BRIDGING BIOLOGICAL NEURONS
and Artificial Activations

LIF Models, FFNNs, and Neuromorphic Motivation

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MATH 456 • December 10, 2025

Why Should You Care?

The energy crisis in artificial intelligence

CHATGPT TRAINING

190,000 kWh

175 billion parameters

HUMAN BRAIN

20 Watts (≈0.48 kWh)

~86 billion neurons

The brain computes differently.

Real neurons use sparse, event-driven spikes, NOT continuous activations.

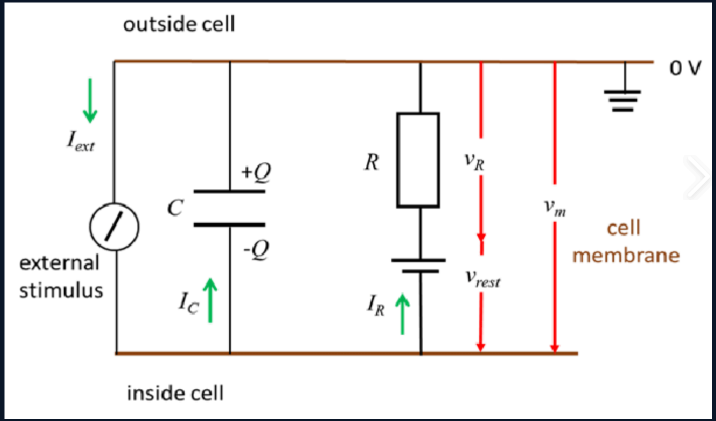
Can we build neural networks that compute more like biology?

This Project:

LIF Neuron → FFNN vs SNN → 76× Efficiency Gain

The Leaky Integrate-and-Fire Model

A mathematical description of biological neuron dynamics



THE GOVERNING ODE

$$C_m \frac{dV}{dt} = -\, g_L(V - E_L) + I_{\text{in}},$$

Three competing forces:

1. Leak: Pulls voltage toward rest
2. Input: Drives voltage up
3. Capacitance: Resists change

Variable Definitions:

V(t): membrane voltage
I_{in}: external input current
$g_L(V - E_L)$: Leak current
$C_m \frac{dV}{dt}$: Capacitive charging term

Spike Rule: When V ≥ V_{th} → Fire spike → Reset to V (E_L)

