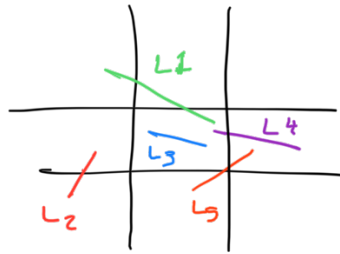


Serie de Ejercicios

- Alumno: Alfonso Murrieta Villegas
- Computación Gráfica e Interacción Humano Computador

1. Sea el código de asignación $\uparrow \leftarrow \downarrow \rightarrow$ y las líneas, resolver por método de Cohen Sutherland



\Rightarrow

1100	1000	1001
0100	0000	0001
0110	0010	0011

$$L1) \begin{array}{r} 1100 \\ 0000 \\ \hline \end{array}$$

$$\text{OR } 1100$$

$$\text{AND } 0000$$

No se tienen valores
 \therefore se dibuja //

$$L2) \begin{array}{r} 0100 \\ 0110 \\ \hline \end{array}$$

$$0110$$

$$0100$$

$$\text{AND} \neq 0$$

\therefore se descarta
L2 //

$$L3) \begin{array}{r} 0000 \\ 0000 \\ \hline \end{array}$$

$$0000$$

\therefore se dibuja
L3 //

$$L4) \begin{array}{r} 0000 \\ 0001 \\ \hline \end{array}$$

$$\text{OR } 0001$$

$$\text{AND } 0000$$

$$\text{AND} = 0$$

No se tienen.

$$L5) \begin{array}{r} 0001 \\ 0010 \\ \hline \end{array}$$

$$\text{OR } 0011 \leftarrow \text{OR} \neq 0$$

$$0000$$

$$\text{AND} = 0$$

No se tienen valores de límites

límites de valores
 \therefore se dibuja

\therefore se dibuja

2.- Sea la ventana de recorte con valores $X_{\min}=-1$, $Y_{\min}=-5$, $X_{\max}=5$, $Y_{\max}=6$, el código de asignación $\rightarrow \downarrow \leftarrow \uparrow$ y las líneas con vértices:

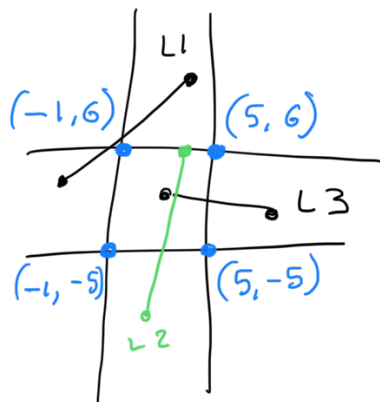
L1(-3,4), (3,8)

L2(2,-6), (4,6)

L3(6,2), (3,4)

Obtener los puntos de recorte de cada línea por método de Cohen Sutherland.

Trazado de rectas



Código y matriz

0011	0001	1001
0010	0000	1000
0110	0100	1100

L1) $\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}$

OR $\begin{array}{cccc} 0 & 0 & 1 & 1 \end{array}$

OR $\neq 0$

AND $\begin{array}{cccc} 0 & 0 & 0 & 0 \end{array}$

AND = 0

// Puntos de recorte

(ventana)

$$X_{12q} = X_{inicial} + m_{12q} (X_{final} - X_{inicial})$$

$$-1 = -3 + m_{12q} (3 - (-3))$$

$$2 = m_{12q} (6)$$

$$\frac{1}{3} = m_{12q} \text{ continua}$$

$$Y_{12q} = Y_{inicial} + m_{12q} (Y_{final} - Y_{inicial})$$

$$y_{29} = 4 + \frac{1}{3}(8-4)$$

$$y_{129} = 4 + \frac{4}{3} = \frac{16}{3}$$

$$P(-1, 16/3) //$$

$$x_{der} = x_{inicial} + m_{der}(x_{final} - x_{inicial})$$

$$5 = -3 + m_{der}(3 - (-3))$$

$$8 = m_{der}(6) ; m_{der} = \frac{8}{6}$$

Pendiente > 1 \therefore No se continua //

$$y_{inf} = y_{inicial} + m_{inf}(y_{final} - y_{inicial})$$

$$-9 = 4 + m_{inf}(8-4)$$

$$m_{inf} = \frac{-9}{4} \therefore \text{No se } \underline{\text{continua}} //$$

