian schemes for multi-material compressible flows: From first-order to high-orders. part ii: The two-dimensional case. Journal of Computational Physics, 312: 416–442, 2016. ISSN 0021-9991. doi: https://doi.org/10.1016/j.jcp.2016.01.037. URL `https://www.sciencedirect.com/science/article/pii/S0021999116000802`.