Lecture 7: Camera Models

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Stanford Vision Lab

What we will learn today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras

Reading:

[FP] Chapters 1 – 3

[HZ] Chapter 6

What we will learn today?

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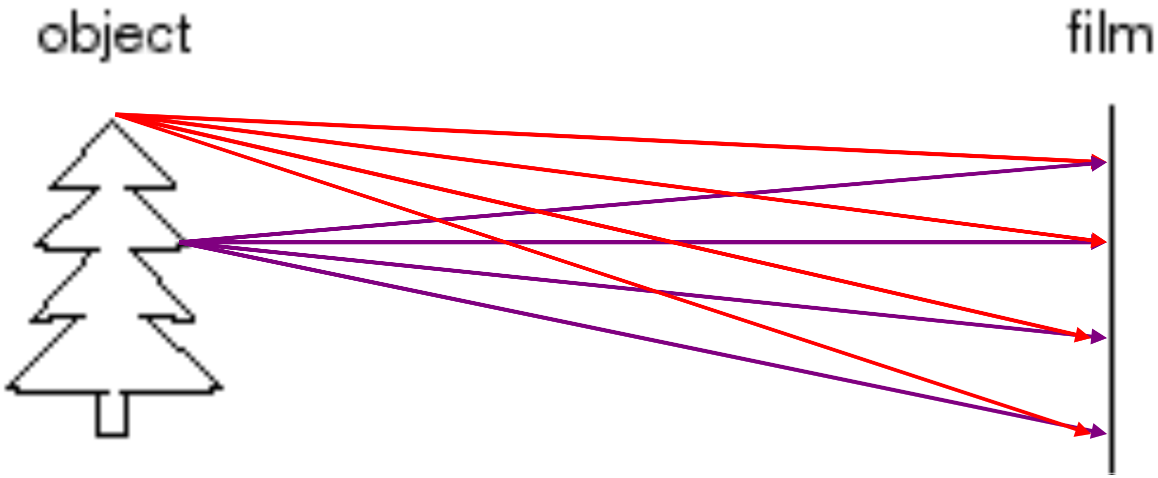
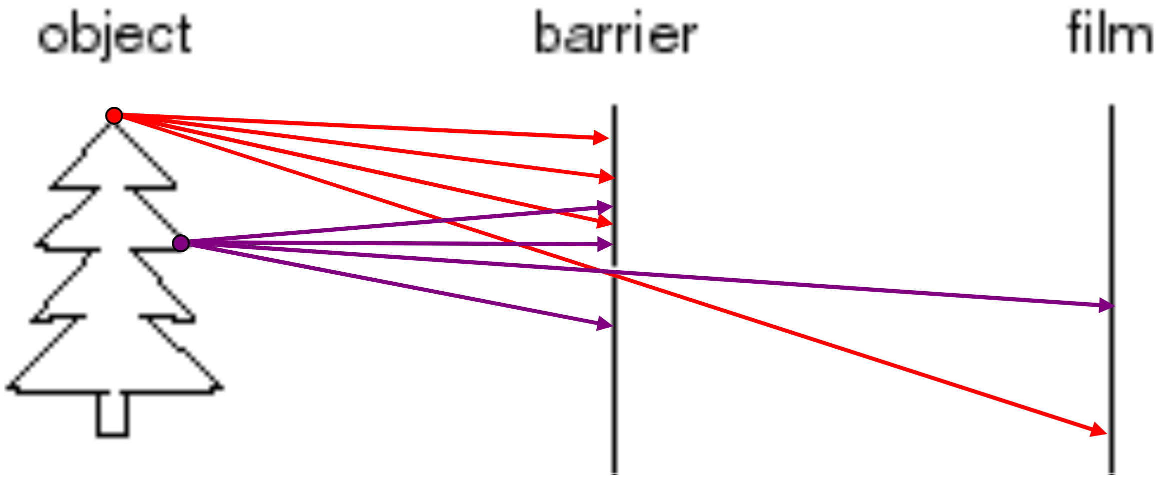
[HZ] Chapter 6

How do we see the world?



- Let’s design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



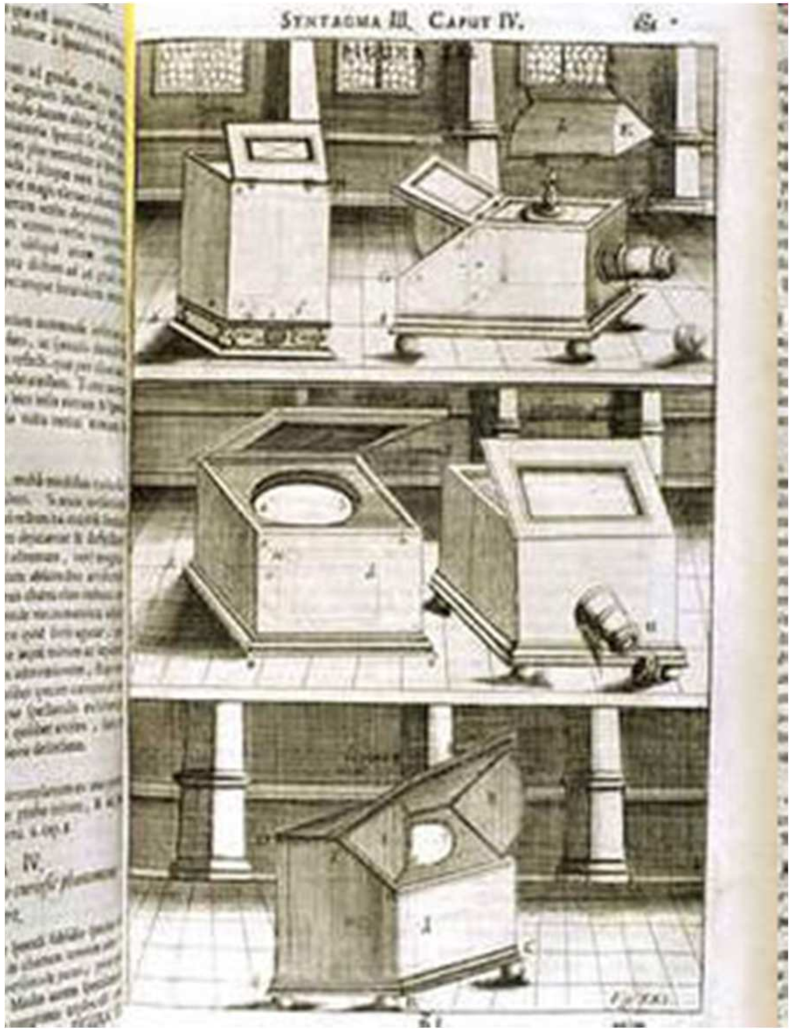
- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**

Some history…

Milestones:

• Leonardo da Vinci (1452-1519): first record of camera *obscura*

• Johann Zahn (1685): first portable camera



Some history…

Milestones:

- Leonardo da Vinci (1452-1519): first record of camera *obscura*
- Johann Zahn (1685): first portable camera
- Joseph Nicephore Niepce (1822): first photo - birth of photography

- Daguerréotypes (1839)
- Photographic Film (Eastman, 1889)
- Cinema (Lumière Brothers, 1895)
- Color Photography (Lumière Brothers, 1908)

Photography (Niepce, “La Table Servie,” 1822)

Some history…

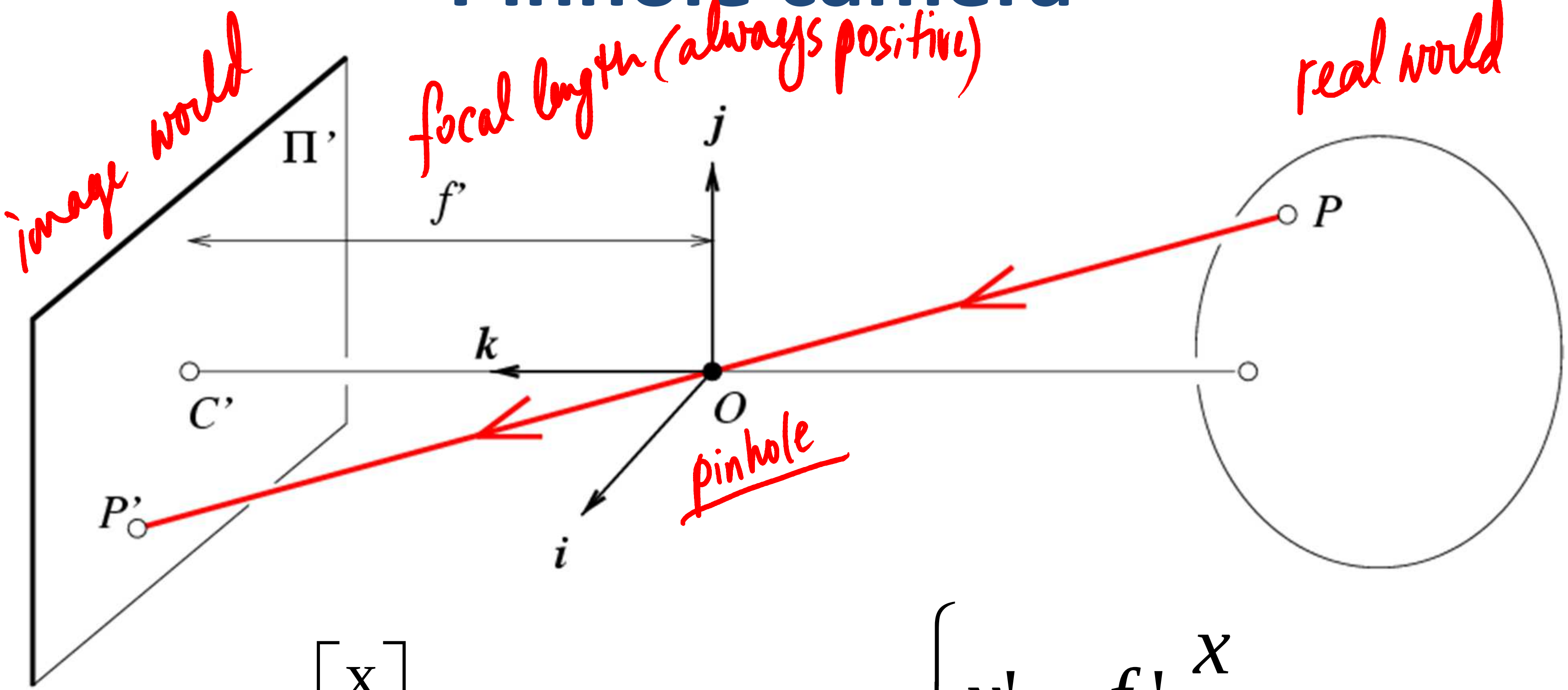
Motzu
(468-376 BC)

Oldest existent book on geometry in China

Aristotle
(384-322 BC)
Also: Plato, Euclid

Al-Kindi (c. 801–873)
Ibn al-Haitham
(965-1040)

