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1.1 Questions

1. Typo in isotropic case of the stress tensor, Eq. (2.5)? Should this be $\boldsymbol{b} - \frac{1}{3}\operatorname{tr}(\boldsymbol{b})\mathbb{I}$?

2. How should the stress function be formulated? I have seen various formulations

   - $\boldsymbol{\sigma} = 2\frac{\rho}{\rho_0} F \frac{\partial e}{\partial \boldsymbol{C}} F^{\mathsf{T}}$ On this implementation in particular, why ρ_0 in the denominator? In [5, Eq. 17] the authors use this formulation without ρ_0. Does this formulation have a name like the Murnaghan stress tensor?
   - Murnaghan stress tensor

   $$\boldsymbol{\sigma} = 2\rho \frac{\partial e}{\partial \boldsymbol{G}} \boldsymbol{G}, \quad G = \left(F^{\mathsf{T}}\right)^{-1} F^{-1}. \tag{1}$$

   This formulation is used in [4, Sec. 3.2], [1, Eq. (14)], [3, p. 176].

3. Uniaxial tension test case [2]. I believe that $\boldsymbol{\sigma}$ needs to be reformulated in terms of the reduced invariants.

4. Issues with momentum conservation (both cartesian directions for all closure models.)

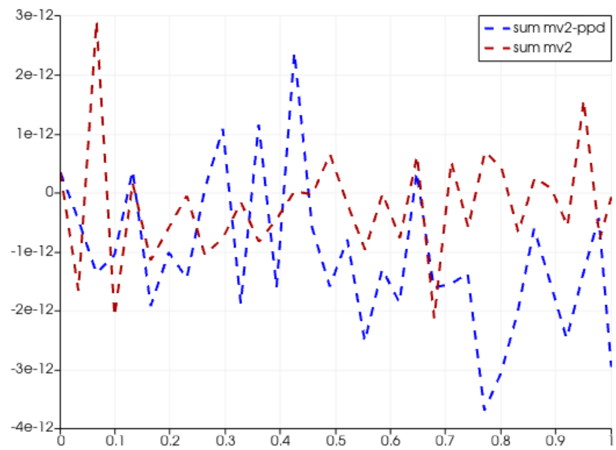

Figure 1: Exact momentum conservation for IVTF1.

5. Bonet Burton human leg impact example. Can I run this problem using rectangle shapes in 2D to represent the human leg and wooden beam? Otherwise I am not sure how to handle collapse of air cells between objects initially not in contact.

1.2 Notes

First a summary of the discussion addressing the above questions.

1. Either is fine. $\operatorname{tr}(\boldsymbol{b})$ is the same as j_1 so both forms are equivalent.

2. The Murnaghan stress tensor is fine to be used in the isotropic case since $\boldsymbol{\sigma}$ is symmetric. The problem is that bC does not commute with $\boldsymbol{G}_i$ in the isotropic case, for example for symmetrix matrices a and G:

   $$(aG)^{\mathsf{T}} = G^{\mathsf{T}} a^{\mathsf{T}} = Ga \neq aG.$$

   We had a discussion as well on $\partial e / \partial C$ being homogeneous which means it is a polynomial with respect to $\boldsymbol{G}_i$ which isn't true.

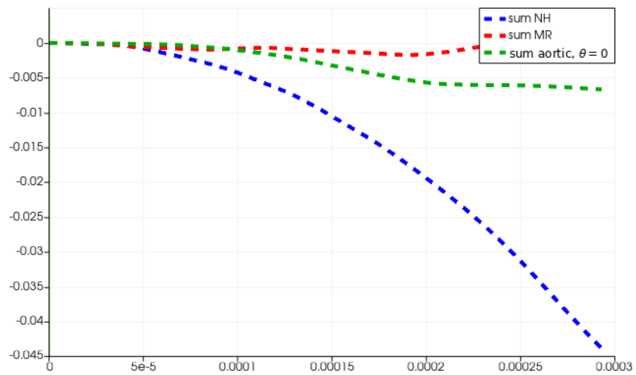

Figure 2: Momentum conservation error for NH, MR, and uniaxial $\theta = 0$.

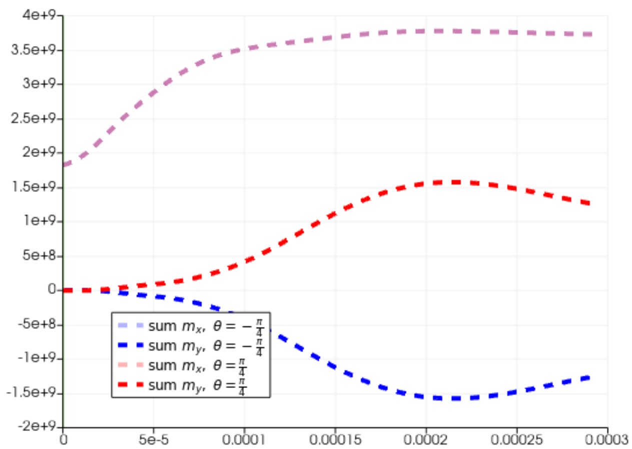

Figure 3: Momentum conservation error for uniaxial closure.

3. The Uniaxial test case from [2] is derived in the incompressible case where $\det \boldsymbol{C} = 1$ and so $\boldsymbol{c} = \boldsymbol{C}$. Because of this, hyperbolicity is proved in the incompressible case but not guaranteed in the compressible case. Hence this is a simplifying assumption for our model, but there is no scaling factor $\det \boldsymbol{C}$ that should arise since we adopt the same assumption.

4. Momentum conservation errors are likely due to the boundary conditions. In particular, the vacuum boundary conditions in the elastic implementation are enforced by setting $\boldsymbol{\sigma} = \mathbf{0}$ on the boundary. Hence the only nonzero entry in the flux at the boundary will be in the update to the specific volume since this is solely dependent on the velocity. To check if boundary conditions are the issue, can also run a test case on a large outer square domain with uniform conditions and then prescribe a velocity on a small subset of the domain. Should get exact mass conservation until that small section interacts with the boundary. Also consider running the elastic isentropic vortex.

5. Rectangle mesh is fine. Can also use concrete instead of wood since we have those

paramters:

$$\begin{aligned} \gamma &= 4.2 & p_\infty &= 23.9 \text{ (gPa)} \\ \rho &= 2400 \text{ (kg/m}^3\text{)} & \mu &= 13 \text{ (gPa)}. \end{aligned}$$

References

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