Key Parameters

τ_m = 10 ms
Membrane time constant

g_L= 10 nS
Leak conductance

E_L = −75 mV
Resting potential

V_{th} = −55 mV
Spike threshold

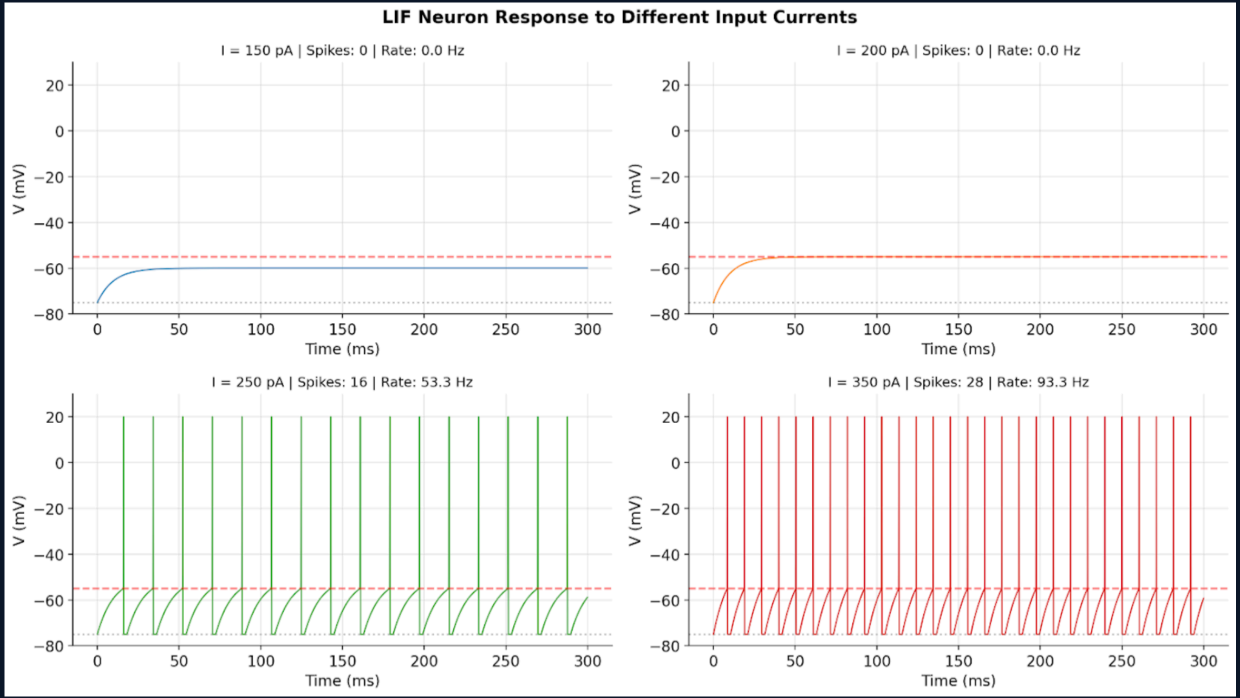
C_m = 100 pF
Membrane capacitance

τ_{ref} = 2 ms
Refractory period

Key insight: Only 5 FLOPs per ms simulation → extremely efficient!

Simulating LIF

How input current determines neuron behavior



150 pA
Subthreshold → 0 Hz

200 pA
At rheobase → 0 Hz

250 pA
Spiking → 53.3 Hz

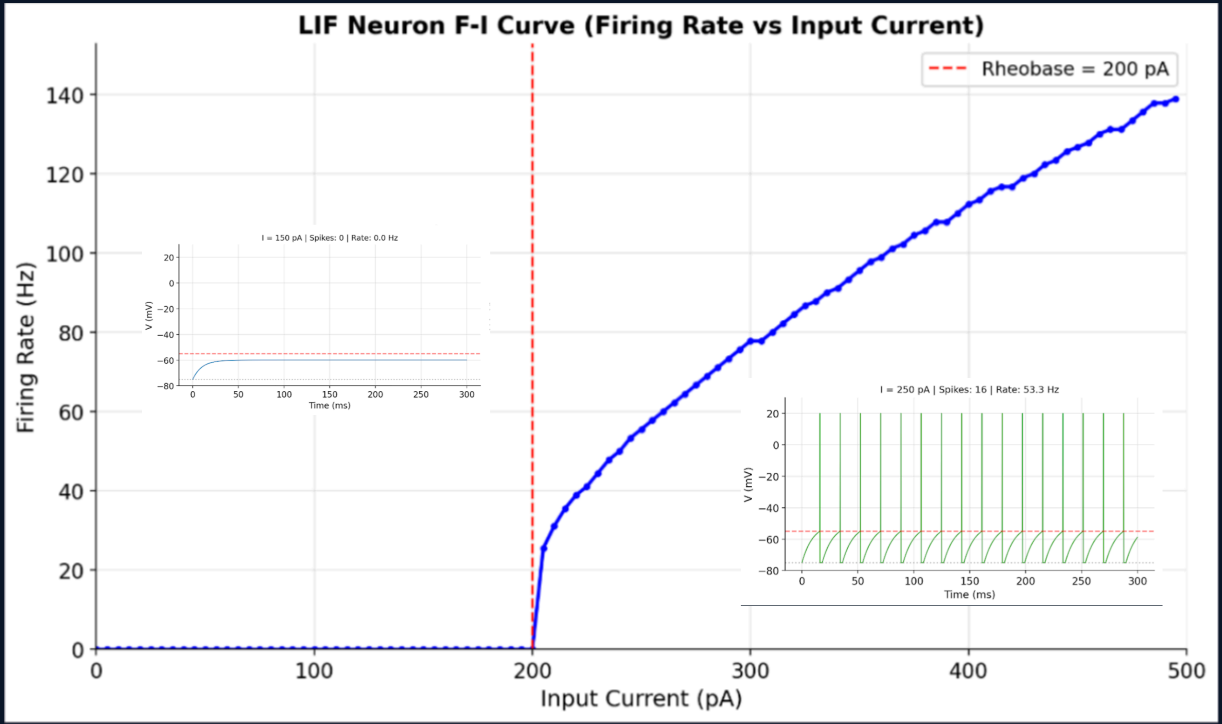
350 pA
Fast spiking → 93.3 Hz

Sharp transition at rheobase (200 pA)