Pinhole camera



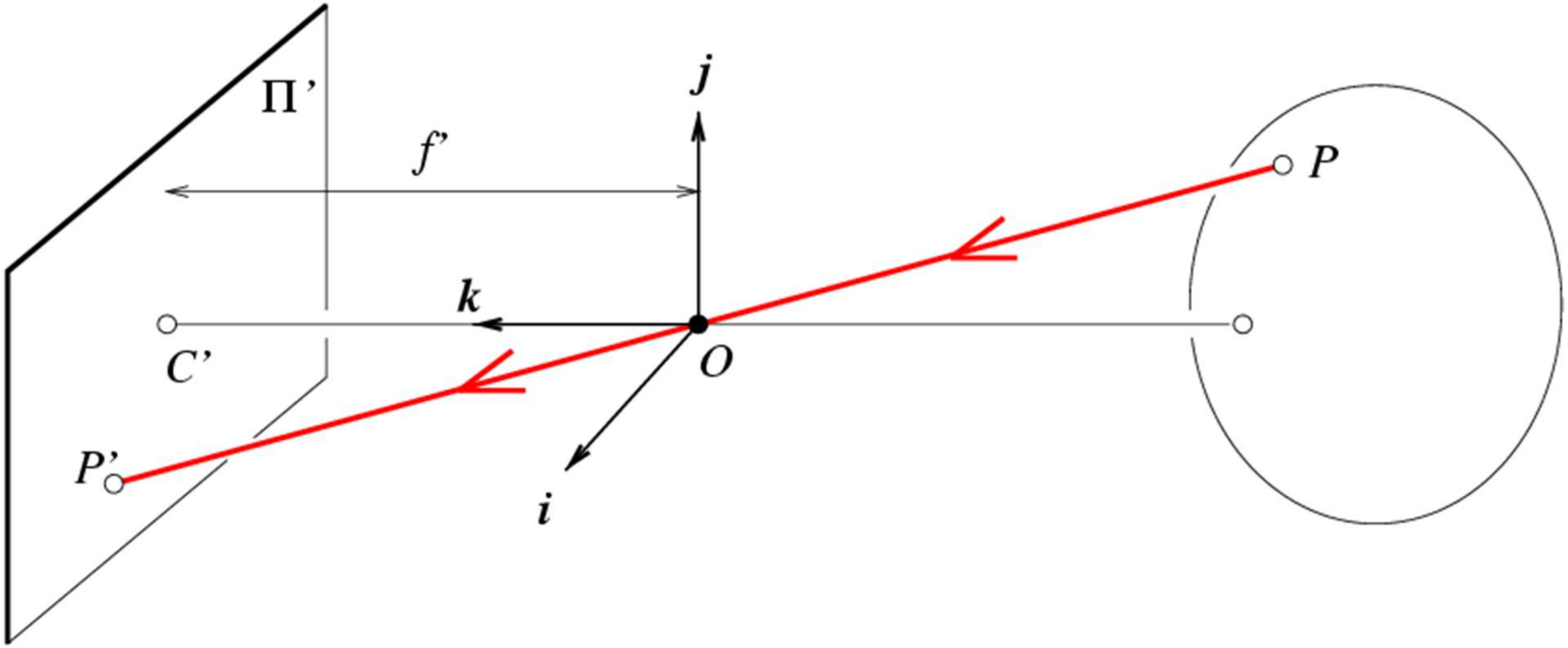
$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

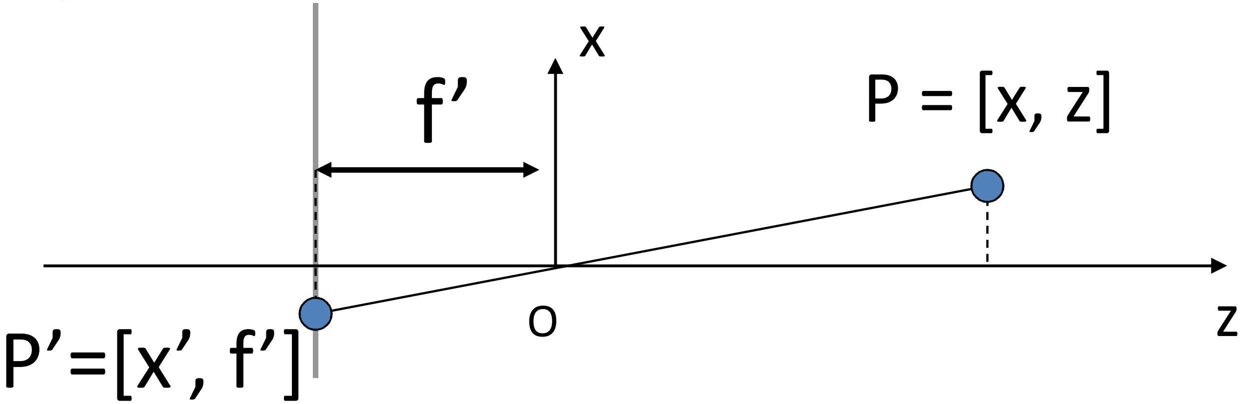
$$\begin{cases} x' = f' \dfrac{x}{z} \\ y' = f' \dfrac{y}{z} \end{cases}$$

Note: z is always negative.

Derived using similar triangles

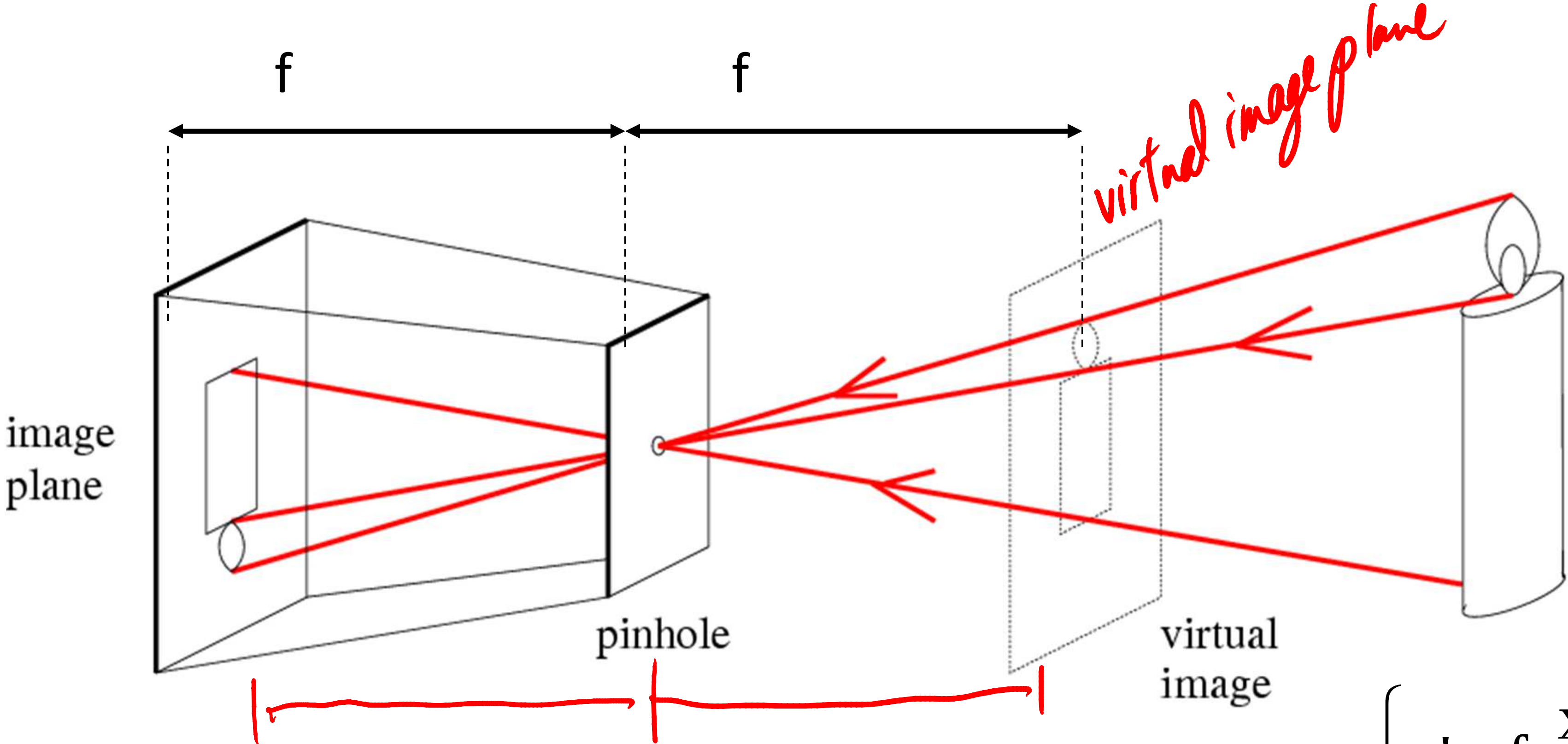
Pinhole camera





$$\frac{x'}{f'} = \frac{x}{z}$$

Pinhole camera

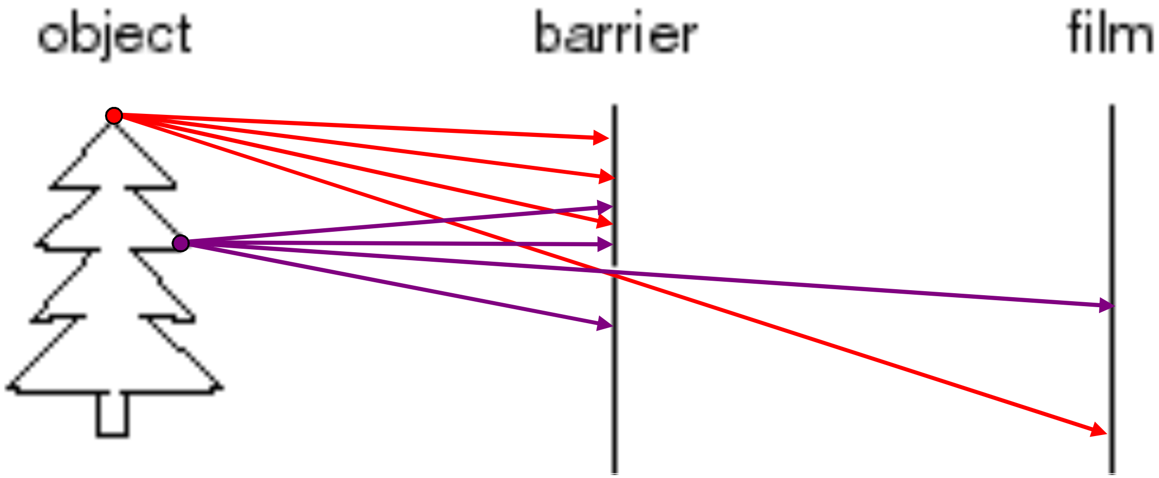


$$\begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$

- Common to draw image plane *in front* of the focal point
- Moving the image plane merely scales the image.

Pinhole camera

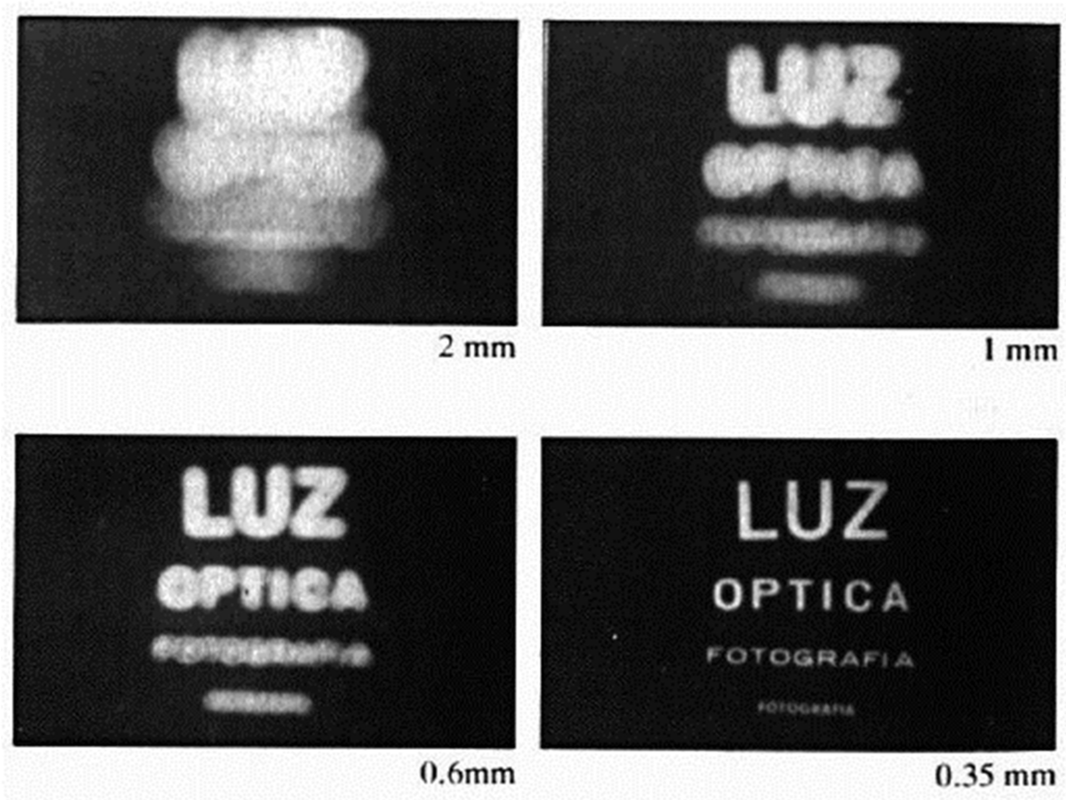
Is the size of the aperture important?