$$y_{sup} = y_{inicial} + m_{sup}(y_{final} - y_{inicial})$$

$$6 = 4 + m_{sup}(8-4)$$

$$\frac{2}{4} = m_{sup} \quad \text{continua}$$

$$x_{sup} = x_{inicial} + m_{sup}(x_{final} - x_{inicial})$$

$$x_{sup} = -3 + \frac{1}{2}(3 - (-3)) = -3 + \frac{6}{2} = 0$$

$$P(0, 6) //$$

$$\begin{array}{l|l} 12) & \begin{array}{l} 0100 \\ \hline 0000 \\ \hline 0100 \\ \hline 0R \neq 0 \\ \hline 0000 \end{array} \\ \hline & \begin{array}{l} y_{inf} = y_{inicial} + m_{inf}(y_{final} - y_{inicial}) \\ -5 = -6 + m_{inf}(6 - (-6)) \\ m_{inf} = \frac{1}{12} \quad \text{continua} \end{array} \end{array}$$

AND 00-
AND=0
Puntos

$$x_{inf} = x_{inicial} + m_{inf}(x_{final} - x_{inicial})$$

$$x_{inf} = 2 + \frac{1}{2}(4-2) = 2 + \frac{1}{2} = \frac{5}{2}$$

$$\therefore P(-5, \frac{5}{2}) //$$

$$y_{sup} = y_{inicial} + m_{sup}(y_{final} - y_{inicial})$$

$$6 = -6 + m_{sup}(6 - (-6))$$

$$m_{sup} = 1 \rightarrow \text{continua}$$

$$x_{sup} = x_{inicial} + m_{sup}(x_{final} - x_{inicial})$$

$$x_{sup} = 2 + 1(4-2)$$

$$x_{sup} = 2 + 2 = 4$$

$$\therefore P(4, 6) //$$

$$x_{izq} = x_{inicial} + m_{izq}(x_{final} - x_{inicial})$$

$$-1 = 2 + m_{izq}(4-2)$$

$$m_{izq} = -\frac{3}{2}$$

$$\therefore \text{No continua} //$$

$$x_{der} = x_{inicial} + m_{der}(x_{final} - x_{inicial})$$

$$5 = 2 + m_{der}(4-2)$$

$$m_{der} = \frac{3}{2}$$

$$\therefore \text{No continua} //$$

L3) 0000 1 $x_{inf} = x_{inicial} + m_{inf}(x_{final} - x_{inicial})$
 1000 1
 1000 1 $-5 = 4 + m_{inf}(2-4)$

OR $\neq 0$
AND 0000
AND $= 0$

$$m_{inf} = \frac{9}{2}$$

\therefore No continua

$$y_{sup} = y_{inicial} + m_{sup}(x_{final} - x_{inicial})$$

$$6 = 4 + m_{sup}(2 - 4)$$

$$m_{sup} = -1$$

\therefore No continua

$$x_{izq} = x_{inicial} + m_{izq}(x_{final} - x_{inicial})$$

$$-1 = 3 + m_{izq}(6 - 3)$$

$$m_{izq} = \frac{-4}{3} \therefore$$
 No continua

$$x_{der} = x_{inicial} + m_{der}(x_{final} - x_{inicial})$$

$$5 = 3 + m_{der}(6 - 3)$$

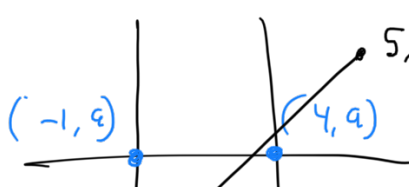
$$m_{der} = \frac{2}{3} \text{ continua}$$

$$y_{der} = y_{inicial} + m_{der}(x_{final} - x_{inicial})$$

$$y_{der} = 4 + \frac{2}{3}(2 - 4) = \frac{8}{3}$$

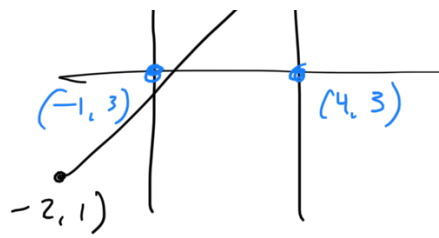
$$\therefore P(5, \frac{8}{3})$$

3.- Sea la ventana formada por los vértices extremos $(-1,3)$ y $(4,9)$ y la línea formada por los puntos inicial $(-2,1)$ y Punto final $(5,10)$ calcular puntos de recorte por método de Liang Barsky



