Lecture 7: Camera Models

Professor Fei-Fei Li

Stanford Vision Lab

What we will learn today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras

Reading:

[FP] Chapters 1 – 3

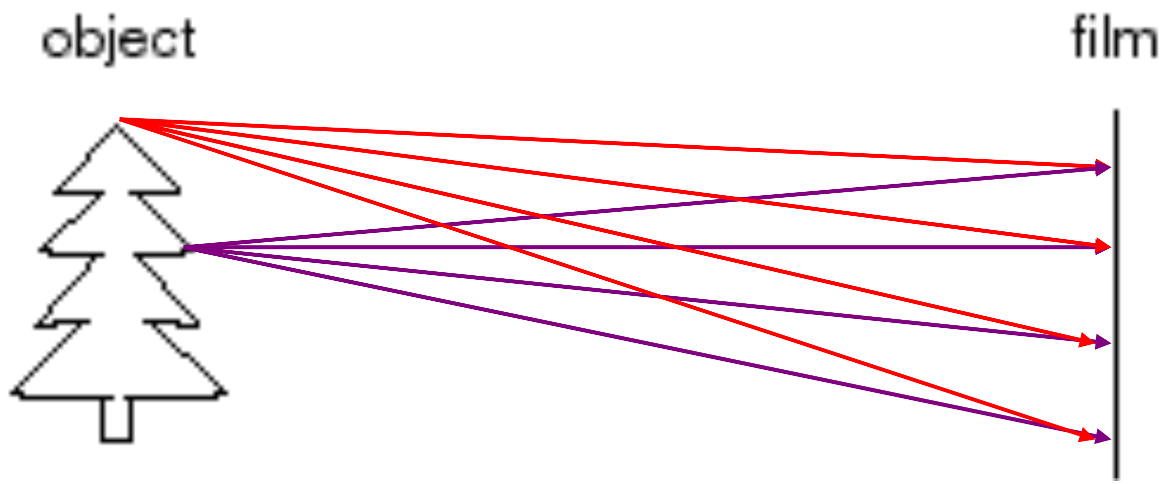
[HZ] Chapter 6

What we will learn today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras

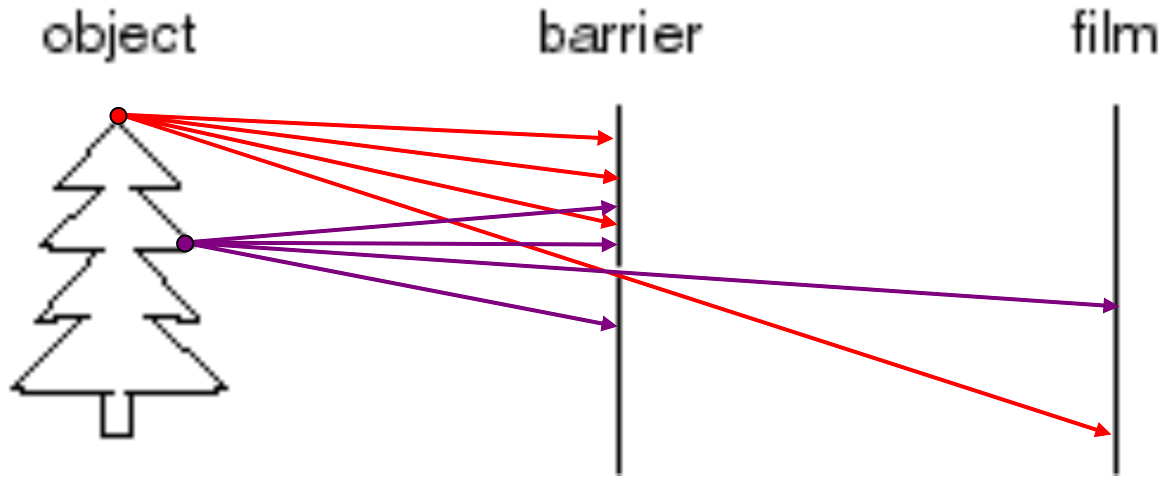
Reading:
[FP] Chapters 1 – 3
[HZ] Chapter 6

How do we see the world?



- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera

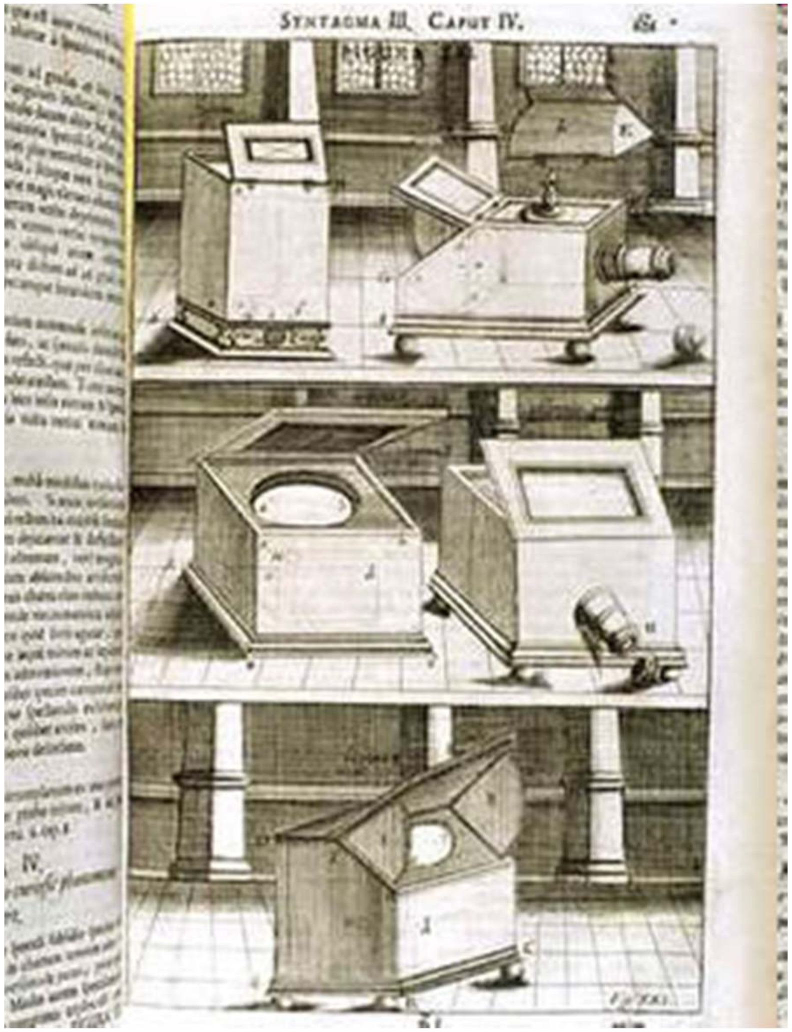


- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**

Some history…

Milestones:

- Leonardo da Vinci (1452-1519): first record of camera *obscura*
- Johann Zahn (1685): first portable camera



Some history…

Milestones:

- Leonardo da Vinci (1452-1519): first record of camera *obscura*
- Johann Zahn (1685): first portable camera
- Joseph Nicephore Niepce (1822): first photo - birth of photography

- Daguerréotypes (1839)
- Photographic Film (Eastman, 1889)
- Cinema (Lumière Brothers, 1895)
- Color Photography (Lumière Brothers, 1908)

Photography (Niepce, "La Table Servie," 1822)

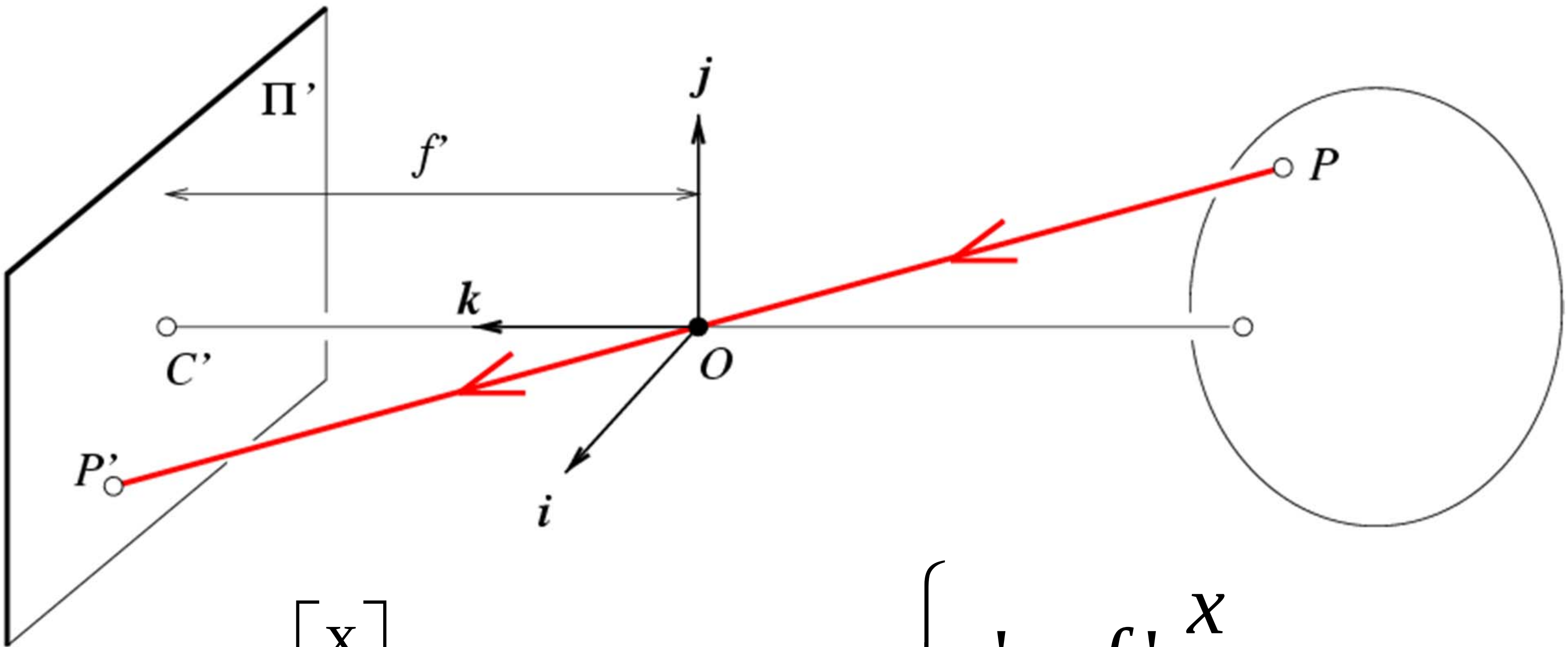
Some history…

Motzu
(468-376 BC)
Oldest existent book on geometry in China

Aristotle
(384-322 BC)
Also: Plato, Euclid

Al-Kindi (c. 801–873)
Ibn al-Haitham
(965-1040)

Pinhole camera



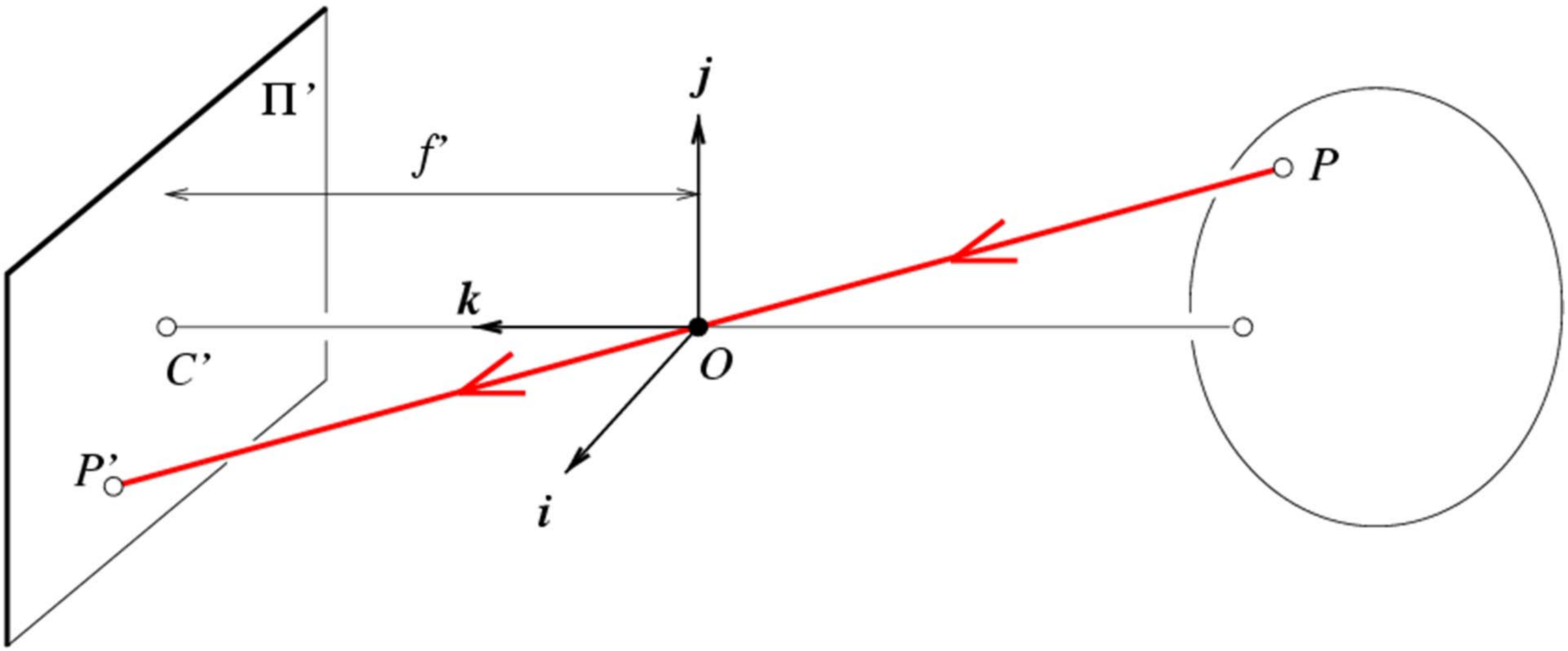
$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