Cameras & Lenses

Shrinking aperture size

- Rays are mixed up



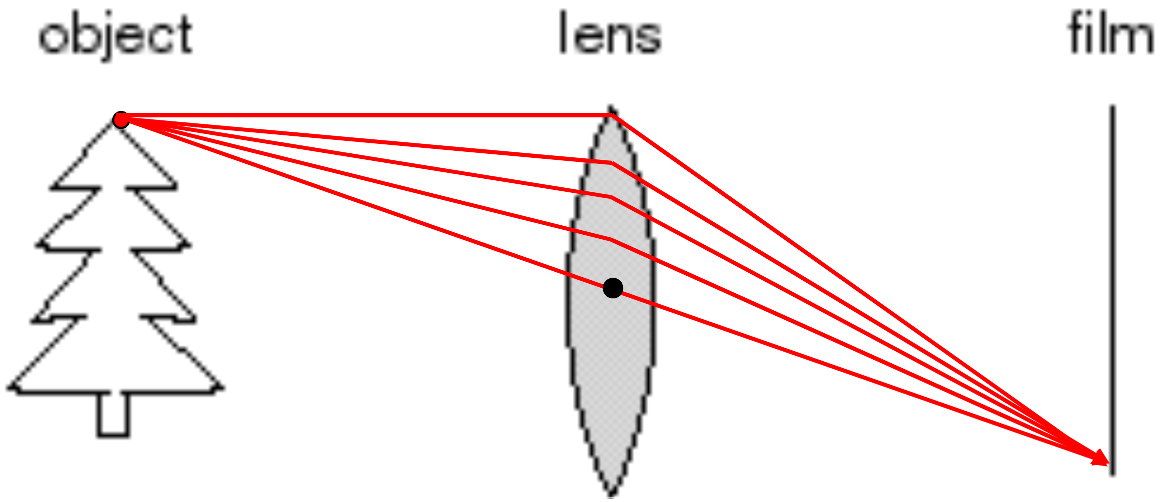
-Why the aperture cannot be too small?

-Less light passes through

-Diffraction effect

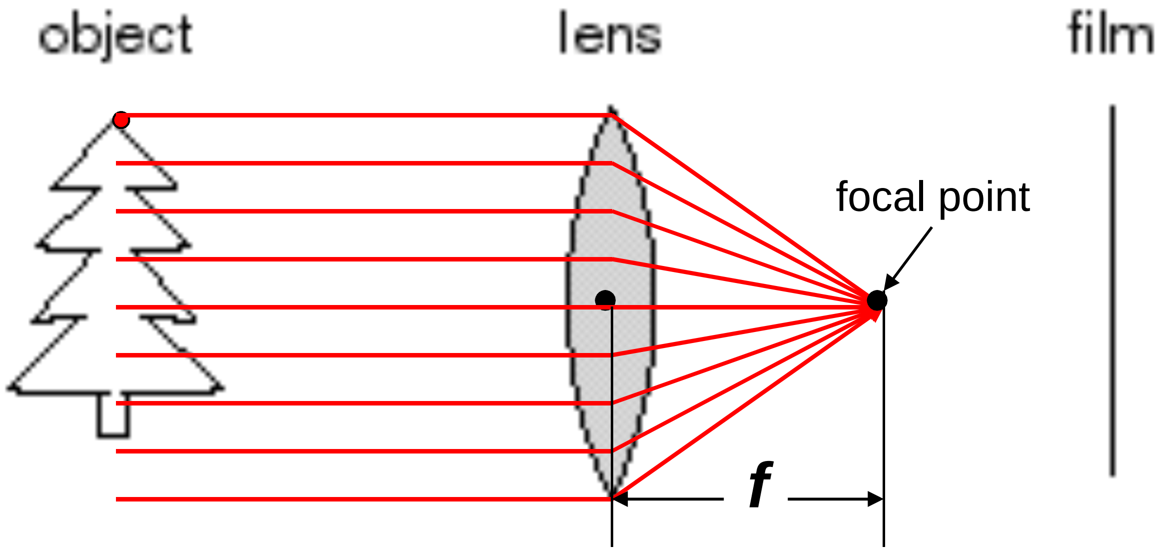
Adding lenses!

Cameras & Lenses



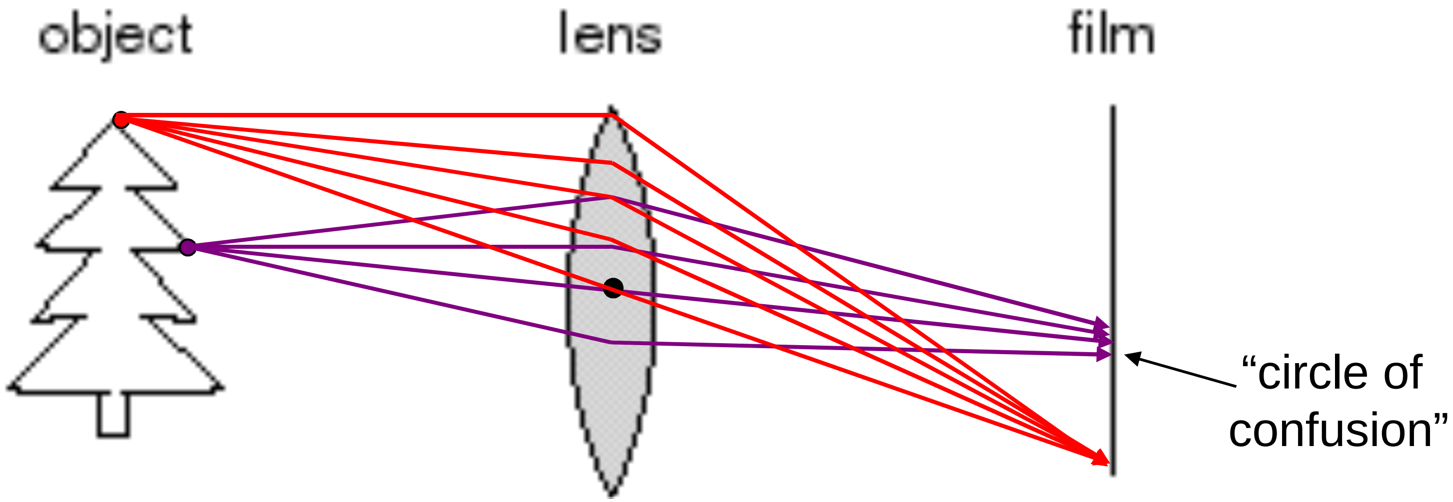
- A lens focuses light onto the film

Cameras & Lenses



- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

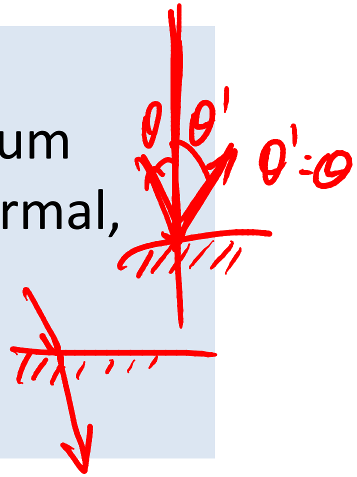
Cameras & Lenses



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    [other points project to a “circle of confusion” in the image]

Cameras & Lenses

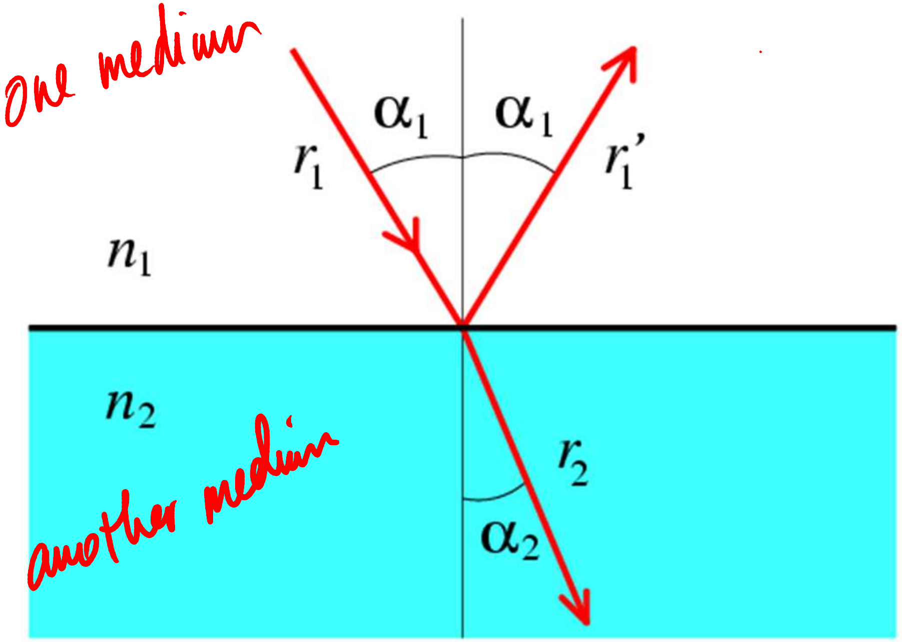
- Laws of geometric optics
  - Light travels in straight lines in homogeneous medium
  - Reflection upon a surface: incoming ray, surface normal, and reflection are co-planar
  - Refraction: when a ray passes from one medium to another



Snell's law

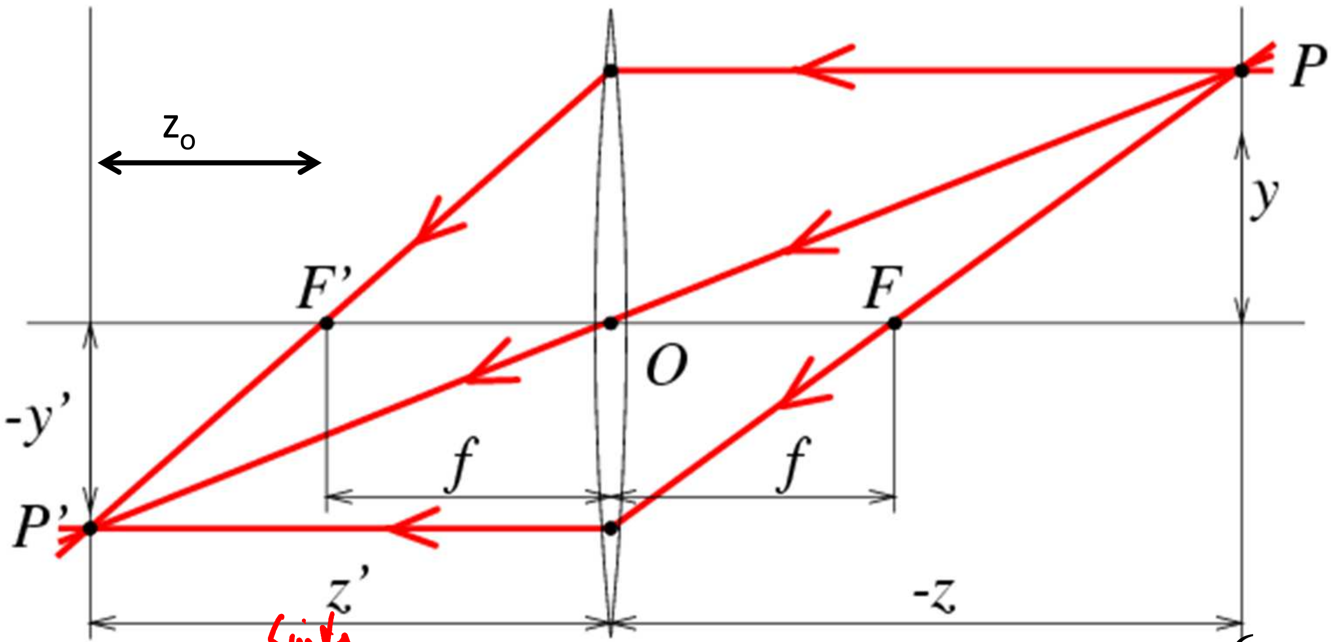
$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

