Office Hours

8/28

$$\ell(\sigma^2) = -\frac{n}{2}\log(2\pi\sigma^2) - \frac{\sum_i (y_i - \beta_0 - \beta_1 x_i)^2}{2\sigma^2} \quad (\sigma^2)^{-1}$$

$$\frac{\partial \ell}{\partial \sigma^2} = -\frac{n}{2}\frac{\cancel{2\pi}}{\cancel{2\pi}\sigma^2} - \frac{\sum_i (y_i - \beta_0 - \beta_1 x_i)^2}{2}(-1)(\sigma^2)^{-2}$$

$$= \frac{-n}{2\sigma^2} + \frac{\sum_i (y_i - \beta_0 - \beta_1 x_i)^2}{2\sigma^4} \overset{!}{=} 0$$

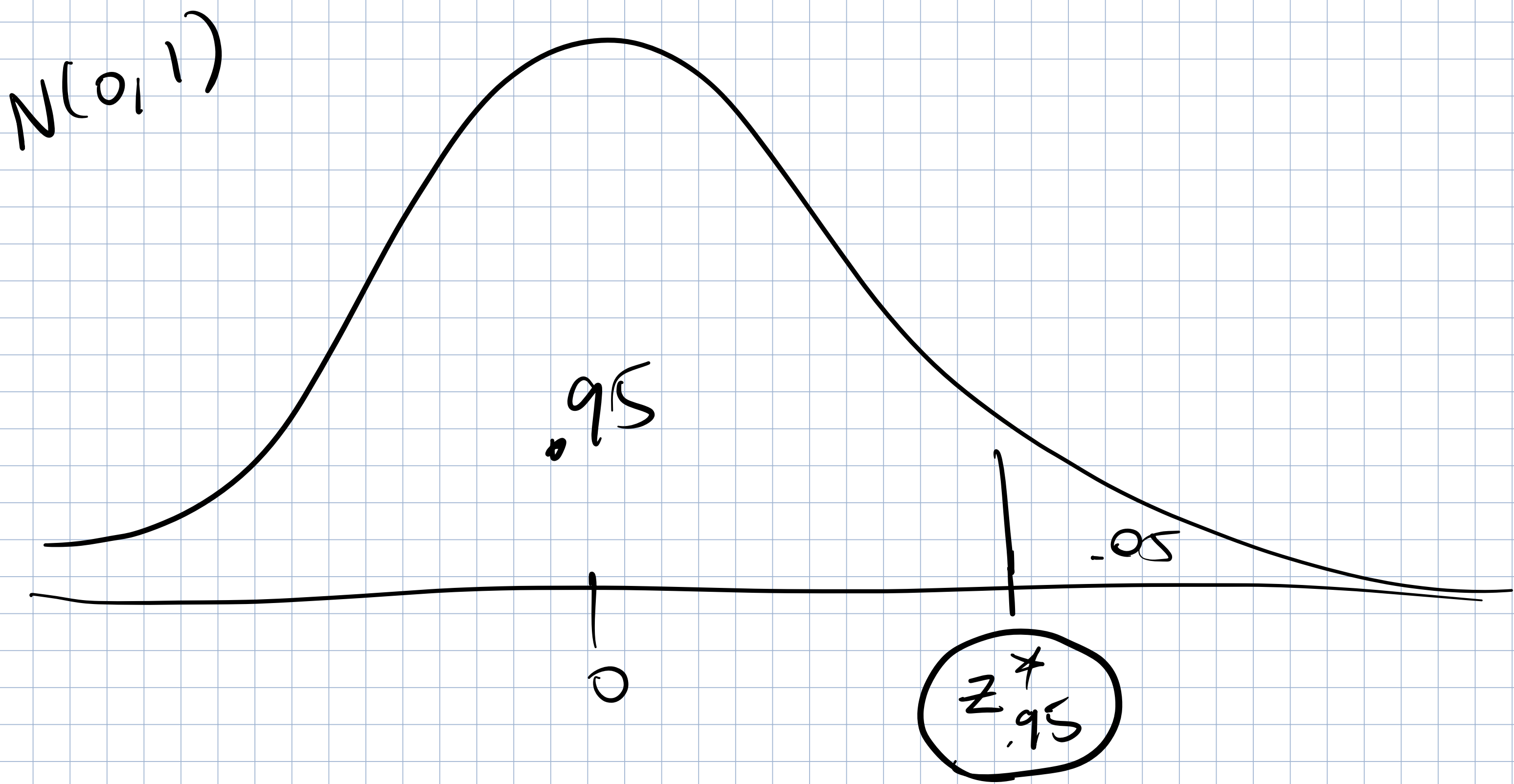
4.

