1C Adding and subtracting algebraic fractions

Learning intentions

- To know how to find the lowest common denominator of algebraic fractions
- To be able to combine numerators using expansion and addition of like terms
- To be able to add and subtract algebraic fractions

The sum or difference of two or more algebraic fractions can be simplified in a similar way to numerical fractions with the use of a common denominator.

Electricians, electrical and electronic engineers work with algebraic fractions when modelling the flow of electric energy in circuits. The application of algebra when using electrical formulas is essential in these professions.

LESSON STARTER Spot the difference

Here are two sets of simplification steps. One set has one critical error. Can you find and correct it?

$$\frac{2}{3} - \frac{5}{2} = \frac{4}{6} - \frac{15}{6}$$
$$= \frac{-11}{6}$$

$$\frac{x}{3} - \frac{x+1}{2} = \frac{2x}{6} - \frac{3(x+1)}{6}$$
$$= \frac{2x - 3x + 3}{6}$$
$$= \frac{-x + 3}{6}$$

KEY IDEAS

- Add and subtract algebraic fractions by firstly finding the lowest common denominator (LCD) and then combine the numerators.
- Expand numerators correctly by taking into account addition and subtraction signs.
  E.g. $-2(x + 1) = -2x - 2$ and $-5(2x - 3) = -10x + 15$.

BUILDING UNDERSTANDING

1. Expand the following.
   a $2(x - 2)$ b $-(x + 6)$ c $-6(x - 2)$
2. Simplify these by firstly finding the lowest common denominator (LCD).
   a $\frac{1}{2} + \frac{1}{3}$ b $\frac{4}{3} - \frac{1}{5}$ c $\frac{3}{7} - \frac{1}{14}$ d $\frac{5}{3} + \frac{7}{6}$
3. State the lowest common denominator for these pairs of fractions.
   a $\frac{a}{3}, \frac{7a}{4}$ b $\frac{x}{2}, \frac{4xy}{6}$ c $\frac{3xy}{7}, \frac{-3x}{14}$ d $\frac{2}{x}, \frac{3}{2x}$

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Example 8 Adding and subtracting simple algebraic fractions

Simplify the following.

a $\frac{3}{4} - \frac{a}{2}$

b $\frac{2}{5} + \frac{3}{a}$

SOLUTION

a $\frac{3}{4} - \frac{a}{2} = \frac{3}{4} - \frac{2a}{4}$

$= \frac{3 - 2a}{4}$

b $\frac{2}{5} + \frac{3}{a} = \frac{2a}{5a} + \frac{15}{5a}$

$= \frac{2a + 15}{5a}$

EXPLANATION

The LCD of 2 and 4 is 4. Express each fraction as an equivalent fraction with a denominator of 4. Subtract the numerators.

The LCD of 5 and a is $5a$.

Add the numerators.

Now you try

Simplify the following.

a $\frac{5}{6} - \frac{a}{3}$

b $\frac{3}{4} + \frac{2}{a}$

Example 9 Adding and subtracting more complex algebraic fractions

Simplify the following algebraic expressions.

a $\frac{x + 3}{2} + \frac{x - 2}{5}$

b $\frac{2x - 1}{3} - \frac{x - 1}{4}$

SOLUTION

a $\frac{x + 3}{2} + \frac{x - 2}{5} = \frac{5(x + 3)}{10} + \frac{2(x - 2)}{10}$

$= \frac{5(x + 3) + 2(x - 2)}{10}$

$= \frac{5x + 15 + 2x - 4}{10}$

$= \frac{7x + 11}{10}$

EXPLANATION

LCD is 10.

Use brackets to ensure you retain equivalent fractions.

Combine the numerators, then expand the brackets and simplify.

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b $\dfrac{2x-1}{3} - \dfrac{x-1}{4} = \dfrac{4(2x-1)}{12} - \dfrac{3(x-1)}{12}$

$= \dfrac{4(2x-1) - 3(x-1)}{12}$

$= \dfrac{8x - 4 - 3x + 3}{12}$

$= \dfrac{5x-1}{12}$

Express each fraction with the LCD of 12.

Combine the numerators.

Expand the brackets: $4(2x-1) = 8x - 4$ and $-3(x-1) = -3x + 3$.
Simplify by collecting like terms.

Now you try

Simplify the following algebraic expressions.

a $\dfrac{x+1}{3} + \dfrac{x-2}{2}$

b $\dfrac{3x-2}{2} - \dfrac{x-2}{5}$

Example 10 Adding and subtracting with algebraic denominators

Simplify the algebraic expression $\dfrac{3}{x-6} - \dfrac{2}{x+2}$.

SOLUTION

$\dfrac{3}{x-6} - \dfrac{2}{x+2} = \dfrac{3(x+2)}{(x-6)(x+2)} - \dfrac{2(x-6)}{(x-6)(x+2)}$

$= \dfrac{3(x+2) - 2(x-6)}{(x-6)(x+2)}$

$= \dfrac{3x + 6 - 2x + 12}{(x-6)(x+2)}$

$= \dfrac{x + 18}{(x-6)(x+2)}$

EXPLANATION

$(x-6)(x+2)$ is the lowest common multiple of $(x-6)$ and $(x+2)$.
Combine the numerators and then expand the brackets.

Recall that $-2 \times (-6) = 12$.

Collect like terms to simplify.

Now you try

Simplify the expression $\dfrac{4}{x-5} - \dfrac{3}{x+1}$.

