1B Multiplying and dividing algebraic fractions

Learning intentions
- To understand that expressions in algebraic fractions need to be in factorised form in order to cancel common factors
- To know that it is helpful to cancel common factors in fractions before multiplying or dividing
- To be able to multiply and divide fractions involving algebraic expressions

Since pronumerals represent numbers, the rules for algebraic fractions are the same as those for simple numerical fractions. This includes processes such as cancelling common factors, adding or subtracting with a lowest common denominator (LCD) and dividing, by multiplying by the reciprocal of the fraction that follows the division sign. In this section we focus on multiplying and dividing algebraic fractions.

The study of air-conditioning uses algebraic fractions to model airflow, air temperatures and humidity. The mechanical engineers who design ventilation systems, and the electricians who install and repair them, all require algebraic skills.

LESSON STARTER   Describe the error

Here are three problems involving algebraic fractions. Each simplification contains one critical error. Find and describe the errors, then give the correct answer.

a $\dfrac{6x - \overset{2}{8}}{\underset{1}{4}} = \dfrac{6x - 2}{1}$
$= 6x - 2$

b $\dfrac{2a}{9} \div \dfrac{2}{3} = \dfrac{2a}{9} \times \dfrac{2}{3}$
$= \dfrac{4a}{27}$

c $\dfrac{3b}{7} \div \dfrac{2b}{3} = \dfrac{3b}{7} \times \dfrac{3b}{2}$
$= \dfrac{9b^2}{14}$

KEY IDEAS

■ Simplify **algebraic fractions** by factorising expressions where possible and cancelling common factors.

■ For multiplication, cancel common factors and then multiply the numerators together and the denominators together.

■ For division, multiply by the **reciprocal** of the fraction that follows the division sign. The reciprocal of a is $\dfrac{1}{a}$ and the reciprocal of $\dfrac{a}{b}$ is $\dfrac{b}{a}$.

BUILDING UNDERSTANDING

1 Simplify to find the answer in simplest form.

   a $\dfrac{2}{3} \times \dfrac{6}{4}$ **b** $\dfrac{3}{4} \times \dfrac{10}{9}$ **c** $\dfrac{4}{7} \div \dfrac{2}{7}$ **d** $\dfrac{3}{6} \div \dfrac{6}{9}$

2 What is the reciprocal of each fraction?

   a $\dfrac{3}{2}$ **b** $\dfrac{7a}{3}$ **c** $\dfrac{-4xy}{7t}$ **d** $\dfrac{-8x^2a}{b^2c}$

3 Simplify by cancelling common factors.

   a $\dfrac{10x}{2}$ **b** $\dfrac{24x}{6}$ **c** $\dfrac{5a}{20}$ **d** $\dfrac{7}{21a}$

Example 6   Cancelling common factors

Simplify by cancelling common factors.

a $\dfrac{8a^2b}{2a}$ **b** $\dfrac{3-9x}{3}$

SOLUTION

a $\dfrac{8a^2b}{2a} = \dfrac{8^4 \times \cancel{a}^1 \times a \times b}{\cancel{2}_1 \times \cancel{a}_1}$

   $= 4ab$

b $\dfrac{3-9x}{3} = \dfrac{\cancel{3}^1(1-3x)}{\cancel{3}^1}$

   $= 1 - 3x$

EXPLANATION

Cancel the common factors 2 and a.

Factorise the numerator, then cancel the common factor of 3.

Now you try

Simplify by cancelling common factors.

a $\dfrac{9ab^2}{3b}$ **b** $\dfrac{5-10x}{5}$

Example 7   Multiplying and dividing algebraic fractions

Simplify the following.

a $\dfrac{2}{a} \times \dfrac{a+2}{4}$ **b** $\dfrac{2x-4}{3} \div \dfrac{x-2}{6}$

SOLUTION

a $\dfrac{\cancel{2}^1}{a} \times \dfrac{a+2}{\cancel{4}_2} = \dfrac{a+2}{2a}$

EXPLANATION

Cancel the common factor of 2 and then multiply the numerators and the denominators. a cannot be cancelled as it is not a common factor in $a+2$.

b $\dfrac{2x-4}{3} \div \dfrac{x-2}{6} = \dfrac{2x-4}{3} \times \dfrac{6}{x-2}$

$\qquad\qquad\qquad = \dfrac{2(x-2)^1}{3^1} \times \dfrac{6^2}{(x-2)^1}$

$\qquad\qquad\qquad = 4$

Multiply by the reciprocal of the second fraction.
Factorise $2x-4$ and cancel the common factors.