$$C_m \frac{dV}{dt} = -g_L(V - E_L) + I_{\text{in}}$$

The F-I Curve: Input → Output Relationship

How neurons encode stimulus strength into firing rate



RHEOBASE DERIVATION

1. At steady state: $dV/dt = 0$
2. Solve: $V_{ss} = E_L + I/g_L$
3. For spiking: $V_{ss} \geq V_{th}$
4. **$I_{rheo} = g_L(V_{th} - E_L)$**

Substituting our parameters:

$$I_{rheo} = 10 \times (-55 - (-75))$$

= 200 pA

Key Curve Features:

- Sharp threshold at 200 pA
- Roughly linear above threshold
- Saturates due to refractory period

Why do we need Non-dimensionalization?

Reducing complexity through mathematical scaling

Characteristic Scales

Voltage Scale:

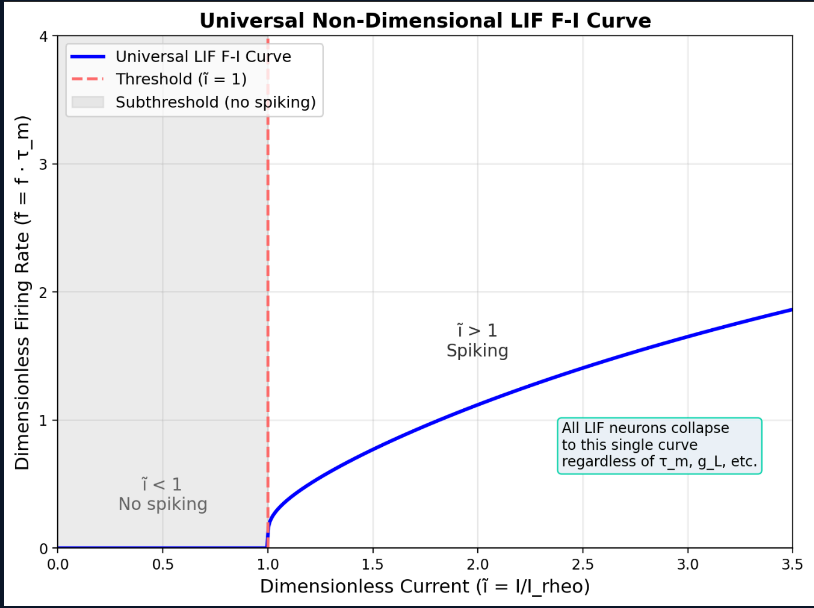
$\Delta V = V - E = 20\ \text{mV}$

Time Scale:

$\tau = 10\ \text{ms}$

Current Scale:

$I = g \times \Delta V = 200\ \text{pA}$



NON-DIMENSIONAL LIF EQUATION

$$\frac{d\tilde{v}}{d\tilde{t}} = -\tilde{v} + \tilde{i}$$

Why This Matters

The entire system reduces to a **single control parameter**:

$\tilde{i} < 1 \rightarrow$ No spiking

$\tilde{i} > 1 \rightarrow$ Spiking

Dimensionless Parameters:

