1A Review of algebra CONSOLIDATING

Learning intentions
- To review the key words of algebra: term, coefficient, expression, equation
- To review how to combine like terms under addition and subtraction
- To review how to multiply and divide algebraic terms and apply the distributive law to expand brackets
- To review how to factorise an expression using the highest common factor
- To be able to substitute values for pronumerals and evaluate expressions

Algebra involves the use of pronumerals (or variables), which are letters representing numbers. Combinations of numbers and pronumerals form terms (numbers and pronumerals connected by multiplication and division), expressions (a term or terms connected by addition and subtraction) and equations (mathematical statements that include an equals sign). Skills in algebra are important when dealing with the precise and concise nature of mathematics. The complex nature of many problems in finance and engineering usually result in algebraic expressions and equations that need to be simplified and solved.


Stockmarket traders rely on financial modelling based on complex algebraic expressions. Financial market analysts and computer systems analysts require advanced algebraic skills.

LESSON STARTER Mystery problem

Between one school day and the next, the following problem appeared on a student noticeboard.

Prove that $8 - x^2 + \frac{3x - 9}{3} + 5(x - 1) - x(6 - x) = 0$.

- By working with the left-hand side of the equation, show that this equation is true for any value of x.
- At each step of your working, discuss what algebraic processes you have used.

KEY IDEAS

- Key words in algebra:
  - **term:** $5x$, $7x^2y$, $\frac{2a}{3}$, 7 (a constant term)
  - **coefficient:** -3 is the coefficient of x^2 in $7 - 3x^2$; 1 is the coefficient of y in $y + 7x$.
  - **expression:** $7x$, $3x + 2xy$, $\frac{x + 3}{2}$, $\sqrt{2a^2 - b}$
  - **equation:** $x = 5$, $7x - 1 = 2$, $x^2 + 2x = -4$
- Expressions can be evaluated by substituting a value for each pronumeral (variable).
  - Order of operations are followed: First brackets, then indices, then multiplication and division, then addition and subtraction, working then from left to right.

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- **Like terms** have the same pronumeral part and, using addition and subtraction, can be collected to form a single term.
  For example, $3x - 7x + x = -3x$
  $6a^2b - ba^2 = 5a^2b$
  Note that $a^2b = ba^2$
- The symbols for multiplication (×) and division (÷) are usually not shown.
  $7 \times x \div y = \frac{7x}{y}$
  $-6a^2b \div (ab) = \frac{-6a^2b}{ab}$
  $= -6a$
- The **distributive law** is used to expand brackets.
  - $a(b + c) = ab + ac$ $\quad 2(x + 7) = 2x + 14$
  - $a(b - c) = ab - ac$ $\quad -x(3 - x) = -3x + x^2$
- **Factorisation** involves writing expressions as a product of factors.
  - Many expressions can be factorised by taking out the highest common factor (HCF).
    $15 = 3 \times 5$
    $3x - 12 = 3(x - 4)$
    $9x^2y - 6xy + 3x = 3x(3xy - 2y + 1)$
- Other general properties are:
  - **associative** $\quad a \times (b \times c) = (a \times b) \times c$ or $a + (b + c) = (a + b) + c$
  - **commutative** $\quad ab = ba$ or $a + b = b + a$ $\left(\text{Note: } \frac{a}{b} \neq \frac{b}{a} \text{ and } a - b \neq b - a.\right)$
  - **identity** $\quad a \times 1 = a$ or $a + 0 = a$
  - **inverse** $\quad a \times \frac{1}{a} = 1$ or $a + (-a) = 0$

BUILDING UNDERSTANDING

1 Which of the following is an equation?

A $3x - 1$ **B** $\frac{x + 1}{4}$ **C** $7x + 2 = 5$ **D** $3x^2y$

2 Which expression contains a term with a coefficient of -9?

A $8 + 9x$ **B** $2x + 9x^2y$ **C** $9a - 2ab$ **D** $b - 9a^2$

3 State the coefficient of a^2 in these expressions.

a $a + a^2$ **b** $\frac{3}{2} - 4a^2$ **c** $1 - \frac{a^2}{5}$ **d** $-\frac{7a^2}{3} - 1$

4 Decide whether the following pairs of terms are like terms.

a xy and $2yx$ **b** $7a^2b$ and $-7ba^2$ **c** $-4abc^2$ and $8ab^2c$

5 Evaluate:

a $(-3)^2$ **b** $(-2)^3$ **c** -2^3 **d** -3^2

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Example 1 Collecting like terms

Simplify by collecting like terms.

a $7a + 3a$ b $3a^2b - 2a^2b$ c $5xy + 2xy^2 - 2xy + 3y^2x$

SOLUTION

a $7a + 3a = 10a$

Keep the pronumeral and add the coefficients.

b $3a^2b - 2a^2b = a^2b$

$3a^2b$ and $2a^2b$ have the same pronumeral part, so they are like terms. Subtract coefficients and recall that $1a^2b = a^2b$.

c $5xy + 2xy^2 - 2xy + 3y^2x = 3xy + 5xy^2$

Collect like terms, noting that $3y^2x = 3xy^2$. The $+$ or $-$ sign belongs to the term that directly follows it.