Now you try

Simplify the following.

a $\dfrac{6}{a} \times \dfrac{a+1}{12}$

b $\dfrac{3x-12}{2} \div \dfrac{x-4}{4}$

Exercise 1B

FLUENCY		1, 2–5(½)	2–5(½)	2–5(⅓)

1 Simplify by cancelling common factors.

Example 6a   **a**   **i** $\dfrac{6a^2b}{2a}$ **ii** $\dfrac{10xy^2}{5y}$

Example 6b   **b**   **i** $\dfrac{4-8x}{4}$ **ii** $\dfrac{5-5x}{5}$

Example 6a   **2** Simplify by cancelling common factors.

a $\dfrac{35x^2}{7x}$ **b** $\dfrac{-14x^2y}{7xy}$ **c** $\dfrac{-36ab^2}{4ab}$ **d** $\dfrac{8xy^3}{-4xy^2}$

e $\dfrac{-15pq^2}{30p^2q^2}$ **f** $\dfrac{-20s}{45s^2t}$ **g** $\dfrac{-48x^2}{16xy}$ **h** $\dfrac{120ab^2}{140ab}$

Example 6b   **3** Simplify by cancelling common factors.

a $\dfrac{4x+8}{4}$ **b** $\dfrac{6a-30}{6}$ **c** $\dfrac{6x-18}{2}$ **d** $\dfrac{5-15y}{5}$

e $\dfrac{-2-12b}{-2}$ **f** $\dfrac{21x-7}{-7}$ **g** $\dfrac{9t-27}{-9}$ **h** $\dfrac{44-11x}{-11}$

i $\dfrac{x^2+2x}{x}$ **j** $\dfrac{6x-4x^2}{2x}$ **k** $\dfrac{a^2-a}{a}$ **l** $\dfrac{7a+14a^2}{21a}$

Example 7a   **4** Simplify the following.

a $\dfrac{3}{x} \times \dfrac{x-1}{6}$ **b** $\dfrac{x+4}{10} \times \dfrac{2}{x}$ **c** $\dfrac{-8a}{7} \times \dfrac{7}{2a}$

d $\dfrac{x+3}{9} \times \dfrac{4}{x+3}$ **e** $\dfrac{y-7}{y} \times \dfrac{5y}{y-7}$ **f** $\dfrac{10a^2}{a+6} \times \dfrac{a+6}{4a}$

g $\dfrac{2m+4}{m} \times \dfrac{m}{m+2}$ **h** $\dfrac{6-18x}{2} \times \dfrac{5}{1-3x}$ **i** $\dfrac{b-1}{10} \times \dfrac{-5}{b-1}$

Example 7b **5** Simplify the following.

a $\dfrac{x}{5} \div \dfrac{x}{15}$

b $\dfrac{x+4}{2} \div \dfrac{x+4}{6}$

c $\dfrac{6x-12}{5} \div \dfrac{x-2}{3}$

d $\dfrac{3-6y}{8} \div \dfrac{1-2y}{2}$

e $\dfrac{2}{a-1} \div \dfrac{3}{2a-2}$

f $\dfrac{2}{10x-5} \div \dfrac{10}{2x-1}$

g $\dfrac{5}{3a+4} \div \dfrac{15}{-15a-20}$

h $\dfrac{2x-6}{5x-20} \div \dfrac{x-3}{x-4}$
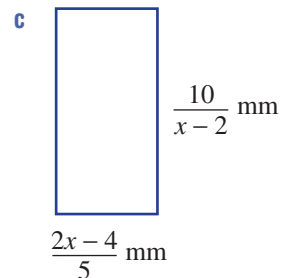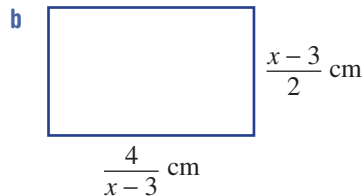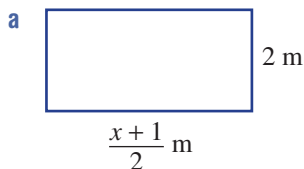
i $\dfrac{t+1}{9} \div \dfrac{-t-1}{3}$

PROBLEM-SOLVING
6   6, 7(¹/₂)   7–8(¹/₂)