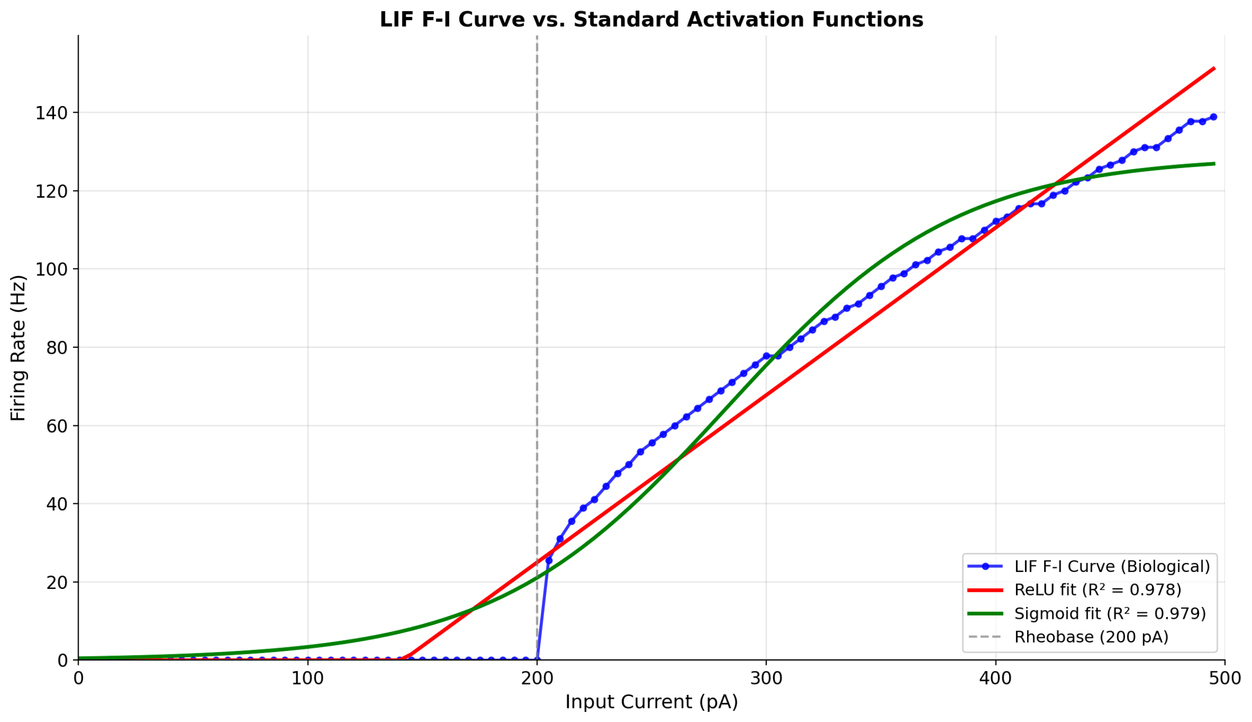
$\tilde{v}_{reset}$ = 1.00 $\tilde{v}_{reset}$= 0.00

$\tilde{v}_{ref}$ = 0.20

All LIF neurons with same ĩ behave identically, regardless of actual parameter values

Can Standard Activations Match LIF Behavior?

Fitting ReLU and Sigmoid to the biological F-I curve



ReLU Fit

$f(x) = a \times \max(0, x - b)$

$R^2 = 0.9784$

✓ Sharp threshold ✗ No saturation

Sigmoid Fit

$f(x) = L/(1 + e-k(x-x_0))$

$R^2 = 0.9788$

✓ Saturates ✗ Gradual threshold

KEY FINDING

Neither activation captures both:

1. Sharp threshold at rheobase
2. Saturation at high currents

Why saturation? Refractory period limits max firing rate. Membrane time constant limits voltage growth speed.