α_1 = incident angle
α_2 = refraction angle
n_i = index of refraction



Assumption

Thin Lenses



$$z' = f + z_o$$

$$f = \frac{R}{2(n-1)}$$

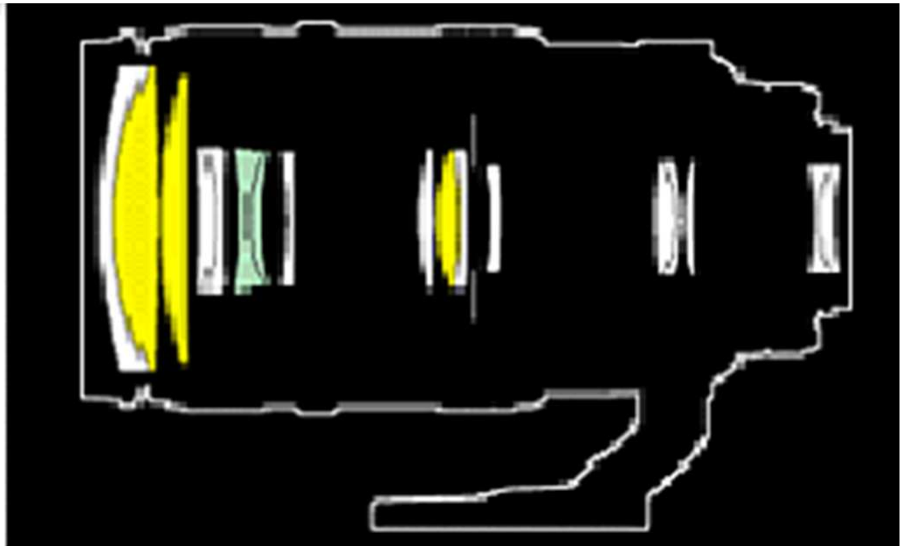
Snell's law:

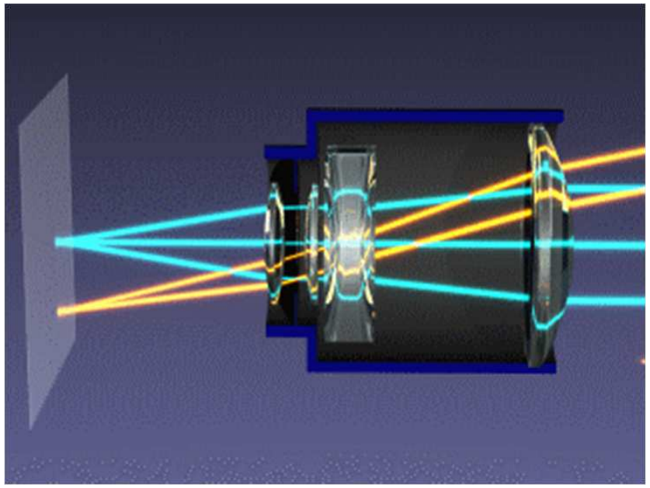
$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$

$$\begin{cases} \text{Small angles:} \\ n_1 \alpha_1 \approx n_2 \alpha_2 \\ n_1 = n \text{ (lens)} \\ n_1 = 1 \text{ (air)} \end{cases}$$

$$\begin{cases} x' = z' \dfrac{x}{z} \\ y' = z' \dfrac{y}{z} \end{cases}$$

Cameras & Lenses



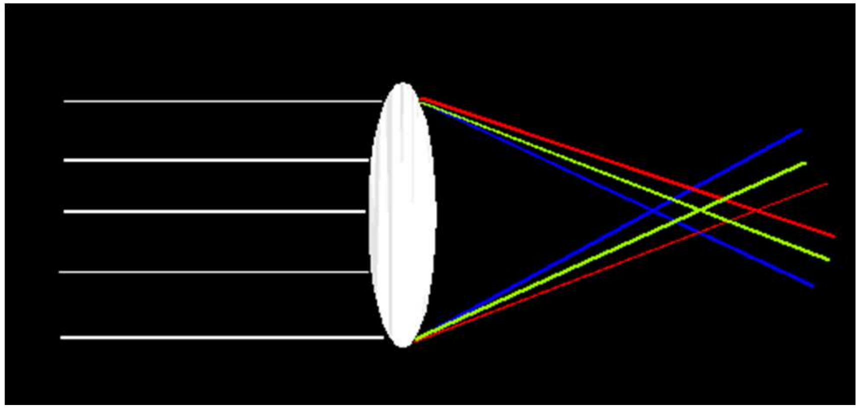


Source wikipedia

Issues with lenses: Chromatic Aberration

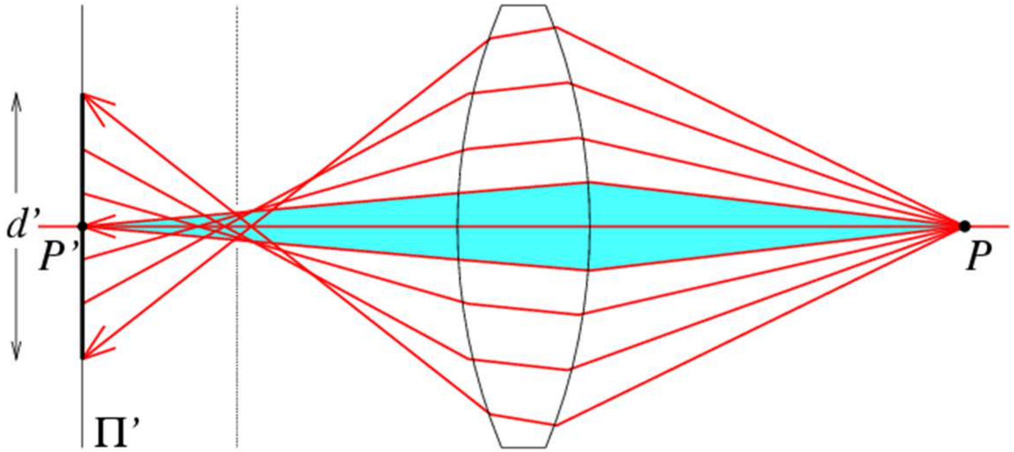
- Lens has different refractive indices for different wavelengths: causes color fringing

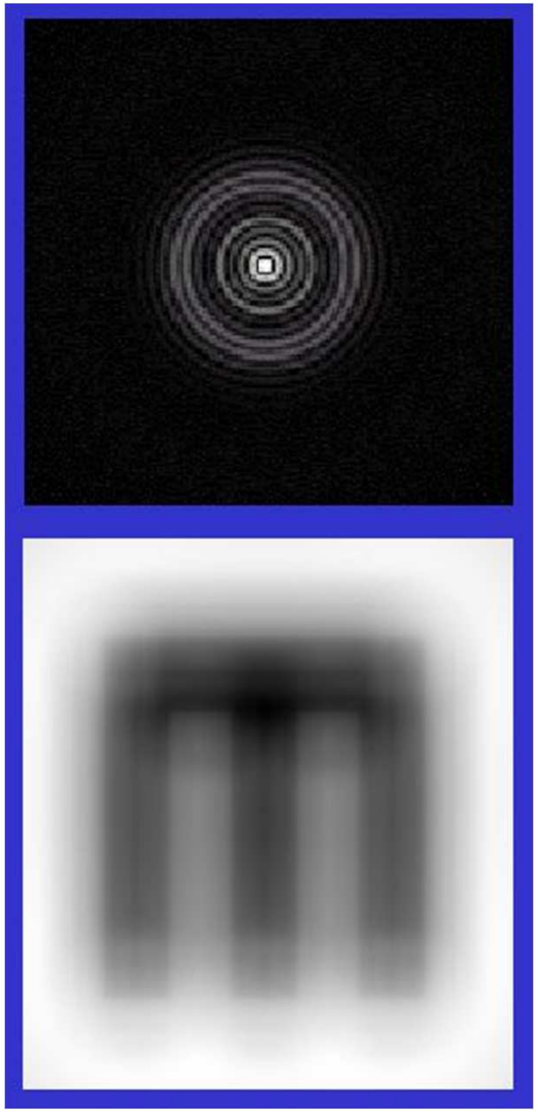
$$f = \frac{R}{2(n-1)}$$



Issues with lenses: Chromatic Aberration

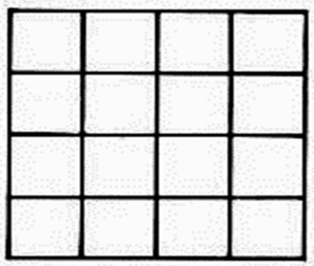
- Rays farther from the optical axis focus closer



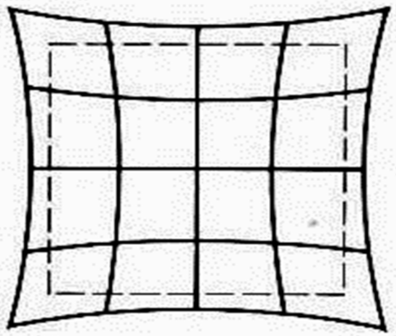


Issues with lenses: Chromatic Aberration

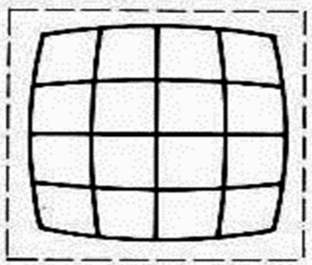
– Deviations are most noticeable for rays that pass through the edge of the lens



No distortion



Pin cushion



Barrel (fisheye lens)

Image magnification decreases with distance from the optical axis

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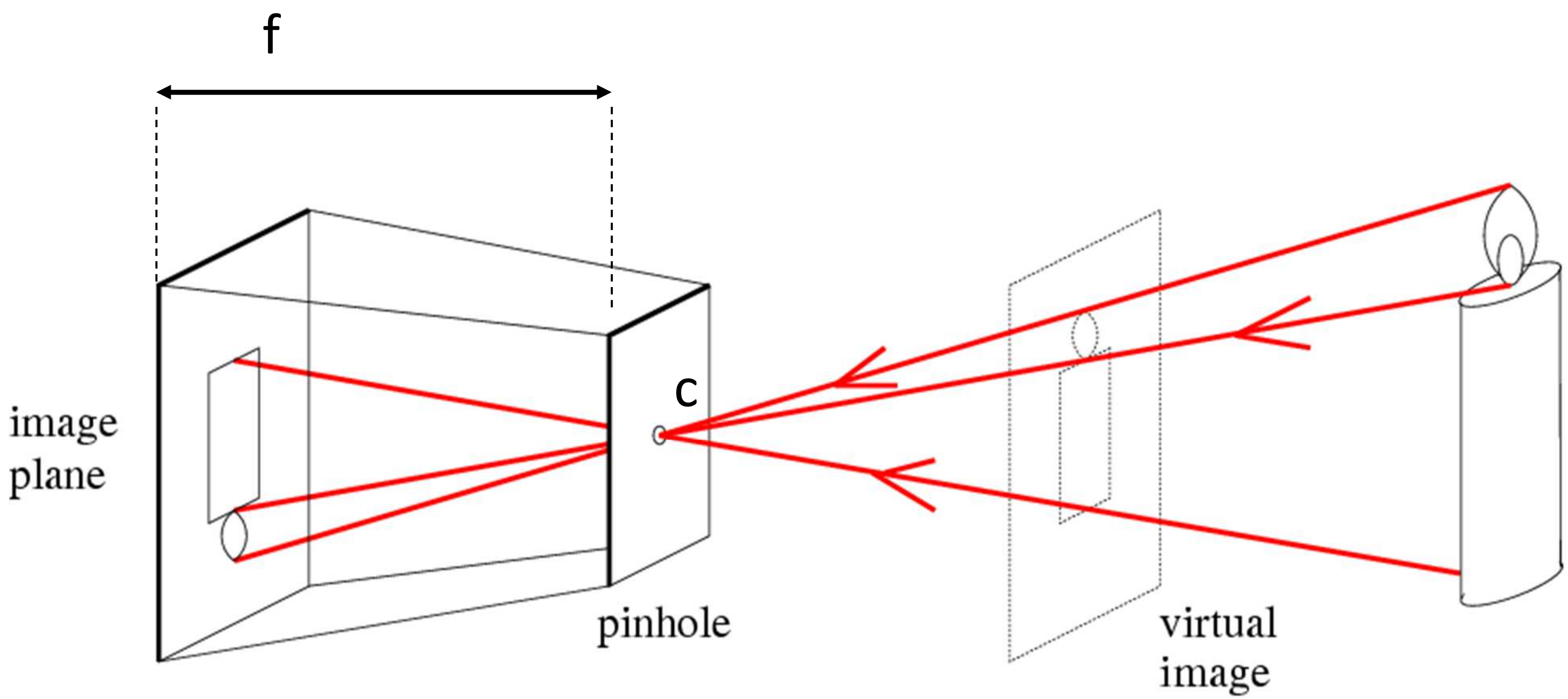
Pinhole camera



$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

f = focal length

c = center of the camera

$$\Re^3 \xrightarrow{E} \Re^2$$

Pinhole camera

Is this a linear transformation?

$$\overset{P}{(x, y, z)} \rightarrow \overset{P'}{(f \frac{x}{z}, f \frac{y}{z})}$$

No — division by z is nonlinear!

How to make it linear?

Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

- Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

Perspective Projection Transformation:

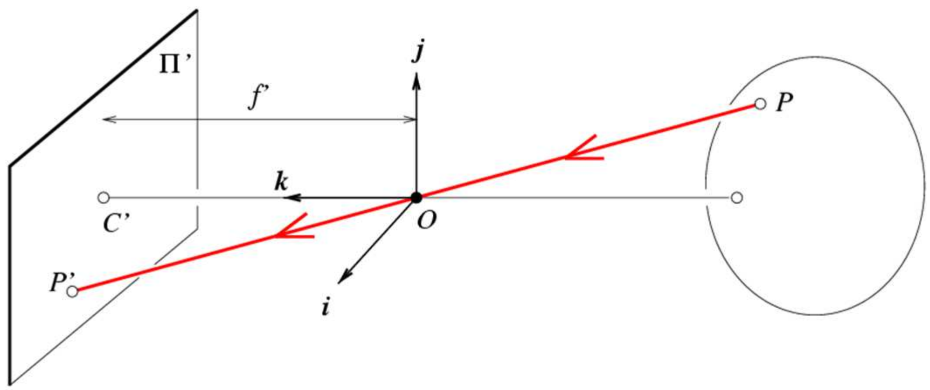
$$P' = \overset{P'}{\begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix}} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{M}} \overset{P}{\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}$$

“Projection matrix”

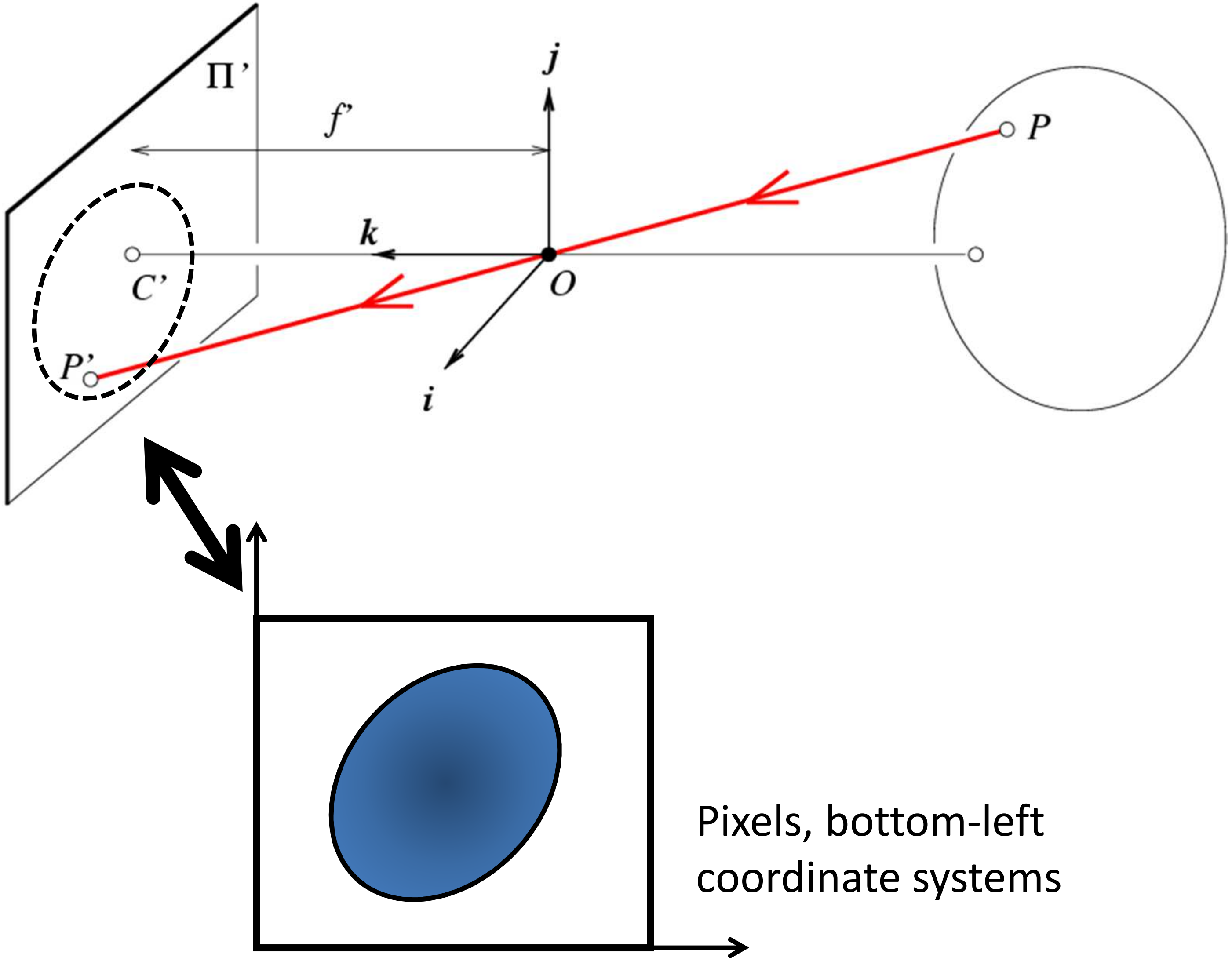
$$P' = M\,P$$

$$\Re^4 \xrightarrow{\text{H}} \Re^3$$

$$P_i' = \begin{bmatrix} f\,\dfrac{x}{z} \\ f\,\dfrac{y}{z} \end{bmatrix}$$

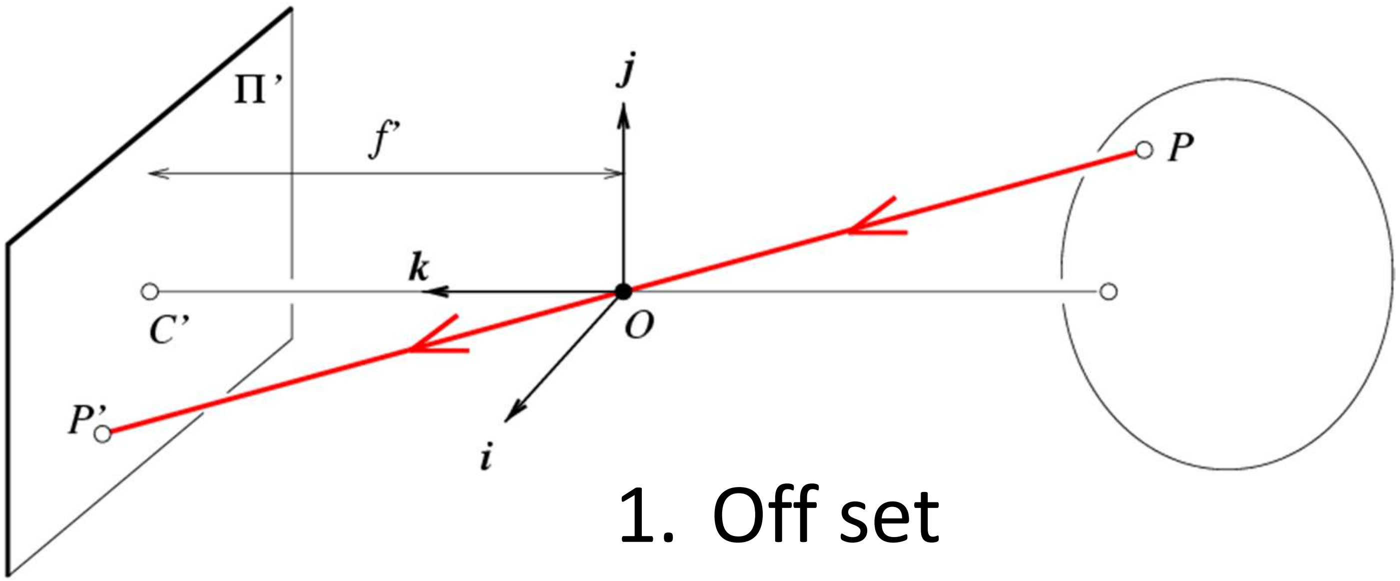


From retina plane to images

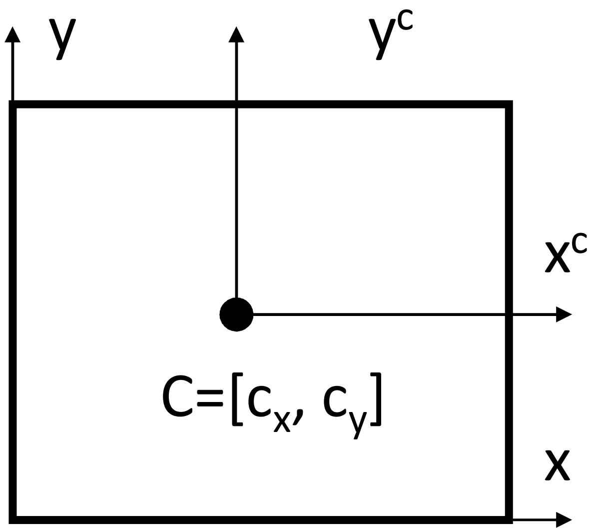


Pixels, bottom-left coordinate systems

From retina plane to images

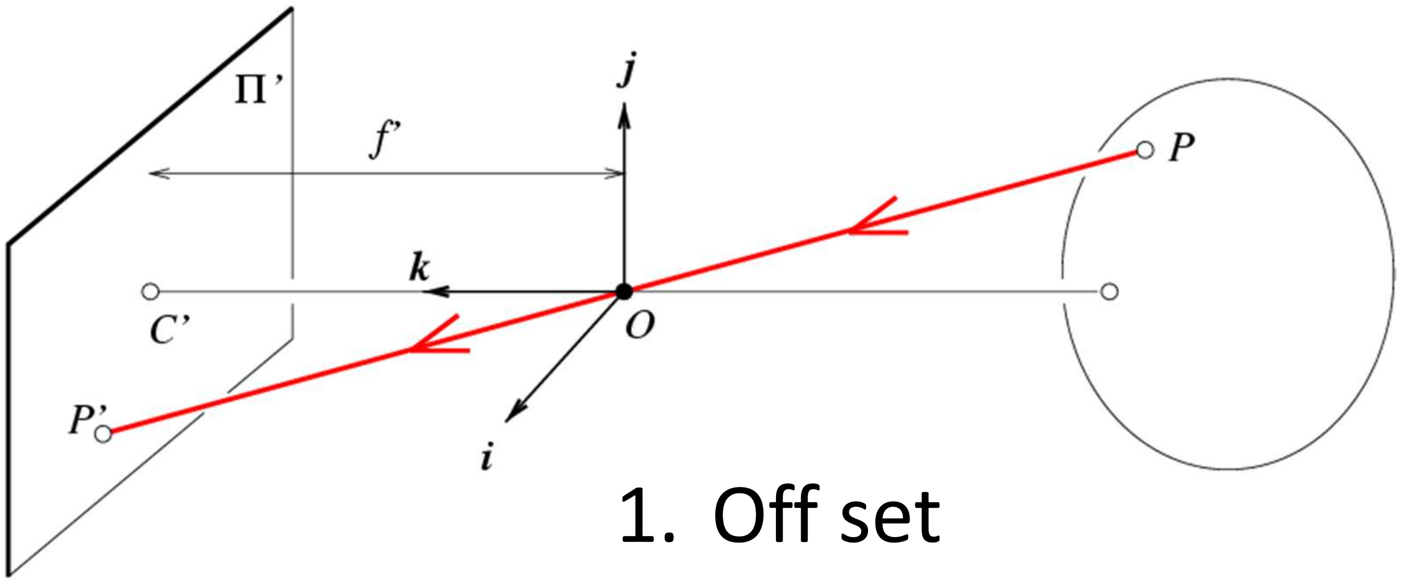


1. Off set

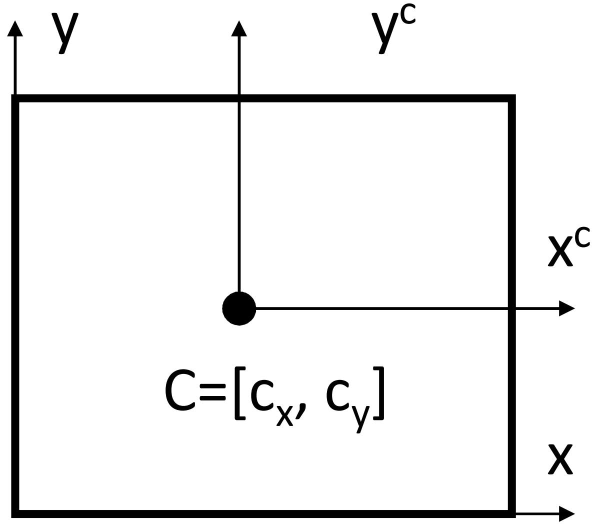


$$(x, y, z) \rightarrow (f\frac{x}{z} + c_x, f\frac{y}{z} + c_y)$$

From retina plane to images



1. Off set
2. From metric to pixels



$$(x, y, z) \rightarrow (\underbrace{\boxed{f\ k}}_{\alpha}\frac{x}{z}+c_x, \underbrace{\boxed{f\ l}}_{\beta}\frac{y}{z}+c_y)$$

