CS231A Section 4
Camera Models
(Problem Set 2)

10/21/2011

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Announcement

- Extra office hour for PS2:

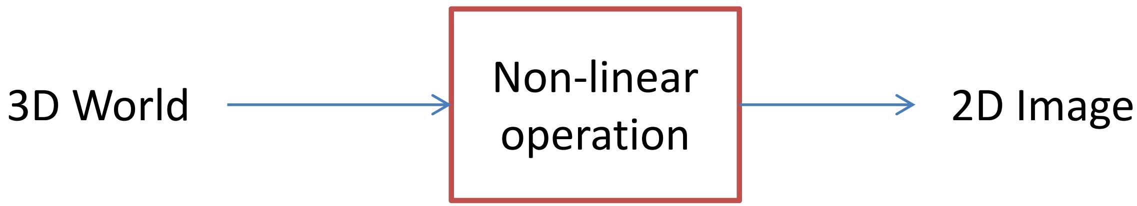
  10/27, 8 - 10pm, Gates 104

Topics

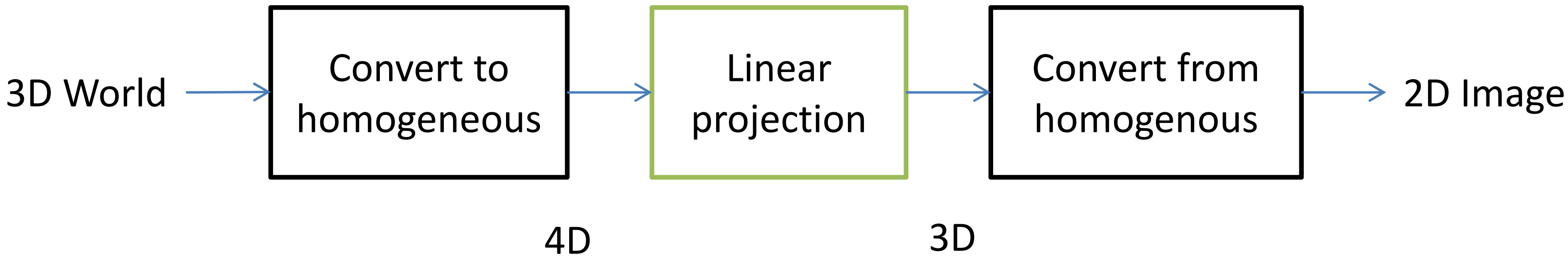
- Homogeneous Coordinates
  - Why is it helpful?
- Transformations
  - Rotations
  - Affine
  - Homography
  - Solving for an affine transform matrix
- Camera matrix
- Fundamental and Essential Matrix

Homogeneous Coordinates

- Why?
- Without homogeneous



- With homogeneous



Homogeneous Coordinates

- Convert to homogeneous

$$(x,y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \text{ (image)} \qquad (x,y,z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \text{ (scene)}$$

- Convert from homogeneous

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

- Keep track of dimensions
  - Usually, 3 (world) to 2 (image)
  - Homogeneous: 4(world) to 3(image)
  - X' = MX
- **Hint

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w) \qquad \begin{bmatrix} cx \\ cy \\ cz \\ cw \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous Coordinates of Lines

- The homogeneous coordinate of a line is $\mathrm{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$
- The dot product of the homogeneous coordinates of a line and a point on it is zero

If $\mathrm{x} = [\,x_1, x_2]^T \in \mathrm{l}$

$$ax + by + c = 0 \quad \Rightarrow \quad \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}^T \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

- Cross product of two lines gives the homogeneous coordinate of their point of intersection $\mathrm{x} = \mathrm{l} \times \mathrm{l}'$
- Points at infinity (ideal points): $\mathrm{x}_\infty = [\mathrm{x}_1 \;\; \mathrm{x}_2 \;\; 0]^\mathrm{T}$
- Line at infinity (ideal line): set of ideal points, $\mathrm{l}_\infty = [0 \;\; 0 \;\; 1]^\mathrm{T}$