$E(\hat{\beta}_0) = \beta_0$

$E(\hat{\beta}_1) = \beta_1$

why?

b/c of Gauss-Markov



CI: α sig. level

Slope:

$$\hat{\beta}_1 \pm t^*_{1-\frac{\alpha}{2},\, \underline{\underline{n-2}}}\ \widehat{SE}(\hat{\beta}_1)$$

Diff. b/w z & t

$$Z = \frac{\hat{\beta}_1 - E(\hat{\beta}_1)}{\underline{\underline{SD}}(\hat{\beta}_1)} = \frac{\hat{\beta}_1 - \beta_1}{SE(\hat{\beta}_1)} = \frac{\hat{\beta}_1 - \beta_1}{\sigma / \sqrt{SSX}}$$

$$\underline{Z \sim N(0,1)}$$

Since σ is unknown, we must replace it w/ its estimate $\hat{\sigma}$:

$$\boxed{t = \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}/\sqrt{SSX}}} \sim t_{n-2}$$

why we go for t-stats instead of z.

Def: A RV u follows a t_{df} dist iff

$$u := \frac{Z}{\sqrt{V/df}},$$

where
$Z \sim N(0,1)$
$V \sim \chi^2_{df}$
$Z \perp\!\!\!\perp V$.

$\Rightarrow u \sim t_{df}$

$$t = \underbrace{\frac{\hat{\beta}_1 - \beta}{\sigma / \sqrt{SSX}}}_{N(0,1)} \underbrace{\frac{\sigma}{\hat{\sigma}}}$$

$$\hat{\sigma}^2 = \frac{\sum_i e_i^2}{\boxed{n-2}}$$

$$\frac{\sum_i e_i^2}{\sigma^2} \sim \chi^2_{n-2}$$

$$= \frac{N(0,1)}{\hat{\sigma}/\sigma} = \frac{N(0,1)}{\sqrt{\hat{\sigma}^2/\sigma^2}}$$

$$= \frac{N(0,1)}{\sqrt{\frac{(n-2)\hat{\sigma}^2}{\sigma^2} / (n-2)}}$$

$$= \frac{N(0,1)}{\sqrt{\frac{\sum_i e_i^2}{\sigma^2} / (n-2)}}$$

$$= \frac{N(0,1)}{\sqrt{\chi^2_{n-2} / n-2}} = \frac{Z}{\sqrt{V/df}}$$

$$e_1 = -\sum_{i=2}^{n} e_i$$

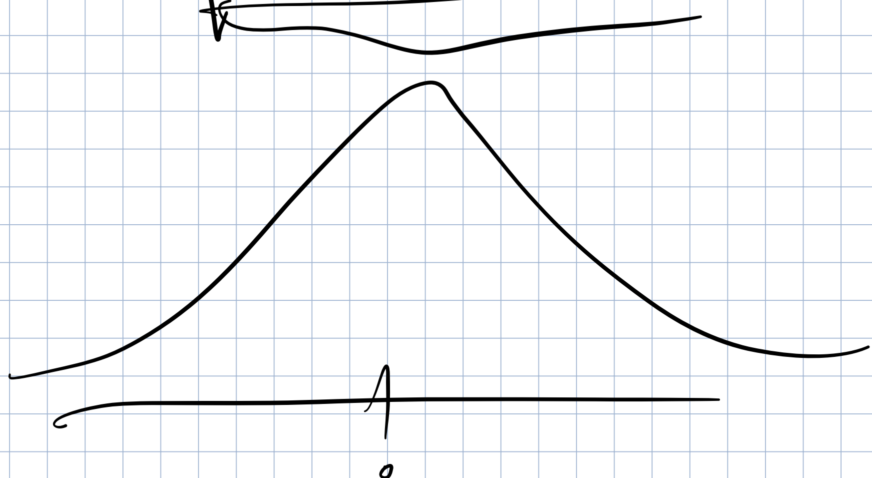
$$e_1 + \sum_{i=2}^{n} e_i = 0$$

never observed

$$\sum_{i=1}^{n} e_i = 0 \quad \text{vs} \quad \sum_{i=1}^{n} \varepsilon_i = ?$$

$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$$

$$\sum_i \varepsilon_i \sim \boxed{N(0, n\sigma^2)}$$



$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$$

$$\underbrace{\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_n} \sim N(\underline{0}, \underline{n\sigma^2})$$

$$E\left(\sum_i \varepsilon_i\right) = \sum_i \cancelto{0}{E(\varepsilon_i)} = 0$$

$$\mathrm{Var}\left(\sum_i \varepsilon_i\right) \overset{\perp}{=} \sum_i \mathrm{Var}(\varepsilon_i)$$

$$= \sum_i \sigma^2 = n\sigma^2$$

$$E(\underline{e_i}) = E(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$$

$$= E(y_i) - E(\hat{\beta}_0) - E(\hat{\beta}_1 x_i)$$

$\downarrow$ $\quad$ $\downarrow$ b/c of GM $\quad$ $\downarrow$

$$= \cancel{\beta_0} + \cancel{\beta_1 x_i} - \cancel{\beta_0} - \cancel{\beta_1 x_i}$$

$$= 0.$$

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$E(y_i) = \underbrace{\beta_0 + \beta_1 x_i}\; + \cancel{E(\varepsilon_i)} \xrightarrow{} 0$$

$$\sum_i (y_i - \beta_0 - \beta_1 x_i)^2$$

↳ minimize w.r.t $\underline{\beta_0}$ & $\underline{\beta_1}$

(iv)

If $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$

$\Rightarrow \; y_i \overset{iid}{\sim} N(\beta_0 + \beta_1 x_i, \sigma^2)$