CS5242 Assignment Report

November 19, 2017

Partners: Kyaw Zaw Lin(E0218306)
Rahul Soni(E???????)

1 Assignment 1

Gibbs sampling is one of the MCMC algorithms. Given a set of random variables, $\{z_i : i = 1, ..., M\}$, and time steps $\tau = 1, ...T$,

- $z_1^{\{\tau+1\}} \sim p(z_1|z_2^{\{\tau\}}, ..., z_M^{\{\tau\}})$
- $z_2^{\{\tau+1\}} \sim p(z_2|z_2^{\{\tau+1\}}, ..., z_M^{\{\tau\}})$
- ...
- $z_M^{\{\tau+1\}} \sim p(z_M|z_2^{\{\tau+1\}}, ..., z_{M-1}^{\{\tau+1\}})$

In other words, at each time step, we sample from the full condtional of one variable, also written $p(z_i|z_{-i})$, which is the probability of z_i given all others. Then we update the state of variable and proceed until time step T. The initial samples are discarded until the Markov chain has burned in, or entered its stationary distribution. In our implementation, we discard initial 25% of the samples.
For image denoising, we apply gibbs sampling on the pairwise Ising CRF model where pairwise potentials $\psi_{st}(x_s, x_t)$ are given by $exp(Jx_sx_t)$ where $x_s, x_t \in \{-1, +1\}$.

$$p(x_t|x_{-t}, \theta) \propto \prod_{s \in nbr(t)} \psi_{st}(x_s, x_t) \tag{1}$$

The model can be further extended by incorporting local evidence $\psi_t(x_t)$.

$$p(x_t|x_{-t}, y, \theta) \propto \psi_t(x_t) \prod_{s \in nbr(t)} \psi_{st}(x_s, x_t) \tag{2}$$

By using gaussian observation model where $\psi_t(x_t) = \mathcal{N}(y_t|x_t, \sigma^2)$, and normalizing we obtain the prbability of a particular pixel being noise:

$$\begin{aligned} p(x_t = +1|x_{-t}, y, \theta) &= \frac{\psi_t(+1)\prod_{s\in nbr(t)} \psi_{st}(x_t = +1, x_s)}{\psi_t(+1)\prod_{s\in nbr(t)} \psi_{st}(x_t = +1, x_s) + \psi_t(-1)\prod_{s\in nbr(t)} \psi_{st}(x_t = -1, x_s)} \\ &= \frac{\psi_t(+1)exp[J\sum_{s\in nbr(t)} x_s]}{\psi_t(+1)exp[J\sum_{s\in nbr(t)} x_s] + \psi_t(-1)exp[-J\sum_{s\in nbr(t)} x_s]} \end{aligned} \tag{3}$$

References