Now you try

Simplify by collecting like terms.

a $4a + 13a$ b $5ab^2 - 2ab^2$ c $3xy + 4x^2y - xy + 2yx^2$

Example 2 Multiplying and dividing expressions

Simplify the following.

a $2h \times 7l$ b $-3p^2r \times 2pr$ c $-\frac{7xy}{14y}$

SOLUTION

a $2h \times 7l = 14hl$

Multiply the coefficients and remove the $\times$ sign.

b $-3p^2r \times 2pr = -6p^3r^2$

Remember the basic index law: When you multiply terms with the same base you add the powers.

c $-\frac{7xy}{14y} = -\frac{x}{2}$

Cancel the highest common factor of 7 and 14 and cancel the y.

Now you try

Simplify the following.

a $3a \times 6b$ b $-2x^2y \times 5xy$ c $-\frac{4ab}{8a}$

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Example 3 Expanding the brackets

Expand the following using the distributive law. Simplify where possible.

a $2(x + 4)$ **b** $-3x(x - y)$ **c** $3(x + 2) - 4(2x - 4)$

SOLUTION

a $2(x + 4) = 2x + 8$

b $-3x(x - y) = -3x^2 + 3xy$

c $3(x + 2) - 4(2x - 4) = 3x + 6 - 8x + 16$
$= -5x + 22$

EXPLANATION

$2(x + 4) = 2 \times x + 2 \times 4$

Note that $x \times x = x^2$ and $-3 \times (-1) = 3$.

Expand each pair of brackets and simplify by collecting like terms.

Now you try

Expand the following using the distributive law. Simplify where possible.

a $3(x + 2)$ **b** $-2x(x - y)$ **c** $2(x + 3) - 3(2x - 1)$

Example 4 Factorising simple algebraic expressions

Factorise:

a $3x - 9$ **b** $2x^2 + 4x$

SOLUTION

a $3x - 9 = 3(x - 3)$

b $2x^2 + 4x = 2x(x + 2)$

EXPLANATION

HCF of $3x$ and 9 is 3.

Check that $3(x - 3) = 3x - 9$.

HCF of $2x^2$ and $4x$ is $2x$.

Check that $2x(x + 2) = 2x^2 + 4x$.

Now you try

Factorise:

a $2x - 10$ **b** $3x^2 + 9x$

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Example 5 Evaluating expressions

Evaluate $a^2 - 2bc$ if $a = -3$, $b = 5$ and $c = -1$.

SOLUTION	EXPLANATION
$a^2 - 2bc = (-3)^2 - 2(5)(-1)$	Substitute for each pronumeral:
$= 9 - (-10)$	$(-3)^2 = -3 \times (-3)$ and $2 \times 5 \times (-1) = -10$
$= 19$	To subtract a negative number, add its opposite.

Now you try

Evaluate $b^2 - 3ac$ if $a = 1$, $b = -2$ and $c = -3$.

Exercise 1A

FLUENCY	1, 2–7(1/2)	2–7(1/2)	2–7(1/3)

1 Simplify by collecting like terms.

Example 1a
- **a** **i** $5a + 9a$ **ii** $7a - 2a$

Example 1b
- **b** **i** $4a^2b - 2a^2b$ **ii** $5x^2y - 4x^2y$

Example 1c
- **c** **i** $4xy + 3xy^2 - 3xy + 2y^2x$ **ii** $6ab + 2ab^2 - 2ab + 4b^2a$

Example 1

2 Simplify by collecting like terms.

- **a** $6a + 4a$
- **b** $8d + 7d$
- **c** $5y - 5y$
- **d** $2xy + 3xy$
- **e** $9ab - 5ab$
- **f** $4t + 3t + 2t$
- **g** $7b - b + 3b$
- **h** $3st^2 - 4st^2$
- **i** $4m^2n - 7nm^2$
- **j** $0.3a^2b - ba^2$
- **k** $4gh + 5 - 2gh$
- **l** $7xy + 5xy - 3y$
- **m** $4a + 5b - a + 2b$
- **n** $3jk - 4j + 5jk - 3j$
- **o** $2ab^2 + 5a^2b - ab^2 + 5ba^2$
- **p** $3mn - 7m^2n + 6nm^2 - mn$
- **q** $4st + 3ts^2 + st - 4s^2t$
- **r** $7x^3y^4 - 3xy^2 - 4y^4x^3 + 5y^2x$

Example 2

3 Simplify the following.

- **a** $4a \times 3b$
- **b** $5a \times 5b$
- **c** $-2a \times 3d$
- **d** $5h \times (-2m)$
- **e** $-6h \times (-5t)$
- **f** $-5b \times (-6l)$
- **g** $2s^2 \times 6t$
- **h** $-3b^2 \times 7d^5$
- **i** $4ab \times 2ab^3$
- **j** $-6p^2 \times (-4pq)$
- **k** $6hi^4 \times (-3h^4i)$
- **l** $7mp \times 9mr$
- **m** $\frac{7x}{7}$
- **n** $\frac{6ab}{2}$
- **o** $-\frac{3a}{9}$
- **p** $-\frac{2ab}{8}$
- **q** $\frac{4ab}{2a}$
- **r** $-\frac{15xy}{5y}$
- **s** $-\frac{4xy}{8x}$
- **t** $-\frac{28ab}{56b}$