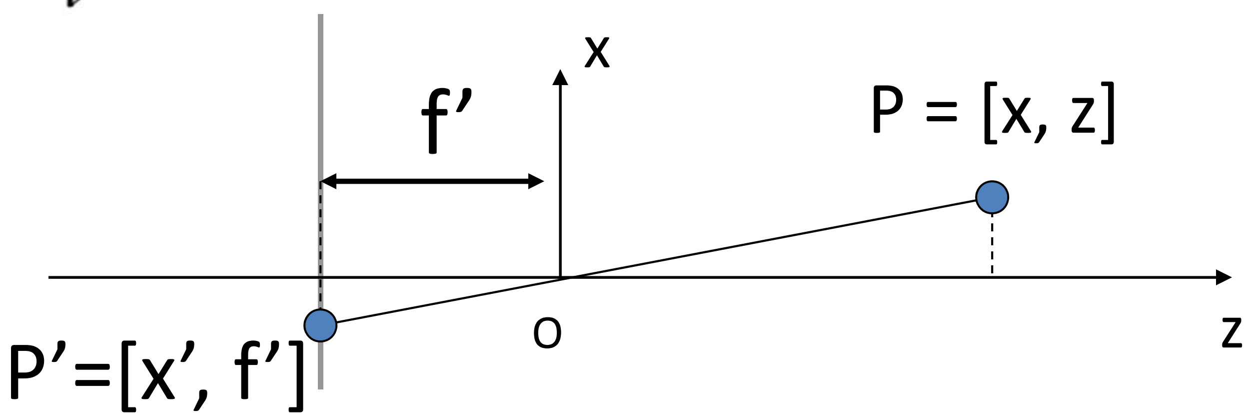
$$\begin{cases} x' = f' \dfrac{x}{z} \\ y' = f' \dfrac{y}{z} \end{cases}$$

Note: z is always negative.

Derived using similar triangles

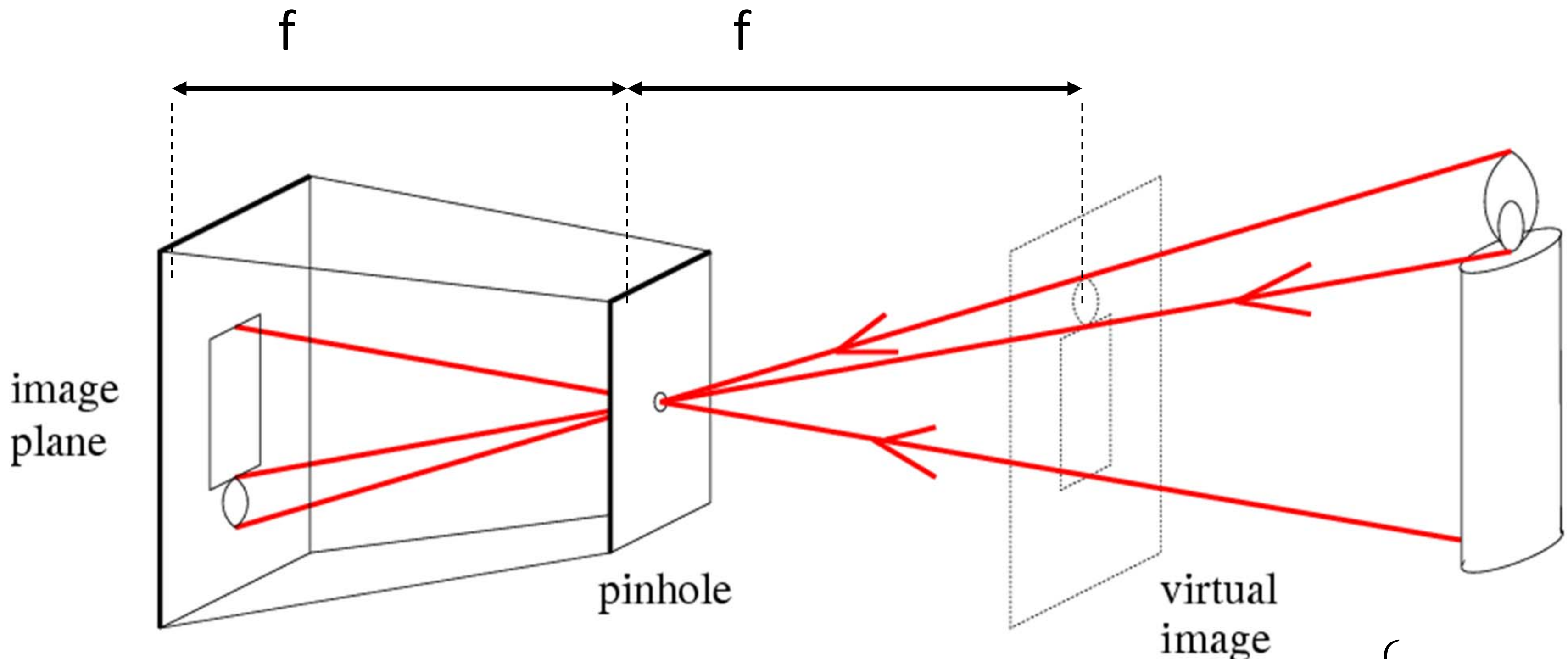
Pinhole camera





$$\frac{x'}{f'} = \frac{x}{z}$$

Pinhole camera

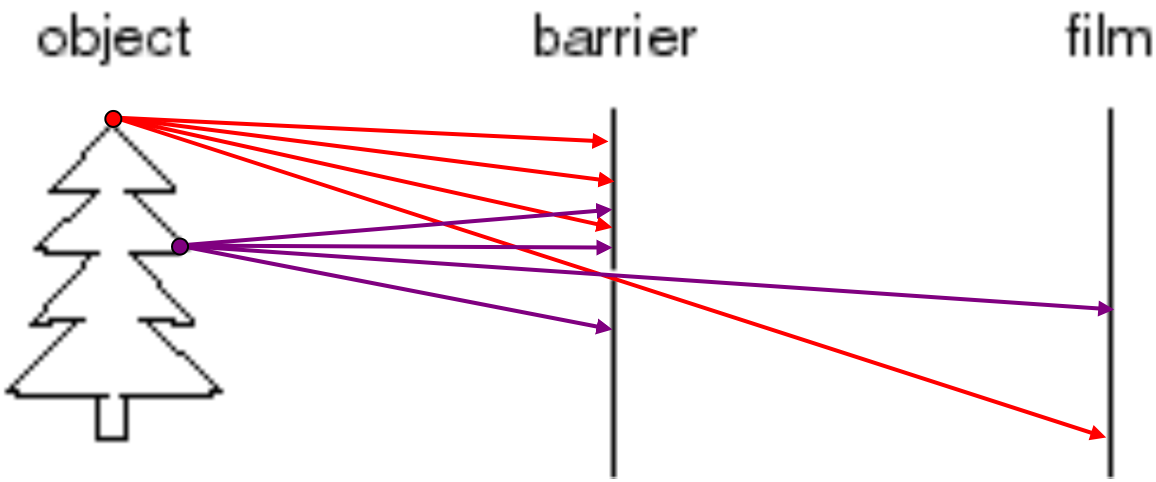


- Common to draw image plane *in front* of the focal point
- Moving the image plane merely scales the image.

$$\begin{cases} x' = f \dfrac{x}{z} \\ y' = f \dfrac{y}{z} \end{cases}$$

Pinhole camera

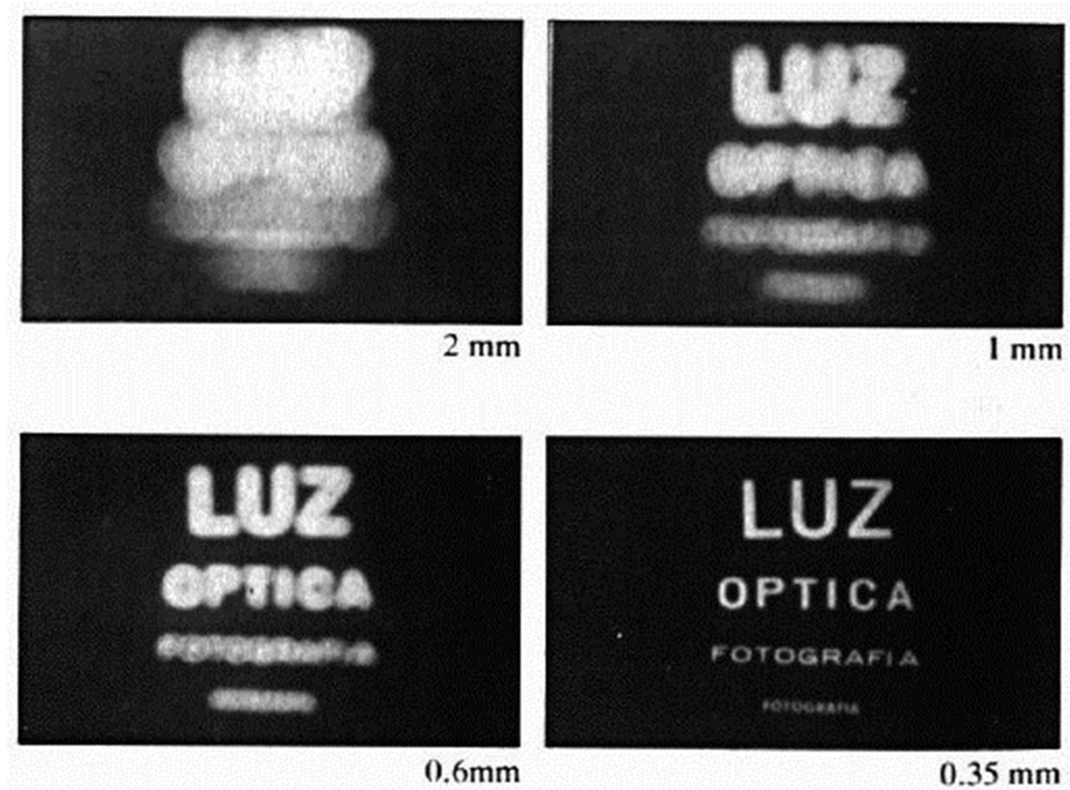
Is the size of the aperture important?