$$y_{inf} = y_{inicial} + m_{inf}(x_{final} - x_{inicial})$$

$$3 = 1 + m_{inf}(10 - 1)$$



$$1 < -m_{inf}(a)$$

$$m_{inf} = \frac{2}{9}$$

$$x_{inf} = x_{inicial} + m_{inf}(x_{fin} - x_{inicial})$$

$$x_{inf} = -2 + \left(\frac{2}{9}\right)(5 - (-2))$$

$$x_{inf} = -\frac{4}{9}$$

$$\therefore P\left(-\frac{4}{9}, 3\right)$$

$$y_{sup} = y_{ini} + m_{sup}(x_{fin} - x_{ini})$$

$$9 = 1 + m_{sup}(10 - 1)$$

$$8 = m_{sup}(9)$$

$$m_{sup} = \frac{8}{9}$$

$$x_{sup} = x_{inicial} + m_{sup}(x_{fin} - x_{ini})$$

$$x_{sup} = -2 + \frac{8}{9}(5 - (-2))$$

$$x_{sup} = \frac{38}{9}$$

$$\therefore P\left(\frac{38}{9}, 9\right)$$

$$x_{129} = x_{ini} + m_{129}(x_{fin} - x_{ini})$$

$$-1 = -2 + m_{129}(5 - (-2))$$

$$1 = m_{129}(7)$$

$$m_{129} = \frac{1}{7}$$

$$y_{129} = y_{inicial} + m_{129}(x_{fin} - x_{ini})$$

$$y_{129} = 1 + \frac{1}{7}(10 - 1) = \frac{16}{7}$$

$$\therefore P\left(-1, \frac{16}{7}\right)$$

$$x_{der} = x_{ini} + t_{der}(x_{fin} - x_{ini})$$

$$4 = -2 + t_{der}(5 - (-2))$$

$$t_{der} = \frac{6}{7}$$

$$y_{der} = y_{ini} + t_{der}(y_{fin} - y_{ini})$$

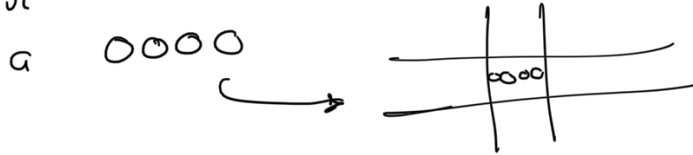
$$y_{der} = 1 + \frac{6}{7}(10 - 1)$$

$$y_{der} = \frac{61}{7}$$

$$\therefore P(4, \frac{61}{7})$$

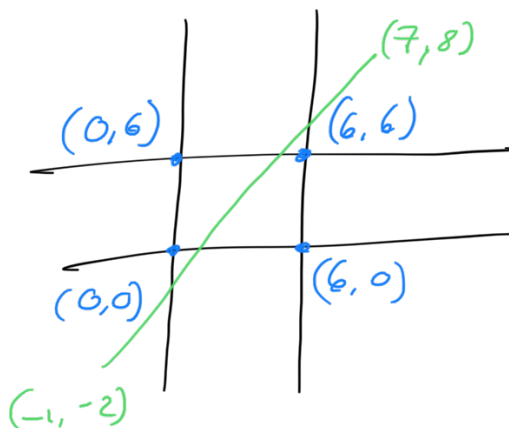
Análisis

\Rightarrow si solo se toman los que pertenecen a



Entonces serían $P(-\frac{4}{a}, 3)$ y $P(4, \frac{61}{7})$

4.- Sea la recta con origen en el punto $(-1, -2)$ y un vértice en $(7, 8)$ Calcular por método de Liang Barsky los puntos de recorte con la ventana formada por los valores $X_{min}=0$, $Y_{min}=0$, $X_{max}=6$, $Y_{max}=6$



$$\begin{aligned} y_{inf} &= y_{ini} + t_{inf}(y_{fin} - y_{ini}) \\ 0 &= -2 + t_{inf}(8 - (-2)) \\ 2 &= t_{inf}(10); t_{inf} = \frac{1}{5} \\ x_{inf} &= x_{ini} + t_{inf}(x_{fin} - x_{ini}) \\ x_{inf} &= -1 + \frac{1}{5}(7 - (-1)) \end{aligned}$$