Feed-Forward Networks: Dense Computation

ARCHITECTURE

784 → 256 → 256 → 10

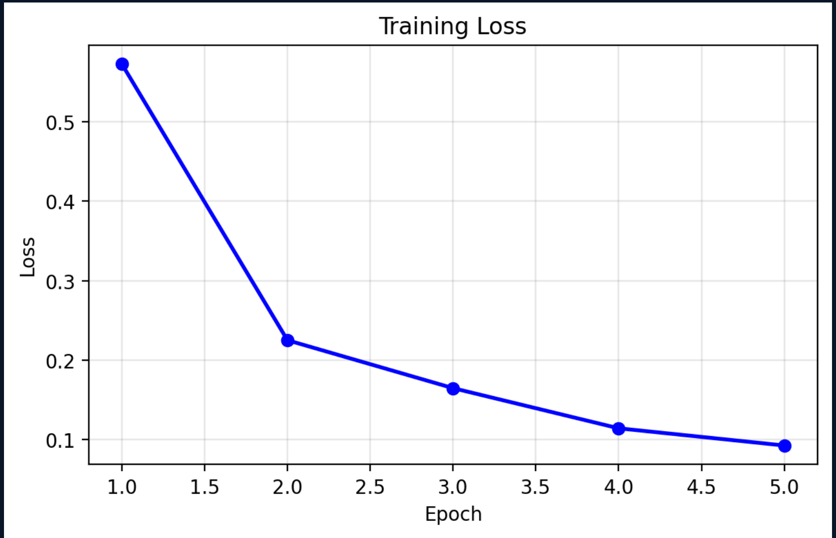
269,322
Parameters

537,600
FLOPs/sample

Accuracy: **93.55%**

Active Neurons: 58%

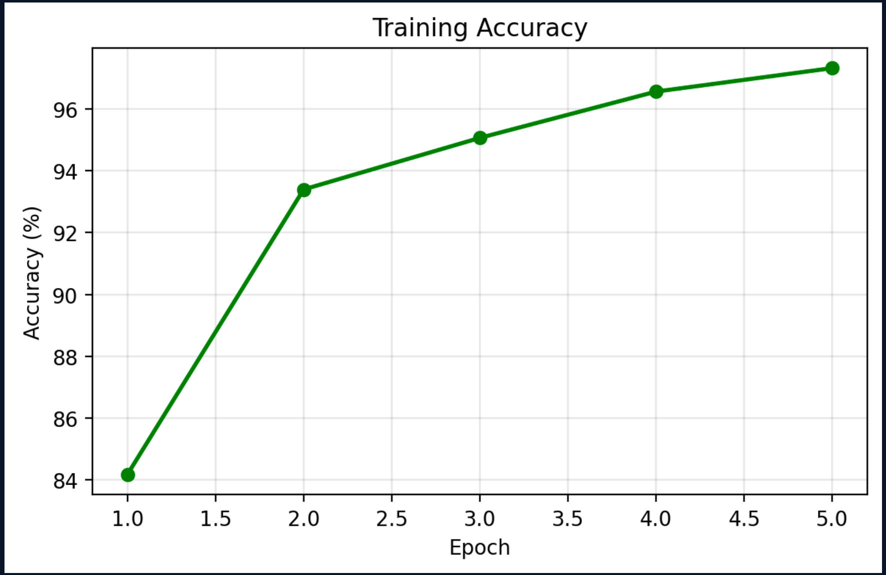
Training: **26.88 billion FLOPs**



THE PROBLEM

42% of neurons output zero, but the network still computes ALL weighted sums first.

Zero output ≠ zero computation



Spiking Networks: Computation Only When It Matters

LIF neurons in a network — event-driven and sparse

SAME ARCHITECTURE, DIFFERENT NEURONS

784 → LIF(256) → LIF(256) → LIF(10)