Cameras & Lenses

Shrinking aperture size

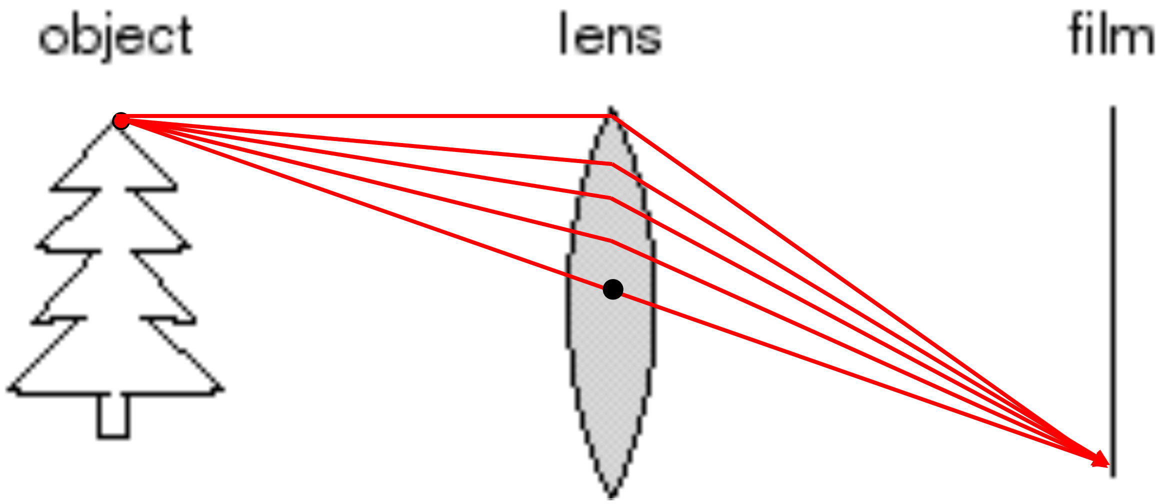
- Rays are mixed up



-Why the aperture cannot be too small?
  - -Less light passes through
  - -Diffraction effect

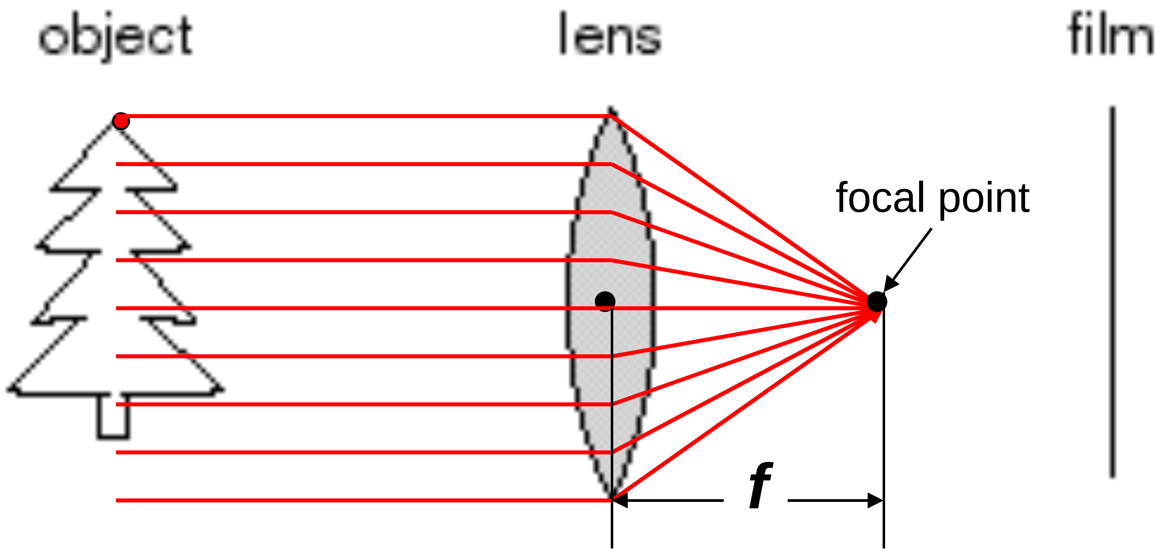
Adding lenses!

Cameras & Lenses



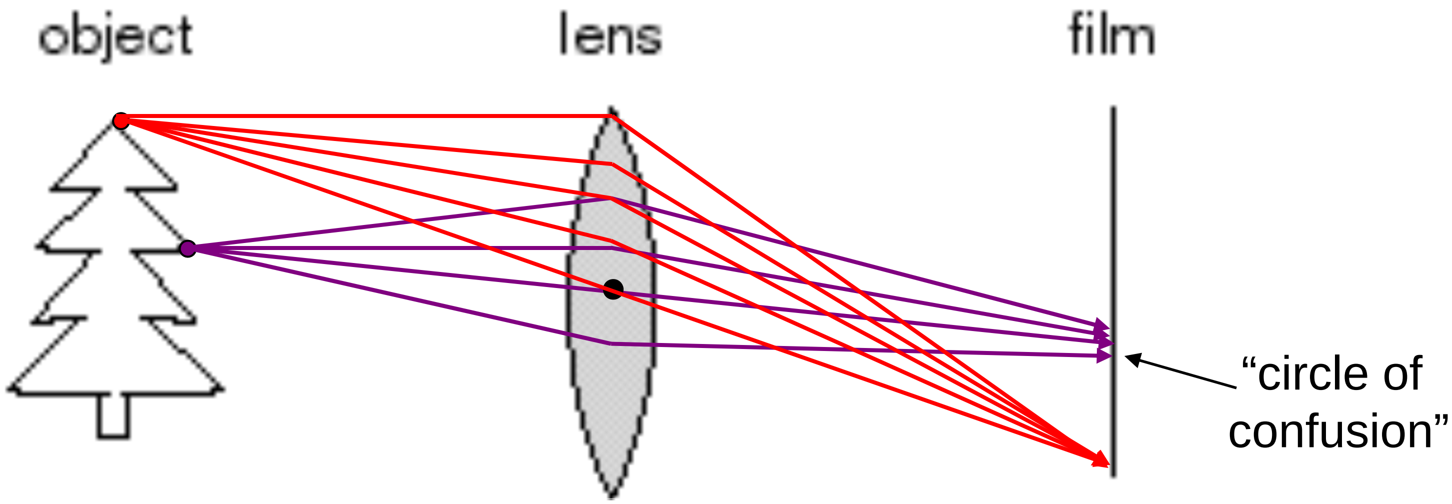
- A lens focuses light onto the film

Cameras & Lenses



- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

Cameras & Lenses



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    [other points project to a “circle of confusion” in the image]

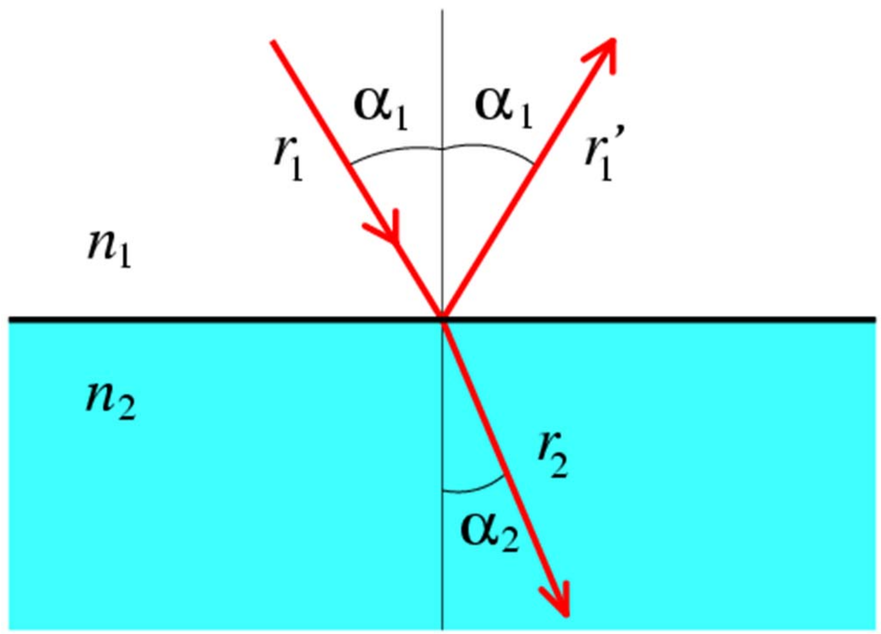
Cameras & Lenses

- Laws of geometric optics
  - Light travels in straight lines in homogeneous medium
  - Reflection upon a surface: incoming ray, surface normal, and reflection are co-planar
  - Refraction: when a ray passes from one medium to another

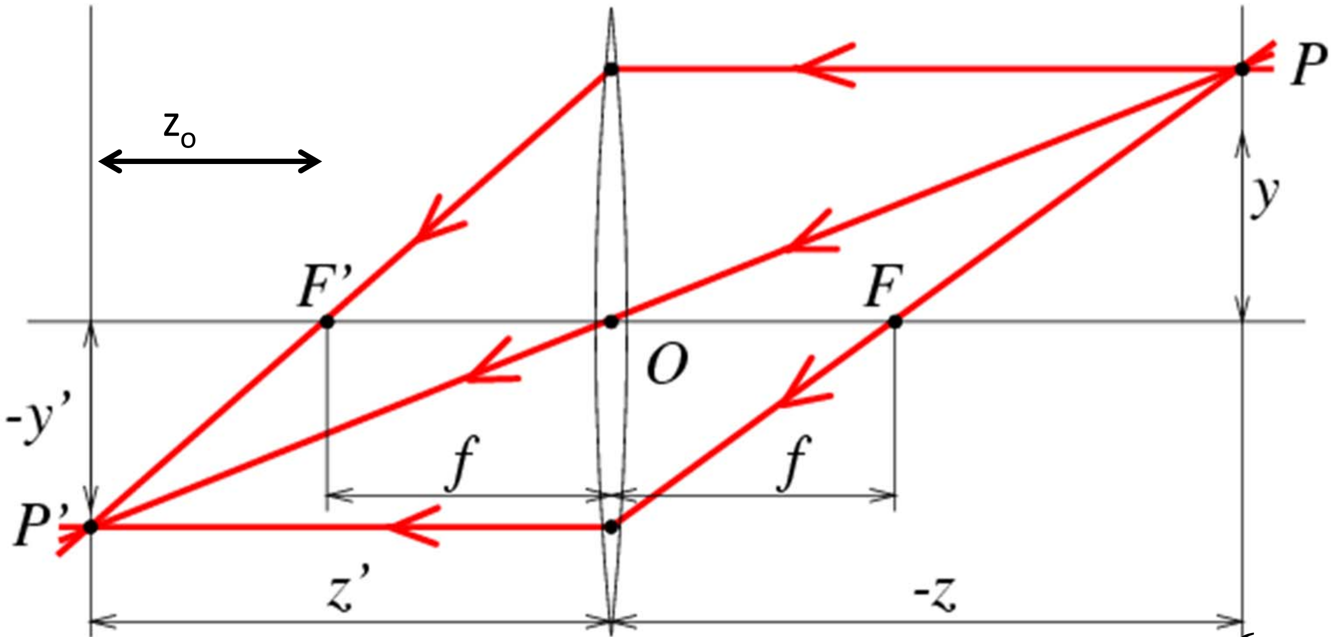
Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

α_1 = incident angle
α_2 = refraction angle
n_i = index of refraction



Thin Lenses



$$z' = f + z_o$$

$$f = \frac{R}{2(n-1)}$$

Snell's law:

$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$

Small angles:

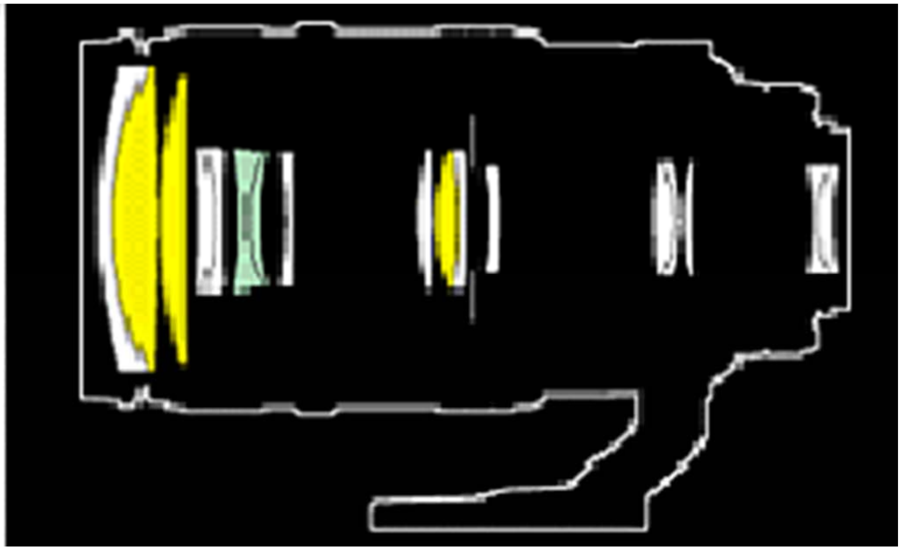
$n_1 \alpha_1 \approx n_2 \alpha_2$

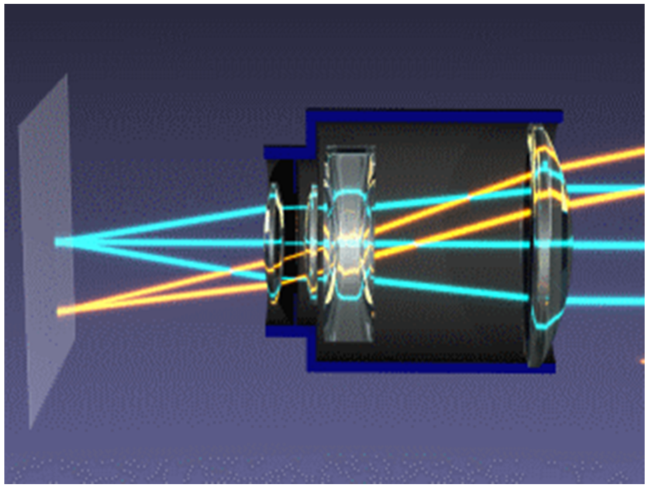
$n_1 = n$ (lens)