$$X_{inf} = \frac{3}{5} \therefore P\left(\frac{3}{5}, 0\right) //$$

$$y_{sup} = y_{in1} + m_{sup}(y_{fin} - y_{in1})$$

$$6 = -2 + m_{sup}(8 + 2)$$

$$m_{sup} = \frac{4}{5}$$

$$x_{sup} = x_{in1} + m_{sup}(x_{final} - x_{in1})$$

$$x_{sup} = -1 + \frac{4}{5}(7 - (-1)) = \frac{27}{5}$$

$$\therefore P\left(\frac{27}{5}, 6\right) //$$

$$x_{2g} = x_{in1} + m_{2g}(x_{final} - x_{in1})$$

$$0 = -1 + m_{2g}(7 + 1)$$

$$1 = m_{2g}(8); m_{2g} = \frac{1}{8}$$

$$y_{2g} = y_{in1} + m_{2g}(y_{fin} - y_{in1})$$

$$y_{2g} = -2 + \frac{1}{8}(8 + 2)$$

$$y_{2g} = -\frac{3}{4} \therefore P\left(0, -\frac{3}{4}\right) //$$

$$x_{der} = x_{in1} + m_{der}(x_f - x_i)$$

$$6 = -1 + m_{der}(7 + 1)$$

$$m_{der} = \frac{7}{8}$$

$$y_{der} = y_{in1} + m_{der}(y_f - y_i)$$

$$y_{der} = -2 + \frac{7}{8}(8 + 2)$$

$$y_{der} = \frac{27}{4} \therefore P\left(6, \frac{27}{4}\right)$$

der - ...

Análisis

=> Los puntos válidos o de recorte
serían

$$P\left(\frac{3}{5}, 0\right) \text{ y } P\left(\frac{27}{5}, 6\right)$$

5.- Calcular por medio del algoritmo de Bresenham la línea que se dibuja para Punto Inicial(0,0) y punto final(5,7)

$$\begin{array}{lll} \text{Inicial (0,0)} & \Delta x = 5 & 2\Delta y = 14 \\ \text{Final (5,7)} & \Delta y = 7 & \end{array}$$

$$\therefore m = \frac{\Delta y}{\Delta x} = \frac{7}{5}$$

$$\text{if } ① 0 \leq |m| \leq 1 \Rightarrow 0 \leq 1.4 \leq 1 \quad \times$$

$$② x_i \leq x_f \Rightarrow 0 < 5 \quad \checkmark$$

Una de las condiciones no cumple
 \therefore No se puede dibujar la línea por
Bresenham

6.- Obtener la línea por método de Bresenham que se dibuja si se tienen el punto final(5,1) y punto inicial(1,4)

$$1 \wedge 1 = 3$$

$$m = \frac{1-7}{5-1} = \frac{-2}{4} \quad |\Delta x| = 4$$

Condiciónes

$$\textcircled{1} x_i < x_f \Rightarrow 1 < 5$$

$$\textcircled{2} 0 \leq |m| \leq 1 \Rightarrow 0 \leq \frac{3}{4} \leq 1$$

Nota:

Caso especial

(Pendiente negativa)

Resta en y

$$P(x_0, y_0) = (1, 4)$$

$$P_0 = 2|\Delta y| - |\Delta x| = 2(3) - 4 = \underline{2}$$

$$\rightarrow j=0 \quad \left. \begin{array}{l} j < dx \\ 0 < 4 \end{array} \right\} P_0 \geq 0$$

$$P_1(x_0 + 1, y_0 - 1) = \underline{(2, 3)}$$

$$P_1 = P_0 + 2|\Delta y| - 2|\Delta x| = 2 + 2|3| - 2|4| = \underline{0}$$

$$\rightarrow j=1 \quad \left. \begin{array}{l} j < dx \\ 1 < 4 \end{array} \right\} P_1 \geq 0$$

$$P_2(x_1 + 1, y_1 - 1) = \underline{(3, 2)}$$

$$P_2 = P_0 + 2|\Delta y| - 2|\Delta x| = 2 + 2|3| - 2|4| = \underline{-2}$$

$$\rightarrow j=2 \quad \left. \begin{array}{l} j < dx \\ 2 < 4 \end{array} \right\} P_2 < 0$$