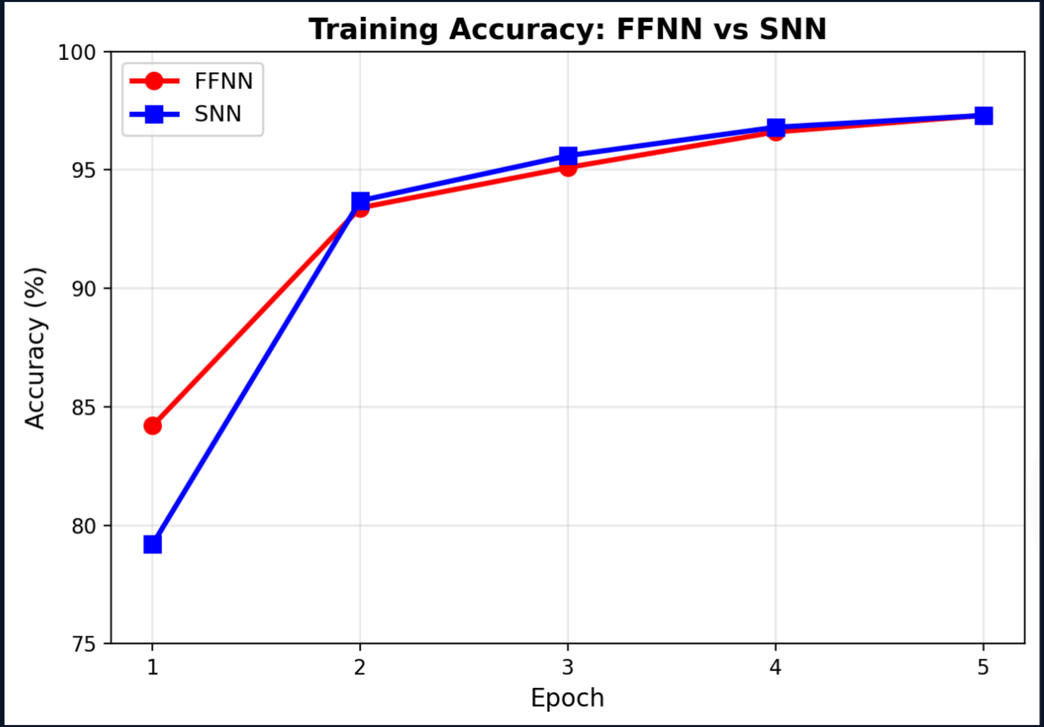
T = 25 timesteps, β = 0.95

Key Difference

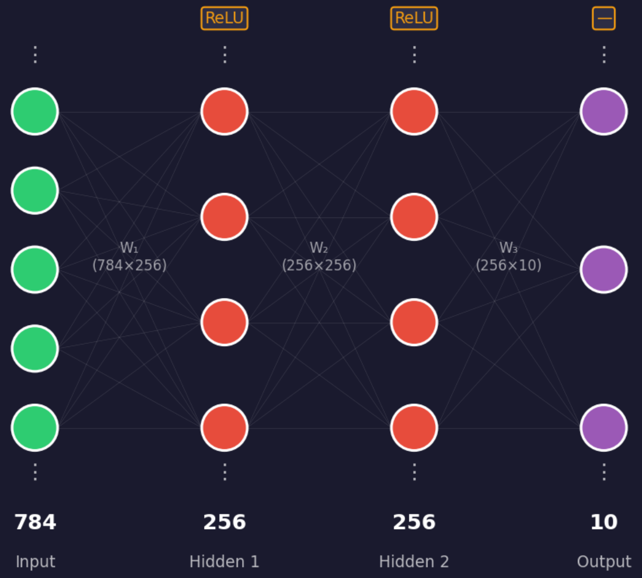
Neurons communicate through discrete spikes, not continuous values. Only neurons that spike trigger downstream computation.

92.95%
Test Accuracy

88.9%
Neurons Silent



FFNN (Feed-Forward Neural Network)



Communication: Continuous Values (ℝ)

SNN (Spiking Neural Network)

Communication: Discrete Spikes (0 or 1)