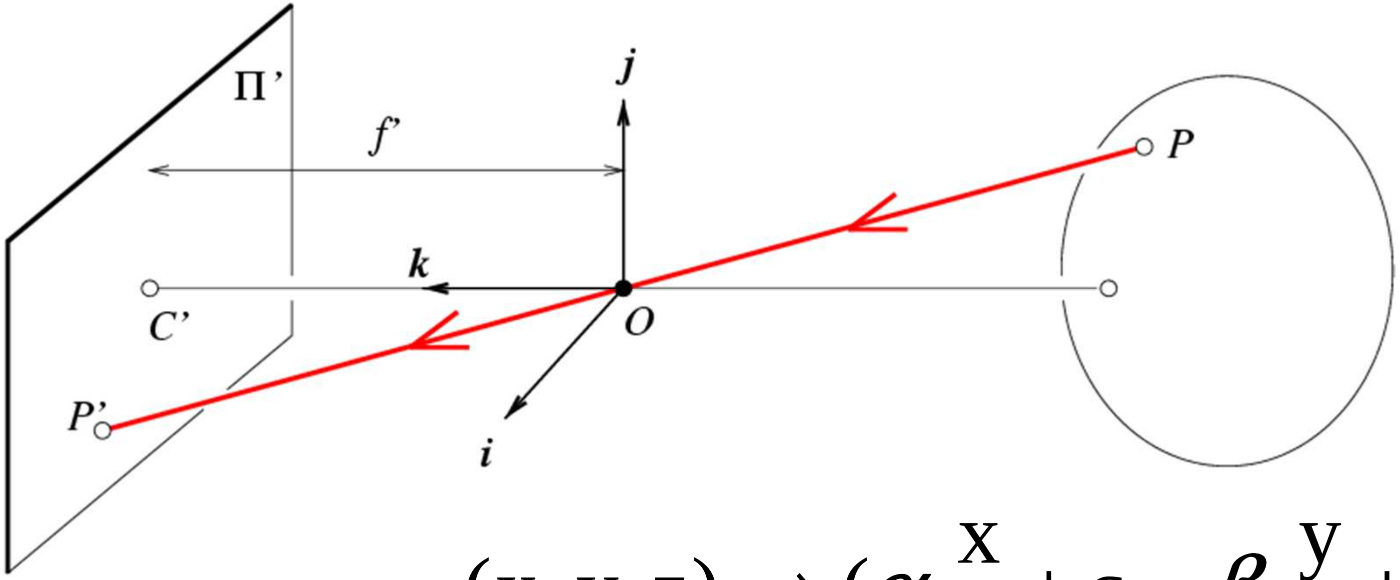
Units: k,l : pixel/m

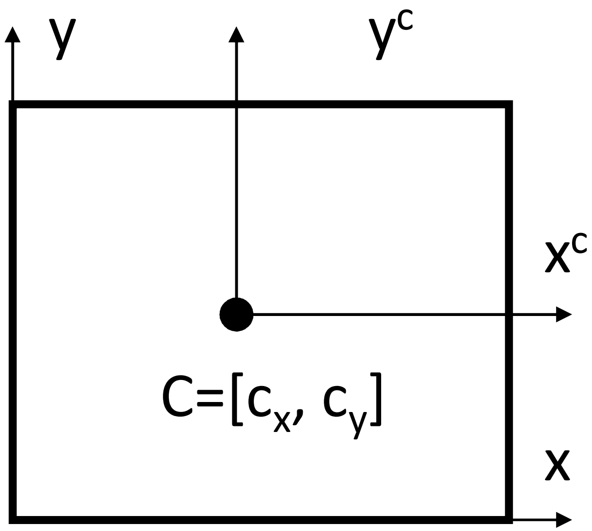
f : m

Non-square pixels

$\alpha, \ \beta$: pixel

From retina plane to images

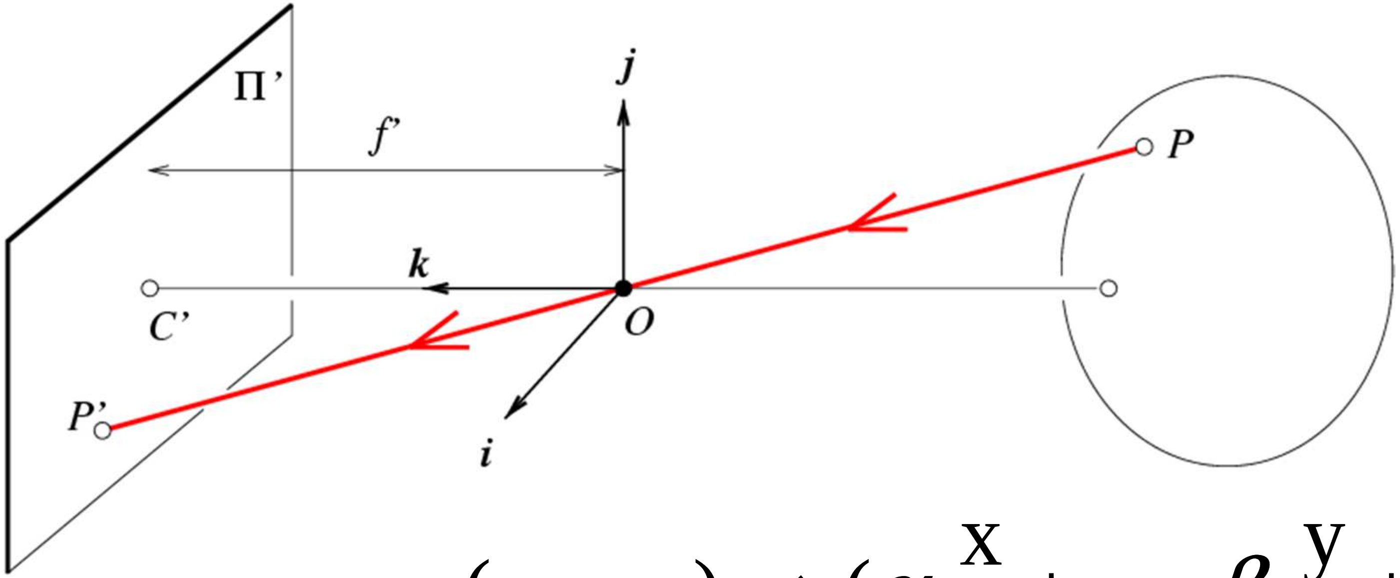


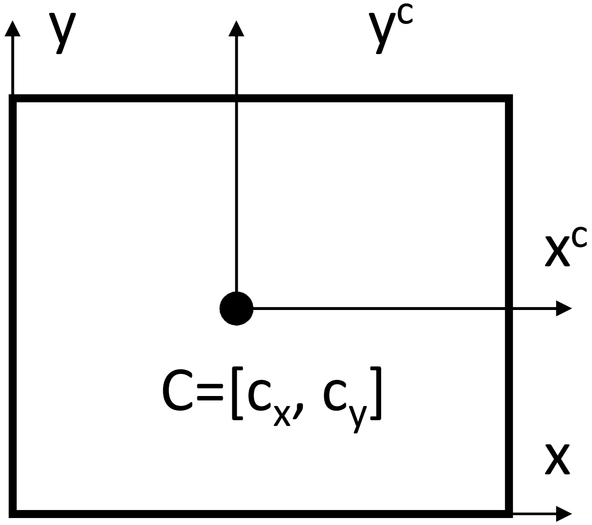


$$(\mathrm{x},\mathrm{y},\mathrm{z}) \to (\alpha\frac{\mathrm{x}}{\mathrm{z}}+\mathrm{c}_\mathrm{x},\ \beta\frac{\mathrm{y}}{\mathrm{z}}+\mathrm{c}_\mathrm{y})$$

Matrix form?

Camera matrix

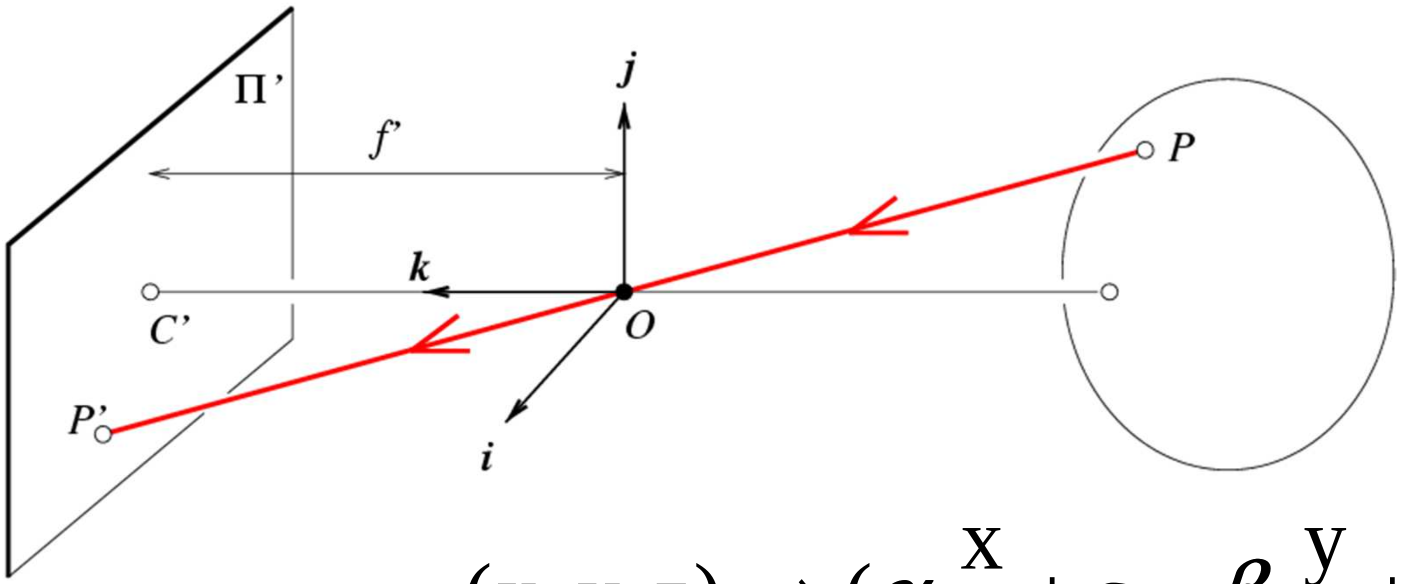


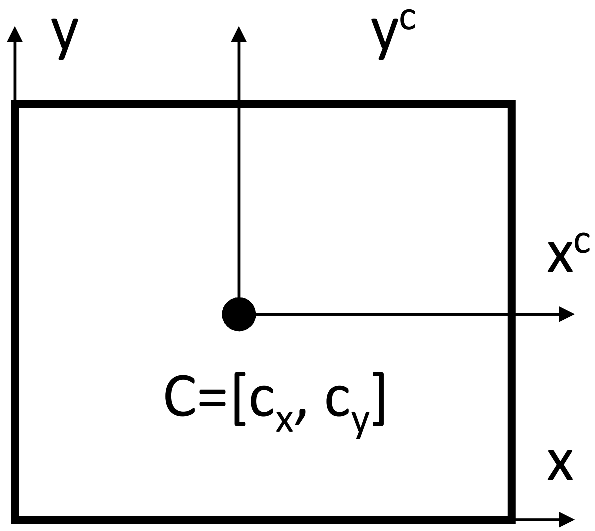


$$(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)$$

$$P' = \begin{bmatrix} \alpha x + c_x z \\ \beta y + c_y z \\ z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}^{P}$$