$n_1 = 1$ (air)

$$\begin{cases} x' = z' \dfrac{x}{z} \\ y' = z' \dfrac{y}{z} \end{cases}$$

Cameras & Lenses



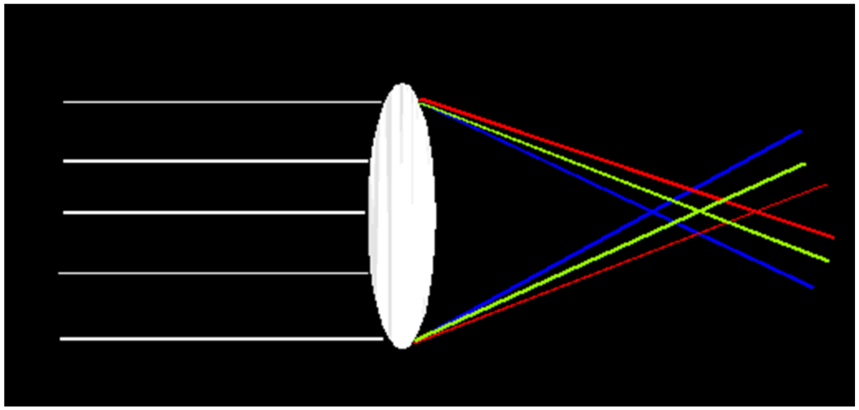


Source wikipedia

Issues with lenses: Chromatic Aberration

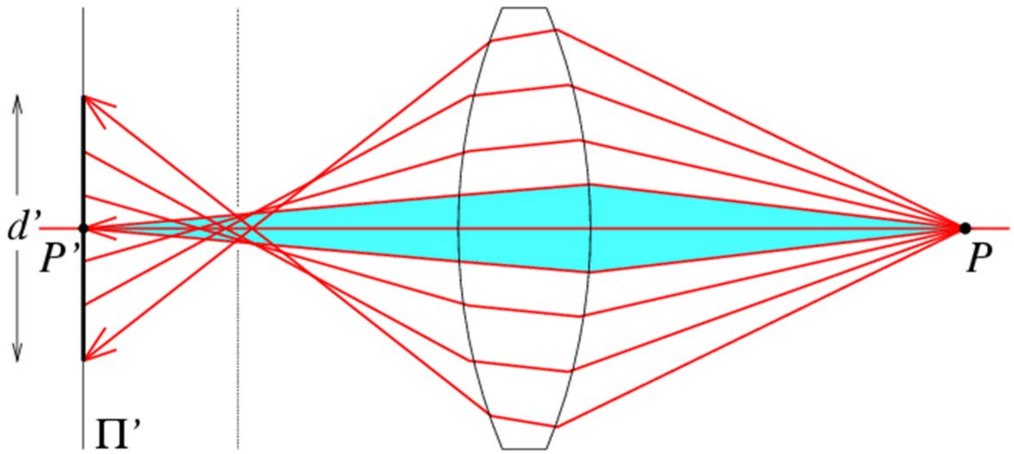
- Lens has different refractive indices for different wavelengths: causes color fringing

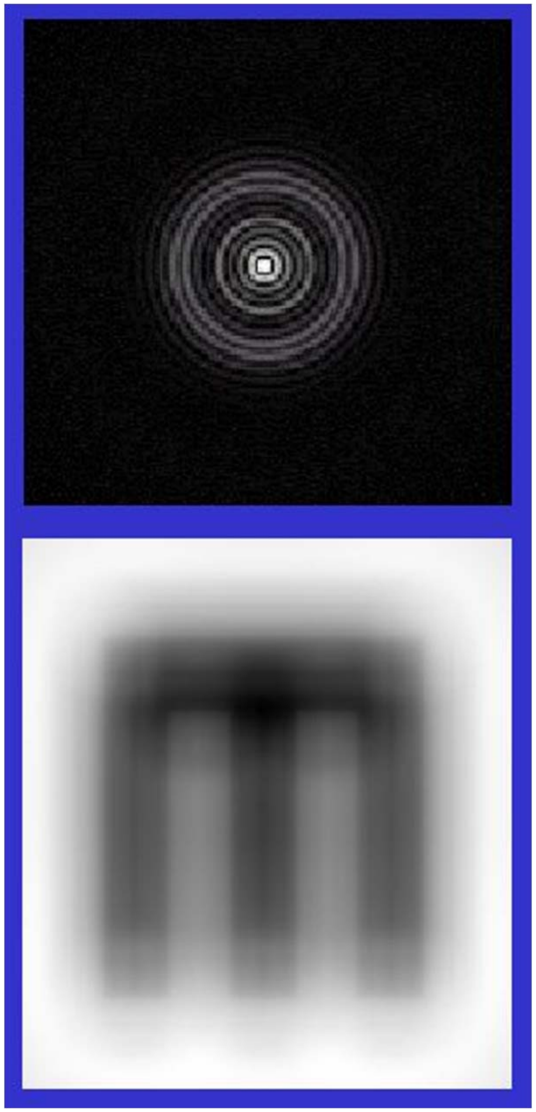
$$f = \frac{R}{2(n-1)}$$



Issues with lenses: Chromatic Aberration

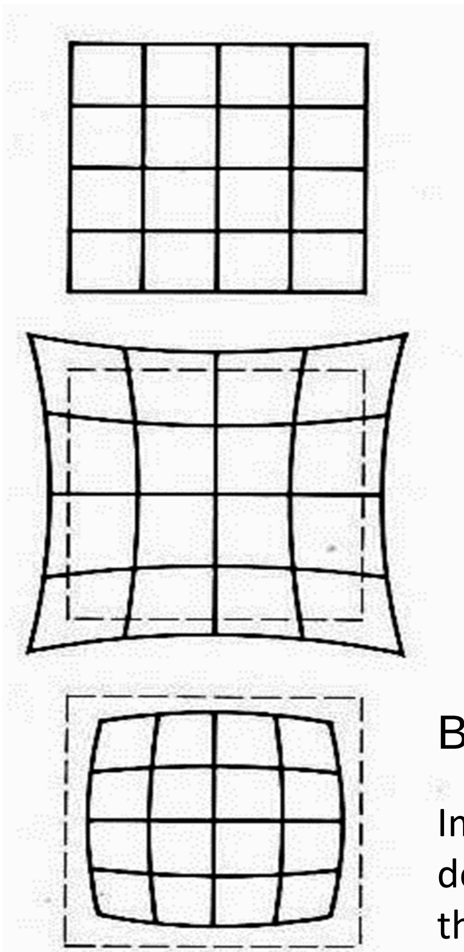
- Rays farther from the optical axis focus closer





Issues with lenses: Chromatic Aberration

– Deviations are most noticeable for rays that pass through the edge of the lens



No distortion

Pin cushion

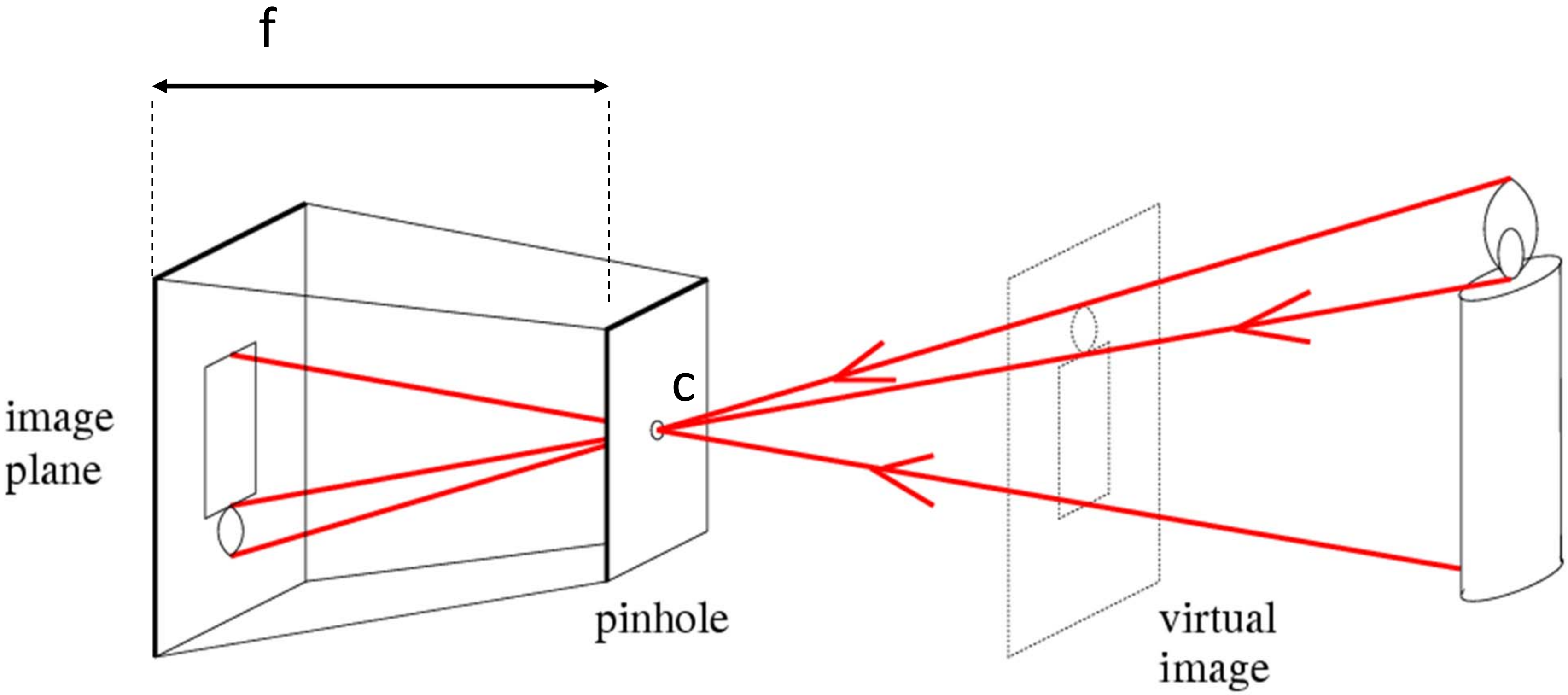
Barrel (fisheye lens)

Image magnification decreases with distance from the optical axis

What we will learn today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras

Pinhole camera



$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

$$\Re^3 \xrightarrow{E} \Re^2$$

f = focal length

c = center of the camera

Pinhole camera

Is this a linear transformation?

$$(x, y, z) \to (f\frac{x}{z}, f\frac{y}{z})$$

No — division by z is nonlinear!

How to make it linear?

Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

- Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

Perspective Projection Transformation:

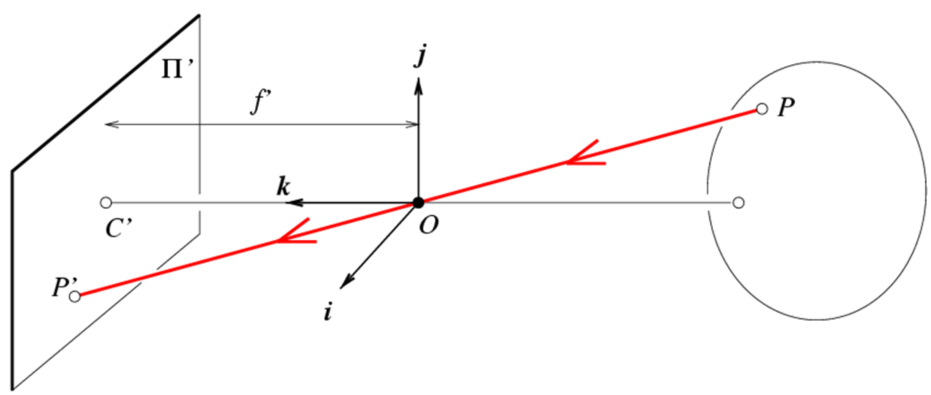
$$P' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{M}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

"Projection matrix"

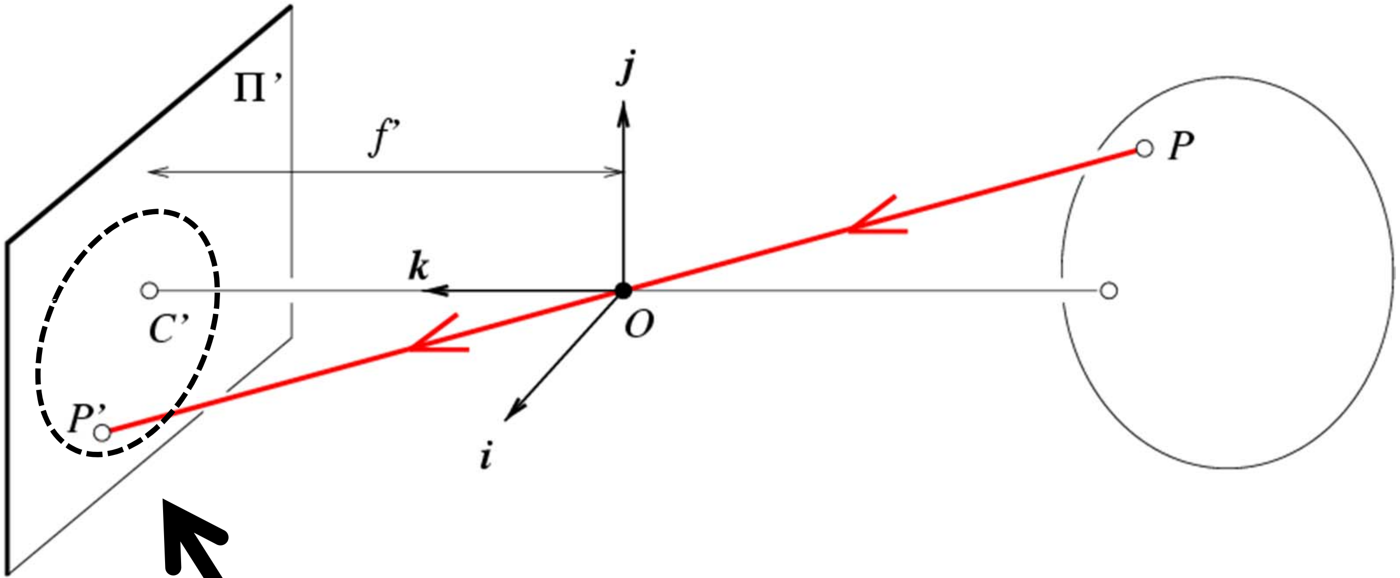
$$P' = M\ P$$

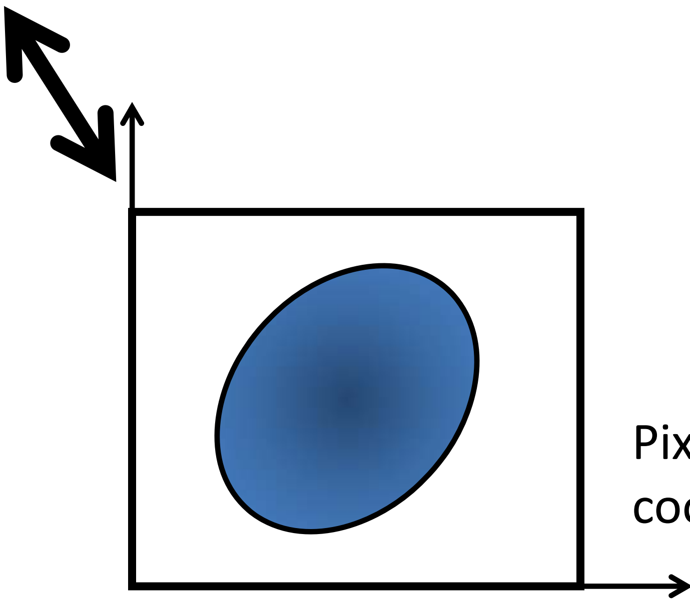
$$\Re^4 \xrightarrow{\text{H}} \Re^3$$

$$P_i' = \begin{bmatrix} f\,\dfrac{x}{z} \\ f\,\dfrac{y}{z} \end{bmatrix}$$



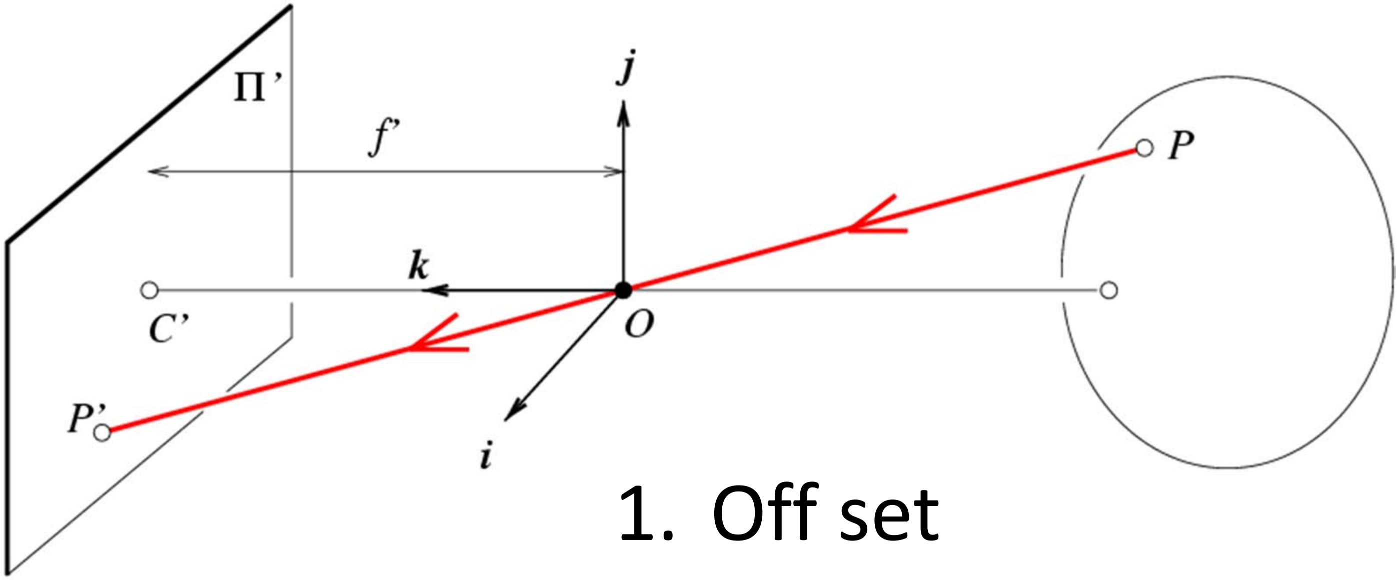
From retina plane to images



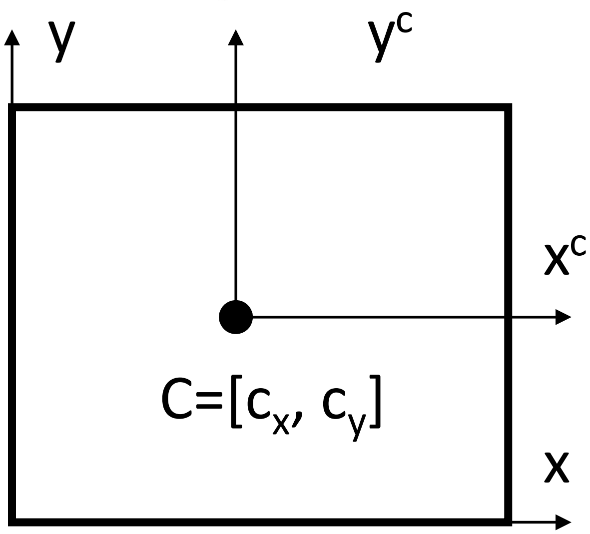


Pixels, bottom-left coordinate systems

From retina plane to images

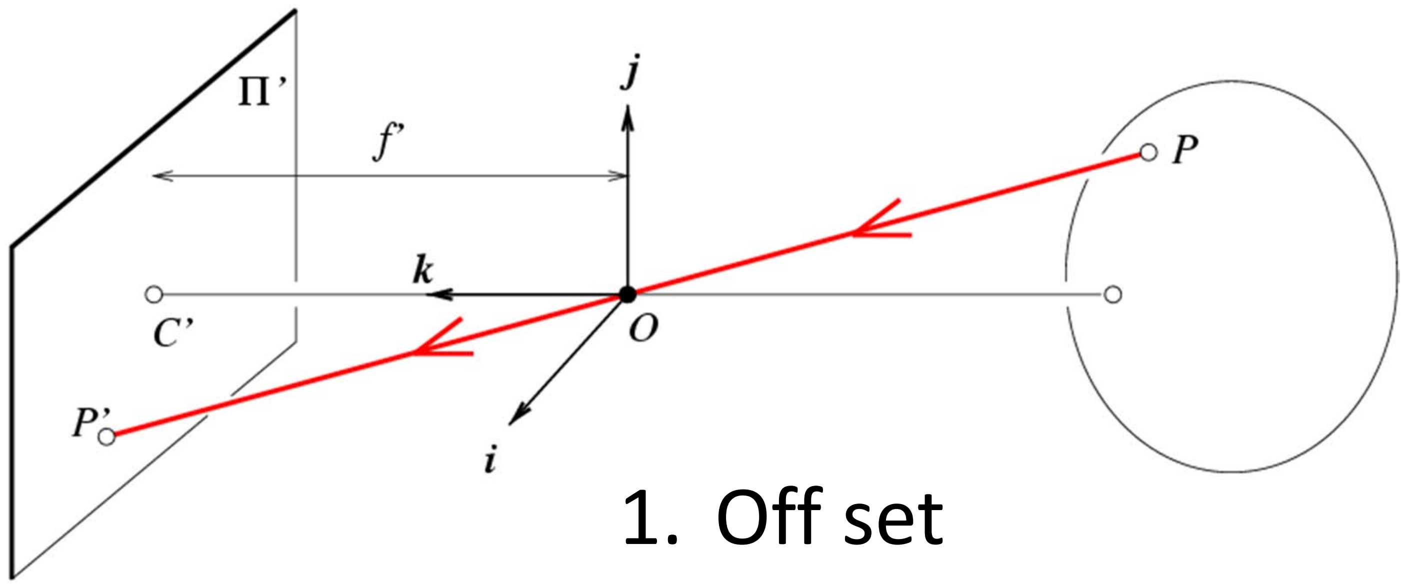


1. Off set

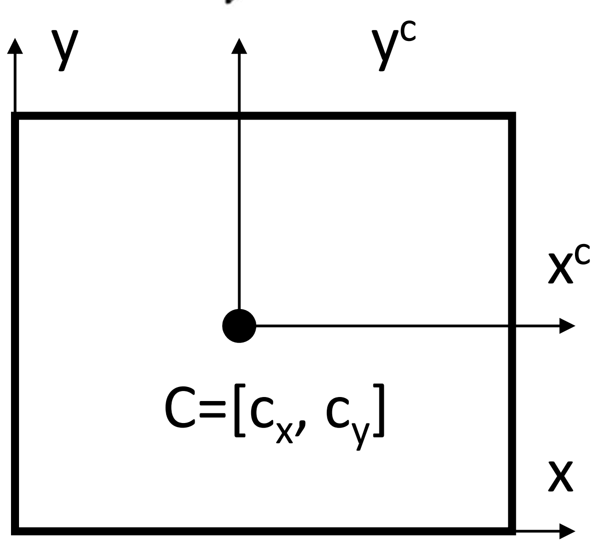


$$(x, y, z) \rightarrow (f\frac{x}{z} + c_x, f\frac{y}{z} + c_y)$$

From retina plane to images



1. Off set
2. From metric to pixels



$$(x, y, z) \rightarrow (\underbrace{f\ k}_{\alpha}\frac{x}{z} + c_x, \underbrace{f\ l}_{\beta}\frac{y}{z} + c_y)$$