$$P_3(x_2 + 1, y_2) = \underline{(4, 2)}$$

$$P_3 = P_2 + 2|\Delta y| = -2 + 2|3| = 4$$

$$\rightarrow j=3 \quad 3 < 4 \quad \{ P_3 \geq 0$$

$$P_4 (x_3 + 1, y_3 - 1) = (5, 1)$$

$$P_4 = P_3 + 2|\Delta y| - 2|\Delta x| = 4 + 2/3 - 2/4 = 2$$

$$\rightarrow j=4 \quad 4 < 4 \quad \therefore \text{Termina}$$

$$\text{Puntos } (x_0, y_0) = (1, 4)$$

$$(x_1, y_1) = (2, 3)$$

$$(x_2, y_2) = (3, 2)$$

$$(x_3, y_3) = (4, 1)$$

$$(x_4, y_4) = (5, 1)$$

7.- Obtener por método de Bresenham la circunferencia con centro en el origen y radio 8

$$r=8 \quad \text{Simetrías: } (0, -8), (8, 0), (-8, 0)$$

$$P_0(0, r) = (0, 8)$$

$$x_k \geq y_k \\ (\text{Para finalizar})$$

$$p_0 = 1 - r = 1 - 8 = -7$$

$$p_0 < 0$$

$$P_1 = (x_0 + 1, y_0) = (1, 8)$$

$$p_1 = p_0 + 2x_0 + 3 = -7 + 2(0) + 3 = -4$$

$$p_1 < 0$$

$$(7, 8)$$

$$p_2 = (x_1 + 1, y_1) = (1, 0)$$

$$p_2 = p_1 + 2(x_1) + 3 = -4 + 2(1) + 3 = 1$$

$$p_2 \geq 0$$

$$p_3 = (x_2 + 1, y_2 - 1) = (3, 7)$$

$$p_3 = p_2 + 2(x_2) - 2(y_2) + 5 = -6$$

$$p_3 < 0$$

$$p_4 = (x_3 + 1, y_3) = (4, 7)$$

$$p_4 = p_3 + 2(x_3) + 3 = -6 + 2(3) + 3 = 3$$

$$p_4 \geq 0$$

$$p_5 = (x_4 + 1, y_4 - 1) = (5, 6)$$

$$p_5 = p_4 + 2x_4 - 2y_4 + 5 = 3 + 2(4) - 2(7) + 5 = 2$$

$$p_5 \geq 0$$

$$p_6 = (x_5 + 1, y_5 - 1) = (6, 5)$$

$$p_6 = p_5 + 2x_5 - 2y_5 + 5 = 2 + 2(5) - 2(6) + 5 = 5$$

$$x_k \geq y_k \quad 6 \geq 5 \quad \therefore \text{Finaliza}$$

Pontos					
(0, 8)	(1, 8)	(2, 8)	(3, 7)	(4, 7)	(5, 6)
(0, -8)	(1, -8)	(2, -8)	(3, -7)	(4, -7)	(5, -6)
(8, 0)	(-1, 8)	(-2, 8)	(-3, 7)	(-4, 7)	(-5, 6)
(-8, 0)	(-1, -8)	(-2, -8)	(-3, -7)	(-4, -7)	(-5, -6)
	(8, 1)	(8, 2)	(7, 3)	(7, 4)	(6, 5)
	(8, -1)	(8, -2)	(7, -3)	(7, -4)	(6, -5)
	(-8, 1)	(-8, 2)	(-7, 3)	(-7, 4)	(-6, 5)
	(-8, -1)	(-8, -2)	(-7, -3)	(-7, -4)	(-6, -5)

8.- Obtener por método de Bresenham la circunferencia de radio 5 y centro en $(-2,3)$

$$P_0(x_0, y_0) = (0, 5) \quad \text{Simétrico} \\ (0, -5), (5, 0), (-5, 0) \\ p_0 = 1 - r = 1 - 5 = -4$$