Camera matrix

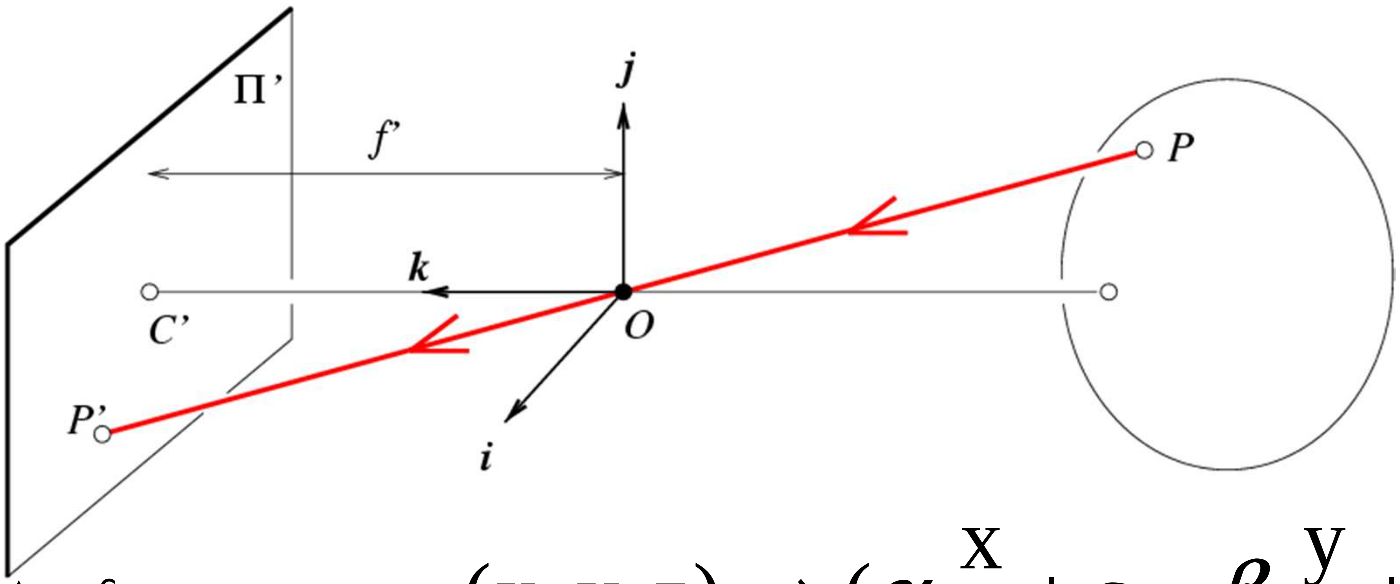


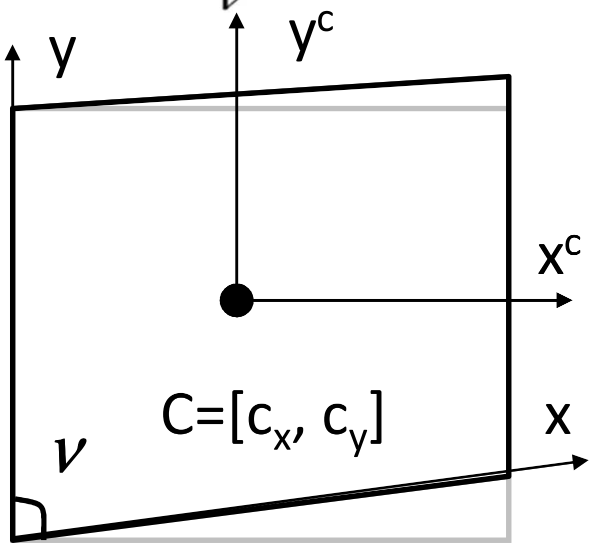


$$(\mathrm{x}, \mathrm{y}, \mathrm{z}) \rightarrow (\alpha \frac{\mathrm{x}}{\mathrm{z}} + \mathrm{c}_\mathrm{x}, \beta \frac{\mathrm{y}}{\mathrm{z}} + \mathrm{c}_\mathrm{y})$$

$$P' = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera matrix



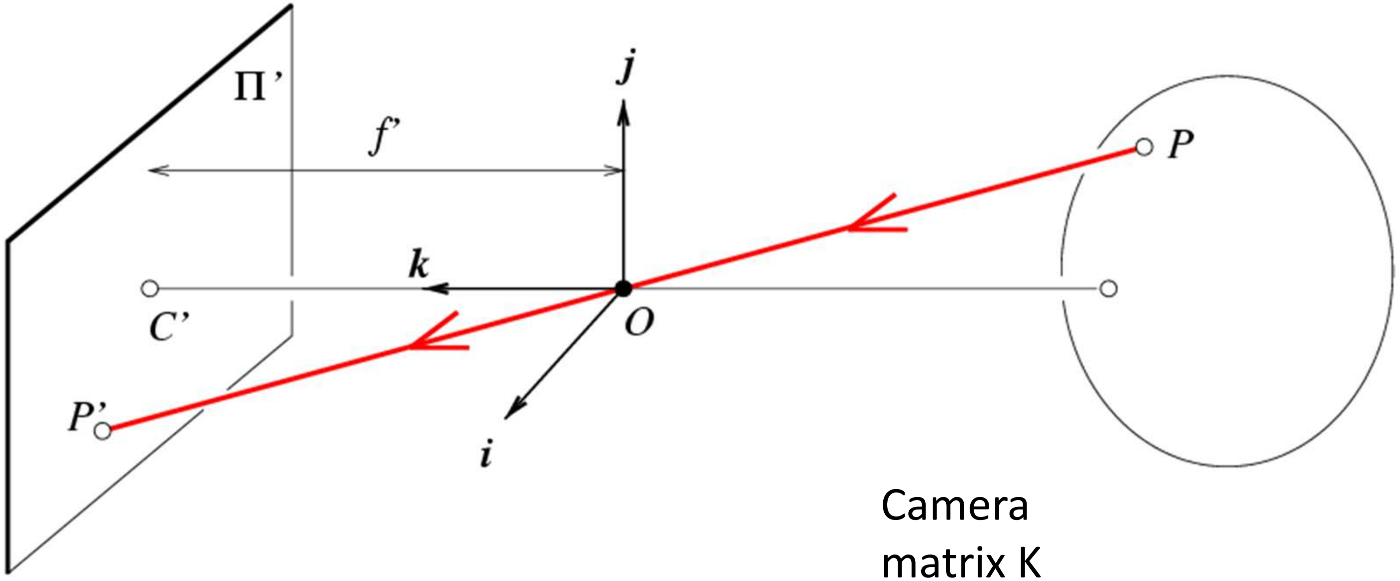


$$(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)$$

Skew parameter

$$P' = \begin{bmatrix} \alpha & \boxed{s} & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera matrix



Camera matrix K

$$P' = M\ P$$

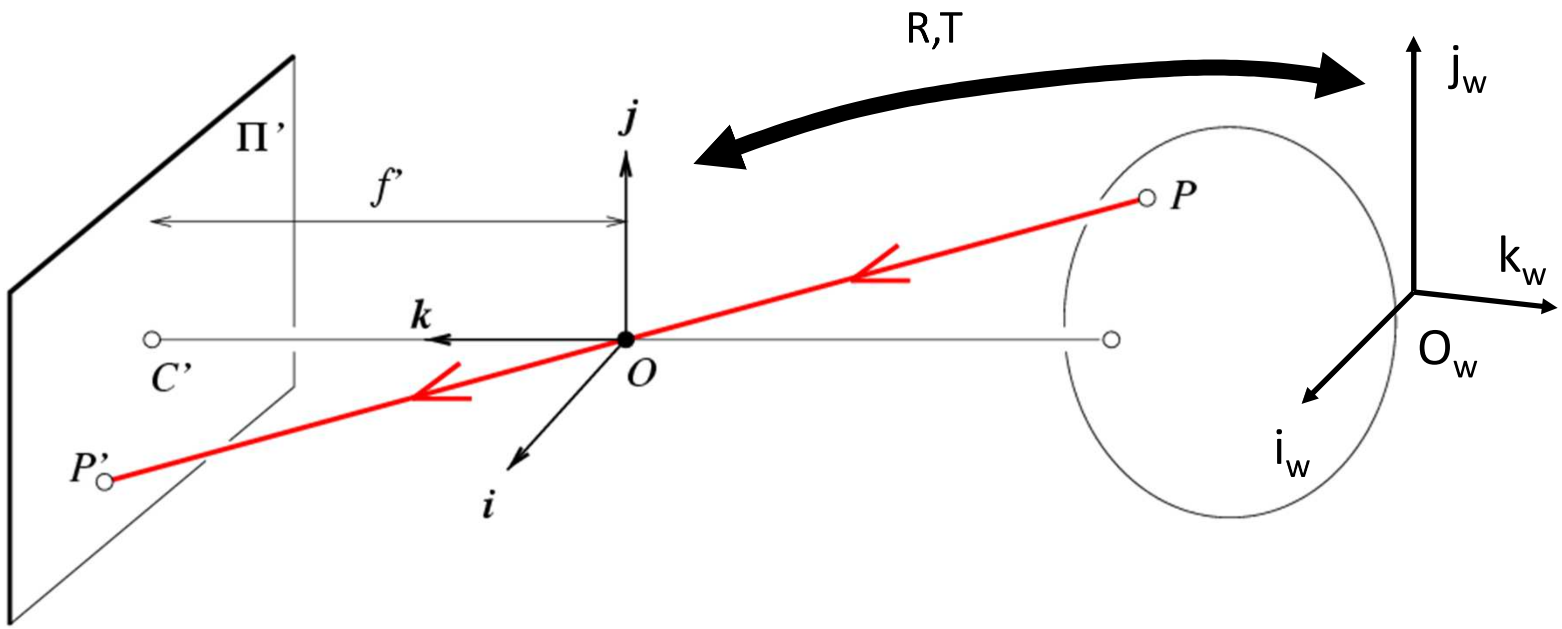
$$= K\begin{bmatrix} I & 0 \end{bmatrix} P$$

intrinsic parameters

$$P' = \begin{bmatrix} \alpha & s & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

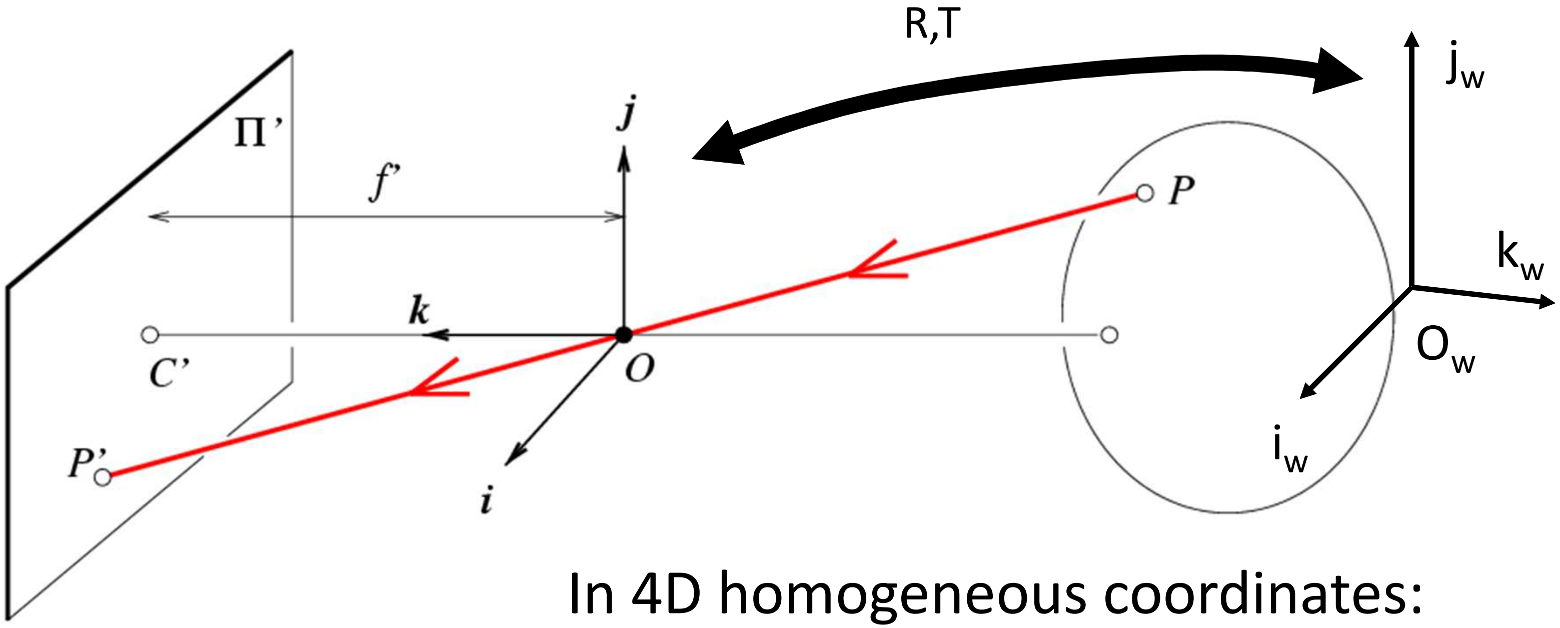
K has 5 degrees of freedom!