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Exercise 1C

FLUENCY 1, 2–4(½) | 2–5(½) | 2–5(⅓)

1 Simplify the following.

Example 8a

a i $\frac{1}{4} - \frac{a}{2}$ ii $\frac{3}{10} - \frac{a}{5}$

Example 8b

b i $\frac{1}{3} + \frac{2}{a}$ ii $\frac{3}{7} + \frac{5}{a}$

Example 8a

2 Simplify the following.

a $\frac{2}{3} + \frac{a}{7}$ **b** $\frac{3}{8} + \frac{a}{2}$ **c** $\frac{3}{10} - \frac{3b}{2}$ **d** $\frac{2}{5} + \frac{4x}{15}$

e $\frac{1}{9} - \frac{2a}{3}$ **f** $\frac{a}{3} - \frac{a}{5}$ **g** $\frac{2x}{5} - \frac{x}{4}$ **h** $\frac{6b}{7} - \frac{b}{14}$

Example 8b

3 Simplify the following.

a $\frac{2}{3} + \frac{5}{a}$ **b** $\frac{3}{4} + \frac{2}{a}$ **c** $\frac{7}{9} - \frac{3}{a}$ **d** $\frac{4}{b} - \frac{3}{4}$

e $\frac{2}{7} - \frac{3}{2b}$ **f** $\frac{3}{2y} - \frac{7}{9}$ **g** $\frac{-4}{x} - \frac{2}{3}$ **h** $\frac{-9}{2x} - \frac{1}{3}$

Example 9a

4 Simplify the following algebraic expressions.

a $\frac{x+3}{4} + \frac{x+2}{5}$ **b** $\frac{x+2}{3} + \frac{x+1}{4}$ **c** $\frac{x-3}{4} + \frac{x+2}{2}$

d $\frac{x+4}{3} + \frac{x-3}{9}$ **e** $\frac{2x+1}{2} + \frac{x-2}{3}$ **f** $\frac{3x+1}{5} + \frac{2x+1}{10}$

g $\frac{x-2}{8} + \frac{2x+4}{12}$ **h** $\frac{5x+3}{10} + \frac{2x-2}{4}$ **i** $\frac{3-x}{14} + \frac{x-1}{7}$

Example 9b

5 Simplify these algebraic fractions.

a $\frac{2x+1}{3} - \frac{x-1}{2}$ **b** $\frac{3x-1}{3} - \frac{2x-3}{4}$ **c** $\frac{x+6}{5} - \frac{x-4}{3}$

d $\frac{x-3}{2} - \frac{2x+1}{7}$ **e** $\frac{7x+2}{7} - \frac{x+2}{3}$ **f** $\frac{10x-4}{3} - \frac{2x+1}{6}$

g $\frac{4-x}{6} - \frac{1-x}{5}$ **h** $\frac{1-3x}{5} - \frac{x+2}{3}$ **i** $\frac{6-5x}{2} - \frac{2-7x}{4}$

PROBLEM-SOLVING 6(½) | 6(½) | 6(½), 7

Example 10

6 Simplify the following algebraic expressions.

a $\frac{5}{x+1} + \frac{2}{x+4}$ **b** $\frac{4}{x-7} + \frac{3}{x+2}$ **c** $\frac{1}{x-3} + \frac{2}{x+5}$

d $\frac{3}{x+3} - \frac{2}{x-4}$ **e** $\frac{6}{2x-1} - \frac{3}{x-4}$ **f** $\frac{4}{x-5} + \frac{2}{3x-4}$

g $\frac{5}{2x-1} - \frac{6}{x+7}$ **h** $\frac{2}{x-3} - \frac{3}{3x+4}$ **i** $\frac{8}{3x-2} - \frac{3}{1-x}$

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7 **a** Write the LCD for these pairs of fractions.

i $\frac{3}{a}, \frac{2}{a^2}$ **ii** $\frac{7}{x^2}, \frac{3+x}{x}$

b Now simplify these expressions.

i $\frac{2}{a} - \frac{3}{a^2}$ **ii** $\frac{a+1}{a} - \frac{4}{a^2}$ **iii** $\frac{7}{2x^2} + \frac{3}{4x}$

REASONING	8	8, 9	8, 9

8 Describe the error in this working, then fix the solution.

$$\frac{x}{2} - \frac{x+1}{3} = \frac{3x}{6} - \frac{2(x+1)}{6}$$
$$= \frac{3x}{6} - \frac{2x+2}{6}$$
$$= \frac{x+2}{6}$$

9 **a** Explain why $2x - 3 = -(3 - 2x)$.

b Use this idea to help simplify these expressions.

i $\frac{1}{x-1} - \frac{1}{1-x}$ **ii** $\frac{3x}{3-x} + \frac{x}{x-3}$ **iii** $\frac{x+1}{7-x} - \frac{2}{x-7}$

ENRICHMENT: Fraction challenges	–	–	10, 11

10 Simplify these expressions.

a $\frac{a-b}{b-a}$ **b** $\frac{5}{a} + \frac{2}{a^2}$

c $\frac{3}{x+1} + \frac{2}{(x+1)^2}$ **d** $\frac{x}{(x-2)^2} - \frac{x}{x-2}$

e $\frac{x}{2(3-x)} - \frac{x^2}{7(x-3)^2}$ **f** $\frac{1}{x} - \frac{1}{y} - \frac{1}{z}$

11 By first simplifying the left-hand side of these equations, find the value of a.

a $\frac{a}{x-1} - \frac{2}{x+1} = \frac{4}{(x-1)(x+1)}$

b $\frac{3}{2x-1} + \frac{a}{x+1} = \frac{5x+2}{(2x-1)(x+1)}$

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