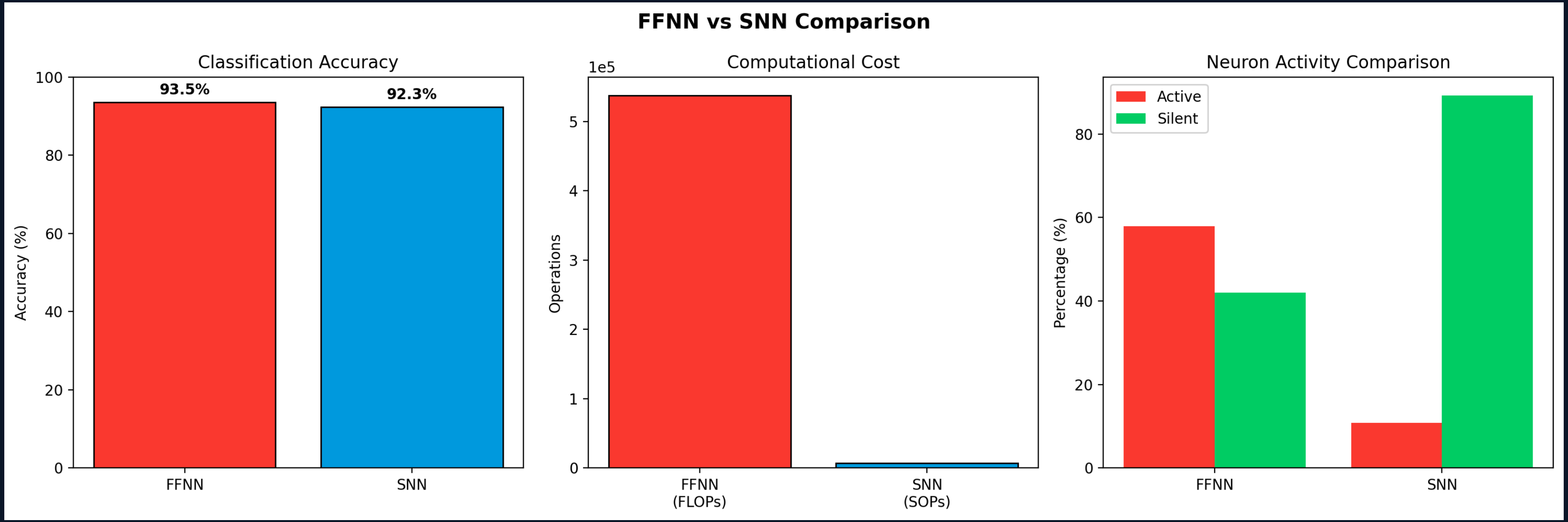
Architecture: 784 → 256 → 256 → 10
Layers: 3 weight layers (2 hidden)
Parameters: 269,322
Activation: ReLU
Loss: CrossEntropyLoss
Optimizer: Adam (lr=0.001)
FLOPs/sample: 537,600
Test Accuracy: 93.55%

76× Fewer Operations

Architecture: 784 → LIF(256) → LIF(256) → LIF(10)
Layers: 3 weight layers (2 hidden)
Parameters: 269,322
Activation: LIF (β=0.95, T=25 steps)
Loss: ce_rate_loss (surrogate gradient)
Optimizer: Adam (lr=0.001)
SOPs/sample: 7,111
Test Accuracy: 92.95%

Results: 76× Fewer Operations, Same Accuracy

Quantitative comparison of FFNN vs SNN on MNIST



76× Efficiency Gain

Same accuracy, fraction of the computation

Why? In SNNs, only neurons that spike contribute. 88.9% stay silent → 88.9% less work.

Conclusions: Bridging Biology and AI

Key Findings

1. The LIF ODE captures essential neuron dynamics with just 5 FLOPs/ms

2. Standard activations (ReLU, sigmoid) miss key LIF features: sharp threshold + saturation

3. SNNs achieve comparable accuracy with 76× fewer operations through spike sparsity

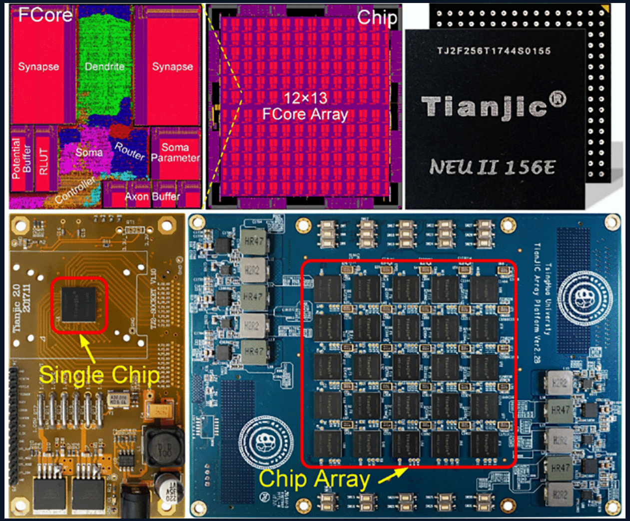
4. Event-driven computation is the key to energy efficiency

Why This Matters

As AI models grow, energy costs become unsustainable. SNNs offer a path toward powerful AND efficient AI systems.

Future Directions

- Neuromorphic hardware (Intel Loihi, IBM TrueNorth)
- Hybrid ANN-SNN architectures
- Scaling SNNs to larger datasets



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