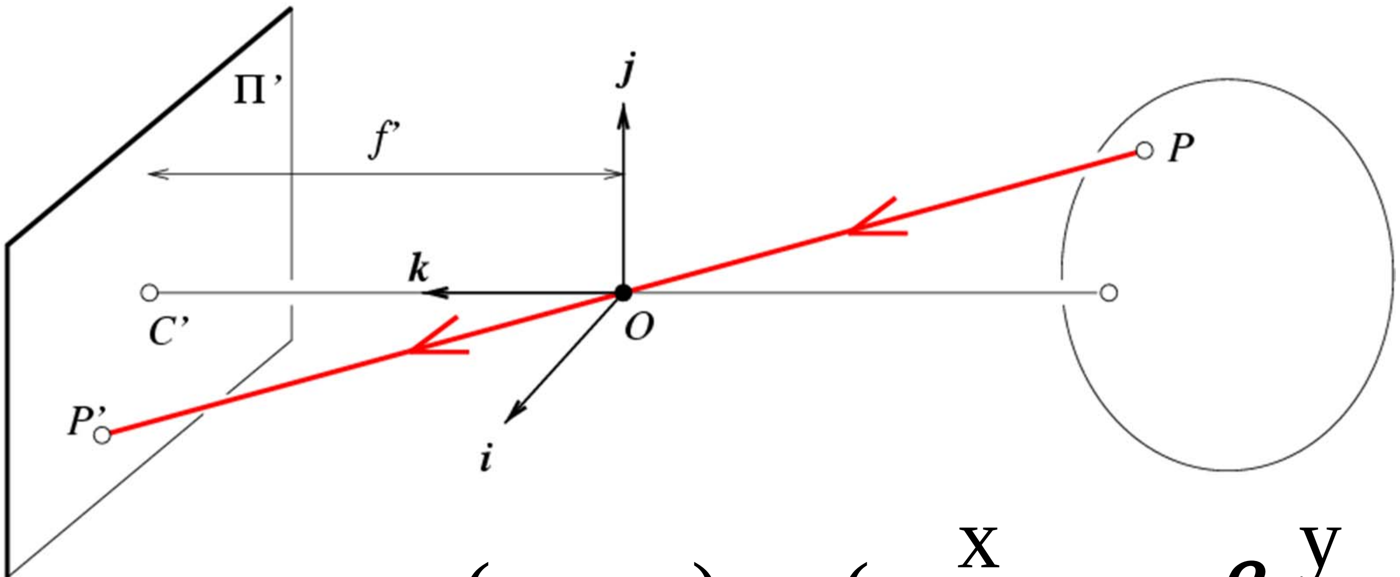
Units: k,l : pixel/m

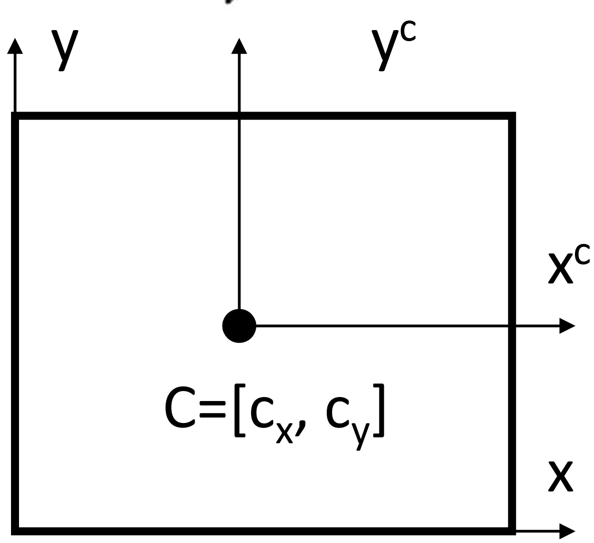
f : m

Non-square pixels

α, β : pixel

From retina plane to images

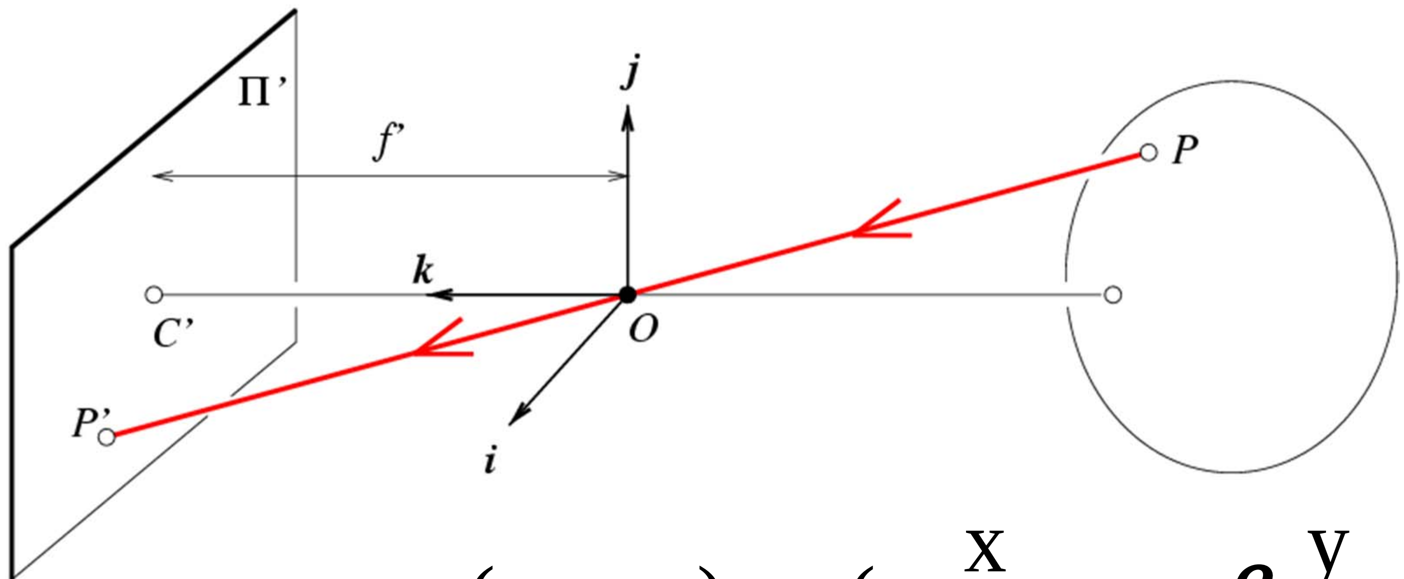


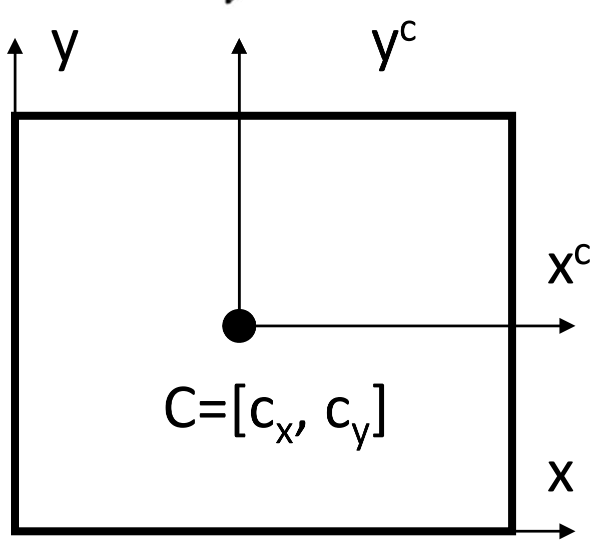


$$(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)$$

Matrix form?

Camera matrix

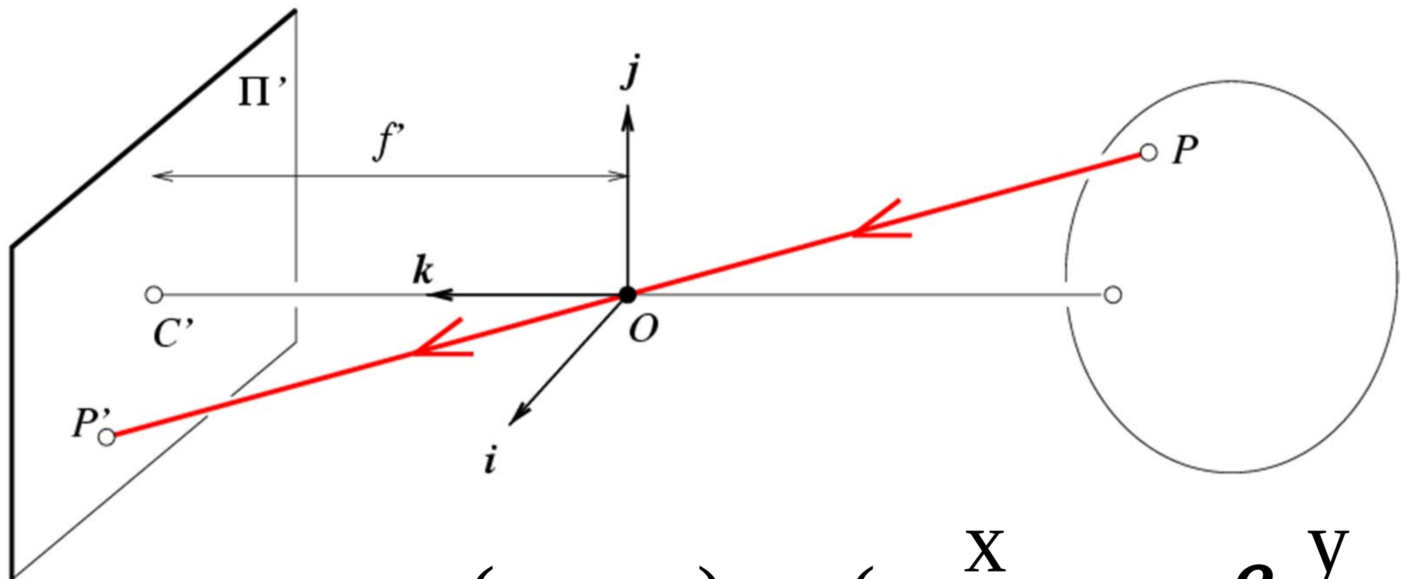


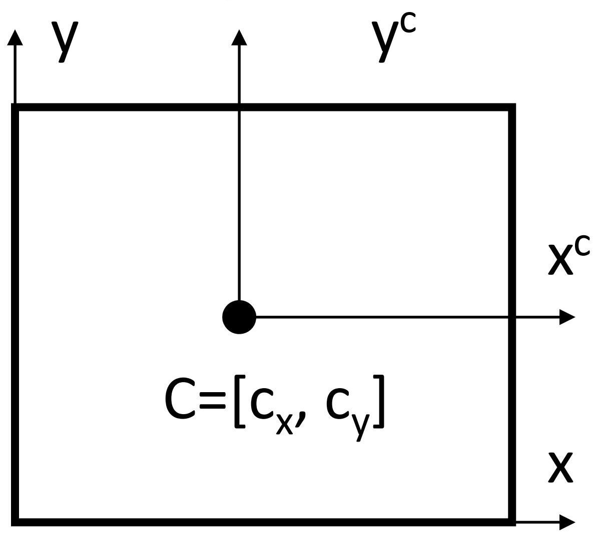


$$(\mathrm{x},\mathrm{y},\mathrm{z}) \rightarrow \left(\boldsymbol{\alpha}\frac{\mathrm{x}}{\mathrm{z}} + \mathrm{c}_{\mathrm{x}}, \boldsymbol{\beta}\frac{\mathrm{y}}{\mathrm{z}} + \mathrm{c}_{\mathrm{y}}\right)$$

$$P' = \begin{bmatrix} \alpha\, x + c_x z \\ \beta\, y + c_y z \\ z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera matrix