Rotation Matrices

- Examples

$$R_z = \begin{bmatrix} cos\theta_z & -sin\theta_z & 0 \\ sin\theta_z & cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_y = \begin{bmatrix} cos\theta_y & 0 & -sin\theta_y \\ 0 & 1 & 0 \\ sin\theta_y & 0 & cos\theta_y \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_x & -sin\theta_x \\ 0 & sin\theta_x & cos\theta_x \end{bmatrix}$$

$$R = R_x R_y R_z$$

- Important Properties
  - $|R| = 1$
  - $R^T R = I$
  - $R^T = R^{-1}$

Homography/Projective/Perspective

- Parallel lines intersect at vanishing points
- World->image

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- Image->image

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

- $|H| \neq 0$

A Quick Question…

- Do these 2 homography matrices yield the same points in the image space x and x'?

$$x = HX \qquad\qquad x' = cHX$$

Homography/Projective/Perspective

- Parallel lines intersect at vanishing points
- World->image

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- Image->image

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

- $|H| \neq 0$

Affine Transform

- Special case: weak perspective – simpler math (less computations)
- Parallel lines remain parallel
- World->image

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- Image->image

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

- $|H| \neq 0$
- w = 1 (no division required!)

Solving For an Affine Transform Matrix

$$\begin{bmatrix} x_1 & x_2 & x_3 & \\ y_1 & y_2 & y_3 & \cdots \\ 1 & 1 & 1 & \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 & X_2 & X_3 & \\ Y_1 & Y_2 & Y_3 & \\ Z_1 & Z_2 & Z_3 & \cdots \\ 1 & 1 & 1 & \end{bmatrix}$$

unknowns

Solving For an Affine Transform Matrix

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \\ 1 & 1 & 1 \end{bmatrix}$$

unknowns

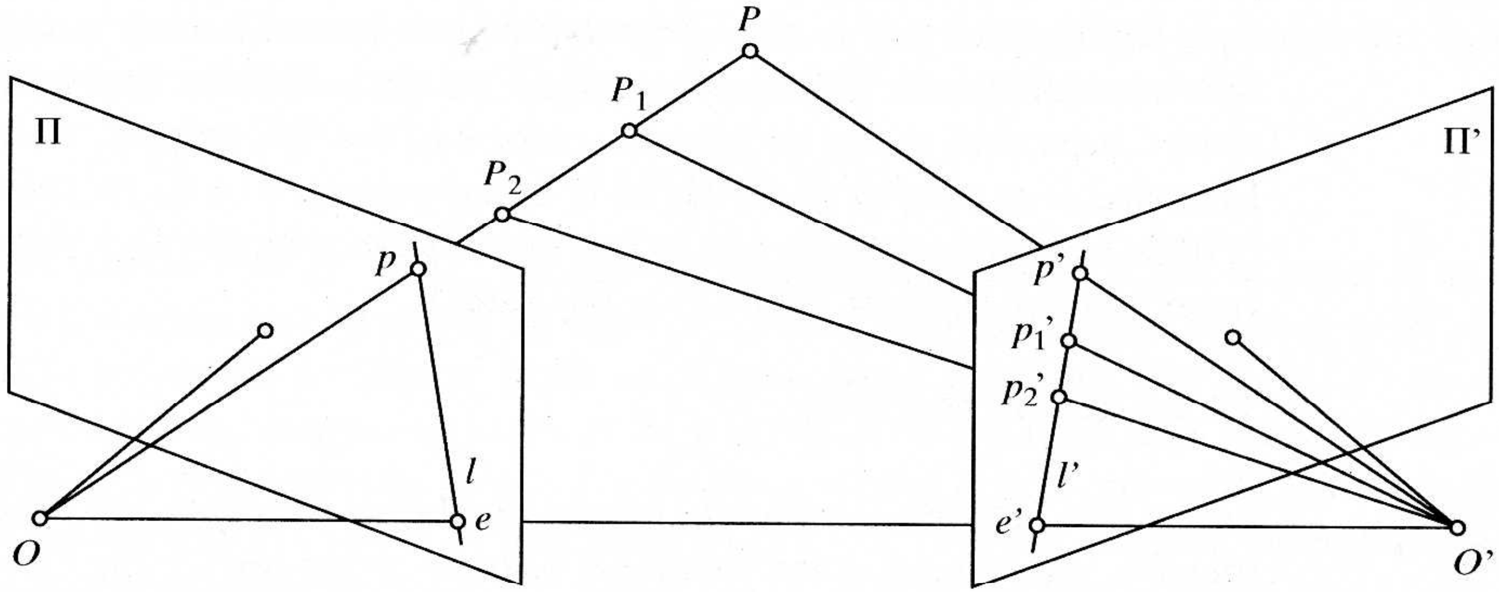
Camera Matrix

- Maps 3D world scene onto 2D image in homogeneous coordinates.
- M= $K[R\ T]$
  - [R T]: rigid transformation
    - From world to camera reference
    - Extrinsic parameters
  - K: camera calibration matrix
    - From camera reference to sensor
    - intrinsic parameters