$$p_0 < 0$$

$$P_1(x_0 + 1, y_0) = (1, 5)$$

$$p_1 = p_0 + 2x_0 + 3 = -4 + 2(0) + 3 = -1$$

$$p_1 < 0$$

$$P_2(x_1 + 1, y_1) = (2, 5)$$

$$p_2 = p_1 + 2x_0 + 3 = -1 + 2(1) + 3 = 4$$

$$p_2 \geq 0$$

$$P_3(x_3 + 1, y_2 - 1) = (3, 4)$$

$$p_3 = p_2 + 2x_2 - 2y_2 + 5 = 4 + 2(2) - 2(5) + 5 = 3$$

$$p_3 \geq 0$$

$$P_4(x_3 + 1, y_3 - 1) = (4, 3)$$

$$p_4 = p_3 + 2x_3 - 2y_3 + 5 = 3 + 6 - 2(4) + 5 = 6$$

$$x_4 \geq y_4 \quad 4 \geq 3 \quad \therefore \text{Finaliza}$$

Simetrías + Desplazamiento $(-2, 3)$

$$\begin{array}{cccc}
 (-2, 8) & (-1, 8) & (0, 8) & (1, 7) \\
 (-2, 2) & (-1, -2) & (0, -2) & (1, -1) \\
 (3, 3) & (-3, 8) & (-4, 8) & (-5, 7) \\
 (-8, 3) & (-3, -2) & (-4, -2) & (-5, -1) \\
 & (3, 4) & (3, 5) & (2, 6) \\
 & (3, 2) & (3, 1) & (2, 0) \\
 & (-7, 4) & (-7, 5) & (-6, 6) \\
 & (7, 2) & (-7, 1) & (-6, 0)
 \end{array}$$

9.- Obtener la transformación [ST1RxT2] para los puntos: $(-3,4)$, $(3,8)$, $(2,-6)$, $(4,6)$, $(3,4)$

Siendo $S_x = \Delta x_1 = 1$, $S_y = -3$, $\Delta y_1 = \Delta x_2 = 2$, $\Delta y_2 = -3$ $R_x = 45^\circ$

$$M_1 = R^T T_2$$

$$\begin{aligned}
 M_1 &= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \\
 &= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 5\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

$$M_2 = T_1 M_1$$

$$M_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 5\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & \frac{2+5\sqrt{2}}{2} \\ \sqrt{2}/2 & \sqrt{2}/2 & \frac{4-\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_3 = SM_2$$

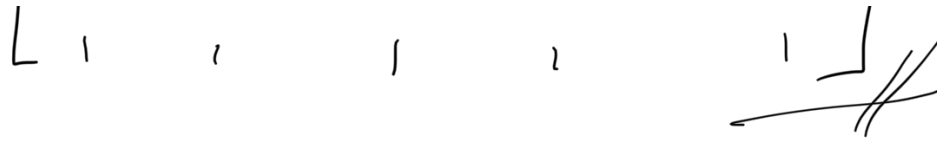
$$M_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & \frac{2+5\sqrt{2}}{2} \\ \sqrt{2}/2 & \sqrt{2}/2 & \frac{4-\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & \frac{2+5\sqrt{2}}{2} \\ -\frac{3\sqrt{2}}{2} & -\frac{3\sqrt{2}}{2} & -3\left(\frac{4-\sqrt{2}}{2}\right) \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_3 P$$

$$\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & \frac{2+5\sqrt{2}}{2} \\ -\frac{3\sqrt{2}}{2} & -\frac{3\sqrt{2}}{2} & -3\left(\frac{4-\sqrt{2}}{2}\right) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 3 & 2 & 4 & 3 \\ 4 & 8 & -6 & 6 & 9 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1-\sqrt{2} & 1 & \frac{2+13\sqrt{2}}{2} & \frac{2+3\sqrt{2}}{2} & 1+2\sqrt{2} \\ -6 & -6-15\sqrt{2} & \frac{-12+15\sqrt{2}}{2} & \frac{12+27\sqrt{2}}{2} & 6-9\sqrt{2} \end{bmatrix}$$



10.- Obtener la transformación $[S1RzS2T1]$ para los puntos $(1,1,1)$, $(3,4,1)$, $(-2,-1,-3)$, $(0,0,-1)$

Siendo $Sx1=Sx2=Sz2=2$, $Sy1=-1$, $Sy2=Sz1=1$, $\Delta x=-1$, $\Delta y=3$, $\Delta z=2$, $Rz=180^\circ$

$$M_1 = S_2 T_1$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_2 = R_2 M_1$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 2 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_3 = S_1 M_2$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 & 2 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 0 & 4 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_3 P$$

$$\begin{bmatrix} -4 & 0 & 0 & 4 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 1 & 4 & -1 & 0 \\ 1 & 1 & -3 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -8 & 12 & -4 \\ 2 & -1 & 7 & 3 \\ 6 & 6 & -2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$