Week7

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1 Probability mass function (PMF) of Y for each possible value of Y (from 1 to k):

Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y .

Probability mass function (PMF) of Y for each possible value of Y (from 1 to k) when n = 3 and k = 5.

```
 # Let's take n = 3 (number of random variables) and k = 5 (range of each random variable).
# Function to calculate the probability mass function (PMF) of Y
pmf_Y <- function(y, n, k) {
  if (y < 1 || y > k) {
    stop("Invalid value of y. It should be between 1 and k.")
  }

  # Calculate the probability mass function (PMF)
  pmf_value <- (1 - ((y - 1) / k)^n)

  return(pmf_value)
}

# Calculate the PMF for Y = 1, 2, ..., k
k <- 5  # Range of each random variable

for (y in 1:k) {
  pmf_value <- pmf_Y(y = y, n = 3, k = k)
  cat("P(Y =", y, ") =", pmf_value, "\n")
}
```

```
## P(Y = 1 ) = 1
## P(Y = 2 ) = 0.992
## P(Y = 3 ) = 0.936
## P(Y = 4 ) = 0.784
## P(Y = 5 ) = 0.488
```

2.Exponential Distribution:

Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

```
# Function to calculate the probability of failure within a given time period
failure_probability <- function(time_period, lambda) {
  if (time_period < 0) {
    stop("Time period should be non-negative.")
```

```r
  }

  # Calculate the probability of failure using the exponential distribution formula
  probability_failure <- 1 - exp(-lambda * time_period)

  return(probability_failure)
}

# Given information
expected_lifetime <- 10  # years
lambda <- 1/expected_lifetime

# Example usage
time_period_5_years <- 5
probability_5_years <- failure_probability(time_period_5_years, lambda)
cat("Probability of failure within", time_period_5_years, "years:", probability_5_years, "\n")
```

```
## Probability of failure within 5 years: 0.3934693
```

```r
time_period_10_years <- 10
probability_10_years <- failure_probability(time_period_10_years, lambda)
cat("Probability of failure within", time_period_10_years, "years:", probability_10_years, "\n")
```

```
## Probability of failure within 10 years: 0.6321206
```

2a: What is the probability that the machine will fail after 8 years?

```r
# Function to calculate the probability of not failing within a given time period
probability_not_failure <- function(time_period, lambda) {
  if (time_period < 0) {
    stop("Time period should be non-negative.")
  }

  # Calculate the probability of not failing using the exponential distribution formula
  probability_not_failure <- exp(-lambda * time_period)

  return(probability_not_failure)
}

# Given information
time_period_8_years <- 8
probability_not_failure_8_years <- probability_not_failure(time_period_8_years, lambda)
probability_failure_8_years <- 1 - probability_not_failure_8_years

cat("Probability of failure after", time_period_8_years, "years:", probability_failure_8_years, "\n")
```

```
## Probability of failure after 8 years: 0.550671
```

```r
# Expected value of the geometric distribution
expected_value <- 1 / lambda
cat("Expected value:", expected_value, "years\n")
```

```
## Expected value: 10 years
```

```r
# Standard deviation of the geometric distribution
standard_deviation <- sqrt((1 - lambda) / (lambda^2))
```

```r
cat("Standard deviation:", standard_deviation, "years\n")
```

```
## Standard deviation: 9.486833 years
```

2b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

```r
# Probability that the machine will fail after 8 years
probability_failure_after_8_years <- 1 - pexp(8, rate = lambda)
cat("Probability of failure after 8 years:", probability_failure_after_8_years, "\n")
```

```
## Probability of failure after 8 years: 0.449329
```

```r
# Expected value of the exponential distribution
expected_value_exponential <- 1 / lambda
cat("Expected value:", expected_value_exponential, "years\n")
```

```
## Expected value: 10 years
```

```r
# Standard deviation of the exponential distribution
standard_deviation_exponential <- 1 / lambda
cat("Standard deviation:", standard_deviation_exponential, "years\n")
```

```
## Standard deviation: 10 years
```

2c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8

years)

```r
# Probability that the machine will fail after 8 years (0 failures in 8 years)
n_trials <- 8
k_failures <- 0
p_failure <- exp(-lambda * 1)
probability_failure_binomial <- dbinom(k_failures, n_trials, p_failure)
cat("Probability of failure after 8 years:", probability_failure_binomial, "\n")
```

```
## Probability of failure after 8 years: 6.72558e-09
```

```r
# Expected value of the binomial distribution
expected_value_binomial <- n_trials * p_failure
cat("Expected value:", expected_value_binomial, "years\n")
```

```
## Expected value: 7.238699 years
```

```r
# Standard deviation of the binomial distribution
standard_deviation_binomial <- sqrt(n_trials * p_failure * (1 - p_failure))
cat("Standard deviation:", standard_deviation_binomial, "years\n")
```

```
## Standard deviation: 0.8299719 years
```

2d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

```r
# Parameters
lambda_poisson <- lambda  # The failure rate per year
```

```r
time_period_years <- 8

# Probability of failure after 8 years
probability_failure_poisson <- 1 - ppois(0, lambda = lambda_poisson * time_period_years)
cat("Probability of failure after 8 years:", probability_failure_poisson, "\n")
```

```
## Probability of failure after 8 years: 0.550671
```

```r
# Expected value of the Poisson distribution
expected_value_poisson <- lambda_poisson * time_period_years
cat("Expected value:", expected_value_poisson, "years\n")
```

```
## Expected value: 0.8 years
```

```r
# Standard deviation of the Poisson distribution
standard_deviation_poisson <- sqrt(lambda_poisson * time_period_years)
cat("Standard deviation:", standard_deviation_poisson, "years\n")
```

```
## Standard deviation: 0.8944272 years
```