Camera & world reference system



- The mapping is defined within the camera reference system
- What if an object is represented in the world reference system?

Camera & world reference system



In 4D homogeneous coordinates:

$$P = \begin{bmatrix} R & T \end{bmatrix} P_w$$

camera

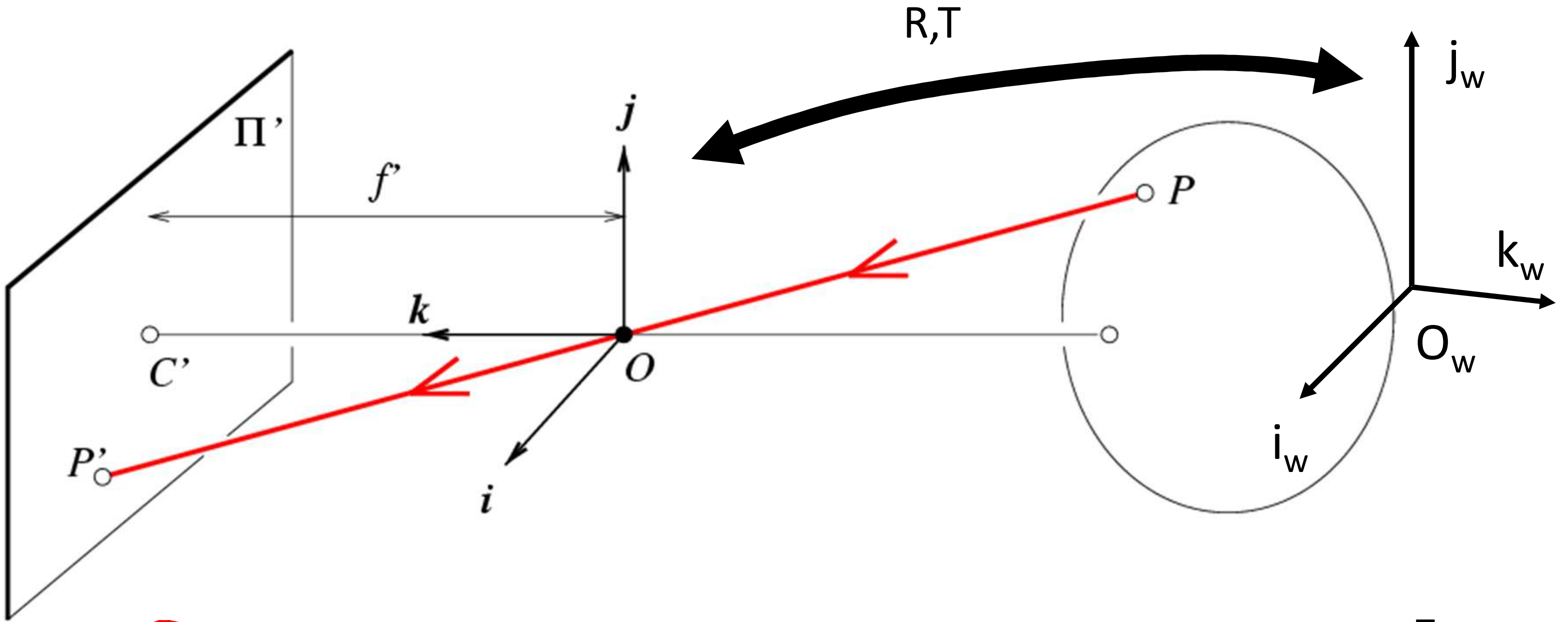
$$P' = M \; P_w = \boxed{K} \boxed{\begin{bmatrix} R & T \end{bmatrix}} P_w$$

"Internal parameters"

real-world transformation

"external" External parameters

Projective cameras



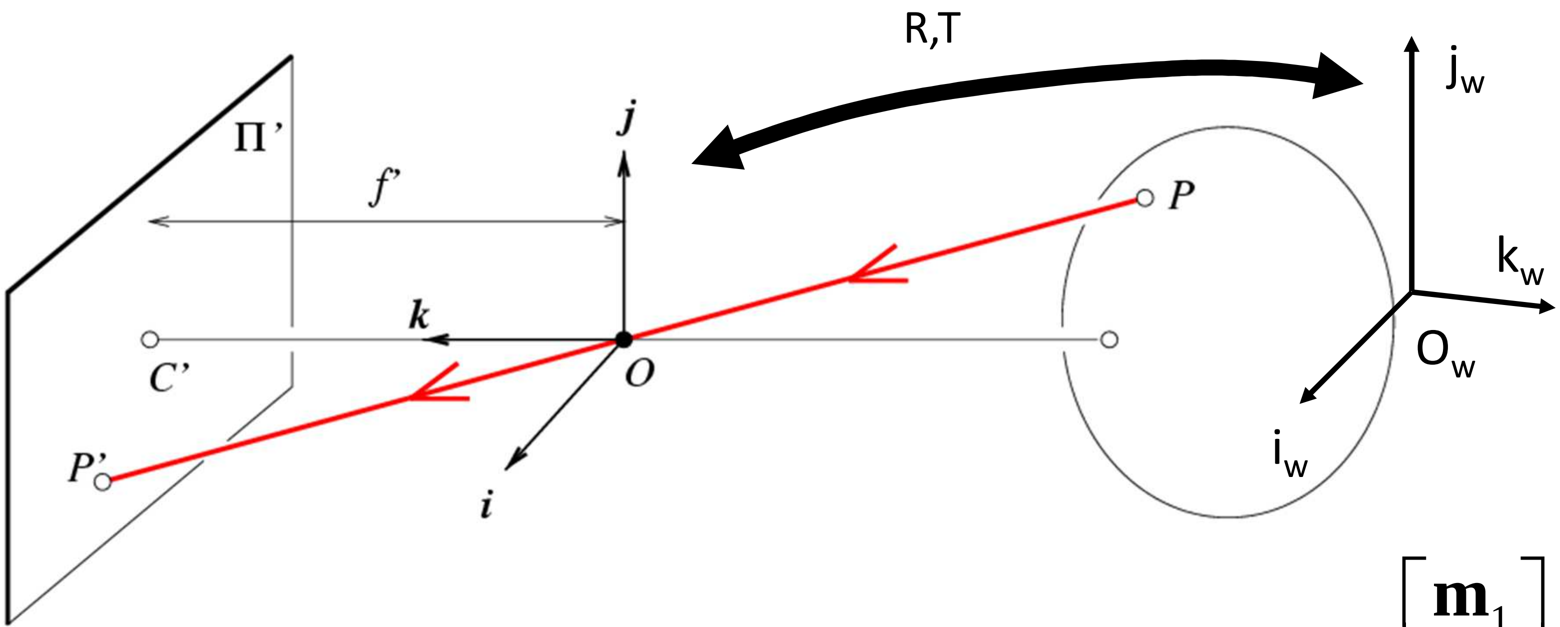
$$P'_{3\times1} = M\,P_w = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4} P_{w\,4\times1}$$

$$K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

How many degrees of freedom?

5 + 3 + 3 =11!

Projective cameras



$$P'_{3\times1} = M\ P_w = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4}\ P_{w4\times1}$$

$$M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$(x, y, z)_w \rightarrow \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w}\right)$$

M is defined up to scale!

Multiplying M by a scalar won't change the image

Theorem (Faugeras, 1993)

$$M = K\begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} K\,R & K\,T \end{bmatrix} = \begin{bmatrix} A & b \end{bmatrix}$$

$$A = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{bmatrix}$$

$$K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

$\alpha = \text{f k};$

$\beta = \text{f l}$

Let $\mathcal{M} = (\mathcal{A} \quad \boldsymbol{b})$ be a 3×4 matrix and let $\boldsymbol{a}_i^T$ $(i = 1, 2, 3)$ denote the rows of the matrix $\mathcal{A}$ formed by the three leftmost columns of $\mathcal{M}$.

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$.
- A necessary and sufficient condition for $\mathcal{M}$ to be a zero-skew perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$(\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0.$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0, \\ (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_1 \times \boldsymbol{a}_3) = (\boldsymbol{a}_2 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3). \end{cases}$$

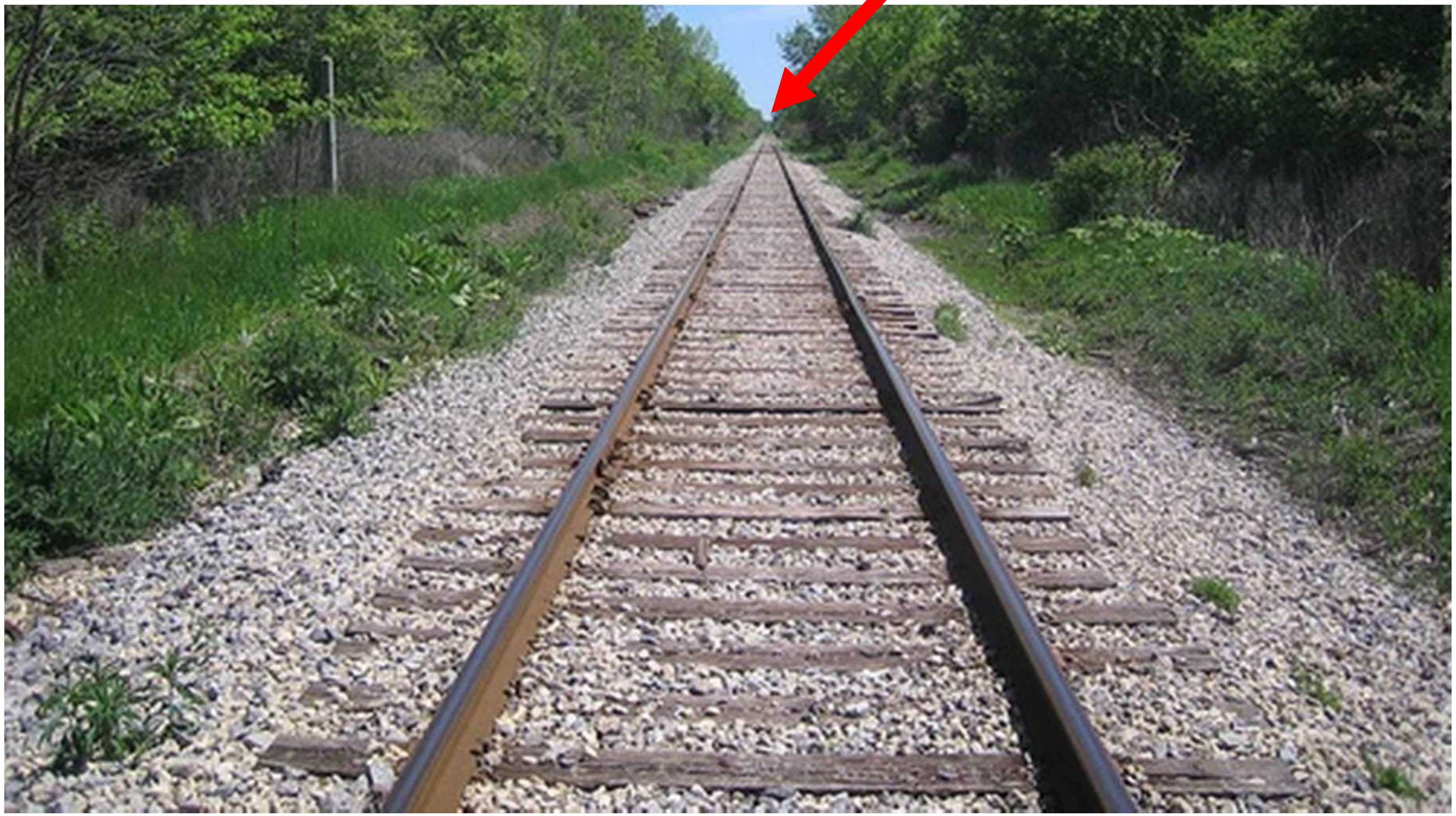
Properties of Projection

- Points project to points
- Lines project to lines

Properties of Projection

- Angles are not preserved
- Parallel lines meet

Vanishing point



What we have learned today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras

Reading:

[FP] Chapters 1 – 3

[HZ] Chapter 6