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Example 3a,b

4 Expand the following, using the distributive law.

a $5(x+1)$ **b** $2(x+4)$ **c** $3(x-5)$
d $-5(4+b)$ **e** $-2(y-3)$ **f** $-7(a+c)$
g $-6(-m-3)$ **h** $4(m-3n+5)$ **i** $-2(p-3q-2)$
j $2x(x+5)$ **k** $6a(a-4)$ **l** $-4x(3x-4y)$
m $3y(5y+z-8)$ **n** $9g(4-2g-5h)$ **o** $-2a(4b-7a+10)$
p $7y(2y-2y^2-4)$ **q** $-3a(2a^2-a-1)$ **r** $-t(5t^3+6t^2+2)$
s $2m(3m^3-m^2+5m)$ **t** $-x(1-x^3)$ **u** $-3s(2t-s^3)$

Example 3c

5 Expand and simplify the following, using the distributive law.

a $2(x+4)+3(x+5)$ **b** $4(a+2)+6(a+3)$
c $6(3y+2)+3(y-3)$ **d** $3(2m+3)+3(3m-1)$
e $2(2+6b)-3(4b-2)$ **f** $3(2t+3)-5(2-t)$
g $2x(x+4)+x(x+7)$ **h** $4(6z-4)-3(3z-3)$
i $3d^2(2d^3-d)-2d(3d^4+4d^2)$ **j** $q^3(2q-5)+q^2(7q^2-4q)$

Example 4

6 Factorise:

a $3x-9$ **b** $4x-8$ **c** $10y+20$
d $6y+30$ **e** x^2+7x **f** $2a^2+8a$
g $5x^2-5x$ **h** $9y^2-63y$ **i** $xy-xy^2$
j $x^2y-4x^2y^2$ **k** $8a^2b+40a^2$ **l** $7a^2b+ab$
m $-5t^2-5t$ **n** $-6mn-18mn^2$ **o** $-y^2-8yz$

Example 5

7 Evaluate these expressions if $a=-4$, $b=3$ and $c=-5$.

a $-2a^2$ **b** $b-a$
c $abc+1$ **d** $-ab$
e $\dfrac{a+b}{2}$ **f** $\dfrac{3b-a}{5}$
g $\dfrac{a^2-b^2}{c}$ **h** $\dfrac{\sqrt{a^2+b^2}}{\sqrt{c^2}}$

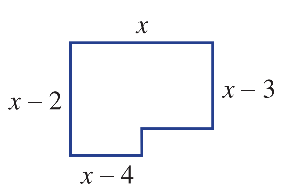
PROBLEM-SOLVING	8	8, 9	9

8 Find an expression for the area of a floor of a rectangular room with the following side lengths. Expand and simplify your answer.

a $x+3$ and $2x$ **b** x and $x-5$

9 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes. (Note: All angles are right angles.)

a



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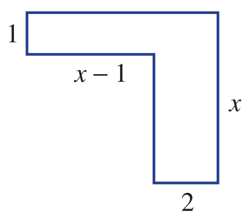

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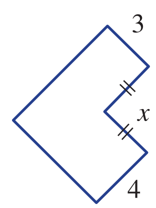

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REASONING	10	10, 11	11, 12

10 When $a = -2$ give reasons why:

- **a** $a^2 > 0$
- **b** $-a^2 < 0$
- **c** $a^3 < 0$

11 Decide whether the following are true or false for all values of a and b. If false, give an example to show that it is false.

- **a** $a + b = b + a$
- **b** $a - b = b - a$
- **c** $ab = ba$
- **d** $\frac{a}{b} = \frac{b}{a}$
- **e** $a + (b + c) = (a + b) + c$
- **f** $a - (b - c) = (a - b) - c$
- **g** $a \times (b \times c) = (a \times b) \times c$
- **h** $a \div (b \div c) = (a \div b) \div c$

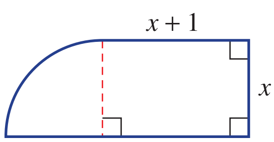
12 **a** Write an expression for the statement 'the sum of x and y divided by 2'.

- **b** Explain why the statement above is ambiguous.
- **c** Write an unambiguous statement describing $\frac{a + b}{2}$.

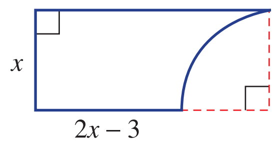
ENRICHMENT: Algebraic circular spaces	–	–	13

13 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes. Your answers may contain π, for example 4π. Do not use decimals.

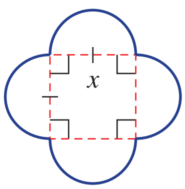
a



b



c



Architecture, building, carpentry and landscaping are among the many occupations that use algebraic formulas to calculate areas and perimeters in daily work.

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