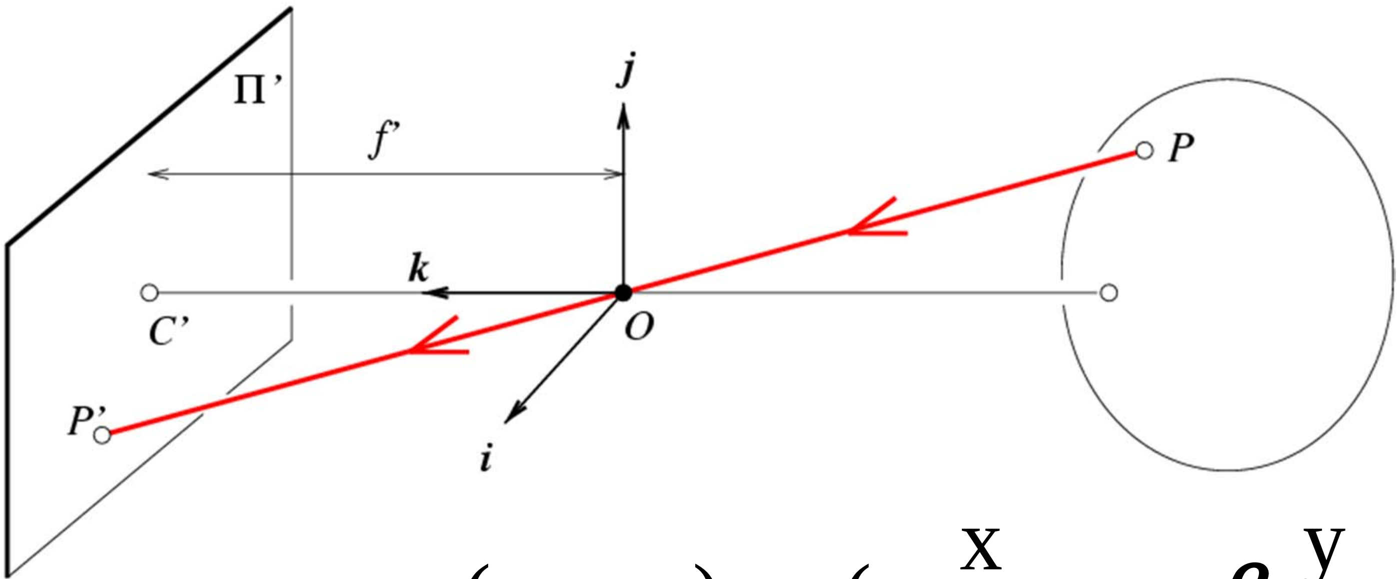


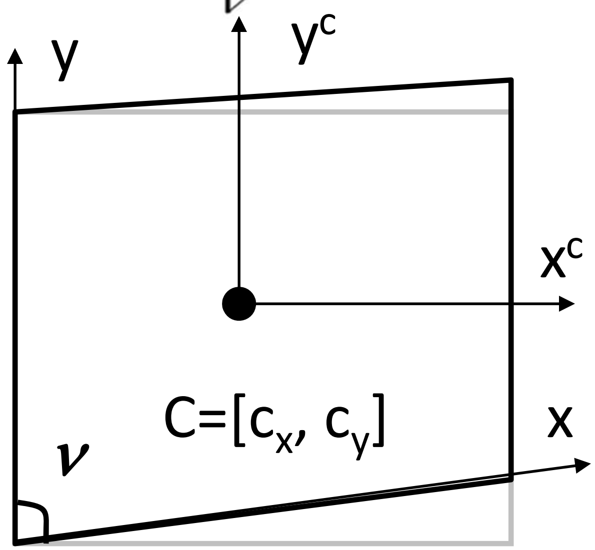


$$(x, y, z) \rightarrow \left(\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y\right)$$

$$P' = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera matrix



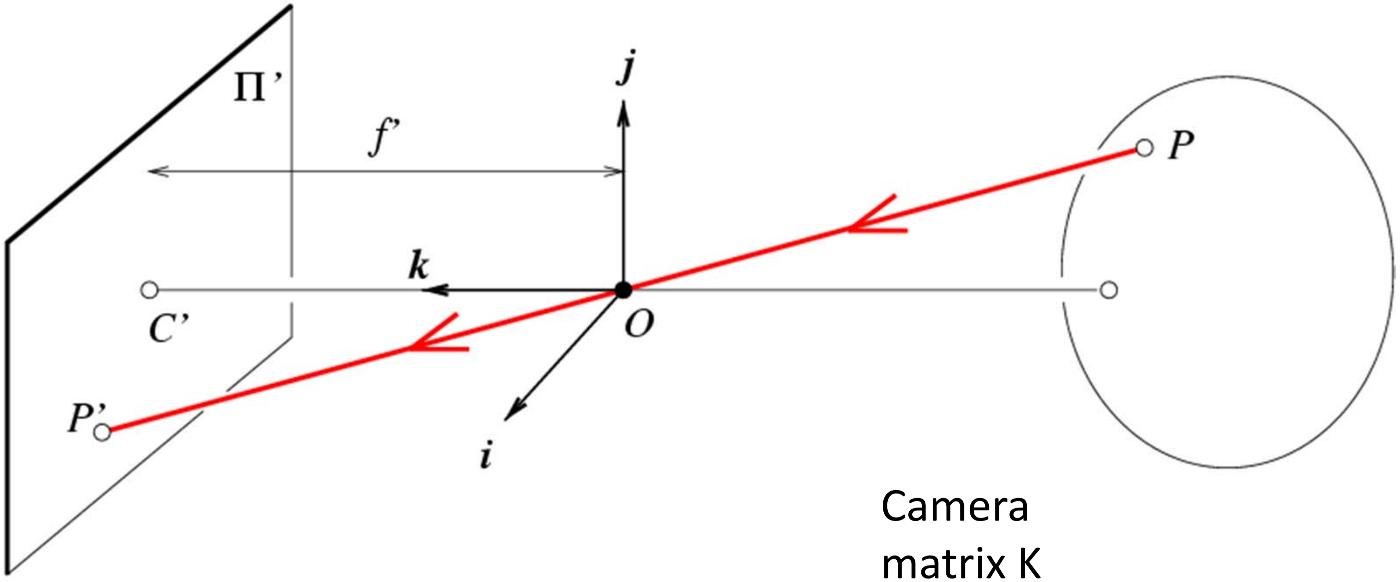


$$(\mathrm{x},\mathrm{y},\mathrm{z}) \rightarrow (\boldsymbol{\alpha}\frac{\mathrm{x}}{\mathrm{z}} + \mathrm{c}_\mathrm{x}, \boldsymbol{\beta}\frac{\mathrm{y}}{\mathrm{z}} + \mathrm{c}_\mathrm{y})$$

Skew parameter

$$P' = \begin{bmatrix} \alpha & \boxed{s} & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera matrix



Camera matrix K

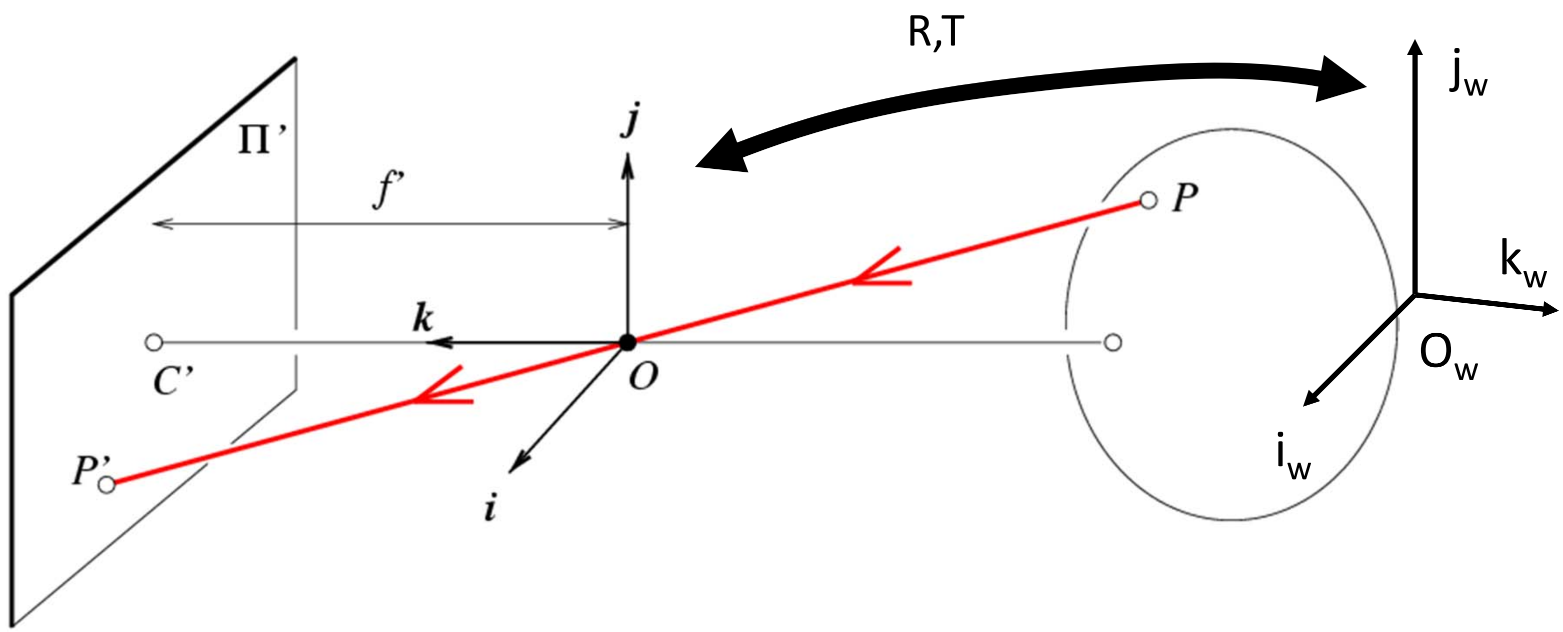
$$P' = M\ P$$

$$= K\begin{bmatrix} I & 0 \end{bmatrix} P$$

$$P' = \begin{bmatrix} \alpha & s & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

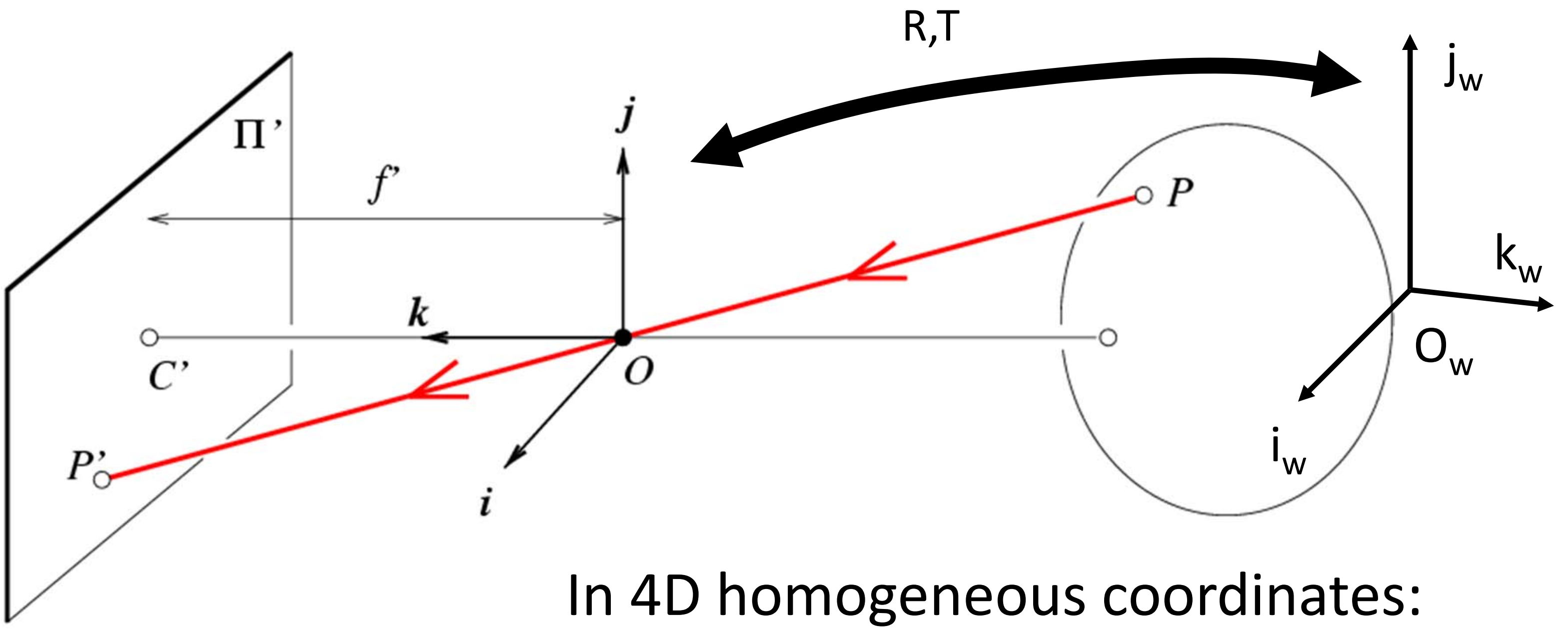
K has 5 degrees of freedom!