Some Notation…

$$[\boldsymbol{a}_\times]\boldsymbol{x} = \boldsymbol{a} \times \boldsymbol{x}$$

Epipolar Geometry



$\overrightarrow{Op}$, $\overrightarrow{O'p'}$, and $\overrightarrow{OO'}$ are coplanar.

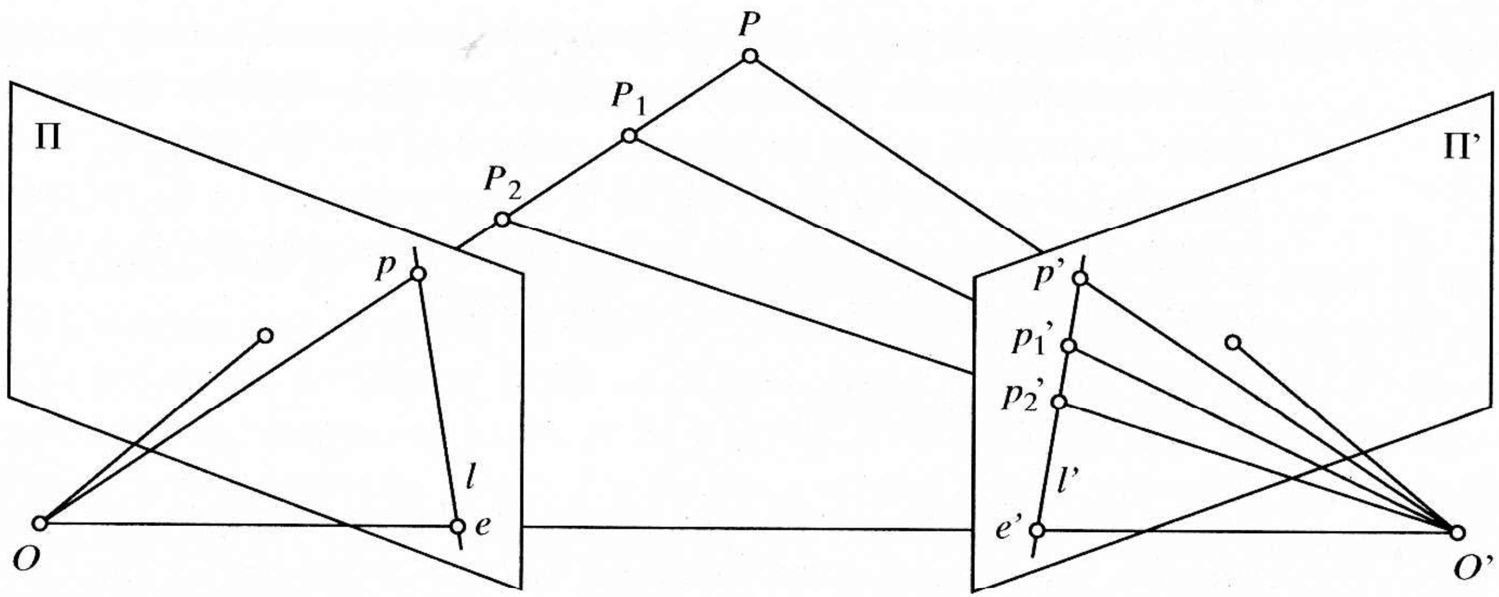
Essential and Fundamental Matrix

$$\overrightarrow{Op} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'p'}] = 0.$$

$$\boldsymbol{p} \cdot [\boldsymbol{t} \times (\mathcal{R}\boldsymbol{p}')]$$

$$\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0 \qquad \mathcal{E} = [\boldsymbol{t}_\times]\mathcal{R}$$

A Simple "Trick"



- The fundamental matrix corresponding to a camera pair, $\mathrm{M} = [I\ 0]$ and $\mathrm{M}' = [A\ a]$ is equal to $[a]_x A$