6 Find a simplified expression for the area of these rectangles.

a

2 m

$\dfrac{x+1}{2}$ m

b

$\dfrac{x-3}{2}$ cm

$\dfrac{4}{x-3}$ cm

c

$\dfrac{10}{x-2}$ mm

$\dfrac{2x-4}{5}$ mm

7 Simplify these expressions.

a $\dfrac{x}{3} \times \dfrac{9x}{5} \times \dfrac{15}{3x}$

b $\dfrac{2}{a} \times \dfrac{a}{5} \times \dfrac{10}{3a}$

c $\dfrac{x-1}{2} \times \dfrac{4x}{2x-2} \times \dfrac{x+3}{5x}$

d $\dfrac{2x-1}{x} \div \dfrac{2x-1}{2} \div \dfrac{1}{2}$

e $\dfrac{2x-3}{5} \div \dfrac{14x-21}{10} \div \dfrac{x}{2}$

f $\dfrac{b^2-b}{b} \div \dfrac{b-1}{b^2} \times \dfrac{2}{b-1}$

8 Write the missing algebraic fraction.

a $\dfrac{x+3}{5} \times \boxed{} = 2$

b $\dfrac{1-x}{x} \times \boxed{} = 3$

c $\boxed{} \div \dfrac{x}{2} = \dfrac{3(x+2)}{x}$

d $\boxed{} \div \dfrac{2x-2}{3} = \dfrac{5x}{x-1}$

e $\dfrac{1}{x} \div \boxed{} \times \dfrac{x-1}{2} = 1$

f $\dfrac{2-x}{7} \times \boxed{} \div \dfrac{5x}{x-1} = x$

REASONING
9(¹/₂)   9(¹/₂), 10   9(¹/₂), 10, 11

9 Recall that $(x-1)^2 = (x-1)(x-1)$. Use this idea to simplify the following.

a $\dfrac{(x-1)^2}{x-1}$

b $\dfrac{3(x+2)^2}{x+2}$

c $\dfrac{4(x-3)^2}{2(x-3)}$

d $\dfrac{4(x+2)}{(x+2)^2}$

e $\dfrac{-5(1-x)}{(1-x)^2}$

f $\dfrac{(2x-2)^2}{x-1}$

10 Prove that the following all simplify to 1.

a $\dfrac{5x + 5}{15} \times \dfrac{3}{x + 1}$

b $\dfrac{3x - 21}{2 - x} \times \dfrac{4 - 2x}{6x - 42}$

c $\dfrac{10 - 5x}{2x + 6} \div \dfrac{20 - 10x}{4x + 12}$

11 a Explain why $\dfrac{x - 1}{2} \times \dfrac{4}{1 - x} = \dfrac{x - 1}{2} \times \dfrac{-4}{x - 1}$.

   b Use this idea to simplify these expressions.

     i $\dfrac{2 - a}{3} \times \dfrac{7}{a - 2}$

     ii $\dfrac{6x - 3}{x} \div \dfrac{1 - 2x}{4}$

     iii $\dfrac{18 - x}{3x - 1} \div \dfrac{2x - 36}{7 - 21x}$

ENRICHMENT: Simplifying with quadratics	–	–	12(½)

12 You may recall that to factorise a monic quadratic of the form $x^2 + bx + c$ we look for factors of c which add to b. So for example: $x^2 - x - 6 = (x - 3)(x + 2)$.

So:
$$\dfrac{x^2 - x - 6}{6} \times \dfrac{3}{x - 3} = \dfrac{\overset{1}{\cancel{(x - 3)}}(x + 2)}{\underset{2}{\cancel{6}}} \times \dfrac{\overset{1}{\cancel{3}}}{\cancel{(x - 3)}^1}$$
$$= \dfrac{x + 2}{2}$$

Now simplify these algebraic fractions which involve quadratics.

a $\dfrac{x^2 - 2x - 8}{4} \times \dfrac{2}{x - 4}$

b $\dfrac{x^2 + 5x + 6}{x + 2} \times \dfrac{x}{x + 3}$

c $\dfrac{x + 1}{x^2 - 4x - 5} \times \dfrac{x - 5}{3}$

d $\dfrac{3x - 27}{4x} \times \dfrac{2x}{x^2 - 7x - 18}$

e $\dfrac{4ab}{a^2 + a} \div \dfrac{b}{a^2 + 2a + 1}$

f $\dfrac{a + 8}{a^2 - 5a - 6} \div \dfrac{a^2 + 5a - 24}{a - 6}$

g $\dfrac{(x - y)^2}{xy} \div \dfrac{x^2 - y^2}{x + y}$

h $\dfrac{y^2 + 4y + 4}{x^2 y} \div \dfrac{(y + 2)^2}{xy^2 + 2xy}$