Camera & world reference system



- The mapping is defined within the camera reference system
- What if an object is represented in the world reference system?

Camera & world reference system



In 4D homogeneous coordinates:

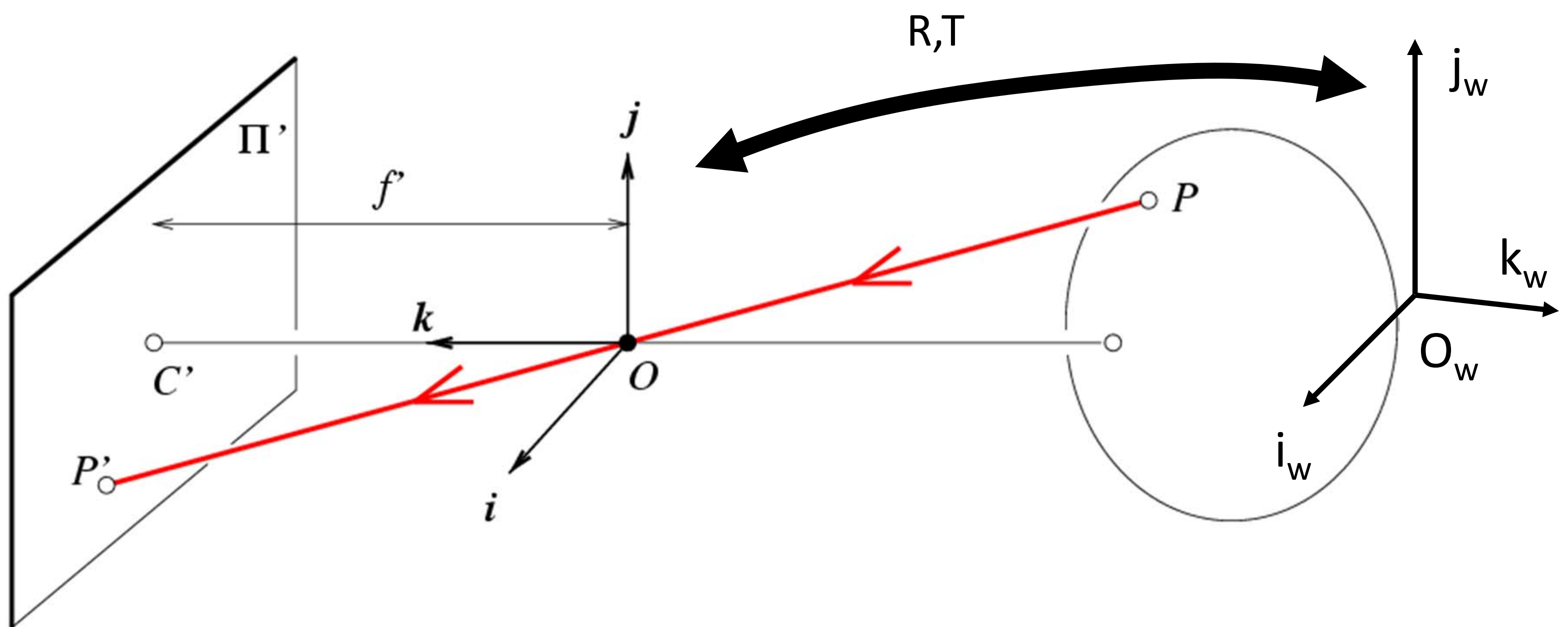
$$P = \begin{bmatrix} R & T \end{bmatrix} P_w$$

$$P' = M\,P_w = \boxed{K}\boxed{\begin{bmatrix} R & T \end{bmatrix}} P_w$$

Internal parameters

External parameters

Projective cameras



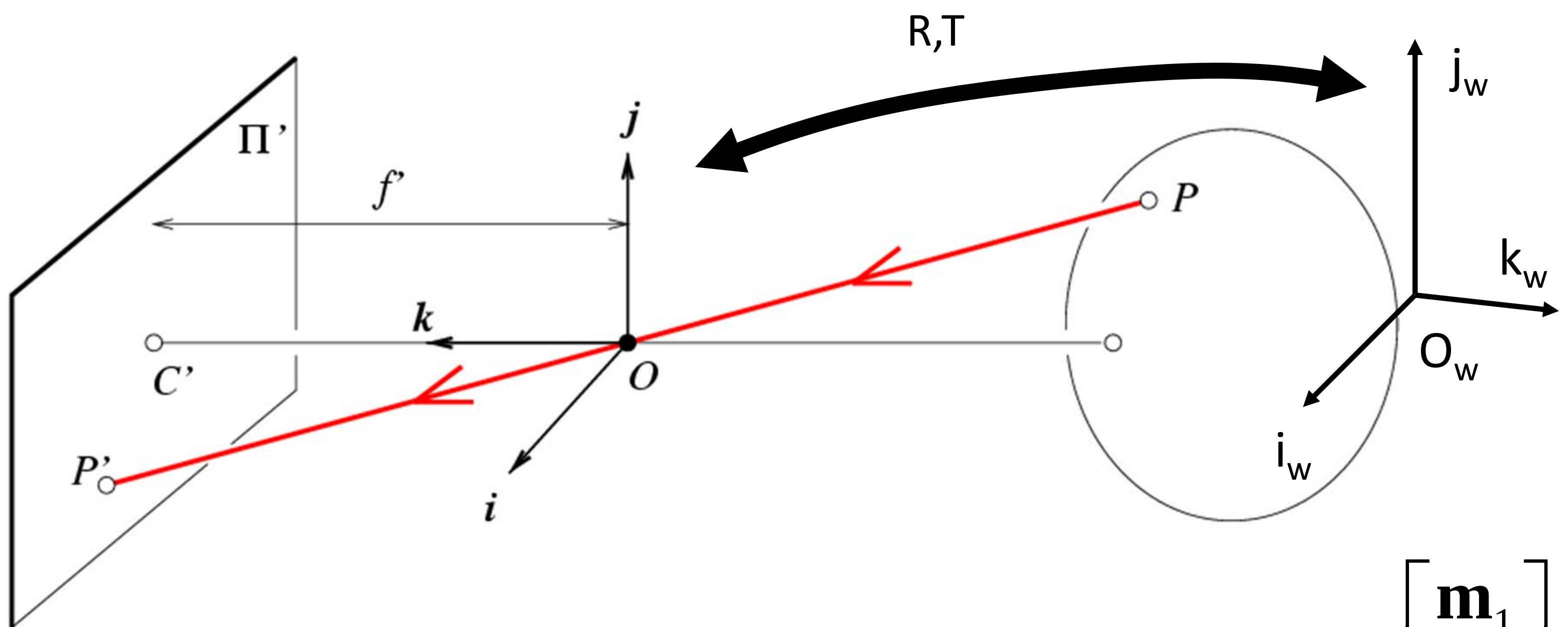
$$P'_{3\times1} = M\ P_w = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4}\ P_{w\,4\times1}$$

$$K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

How many degrees of freedom?

5 + 3 + 3 =11!

Projective cameras



$$P'_{3\times 1} = M\ P_w = K_{3\times 3}\begin{bmatrix} R & T \end{bmatrix}_{3\times 4}\ P_{w4\times 1} \qquad M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$(x, y, z)_w \rightarrow \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w}\right)$$

M is defined up to scale!

Multiplying M by a scalar won't change the image

Theorem (Faugeras, 1993)

$$M = K\begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} K\,R & K\,T \end{bmatrix} = [A \quad b]$$

$$A = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{bmatrix}$$

Let $\mathcal{M} = (\mathcal{A} \quad \boldsymbol{b})$ be a 3×4 matrix and let $\boldsymbol{a}_i^T$ $(i = 1, 2, 3)$ denote the rows of the matrix $\mathcal{A}$ formed by the three leftmost columns of $\mathcal{M}$.

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$.
- A necessary and sufficient condition for $\mathcal{M}$ to be a zero-skew perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$(\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0.$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0, \\ (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_1 \times \boldsymbol{a}_3) = (\boldsymbol{a}_2 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3). \end{cases}$$

$$K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\alpha} = \mathrm{f}\ \mathrm{k};$$
$$\boldsymbol{\beta} = \mathrm{f}\ \mathrm{l}$$

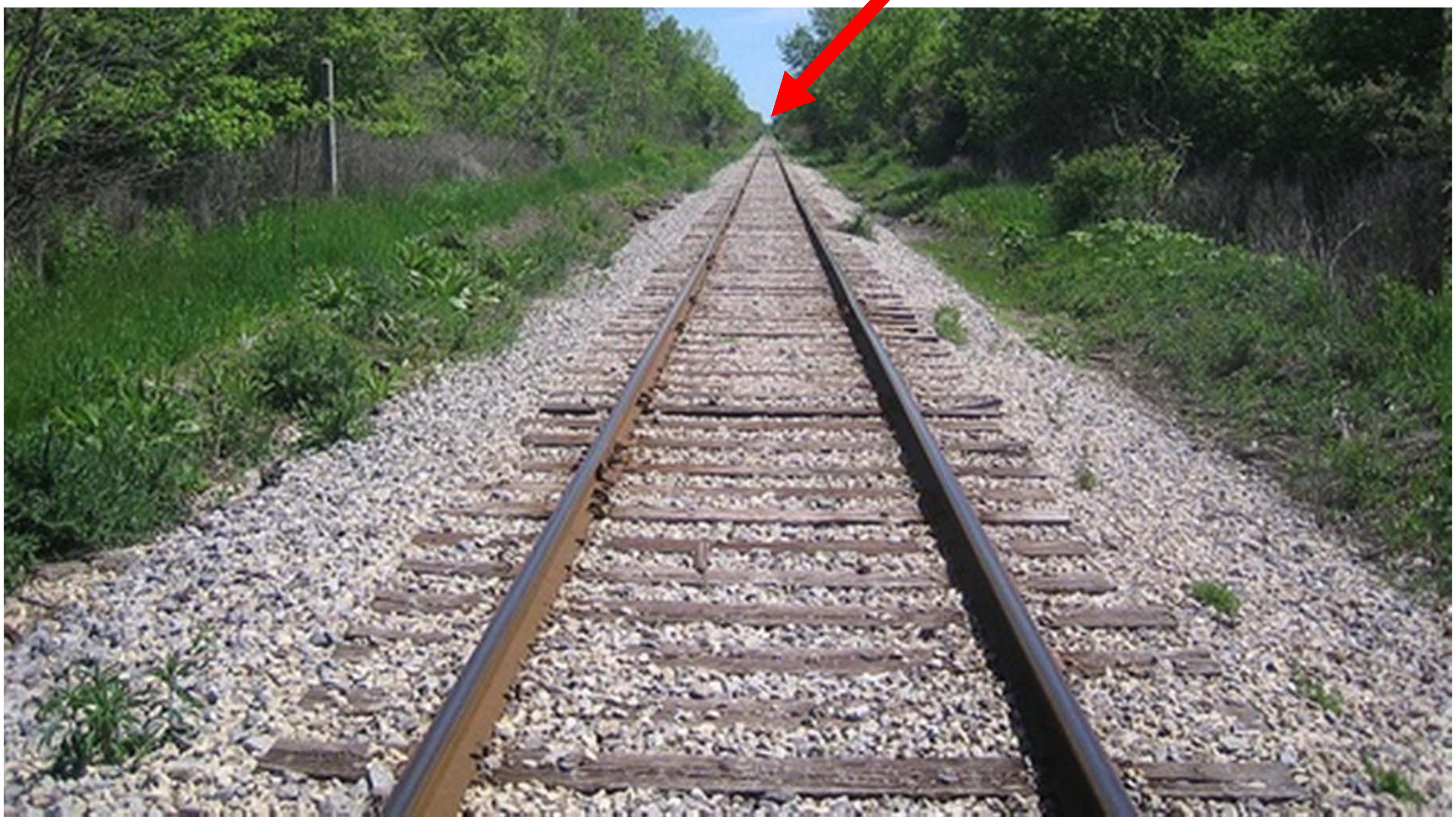
Properties of Projection

- Points project to points
- Lines project to lines

Properties of Projection

- Angles are not preserved
- Parallel lines meet

Vanishing point



What we have learned today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras

Reading:

[FP] Chapters 1 – 3

[HZ] Chapter 6